

## Example Crossover and Mutation Techniques for various Chromosome Representations

Chromosome Representation	Crossover	Mutation
Bits	Single Point	Randomly Flip one or more bits
	Double Point	
	N-Point	
	Uniform	
Permutation (order Based)	Order1	PE (Pair-wise Exchange)
	Order2	C3 (Cycle of Three)
	PMX	SM (Single Move)
	Cycle	TBE (Two Bond Exchange)
	Position Based	
	Cut-and-crossfill	
Reals	Simple Arithmetic Recombination	Change value randomly within the range
	Single Arithmetic Recombination	
	Whole Arithmetic Recombination	

Order1, Order2, PMX, Cycle and Position Based are covered in the Starkweather paper. PE, C3, SM and TBE are also Perturbation functions covered in Simulated Annealing. Uniform, Simple A.R., Single A.R. and Whole A.R. are shown below.

## Uniform Crossover

Parent 1      0 1 1 0 0 1 1 0 1 0 1 0

Parent 2      1 1 0 0 1 0 0 0 1 1 0 1

U              1 0 1 1 1 0 1 0 1 1 1 0      The Uniform bit string is randomly generated

**Rules for generating Child1:** if  $U_i = 0$ , select the  $i$ th bit from Parent1  
if  $U_i = 1$ , select the  $i$ th bit from Parent2

Hence

Child1 is      1 1 0 0 1 1 0 0 1 1 0 0

**Rules for generating Child2:** if  $U_i = 0$ , select the  $i$ th bit from Parent2  
if  $U_i = 1$ , select the  $i$ th bit from Parent1

Hence

Child2 is      0 1 1 0 0 0 1 0 1 0 1 1

**Note** Uniform crossover can be used on most any chromosome representation where you want to select the allele from one parent or the other. It does not have to be bits.

## Simple Arithmetic Average

Select randomly a recombination point in the chromosome,  $i$ .

For child1, copy the first  $i$  alleles of Parent1 and place in Child1.

The rest of Child1 is the arithmetic average of Parent1 and Parent2.

$\alpha$  is usually a predetermined real number from 0..1, or it could be determined randomly.

For Child1 the arithmetic average is  $(\alpha * (\text{parent1 allele}) + (1 - \alpha) * (\text{parent2 allele}))$

The process for Child 2 is analogous. with Parent1 and Parent2 reversed.

Parent1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
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Parent2	0.3	0.2	0.3	0.2	0.3	0.2	0.3	0.2	0.3
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For  $\alpha = 0.5$  and  $i = 6$

Child1	0.1	0.2	0.3	0.4	0.5	0.6	0.5	0.5	0.6
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Child2	0.3	0.2	0.3	0.2	0.3	0.2	0.5	0.5	0.6
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For  $\alpha = 0.4$  and  $i = 6$

Child1	0.1	0.2	0.3	0.4	0.5	0.6	0.46	0.44	0.54
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Child2	0.3	0.2	0.3	0.2	0.3	0.2	0.54	0.56	0.66
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## Single Arithmetic Average

This is the same as the Simple Arithmetic Average except it is applied to only one randomly selected allele.

## Whole Arithmetic Average

This is the same as the Simple Arithmetic Average except it is applied to all of the alleles in the chromosome. Among the three Arithmetic Average crossovers Whole Arithmetic Average is most commonly used.

(Variation: Wainwright) If Parent1 and Parent2 have fitness function values within some epsilon, then use  $\alpha = 0.5$ , otherwise use  $\alpha = 0.6$  and give the more fit parent the higher weight.

Another variation: independent of epsilon, randomly (50% of the time) use  $\alpha = 0.5$  and then 50% of the time randomly select either  $\alpha = 0.1$ , or 0.2, or 0.3 or 0.4.

## Wainwright Variation using Uniform Crossover 01-20-23

Parent1 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9

Parent2 0.3 0.2 0.3 0.2 0.3 0.2 0.3 0.2 0.3

U 1 0 1 1 1 0 1 0 1

The Uniform bit string is randomly generated

**Rules for generating Child1:** if  $U_i = 0$ ,  $(\alpha * (\text{parent1 allele}) + (1 - \alpha) (\text{parent2 allele}))$   
if  $U_i = 1$ ,  $(\alpha * (\text{parent2 allele}) + (1 - \alpha) (\text{parent1 allele}))$

**Rules for generating Child2:** if  $U_i = 0$ ,  $(\alpha * (\text{parent2 allele}) + (1 - \alpha) (\text{parent1 allele}))$   
if  $U_i = 1$ ,  $(\alpha * (\text{parent1 allele}) + (1 - \alpha) (\text{parent2 allele}))$

For  $\alpha = 0.6$

Child1 0.22 0.30 0.40 0.38 0.38 0.44 0.46 0.54 0.54

Child2 0.18 0.26 0.30 0.32 0.42 0.36 0.54 0.44 0.66