

Machine Learning Assignment I

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0.1 Problem 1

Derive a matrix form:

Problem-1

$W \in \mathbb{R}^{n \times n}$ with w_i on the diagonal

$$W = \begin{bmatrix} w_1 & 0 & \dots & 0 \\ 0 & w_2 & & \\ \vdots & & \ddots & \\ 0 & & & w_n \end{bmatrix} \rightarrow \text{diagonal matrix}$$

$n \times n$

$$J(\beta) = \sum_{i=1}^n w_i (x_i^T \beta - y_i)^2 + \lambda \sum_{j=1}^{p-1} \beta_j^2$$

The

$$W (x\beta - y)^T (x\beta - y) + \lambda \beta^T I_0 \beta$$

I_0 Identity matrix with element 1.

$$J = W (x\beta - y)^T (x\beta - y) + \lambda \beta^T I_0 \beta$$

$$= W [\beta^T x^T x \beta - \beta^T x^T y - y^T x \beta + y^T y]$$

$$+ \lambda \beta^T I_0 \beta$$

$$= W [\beta^T x^T x \beta - 2 \beta^T x^T y + y^T y] + \lambda \beta^T I_0 \beta$$

$$= [\beta^T x^T W x \beta - 2 W \beta^T x^T y + W y^T y] + \lambda \beta^T I_0 \beta$$

$$J = \beta^T [x^T W x + \lambda I_0] \beta - 2 W \beta^T x^T y + W y^T y$$

Matrix form of $J(\beta)$

Scanned with CamScanner

Figure 1: Matrix form

0.2 Problem 2

Derive an analytical solution for beta:

Problem-2

$$J' = 2\beta (X^T W X + \lambda I_D) - 2X^T W y = 0$$
$$\Rightarrow \beta (X^T W X + \lambda I_D) = X^T W y$$
$$\boxed{\beta = (X^T W X + \lambda I_D)^{-1} X^T W y}$$

Analytical solution for β in matrix

Scanned with CamScanner

Figure 2: Analytical solution

0.3 Problem 3

Weighted regularized Least Square Model:



Figure 3: Training/Testing Errors versus Lambda

Types of Errors (Lambda = 16)	w1 (5)	w2 (10)	w3 (50)
Error on all testing instance	0.0176	0.0177	0.0184
Error on minority testing instances	0.0133	0.0133	0.0137
on non-minority testing instances	0.0207	0.021	0.0219

Table 1: Testing Errors with Different Minority Weights

0.4 Problem 4

LASSO Model:

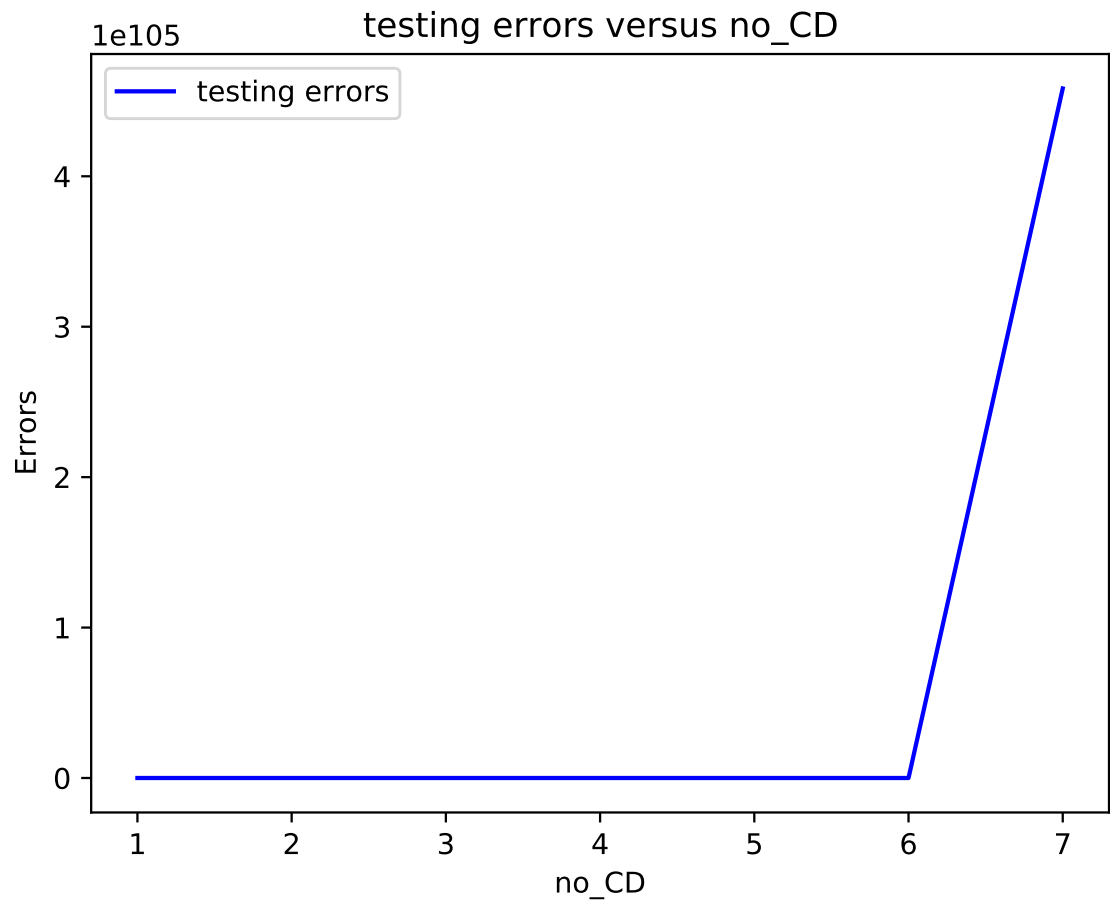


Figure 4: Testing Error versus CD Updates

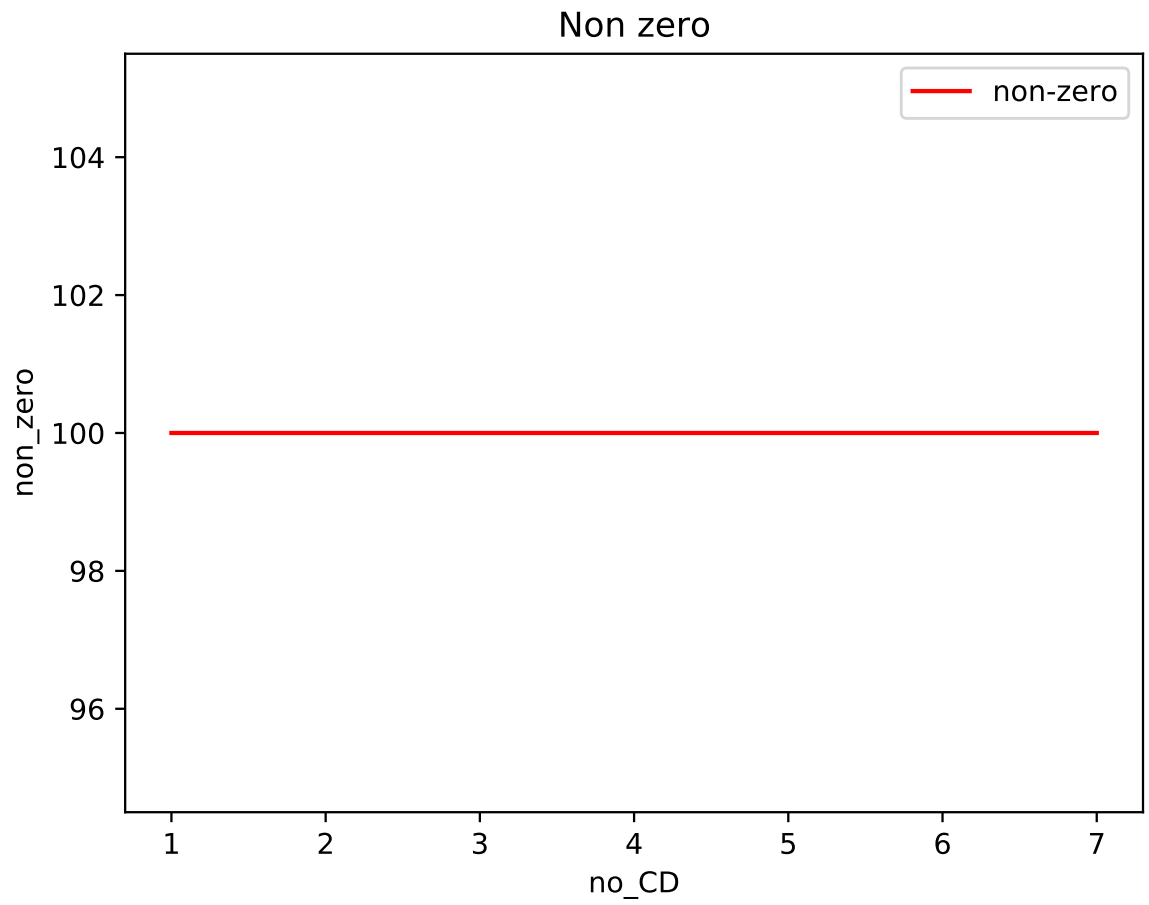


Figure 5: Number of Non-Zero Elements in Beta versus CD Updates