# Machine Learning Assignment III

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## 0.1 Task 1

Derive the AKRR solution

Task 1:

$$\beta = \sum_{i=1}^{n} \alpha_i \phi(x_i)$$

$$\bar{\beta} = \sum_{i=1}^{m} \alpha_i \phi(x_i)$$

$$J = \sum_{i=1}^{n} (x_i^T \beta - y_i)^2 + \lambda \beta^T \beta \quad - (1)$$

So in order to make $\beta$ to $\bar{\beta}$, we need to make all the entries except $m$ entries equal to 0.

$$J = \sum_{i=1}^{m} (x_i^T \bar{\beta} - y_i)^2 + \sum_{i'} (-y_i)^2 + \lambda \bar{\beta}^T \bar{\beta} \quad - (2)$$

$$\frac{\partial J}{\partial \bar{\beta}} = \sum_{i=1}^{m} 2(x_i^T \bar{\beta} - y_i) x_i + 2\lambda \bar{\beta} = 0$$

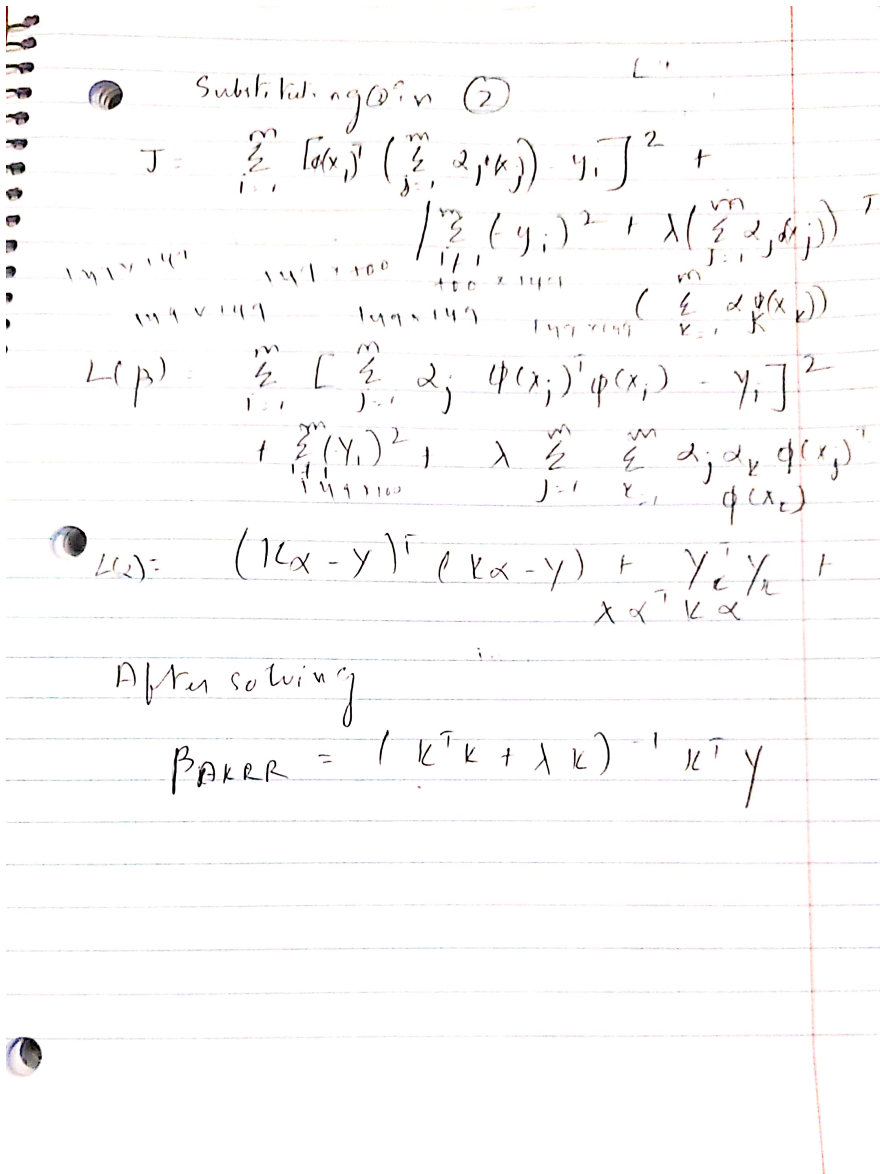
$$\cancel{2}\lambda \bar{\beta} = -\sum_{i=1}^{m} \cancel{2}(x_i^T \bar{\beta} - y_i) x_i$$

$$\bar{\beta} = \sum_{i=1}^{m} \overbrace{\boxed{-\frac{1}{\lambda}(x_i^T \bar{\beta} - y_i)}}^{\alpha_i} \underbrace{x_i}_{\phi(x_i)}$$

$$\bar{\beta} = \sum_{i=1}^{m} \alpha_i \phi(x_i) \quad - (3)$$



Figure 1: solution AKRR



Substituting (3) in (2)

$$J = \sum_{i=1}^{m} \left[\phi(x_i)^T \left(\sum_{j=1}^{m} \alpha_j \phi(x_j)\right) - y_i\right]^2 + \sum_{i=1}^{} (y_i)^2 + \lambda\left(\sum_{j=1}^{m} \alpha_j \phi(x_j)\right)^T \left(\sum_{k=1}^{m} \alpha \phi(x_k)\right)$$

$$L(\beta) = \sum_{i=1}^{m} \left[\sum_{j=1}^{m} \alpha_j \phi(x_j)^T \phi(x_i) - y_i\right]^2 + \sum_{i=1}^{m} (Y_i)^2 + \lambda \sum_{j=1}^{m} \sum_{k=1}^{m} \alpha_j \alpha_k \phi(x_j)^T \phi(x_k)$$

$$L(2) = (K\alpha - y)^T (K\alpha - y) + Y_c^T Y_c + \lambda \alpha^T K \alpha$$

After solving

$$\beta_{AKRR} = (K^T K + \lambda K)^{-1} K^T Y$$



Figure 2: solution AKRR

## 0.2 Task 3

The train and test error across different values of m.

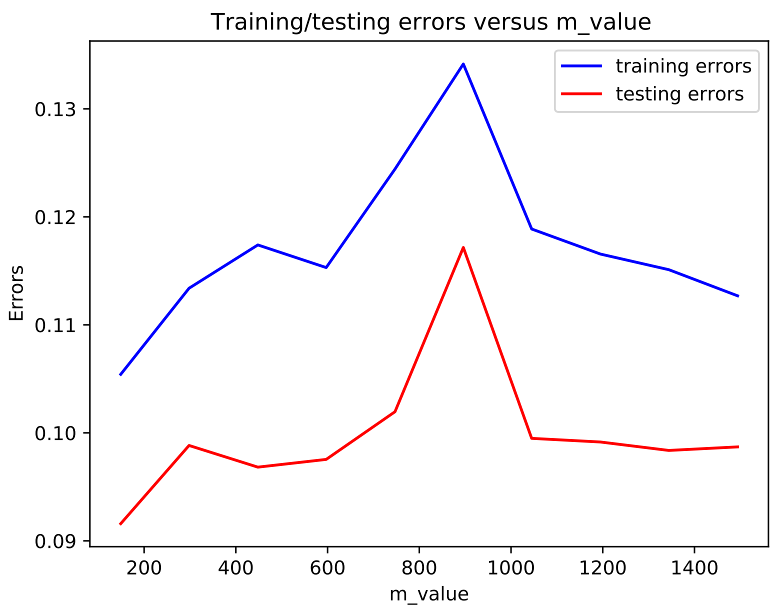



Figure 3: Train and test error