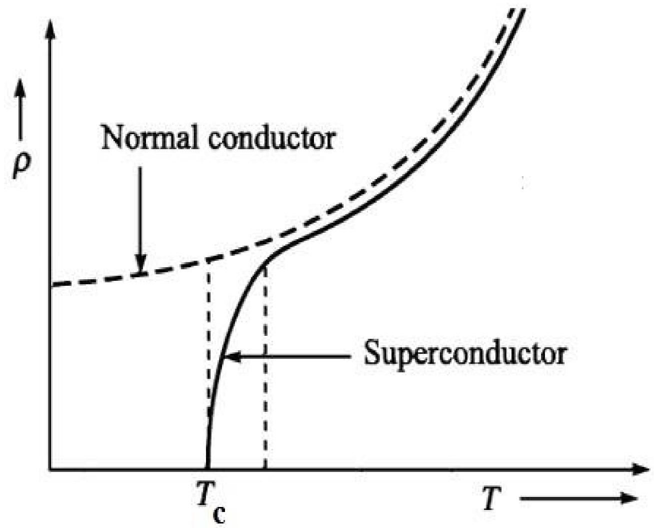
UNIT 5

: INTRODUCTION TO SUPERCONDUCTIVITY

Helium gas–was liquefied at 4.2 K by Dutch Physicist Heike Kamerlingh Onnes in 1908. Superconductivity was discovered by K. Onnes in 1911. Liquid Helium has a temperature of 4.2
K. It was observed that the resistance of mercury dropped from 0.08 Ω at about 4.3 K to less than 3 × 10-6 Ω at 4.2 K. A large number and wide variety of metals, alloys, binary and ternary chemical compounds have been found to show superconductivity at various temperatures.

Superconductivity is a state in which quantum mechanics operates on a macroscopic scale of the order of many atomic distances rather than the usual atomic and subatomic scale. The superconducting state is influenced by the temperature, magnetic field and current. All these three parameters have critical values, above which material enters into normal state. Every superconductor has its own transition temperature (Tc).



Good electrical conductors such as silver, gold, and copper are not good superconductors because the resistivity of these conductors at low temperatures is limited to low resistivity i.e. residual resistivity, value due to scattering of electrons from crystal defects and impurities. Similarly, good superconducting materials like zinc and lead are not good electrical conductors.

: DEFINITIONS

SUPERCONDUCTOR

It is a material that loses all its resistance (offers zero resistance) to the flow of electric current when it is cooled below a certain temperature called the critical temperature or transition temperature Tc.

e.g. Mercury, Zinc, Niobium, etc…

CRITICAL TEMPERATURE (TC)

The temperature at which a material's electrical resistivity drops to absolute zero is called the critical temperature or transition temperature. It is denoted by Tc.

SUPERCONDUCTIVITY

It is a phenomenon in which certain metals, alloys and ceramics conduct electricity without resistance when it is cooled below a certain temperature called the critical temperature.

: PROPERTIES OF SUPERCONDUCTORS

: ELECTRICAL RESISTANCE

The electrical resistance of superconducting material is very low and is of the order of 10-7 Ω.

: EFFECT OF IMPURITIES

When impurities are added to superconducting elements, the superconducting property is not lost but the Tc value is lowered.

: ISOTOPE EFFECT

The critical temperature of a superconductor is found to vary with its isotopic mass. The atomic mass of Hg varies from 199.5 to 203.4. Due to this variation in atomic mass, the transition temperature of isotopes of Hg varies from 4.185 to 4.146 K. They are related as $T_c \propto \frac{1}{\sqrt{M}}$ where M is the isotopic mass.

: MAGNETIC FIELD EFFECT

If a sufficiently strong magnetic field is applied to a superconductor at any temperature below critical temperature Tc, the superconductor is found to undergo a transition from the superconducting state to the normal state. OR This minimum magnetic field required to destroy the superconducting state is called critical magnetic field Hc. $H_c = H_0 [1 - (\frac{T}{T_c})^2]$ where H0 is the applied magnetic field at 0 K.

: EFFECT OF PRESSURE AND STRESS

Certain materials are found to exhibit the superconductivity phenomena on increasing the pressure over them. For e.g. Cs shows superconductivity at Tc = 1.5 K and 110 k bar Also, in Superconductors, an increase in stress results in increase of the Tc value.

: CRITICAL CURRENT DENSITY

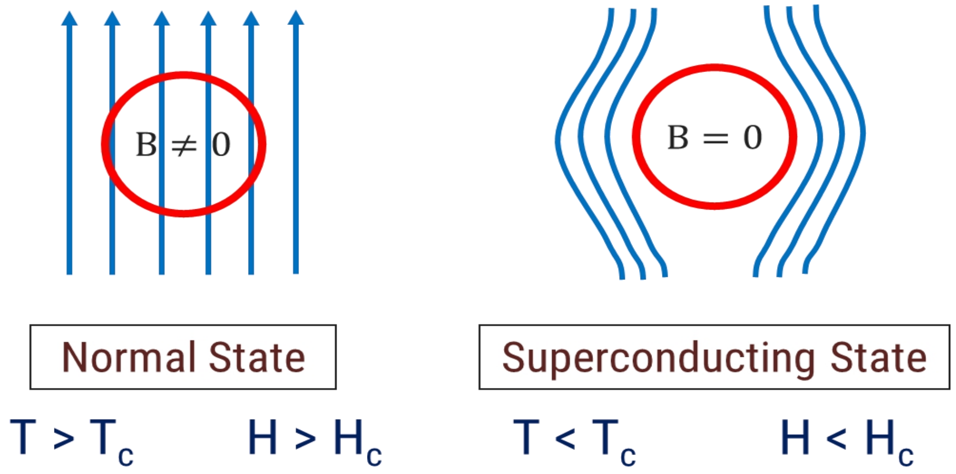
It is defined as the maximum current that can be permitted in a superconducting material without destroying its superconductivity state. OR Minimum current required to destroy the superconducting state is called critical current density (Jc) and this current is called critical current (Ic). The equation relating $J_C A = I_c$ and $I_c = 2 \pi r H_c$.

: PERSISTENT CURRENT

If current is made to flow through a superconducting ring then it is observed that the current flows through the material without any significant loss. This steady flow of current in a superconducting ring without any potential deriving is called the persistent current.

: MEISSNER EFFECT

The complete expulsion of all the magnetic field lines by a superconductor material is called Meissner effect. The Meissner effect is a distinct characteristic of a superconductor from a normal perfect conductor. In addition, this effect is exhibited by the superconducting materials only when the applied field is less than the critical field Hc.



To Prove that χm 1 for superconductors

We know that for a magnetic material the magnetic induction or magnetic flux density B is given by the equation B $\mu 0$ M H where $\mu 0$ is the permeability of free space; M is the intensity of magnetization; H is the applied magnetic field.

But for the superconductors, we know that B = 0, thus the above equation can be written as

$\therefore$ i. e. 0 $\mu 0$ M H

$\therefore$ i. e. 0 M H since $\mu 0 \neq 0$

$\therefore$ i. e. M = H

M
OR H = −1 = $\cancel{\chi m}$

Where m is called as the magnetic susceptibility. This means that for a superconductor, the susceptibility is negative and maximum, i.e. a superconductor exhibits perfect diamagnetism. For all other magnetic materials, the susceptibility values are positive.

EXAMPLES

Q.1 For mercury of mass 202, value of α is 0.5 and Tc is 4.2 K. Find transition temperature for isotope of mercury of mass 200.

Ans. $M_1 = 202$, $T_{c1} = 4.2$ K, $M_2 = 200$, $\alpha = 0.5$, $T_{c2} = ?$

$$T_c M^{\alpha} = \text{constant}$$

$$\therefore T_{c1} M_1^{\alpha} = T_{c2} M_2^{\alpha}$$

$$\therefore T_{c2} = \left(\frac{M_1}{M_2}\right)^{\alpha} T_{c1}$$

$$\therefore T_{c2} = \left(\frac{202}{200}\right)^{0.5} 4.2$$

$$\therefore T_{c2} = 1.004987 \times 4.2$$

$$\therefore T_{c2} = 4.2209 \text{ K}$$

Q.2 The critical temperature of a metal with isotopic mass 199.5 is 4.185 K. Calculate the isotopic mass if critical temperature falls to 4.133 K.

Ans. $M_1 = 199.5$, $T_{c1} = 4.185$ K, $T_{c2} = 4.133$ K, $M_2 = ?$

$$T_c M^{\alpha} = \text{constant}$$

$$\therefore T_{c1} M_1^{\alpha} = T_{c2} M_2^{\alpha}$$

$$\therefore M_2^{\alpha} = \frac{T_{c1}}{T_{c2}} (M_1^{\alpha}) \qquad \text{But } \alpha = 0.5$$

$$\therefore \sqrt{M_2} = \frac{T_{c1}}{T_{c2}} \sqrt{M_1}$$

$$\therefore \sqrt{M_2} = \frac{4.185}{4.133} \sqrt{199.5}$$

$$\therefore \sqrt{M_2} = 1.01258\sqrt{199.5}$$

$$\therefore \sqrt{M_2} = 14.301$$

$$\therefore M_2 = 204.55$$

Q.3 Two isotopes of lead of mass 206 and 210 have T_c values of 7.193 K and 7.125 K respectively. Calculate the isotopes constant for lead (Pb).

Ans. $M_1 = 206$, $M_2 = 210$, $T_{c1} = 7.193$ K, $T_{c2} = 7.125$ K, $\alpha = ?$

$$\therefore T_{c1} M_1^{\alpha} = T_{c2} M_2^{\alpha}$$

$$\therefore \left(\frac{T_{c1}}{T_{c2}}\right) = \left(\frac{M_2}{M_1}\right)^{\alpha}$$

$$\therefore \left(\frac{7.193}{7.125}\right) = \left(\frac{210}{206}\right)^{\alpha}$$

$$\therefore 1.00954 = (1.0941)^{\alpha}$$

$$\therefore \log (1.00954) = \alpha \log (1.0941)$$

$$\therefore 4.12519 \times 10^{-3} = \alpha (1.0941 \times 10^{-3})$$

$$\therefore \alpha = \frac{1.0941 \times 10^{-3}}{4.12519 \times 10^{-3}}$$

$$\therefore \alpha = 0.4941$$

Q.4 A superconductor tin has a critical temperature of 3.7 K in zero magnetic field and a critical field of 0.0306 T at 0 K. Find the critical field at 2 K.

Ans. $H_c(0) = 0.0306$ T, $T_c = 3.7$ K, $T = 2$ K, $H_c = ?$

Now, the critical field at any temperature T K is given as: $H_c = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$

$$\therefore H_c = 0.0306\left[1 - \left(\frac{2}{3.7}\right)^2\right]$$

$$\therefore H_c = 0.0306 \times 0.708$$

$$\therefore H_c = 0.0216 \text{ T}$$

Q.5 The transition temperature for lead is 7.2 K. However, at 5 K, it loses the superconducting property when subjected to a magnetic field of 3.3×10^4 A/m. Find the value of the magnetic field that will allow the metal to remain its superconductivity at 0 K.

Ans. $T_c = 7.2$ K, $T = 5$ K, $H_c = 3.3 \times 10^4$ A/m, $H_c(0) = ?$

$$\therefore H_c = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

$$\therefore H_c(0) = H_c\left[1 - \left(\frac{T}{T_c}\right)^2\right]^{-1}$$

$$\therefore H_c(0) = 3.3 \times 10^4\left[1 - \left(\frac{5}{7.2}\right)^2\right]^{-1}$$

∴ Hc(0) = 6.37 × 104 A/m

Q.6 Calculate the transition temperature of niobium for which the critical field is 1 × 105 A/m at 8 K and 2 × 105 A/m at 0 K.

Ans. Hc = 1 × 105 A/m, T = 8 K, Hc(0)= 1 × 105 A/m, Tc = ?

T 2
∴ Hc = Hc(0) [1 − (T)]
c

Hc T 2
∴ = 1 − ( ) Hc(0) Tc

Hc T 2
∴ √1 − = √( ) Hc(0) Tc

1
T Hc 2
∴ = [1 − ( )] Tc Hc(0)

T
∴ Tc = 1
[1 − (Hc)]2
Hc(0)

8
∴ Tc = 1
[1 − (1 × 105)]2
2 × 105

∴ Tc = 8√2 K

∴ Tc = 11.3 K

Q.7 Critical temperature of a superconductor is 78 K and critical field is 0.518 T at 0 K. Find critical field at 25 K and 58 K.

Ans. T = 25 K T = 58 K

Hc(0) = 0.518 THc(0) = 0.518 T

Tc = 78 K   Tc = 78 K

T   2   T   2
∴ Hc = Hc(0) [1 − (T)]   ∴ Hc = Hc(0) [1 − (T)]
c   c

25   2   58 2
∴ Hc = 0.518 [1 − (78)]   ∴ Hc = 0.518 [1 − (78)]

∴ Hc = 0.46514 T   ∴ Hc = 0.2314 T

Q.8 Calculate the critical current through a long thin superconducting wire of radius 0.5 mm. The critical field is 7.2 × 103 A/m.

Ans. Hc = 7.2 × 103 A/m, r = 0.5 mm = 0.5 × 10-3 m, Ic =?

According to Silsbee's law: Ic = 2π r Hc

∴ Ic = (2π) (0.5 × 10−3)(7.2 × 103)

∴ Ic = 22.608 A

Q.9 Calculate the critical current for a superconducting wire of lead (Pb) having a diameter of 1 mm at 4.2 K. Critical temperature of lead is 7.18 K and Hc(0) is 6.5 × 104 A/m.

Ans. Hc(0) = 6.5 × 104 A/m, Tc = 7.18 K, r = 0.5 mm = 0.5 × 10-3 m, T = 4.2 K, Ic = ?, Hc = ?

T   2
We know, Hc = Hc(0) [1 − (T)]
c

$$\therefore H_c = 6.5 \times 10^4 \left[1 - \left(\frac{4.2}{7.18}\right)^2\right]$$

$$\therefore H_C = 6.5 \times 10^4 \,(1 - 0.342)$$

$$\therefore H_C = 6.5 \times 10^4 \,(0.0657)$$

$$\therefore H_C = 42.75 \times 10^3 \text{ A/m}$$

According to Silsbee's law: $I_c = 2\pi\, r\, H_c$

$$\therefore I_c = (2\pi)\,(0.5 \times 10^{-3})(4.2 \times 10^3)$$

$$\therefore I_c = 134.26 \text{ A}$$

Q.10 Calculate critical current density for a superconducting wire of lead having diameter of 1.5 mm at 5.3 K. The value of critical temperature of lead is 7.8 K and critical magnetic field at 0 K is 6.5×10^4 A/m.

Ans. $H_c(0) = 6.5 \times 10^4$ A/m, $T_c = 7.8$ K, $r = 0.75$ mm $= 0.75 \times 10^{-3}$ m, $T = 5.3$ K, $J_c = ?$, $I_c = ?$, $H_c = ?$

We know, $H_c = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$

$$\therefore H_c = 6.5 \times 10^4 \left[1 - \left(\frac{5.3}{7.8}\right)^2\right]$$

$$\therefore H_c = 3.498 \times 10^4 \text{ A/m}$$

Now, $I_c = 2\pi\, r\, H_c$

$$\therefore I_c = (2\pi)\,(0.75 \times 10^{-3})(3.498 \times 10^4)$$

∴ Ic = 164.75 A

Ic Current density: Jc = A

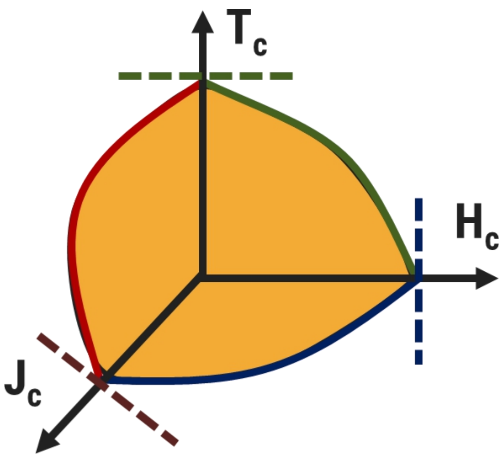
$$\therefore Jc = \frac{Ic}{\pi r2}$$

$$\therefore Jc = \frac{164.75}{\pi\,(0.75)2}$$

∴ Jc = 93.28 × 106 A/m

: THREE IMPORTANT FACTORS TO DEFINE A SUPERCONDUCTING STATE

Critical temperature (Tc)
Critical current density (Jc)
Critical magnetic field (Hc)



Each of the above three parameters is very dependent on the other two properties. The highest values for Hc and Jc occur at 0 K; while the highest value for Tc occurs when H and J are zero. Thus the plot of all these three parameters represents a critical surface. Within the surface, the material is superconducting and outside the surface, the material is said to be in the normal state.

: TYPES OF SUPERCONDUCTORS

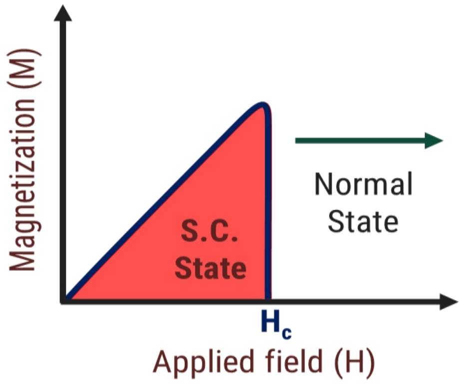
There are two types of super conductors based on their variation in magnetization, due to external magnetic field applied.

Type I superconductor or soft super conductor Type II superconductor or hard superconductor

TYPE I SUPERCONDUCTORS:

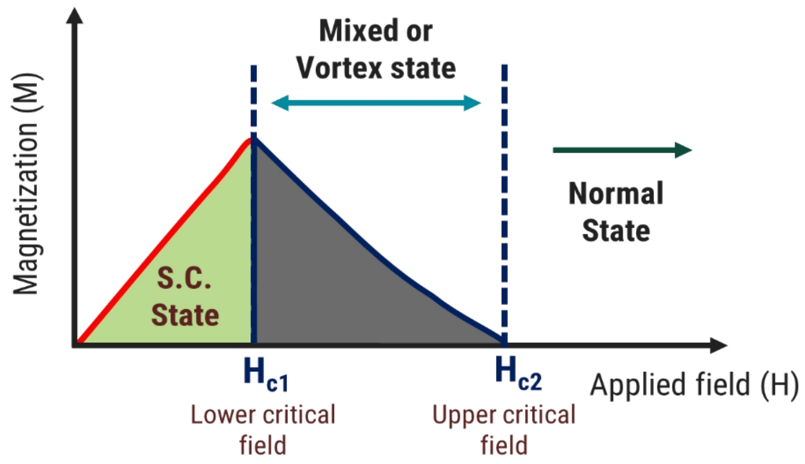
When the super conductor is kept in the magnetic field and if the field is increased the superconductor becomes normal conductor abruptly at critical magnetic field as shown in fig. These types of materials are termed as Type I superconductors.

Below critical field, the specimen excludes all the magnetic lines of force and exhibit perfect Meissner effect. Hence, Type I superconductors are perfect diamagnetic, represented by negative sign in magnetization.



TYPE II SUPERCONDUCTORS:

When the super conductor kept in the magnetic field and if the field is increased, below the lower critical field Hc1, the material exhibit perfect diamagnetism (i.e.) it behaves as a super conductor and above Hc1, the magnetization decreases and hence the magnetic flux starts penetrating through the material. The specimen is said to be in a mixed state between Hc1 and Hc2. above Hc2 (upper critical field) it becomes normal conductor as shown in fig.



The materials which lose its superconducting property gradually due to increase on the magnetic field are called Type II superconductor. Value of Hc2 for type-II superconductor is 100 times or even more as compared to type-I superconductor Because of relatively large magnetic field requirement for type-II superconductor.

I.e. YBCO (Y1Ba2Cu3O7), Nb - Ti (Niobium titanium)

: DIFFERENCE BETWEEN TYPE I AND TYPE II SUPERCONDUCTORS

Type-I superconductors	Type-II superconductors
They are called soft superconductors	They are known as hard superconductors
They exhibit complete Meissner effect	They do not exhibit complete Meissner effect
They show perfect diamagnetic behavior	They do not show perfect diamagnetic behavior

It requires low magnetic field to destroy the superconductivity	It requires large magnetic field to destroy the superconductivity
They have only one critical magnetic field (Hc)	They have two critical magnetic fields, lower critical magnetic field (Hc1) and upper critical magnetic field (Hc2)
There is no mixed or intermediate state in case of these materials	Mixed or intermediate state is present in these materials
They have limited applications because of low field strength. e.g. Pb, Hg, Zn, Nb	They have wider technical applications because of high field strength. e.g. Nb-Sn, Va-Ga, YBCO

: LOW TEMPERATURE AND HIGH TEMPERATURE SUPERCONDUCTORS

LOW TEMPERATURE SUPERCONDUCTORS (LTSC):

Superconductors that require liquid helium as a coolant are called low temperature Superconductors (LTSC).

For LTSC superconductors the temperature is usually well below 20 K (-253 °C). Temperature of liquid helium is 4.2 K above absolute zero (0 K).

HIGH TEMPERATURE SUPERCONDUCTORS (HTSC):

Superconductors having their critical temperature (Tc) value above the temperature of liquid nitrogen are called high temperature superconductors.

In a superconductor if the transition temperature is high i.e. greater than 20K, then it is also called high temperature superconductor. Earlier it was believed that the superconductivity was only in metals.

Surprisingly in 1986, Muller and Bednorz discovered high temperature superconductor in ceramics. They made a particular type of ceramic material from a compound of barium, lanthanum, copper and oxygen (Ba-La-Cu-O).

This compound superconductor showed superconductivity even at a temperature as high as 30 K. Similar materials with higher transition temperature soon followed when Y1Ba2Cu3O7 (YBCO) or the so called 1-2-3 compound was discovered.

The onset of superconductivity for 1-2-3 occurs at 93 K. The highest critical temperature at ambient pressure discovered so far (2009) is 134 K for a doped HgBa2Ca2Cu3O8+δ.

: BCS THEORY

The microscopic theory of superconductivity developed by J. Bardeen, L.N. Cooper and J.R. Scriffer in 1957, successfully explained the effect like zero resistivity, Meissner effect etc. this theory is known as BCS theory.

PRINCIPLE:

This theory states that the electrons experience a special kind of attractive interaction, overcoming the coulomb forces of repulsion between them; as a result cooper pairs (i.e) electro pair are formed. At low temperature, these pairs move without any restriction through the lattice points and the material becomes superconductor. Here the electron-lattice-electron interaction should be stronger than electron-electron interaction.

IMPORTANT FEATURES OF BCS THEORY:

Electrons form pairs (called cooper pair) which propagate throughout the lattice.

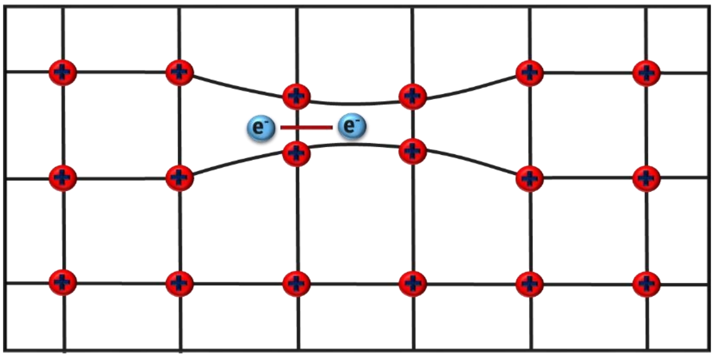
The propagation of cooper pairs is without resistance because the electrons move in resonance with phonons.

ELECTRON-LATTICE-ELECTRON INTERACTION:

When an electron (1st) moves through the lattice, it will be attracted by the core (+ve charge) of the lattice. Due to this attraction, ion core is disturbed and it is called as lattice distortion. The lattice vibrations are quantized in terms of phonons.

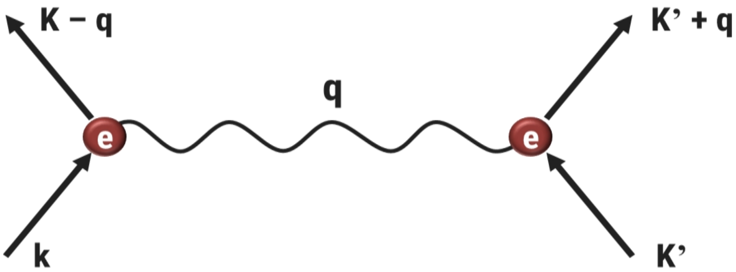
The deformation produces a region of increased positive charge. Thus if another electron (2nd) moves through this region as shown in figure. It will be attracted by the greater concentration of positive charge and hence the energy of the 2nd electron is lowered.

Hence two electrons interact through the lattice or the phonons field resulting in lowering the energy of electrons. This lowering of energy implies that the force between the two electrons is attractive. This type of interaction is called electrons - lattice electron interaction. The interaction is strong only when the two electrons have equal and opposite momenta and spins.



EXPLANATION:

Consider the 1st electron with wave vector k distorts the lattice, here by emitting phonons of wave vector q. This results in the wave vector k - q for the 1st electron. Now if the 2nd electron with wave vector k', seeks the lattice it takes up the energy from the lattice and its wave vector changes k' + q as shown in figure. Two electrons with wave vectors k - q and k' + q form a pair known as cooper pair.



: COOPER PAIRS

The fundamental of BCS theory is that superconductivity occurs when an attractive interaction between two electrons, by mean of phonon exchange takes place. This dominates the usual repulsive Coulomb interaction.

“Two such electrons which interacts alternatively in phonon field are called Cooper Pairs.”

The energy of cooper pairs of electrons in bound state is less than the energy of electron in Free State.

This difference in energy is due to binding energy, which is supplied if the pair is broken. At lower temperature (lower than Tc), the electron-electron lattice interaction is stronger than electron-electron Coulomb repulsive interaction, which leads to pairing up of valence electrons.

This pairing is complete at T = 0 K and completely broken at critical temperature (Tc).

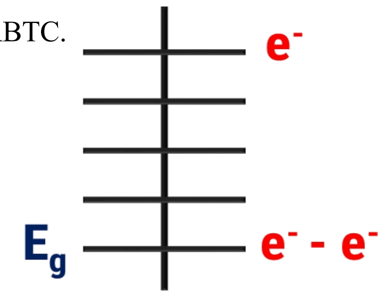
: ENERGY GAP

“The energy difference between the free state of an electron (i.e. energy of individual electron) and the paired state (i.e. energy of paired electron) appears as the energy gap at the fermi surface.”

The normal electron states are above the energy gap and superconducting electron states are below the energy gap (as shown in figure).

The energy gap in superconductors is a function of temperature. As the pairing is complete at T = 0 K, the difference in energy of free electron states is maximum.

BCS theory predicts that the energy gap in superconductors at T = 0 K is Eg = - 3.53 KBTC.



This equation can be verified by the absorption of electromagnetic radiation. Photons with energies equal to or greater than energy gaps can be absorbed.

: COHERENCE LENGTH

The paired electrons are not scattered because of their peculiar property of smoothly riding over the lattice imperfections without exchanging energy with them.

The concept of coherence can be explained by the fact that superconductivity is due to mutual interaction and correlation of electrons that extend upto a considerable distance.

“The maximum distance upto which the state of paired electrons are correlated to produce superconductivity is called Coherence length ($\varepsilon 0$).”

The paired electrons can be thousands of atomic spacing approx. (0.001 nm) apart.

The properties of a superconductor depends on the correlation of electrons within a volume $(\varepsilon 0)3$ (Called the ‘coherence volume’).

This is mainly because, a large no. of electrons in such a volume act together to produce superconductivity with an extremely sharp transition.

: PENETRATION DEPTH

It is a concept proposed by London brothers in the year 1935 that described the Meissner effect.

In a superconductor the London penetration depth (λL) characterizes the distance to which a magnetic field penetrates into a superconductor.

The exponential decay of magnetic field at the interior surface of superconductor.

$$\therefore B(x) = B0\, e^{(-x)\,\lambda}$$

X = distance inside the superconductor from surface B0 = Magnetic field at the surface

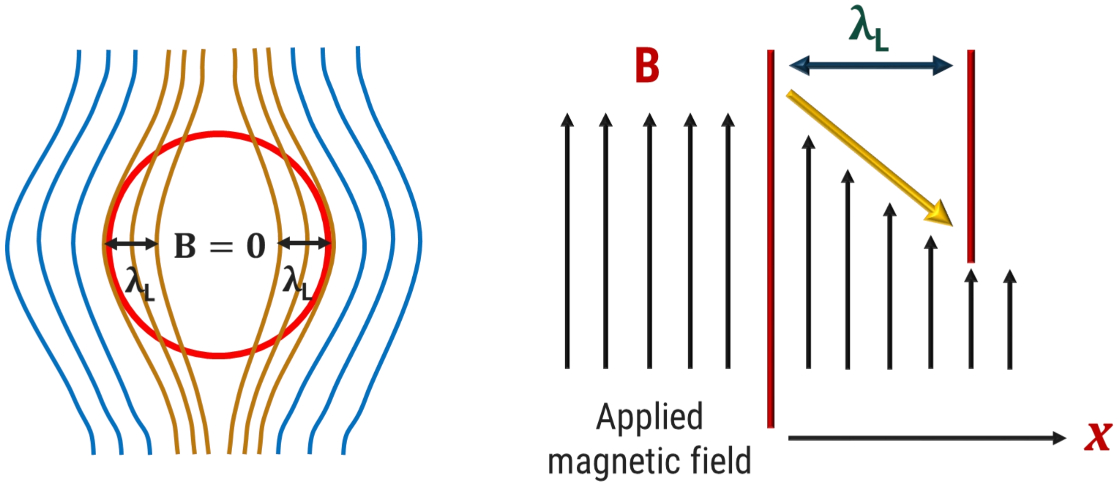
λL = Penetration depth

λL is the distance across which the magnetic field becomes 'e' times weaker.

m
λL can be defined as: λL = √
μ0 ns q2

m = mass of the charge carriers
ns = number super-electron density μ0 = absolute permeability

LONDON EQUATIONS:



Maxwell's equations could not explain the zero resistance and perfect diamagnetism observed as main characteristics of superconductivity.

∴ ∇ × E = − ∂ B
∂t

F. London and H. London (London brothers) suggested that motion of superconducting electrons in the presence of applied electric field E. If a magnetic field is applied to a superconductor which is initially in zero field, the magnetic field is a function of time.

London brothers assume there are two types of electrons:

Normal electrons: T > Tc   Super electrons: T < Tc

When electric field is applied there will be force exerted on electrons: F = q E

$\therefore$ ma = q E

$$\therefore m \frac{\partial v}{\partial t} = q E \ldots \ldots \ldots (1)$$

Due to force there will be motion of charge, so super-current density (J): J = n q v Where, n = concentration of super-electrons, v = drift velocity, q = electric charge

By taking derivation w. r. t. time: $$\frac{\partial J}{\partial t} = n q \frac{\partial v}{\partial t} \ldots \ldots \ldots (2)$$

$$\therefore \frac{\partial J}{\partial t} = n q \frac{q E}{m} \quad \text{From equation (1)}$$

$$\therefore \frac{\partial J}{\partial t} = \frac{n q^2 E}{m} \ldots \ldots \ldots (3)$$

This equation known as first London equation.

Now, when electric field (E) becomes zero means current density (J) is constant By taking curl of both sides of equation (3), we get

$$\therefore \nabla \times \frac{\partial J}{\partial t} = \frac{n q^2}{m} (\nabla \times E)$$

$$\therefore \frac{\partial}{\partial t} (\nabla \times J) = \frac{n q^2}{m} (\nabla \times E)$$

$$\therefore \frac{\partial}{\partial t} (\nabla \times J) = \frac{n q^2}{m} \frac{-\partial B}{\partial t}$$

By taking integration on both sides

$$\therefore \int \frac{\partial}{\partial t}(\nabla \times J) = \frac{n q^2}{m} \int \frac{\partial B}{\partial t}$$

$$\therefore (\nabla \times J) = -\frac{n q^2}{m} B \quad \ldots \ldots \ldots (4)$$

This equation known as second London equation.

The super current density is related to magnetic field 'B' by another Maxwell's equation as,

$$\therefore \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

But for electrostatic current the curl of E becomes zero.

$$\therefore \vec{\nabla} \times \vec{B} = \mu_0 J$$

$$\therefore \frac{1}{\mu_0}(\vec{\nabla} \times \vec{B}) = J \quad \ldots \ldots \ldots (5)$$

Substituting equation (5) in equation (4), we get

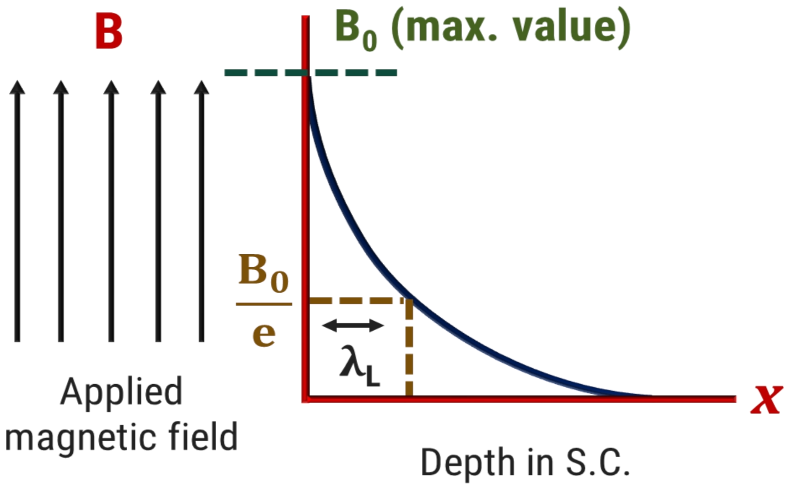
$$\therefore \left(\nabla \times \frac{1}{\mu_0}(\vec{\nabla} \times \vec{B})\right) = -\frac{n q^2}{m} B$$

$$\therefore \frac{m}{\mu_0 n q^2}(\nabla \times (\vec{\nabla} \times \vec{B})) = -B$$

$$\therefore \frac{m}{\mu_0 n q^2}(\nabla \times (\vec{\nabla} \times \vec{B})) + B = 0$$

Where, $\therefore \lambda_L^2 = \dfrac{m}{\mu_0 n_s q^2}$

Above equation defines: London penetration depth (λ_L).



For $x = \lambda L$,

$$\therefore B(x) = B_0 e^{(-x)/\lambda}$$

$$\therefore B(x) = B_0 e^{(-1)}$$

$$\therefore B(x) = \frac{B_0}{e}$$

The field decays to B_0/e at a distance $x = \lambda L$ inside the superconducting material.

Typical values of λ range from 50 to 500 nm.

The penetration depth (λL) is determined by the superfluid density, which is an important quantity that determines Tc in high-temperature superconductors. At a temperature T, penetration depth is given by:

$$\therefore \lambda_T = \lambda_0 \left[1 - \left(\frac{T}{T_c}\right)^4\right]^{-\frac{1}{2}}$$

Where, λ_0 is penetration depth at 0 K. It is given by:

$$\therefore \lambda_L = \left[\frac{m}{\mu_0\, n_s\, q^2}\right]^{\frac{1}{2}}$$

EXAMPLES

Q.1 The number of super-electrons in a superconductor is $10^{28}/m^3$ at critical temperature T_c = 3 K. Calculate the penetration depth at 0 K and 1 K.

Ans. $m = 3.1 \times 10^{-31}$ kg, $\mu_0 = 12.56 \times 10^{-7}$, $q = 1.6 \times 10^{-19}$ C, $n_s = 10^{28}/m^3$, λ_0 =?, λ_T =?

At T = 0 K, the penetration depth is

$$\therefore \lambda_0 = \left[\frac{m}{\mu_0\, n_s\, q^2}\right]^{\frac{1}{2}}$$

$$\therefore \lambda_0 = \left[\frac{9.1 \times 10^{-31}}{12.56 \times 10^{-7} \times 10^{28} \times (1.6 \times 10^{-19})^2}\right]^{\frac{1}{2}}$$

$$\therefore \lambda_0 = 0.530 \times 10^{-7} \text{ m}$$

$$\therefore \lambda_0 = 530 \text{ Å}$$

As, we know that $\lambda_T = \lambda_0 \left[1 - \left(\frac{T}{T_c}\right)^4\right]^{-\frac{1}{2}}$

$$\therefore \lambda_T = 530 \left[1 - \left(\frac{1}{3}\right)^4\right]^{-\frac{1}{2}}$$

$$\therefore \lambda_T = 530 \left[1 - \frac{1}{81}\right]^{-\frac{1}{2}}$$

$$\therefore \lambda_T = \frac{530 \times 9}{0.94}$$

$$\therefore \lambda_T = 533.30 \text{ Å}$$

Q.2 The penetration depth λ of Hg at 3.5 K is about 750 Å. Find the penetration depth at 0 K. Given Tc for Hg = 4.153 K.

Ans. $\lambda_T = 750$ Å, $T = 3.5$ K, $T_c = 4.153$ K, $\lambda_0 = ?$

The penetration depth at temperature (T) is given by

$$\therefore \lambda_T = \lambda_0 \left[1 - \left(\frac{T}{T_c}\right)^4\right]^{-\frac{1}{2}}$$

$$\therefore \lambda_0 = \frac{\lambda_T}{\left[1 - \left(\frac{T}{T_c}\right)^4\right]^{-\frac{1}{2}}}$$

$$\therefore \lambda_0 = \lambda_T \left[1 - \left(\frac{T}{T_c}\right)^4\right]^{\frac{1}{2}}$$

$$\therefore \lambda_0 = 750 \left[1 - \left(\frac{3.5}{4.153}\right)^4\right]^{\frac{1}{2}}$$

$$\therefore \lambda_0 = 750 \left[1 - (0.84)^4\right]^{\frac{1}{2}}$$

$$\therefore \lambda_0 = 750 \left[1 - 0.50\right]^{\frac{1}{2}}$$

$$\therefore \lambda_0 = 750 \left[0.50\right]^{\frac{1}{2}}$$

$$\therefore \lambda_0 = 750 \times 0.71$$

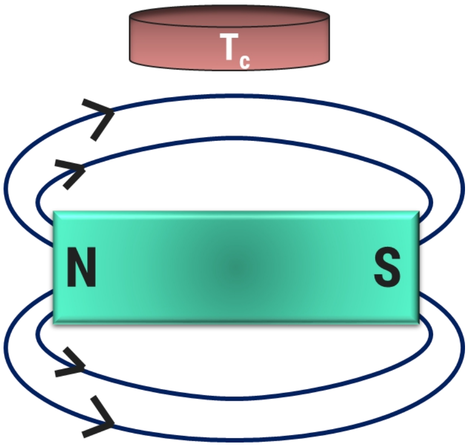
∴ $\lambda 0 = 530$ Å

: APPLICATION OF SUPERCONDUCTORS

: MAGNETIC LEVITATION

It is the process by which an object is suspended above another object with no other support, but magnetic field.

This phenomena is used in Maglev train. In Maglev trains, the levitation is brought due to presence of enormous repulsion among two highly powerful magnetic field when a small magnet is brought close to a superconducting magnet gets repelled.

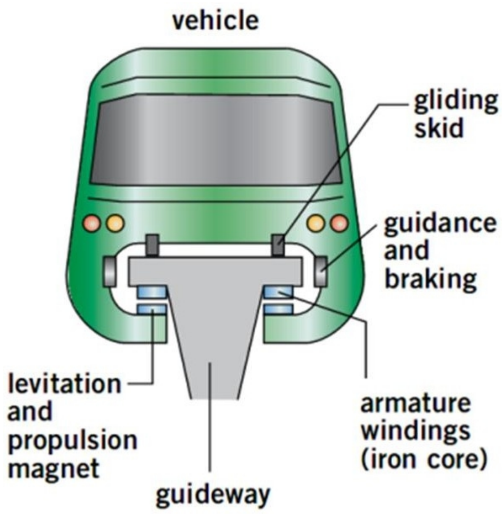


This repulsion takes place due to the induced currents in the superconductor which is being generated by the magnetic field of the magnet. Because of zero resistance property of the superconductor this current persists, and thus the field due to this induced current repels the field due to the magnet.

As a result, the magnet floats freely above the superconductor.

In attractive Electromagnetic suspension (EMS), the electromagnets installed on the train bogies attract the iron rails. The vehicle magnets wrap around the iron guide-ways and the attractive upward force lifts the train.

Electrodynamic suspension uses repulsive force (magnet of same polarity) to levitate the train.



Magnet of same polarity creates a repulsive force between levitation magnet and guideway. Because of superconductor is used, conduction is possible even when there is no power. Maglev trains travel at speed of about 500 km/h. A similar magnetic propulsion system is being used to launch the satellite into orbits directly from the earth without the use of rockets.

: JOSEPHSON EFFECT

Josephson effect was defined by Brian Josephson, in the year 1962.

This is called Josephson junction or SIS junction.

There are two type of Josephson effects: (1) D.C. Josephson effect & (2) A.C. Josephson effect

D.C. JOSEPHSON EFFECT:

When superconductors are separated by a thin insulating layer, the tunneling of electron pairs takes place from one superconductor to another.

The cooper pairs of electrons tunnel through the insulating layer, where their thickness is very small, of an order of 1nm.The supercurrent (J) which is developed across the insulating junction depending on phase difference.

A dc current is noted when there is no potential difference is applied across the junction. The expression for the current for this effect is

$\therefore I_s = I_{max} \sin\theta$

Where theta is the phase difference between the two electrons of cooper pairs.

A.C. JOSEPHSON EFFECT:

When an external magnetic or electric field of potential 'v' is applied across the junction, the zero - voltage current developed across the junction depending on applied field, which is the "AC Josephson effect".

When a potential 'v' is applied across the junction, cooper pairs on both the sides of the junction differs by an energy equals to 2eV, where 2e is the charge on a cooper pair of electrons.

Since in superconductors, current is carried by cooper pairs of electrons, if a copper pair passes across the gap, then it emits a photon of energy (hv) equal to 2eV.

Then, the frequency of emitted radiation is = 2eV/h.

This is the oscillating frequency of sinusoidal current across the gap. Suppose the P.D. across the gap is 1mV, then the frequency will be of the order of 483.6 GHz.

Applied voltage changes the phase difference:

$$\frac{\theta}{t} = \frac{2eV}{h}$$

Due to this change we call it A. C. Josephson effect

The value of current is given as: $I_s = I_{max} \sin\left[\frac{2\pi eV}{h} t + \theta\right]$

EXAMPLES

Q.1 A Josephson junction having a voltage of 8.50 μV across its terminals, then calculate the frequency of the alternating current. [Plank's constant = 6.625×10^{-34} J · sec]

Ans. $V = 8.5 \times 10^{-6}$ V, h = = 6.625×10^{-34} J · sec, e = 1.6×10^{-19}, V =?

Frequency of alternating current, $= \frac{2eV}{h}$

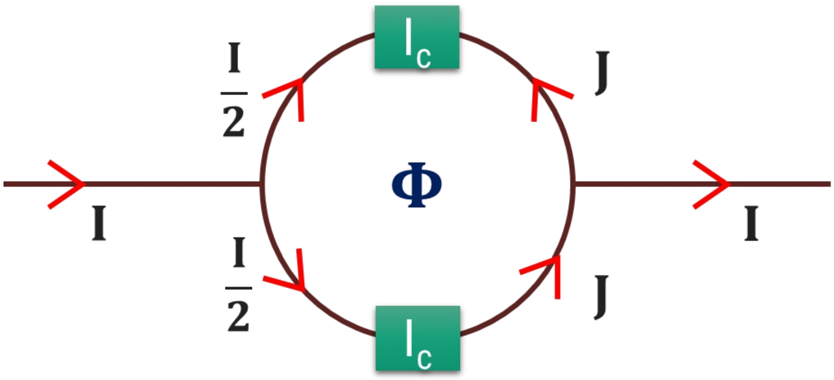
$$\therefore \; = \frac{2 \times (1.6 \times 10^{-19})(8.5 \times 10^{-6})}{6.625 \times 10^{-34}}$$

$$\therefore \; = 4.1 \times 10^{9} \text{ Hz}$$

: SQUID (SUPERCONDUCTING QUANTUM INTERFACE DEVICES)

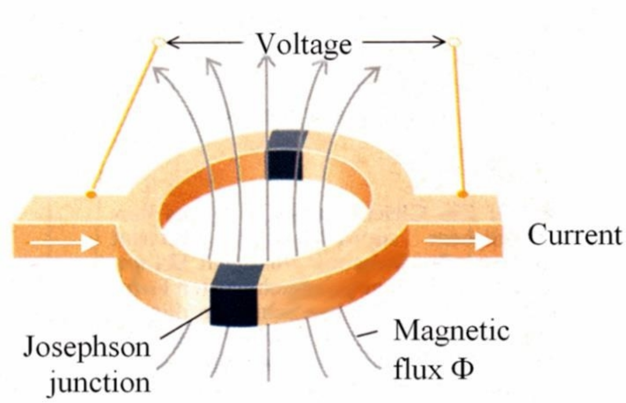
It works on the principal of Josephson effect.

When current is passed through one side of superconductor, it flows equally through the Josephson junction.



CONSTRUCTION:

As shown in the diagram, it is seen that SQUID is formed when two superconductors and insulators are arranged in such a way that they form two parallel Josephson junction.



It is connected to measuring device which can measure nominal changes in voltage and magnetic field across junction.

WORKING:

Current through the SQUID is highly sensitive to magnetic flux through the closed circuit. Evan an extremely small magnetic flux can be detected with the device.

The current through the circuit will have a periodicity which is very sensitive to the magnetic flux passing normally through the closed circuit. As a result, extremely small magnetic flux can be detected with this device.

This device can also be used to detect voltages as small as 10 – 15 V. Magnetic field changes as small as 10-21 T can be detected.

APPLICATION:

Used to measure earth's magnetic field.
It can detect weak magnetic field produced by biological currents like human brain.

Based on these behavior, today we have (1) Magneto-Cardio Graphy (MCG)
(2) Magneto- Plethismo Graphy
Detection of iron deposition in human liver.

They are used in NMR imaging systems.
They are used to detect brain tumors and clots using superconducting-solenoids.

: OTHER APPLICATIONS OF SUPERCONDUCTORS

SUPERCONDUCTING MAGNETS:

Generally, when current flows through coil, it generates magnetic field. If the coil is replaced by a superconducting material, it generates a very large magnetic field.

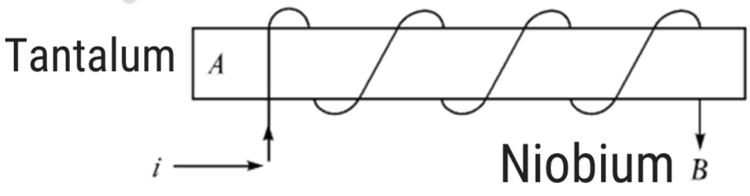
In this case, there is no effect of joule heating or resistive loss. Superconducting magnets are used in medical diagnosis, spectroscopy, magnetic levitation, etc.

Superconducting electrical generators are small in size compared to conventional electric generators and produces more power as compared to ordinary generators.

Superconducting materials are used in no-loss transmission lines and as relays on switching circuit.

CRYOTRON:

Two different superconducting materials are taken A= Tantalum Core and B= Niobium as winding wire.



Taken them as such that: HcA < HcB

If the current passes through winding B, it induces magnetic field.

If this induced magnetic field is greater than HcA, its corresponding state will be destroyed, thereby increasing resistivity and hence decreasing the current.

Thus we can say that current through the winding controls superconducting state of core.

It is useful in circuit breaker, relays etc…

: ASSIGNMENT

Define superconductivity and critical temperature. (GTU: Jan-2019)
Explain mechanism of superconductivity. (GTU: Jan-2019)
Discuss the properties of superconductors. (GTU: Jan-2019, Jan-2020)
Distinguish between Type-I and Type-II superconductors. (GTU: Jan-2020)
Explain the BCS theory for superconductivity. (GTU: May-2019)

Differentiate between soft and hard superconductors. (GTU: May-2019)
What is London penetration depth? Derive its equations. (GTU: May-2019, Jan-2020)
Distinguish between dc and ac Josephson effect.
What is a SQUID? Explain the construction, working and applications of SQUID. (GTU: May- 2019, Jan-2020)
Write a note on cryotron. (GTU: Jan-2020)
The critical temperature of a metal with isotopic mass 199.5 is 4.185 K. Calculate its critical temperature when its isotopic mass is changed to 203.4.
[Ans. 4.145 K]
A lead super conductor with Tc = 7.2 K has a critical magnetic field of 6.5 × 103 Am-1 at absolute zero. What would be the value of critical field at 5 K temperature?
[Ans. 3.36 × 103 Am-1]
The critical current density equal to 1.71 x 108 A/m2 is required to change a superconducting wire of radius 0.5 mm at 4.2 K. If the critical temperature of the material is
7.18 K, calculate the maximum value of the critical magnetic field. [Hint: Hc(0) = ?] (GTU: May-2019)
[Ans. 64.986 × 103 Am-1]
The critical temperature of Nb is 9.15 K. At zero kelvin, the critical field is 0.196 T. Calculate the critical field at 6 K. (GTU: Jan-2019)
[Ans. 0.1117 T]
What is the frequency of the electromagnetic waves radiate from a Josephson junction, if the voltage drop at the junction is 650 μV.
[Ans. 313.95 × 109 Hz]

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