

Effects of Network Topology, Mobility, Interaction, and Memory on Tipping Points

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Submission Id: 1032

ABSTRACT

Understanding the conditions under which social norms reach tipping points is critical for modeling large-scale behavioral change. A tipping point is reached when a committed minority influences the rest of the population to switch from a pre-existing norm. In this paper, we explore how network topology, influencer mobility, interactivity, and adaptive learning affect the acceleration and emergence of tipping points in agent-based coordination games. We investigate two different network topologies (grids and small worlds), neighborhood interactivity, varied mobility rates of influencer nodes, and agent memory capacity. We use extensive simulations to identify favorable conditions as well as the size and distribution of influencer groups needed for norm change. We analyze the norm change process to show the interplay between network structure and influencer mobility on the rate of norm change in the tipping process. Our findings contribute to a deeper understanding of norm change in multiagent systems, offering insights for the design of more effective social interventions and mechanisms of norm enforcement.

CCS CONCEPTS

• Theory of computation → Social networks.

KEYWORDS

Tipping Points, Norm Change, Network Topology, Mobility, Influencers

ACM Reference Format:

Anonymous Author(s). 2025. Effects of Network Topology, Mobility, Interaction, and Memory on Tipping Points. In *Proc. of the 24th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2025)*, Detroit, Michigan, USA, May 19 – 23, 2025, IFAAMAS, 7 pages.

1 INTRODUCTION

Social norms and conventions play a crucial role in shaping collective human behavior. Understanding how these norms emerge, spread, and undergo change is essential for various fields, including sociology, economics, and artificial intelligence [2, 11, 17, 21]. In particular, sudden and significant shifts in established behavioral norms, known as *tipping points*, where a previous minority behavior is adopted by the majority in the population, has received increasing attention because of the disruptive forces they are often associated with [1, 6, 9, 10, 16]. Of particular interest is the size of the minority group of like-minded influencers needed to effect a change in the established convention followed by the population.

The success of such minority groups in effecting norm change, i.e., the presence of tipping points, is consistent with the theory of *critical mass* discussed in evolutionary game theory literature [14, 20]. In the context of multi-agent systems (MAS), the study of norm adoption and tipping points has garnered significant attention, as it provides insight into collective behaviors and how they can be influenced or accelerated under various conditions [3, 13, 15, 17].

In this paper, we explore the impact of several factors on the emergence and acceleration of tipping points in social norms using agent-based simulations. Specifically, we examine how different network topologies (such as lattice and small-world [18] networks), mobility patterns (stationary vs. migratory agents [7]), interaction frameworks (Moore vs. von Neumann neighborhoods on a grid [22], or small-world networks), and learning modes affect the rate and likelihood of tipping points. Our approach introduces systematic variations in these dimensions to better understand how specific configurations can either accelerate or delay the critical mass required for tipping points involving a change of established norms.

While much of the previous work on tipping points in social norms has focused on static networks or singular norm options, we extend this research by allowing for dynamic mobility of influencer agents and the presence of multiple competing norms with varying levels of similarity. Additionally, we incorporate memory into agent decision-making, where agents base their norm adoption not only on immediate observations but also on a history of past interactions [6]. This adds complexity and realism to the model, providing a richer understanding of the conditions that lead to tipping points.

Our contributions are twofold:

- We develop a simulation framework to perform extensive experimentation with a range of influential factors, including network topology, mobility, and memory history.
- We provide empirical results that show how varying these parameters can accelerate tipping points, offering new strategies for inducing rapid norm change in MAS.

This work builds upon existing literature on norm emergence, tipping points, and network science, but it moves beyond prior approaches by integrating dynamic mobility, memory-based decision making, and multiple norm options into a unified framework.

The rest of this paper is organized as follows: Section 2 explores related work, Section 3 outlines the methodology of our agent-based model, Section 4 presents our results, Section 5 discusses our findings, Section 6 concludes our study with a number of takeaways, and Section 7 looks ahead to the future of this research.

2 RELATED WORKS

Our work is heavily influenced by the work of Centola *et al.* [6] who presents results, both from empirical study with human subjects and from simulation based experiments to validate formal predictions about the "critical mass" or proportion of minority influencers

needed to effect a change in established norms. Centola *et al.* found that in contrast to no significant effect of varying population size, varying memory size, i.e., the number of recent interactions influencing agent decisions, have a significant effect on tipping point. They experimented with 10 independent groups where members of each group were randomly paired to select a name for a picture. Choosing the same name gave them higher rewards. Once a convention was established in each group, a minority of 10-30% individuals, using a different naming convention for pictures, was introduced. Data was collected over successive interactions to see if the naming conventions adopted by the population changed to that used by the introduced minority groups.

Under explicit incentives that reward social coordination among peers, the authors observed a tipping point threshold percentage in the mid-20s of influencers need to change then established social convention to a newly introduced convention. This contrasts with prior theoretical models which hypothesized critical mass of as low as 10% to as high as 40% of the population size. Whereas Centola *et al.* considered random pairings within each group and no pairings between group, our work focuses on agents connected in a network topology and allows influencing minority group to move over the network in their attempts to "spread their influence."

Centola showed that the topology of networks influences how social norms spread, especially in the case of complex contagions, where adoption requires reinforcement from multiple neighbors [5]. Centola's work demonstrated that clustered-lattice networks are more conducive to norm adoption compared to random networks because they facilitate repeated interactions among individuals with similar thresholds. These findings highlight the crucial role that network structure plays in shaping the adoption process.

Granovetter's "Threshold Models of Collective Behavior" provided an influential framework for understanding how individual decision-making, which is contingent on social thresholds, combines to create outcomes [12]. Granovetter's model explains why small initial shifts in individual behavior can eventually lead to large-scale social changes, especially when a critical mass is reached. His work offers an essential theoretical lens for studying tipping points in social networks, complementing Centola's empirical findings.

While Granovetter's model primarily focused on binary choices, later research by Young explored how social learning influences the establishment of conventions, emphasizing that small initial groups of adopters can play a disproportionately large role in norm-setting under the right conditions [21]. Young also emphasized the significance of coordination games in social learning, where repeated interactions lead to the emergence of stable outcomes. This line of work aligns closely with Centola's focus on clustered-lattice networks, where local reinforcement among agents can establish enduring behavioral patterns. Both Centola and Young's findings underscore the idea that tipping points are not solely a function of network size or randomness but depend critically on how agents interact and learn from one another over time.

More recent research has focused on the role of minority influence in tipping dynamics. Baccino and Villata investigate how small, committed groups can influence the majority even when their views are initially unpopular [3]. Their findings suggest that the placement of such groups in network structures can greatly

amplify their influence, particularly when they demonstrate high levels of commitment or "loudness." This research highlights an important aspect of norm adoption: the qualitative characteristics of influencers, rather than their sheer number, can be decisive in tipping points. This echoes earlier insights from Centola, who argued that network structure, when combined with strong behavioral reinforcement, plays a pivotal role in norm propagation. They suggest this committed minority thrives especially in small-world or scale-free networks.

Crawford *et al.* investigated the repulsion effect seen so often in real-world interactions [8]. The researchers proposed two models, one for repulsion by social judgment, and another for repulsion of by categorization. They also theorized about the formation of extremist groups and their implications under these models.

Researchers, like Gelfand *et al.*, have also made extensive work of surveying the tipping-point literature and examining it for consensus opinions as well as novel findings [11]. Such work is imperative given the ease of exploration in social network dynamics in the modern day.

Together, these studies provide a comprehensive view of how individual decision-making, network structure, and influencer strategies interact to shape tipping points in social systems. The current research builds on this foundation by investigating how different network topologies, agent mobility, and interaction frameworks can alter tipping dynamics, with a specific focus on the role of influencers in driving collective behavior toward critical thresholds.

3 METHODOLOGY

3.1 Topological Generation

These experiments tested two different topologies: square lattices and small-world networks. They were chosen for their applicability to real-world social networks, and they provide short paths, on average, for easy network traversal. This will be useful for one of our agent types, which we will discuss later.

3.1.1 Lattice. This topology featured $N = n^2$ agents that were placed in an $n \times n$ grid. The connectivity of each node was determined by two boolean parameters: *Torus* and *Moore*. The former indicated whether the lattice should be treated like a torus (i.e. the left and right edges were connected as well as the top and bottom). The latter specified whether each agent was surrounded by a von Neumann neighborhood (4 adjacencies) or a Moore neighborhood (8 adjacencies).

3.1.2 Small world. These networks consisted of N agents connected within a Watts–Strogatz small-world graph. They were built by specifying two parameters: k (the number of nearest neighbors each node was initially connected to in a ring topology) and p (the probability of rewiring an edge). For all of our simulations, $p = .1$.

3.2 Agent Types

The agents in these simulations came in two varieties: ordinary and influencer. While an agent's type stayed the same throughout a simulation, its internal state value, representing its current norm, could change.

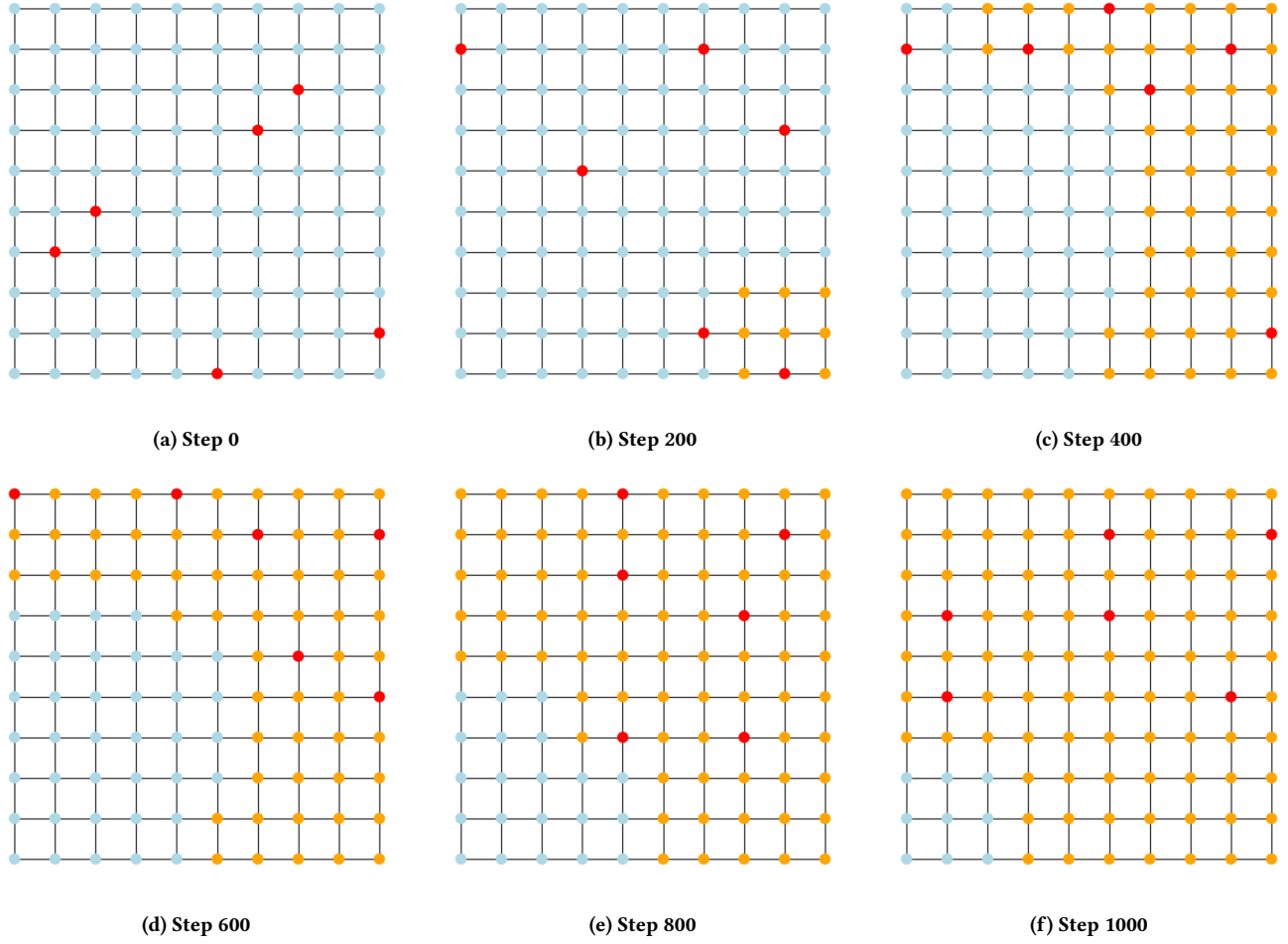


Figure 1: Example simulation ($m = .25$, $M = 5$, $Torus = False$, $Moore = False$)

3.2.1 Ordinary. Most agents at the beginning of each simulation were ordinary. They represented the dominant majority all following the same norm, but could be convinced to switch. To represent the dominant norm, their initial state was set to 0. These agents did not move on their own, but they did interact with their direct neighbors at each time step. The result of this interaction determined the state they assumed for that time step. To reach that result, the agent observed the states of its current neighbors along with the previous M steps of neighbor states in its memory. The agent would then take on the majority norm or remain unchanged if there was a tie.

3.2.2 Influencer. A fraction ($f < .5$) of the agents were given an alternative norm, represented as state 1, along with some special properties. These committed influencers never changed their norm, but they did move throughout the network in hopes of converting other agents. At each time step, an influencer had an $m\%$ chance to swap places with one of its ordinary neighbors, chosen at random.

3.3 Simulation

After generating the topology, the influencers were placed evenly throughout the network according to a Halton sequence with Owen scrambling for sufficiently different placements between simulations. For lattices, a two-dimensional sample was calculated, then each coordinate was rounded to the nearest integer and mapped to the corresponding node on the grid in which to place an influencer. For small worlds, a one-dimensional sample was chosen and directly mapped to the node IDs in the network.

A full simulation using a lattice network is displayed in Figure 1. Red dots represent the influencers, blue dots are ordinary agents that still adhere to the initial norm of the majority, and orange dots are those that have switched to the minority norm. Additionally, Figure 2 shows the initial state of a small-world network.

At each time step, influencers moved according to their mobility rate m , and ordinary agents interacted with their neighborhoods. The adoption rate of the enter population was then recorded, and the cycle would repeat. If the alternative norm (state 1) ever reached

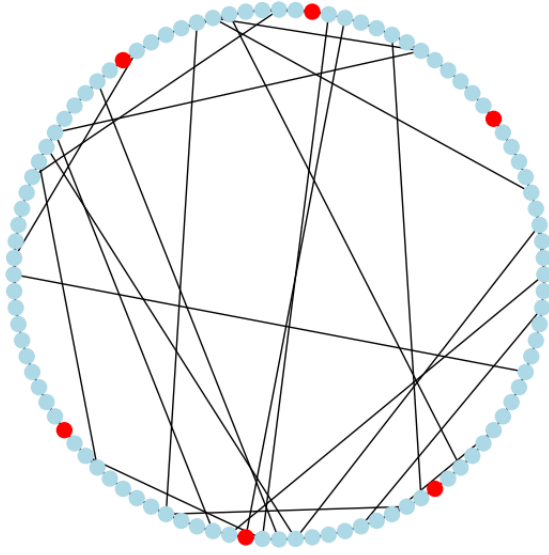


Figure 2: Example of a small-world network ($k = 4$)

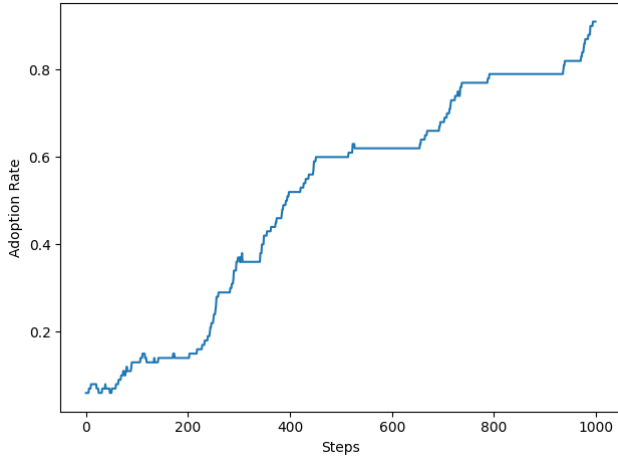


Figure 3: Example adoption rate over time

total ubiquity, the simulation would cease. Figure 3 shows the adoption rate over time of the simulation in Figure 1.

3.4 Sampling

In order to determine an estimate for the true tipping point f^* of each simulation configuration, a Markov chain Monte Carlo (MCMC) sampling method was employed. This is a great method for sampling considering the complexity of the model and the underlying distribution [4]. Given the target configuration, the sampler initialized $f_0 = .25$ and ran a simulation. If $a_i \geq .5$, where a_i was the final adoption rate of simulation i , then f_i would be

reduced as such:

$$f_{i+1} = f_i - .01 * a_i.$$

Otherwise, f_i would increase:

$$f_{i+1} = f_i + .01 * (1 - a_i).$$

The sampler went through a burn-in period initially to ensure the samples were sufficiently close to the true value. This period ended when f had both increased and decreased at least once. The next 25 samples were kept and averaged in order to arrive at an estimate for f^* .

4 RESULTS

All simulations ran for a maximum of 1000 time steps. Additionally, the number of agents N in each simulation was 225 (number of locations on a 15x15 grid).

4.1 Topology

4.1.1 Mobile Influencers. Table 1 shows the estimated tipping point values for many different simulation configurations within the lattice topology. These estimates suggest that f increases with the number of adjacencies in the network. Torus layouts ensure that edge cells have the same number of connections as interior cells. Moore neighborhoods are twice as large as von Neumann neighborhoods. Given $n = 15$, a non-toral grid with von Neumann neighborhoods would only have $2n(n - 1) = 420$ edges, whereas a torus with Moore neighborhoods would have over double with $4n^2 = 900$. Moore neighborhoods asymptotically double the number of edges in a lattice as n grows. This increases the number of influencers needed around an ordinary agent to convert its norm. Additionally, non-toruses require fewer influencers to convert those near the edge. Persuaded agents form a much more stable block than a coalition in the center of the grid.

Table 1: Lattice tipping points (mobile influencers)

m (%)	M	Torus	Moore	f (%)
10	0			5.93
10	0	✓		6.87
10	0		✓	8.00
10	0	✓	✓	9.81
10	5			6.62
10	5	✓		7.83
10	5		✓	9.00
10	5	✓	✓	11.38
25	0			3.66
25	0	✓		5.15
25	0		✓	5.35
25	0	✓	✓	9.49
25	5			6.16
25	5	✓		6.75
25	5		✓	7.92
25	5	✓	✓	12.12

Table 2 shows a number of small-world configurations along with the estimated tipping points. These thresholds are quite similar

to the lattice tipping points. Despite their differences in topology, the tested network configurations are comparable in size to the square grids, which could drive this similarity. In addition, f clearly increases with k , as was the case with lattices. This is also linked to the number of edges in the graph, which k directly controls. The effects of mobility rate and memory are somewhat difficult to see, so they will be analyzed further.

Table 2: Small-world tipping points (mobile influencers)

m (%)	M	k	f (%)
10	0	2	4.72
10	0	4	4.80
10	0	6	6.64
10	0	8	8.40
10	0	10	9.47
10	5	2	4.75
10	5	4	5.22
10	5	6	7.74
10	5	8	10.03
10	5	10	11.19
25	0	2	2.83
25	0	4	2.95
25	0	6	5.11
25	0	8	6.77
25	0	10	9.05
25	5	2	2.68
25	5	4	4.61
25	5	6	7.99
25	5	8	10.03
25	5	10	12.45

4.1.2 Stationary Influencers. We wanted to see if the influencers could convert the majority while not moving from their initial positions. To do this we set $m = 0$. We thought this condition would greatly inhibit the influencers’ chances and cause the tipping points to balloon to nearly 50%.

Table 3 shows the results for such simulations in square lattices. As expected, the thresholds were much higher than the mobile case. It appears that memory had little to no effect on the configuration’s tipping point. The topological parameters did seem to slightly affect the outcome, where toruses increased the tipping point threshold and Moore neighborhoods decreased it. It appears stationary influencers benefit from more connections in the network, but still lose out on the ability to corner the edge nodes with toruses.

Table 4 shows similarly high results for small-world networks. Essentially, larger k -values create larger networks, which are more favorable for stationary influencers. Though this difference is only apparent when k is quite small. Additionally, memory seems to have an insignificant effect on f .

4.2 Mobility

We then wanted to expand upon the analysis of this mobility rate. We selected a baseline case for both topologies and vary only m

Table 3: Lattice tipping points (stationary influencers)

M	<i>Torus</i>	<i>Moore</i>	f (%)
0			35.24
0	✓		37.98
0		✓	30.38
0	✓	✓	33.88
5			35.39
5	✓		38.16
5		✓	31.18
5	✓	✓	33.70

Table 4: Small-world tipping points (stationary influencers)

M	k	f (%)
0	2	35.27
0	4	30.50
0	6	30.78
0	8	30.12
0	10	30.12
5	2	34.95
5	4	30.37
5	6	29.68
5	8	30.39
5	10	29.71

in our simulations. Our base lattice simulation featured a memory of 5, von Neumann neighborhoods, and no toral connections. The standard small-world simulation also set $M = 5$, with $k = 4$.

According to Figure 4, immobile influencers do not tend to succeed in their conversion attempts, as their tipping-point threshold hovers above 30%. This corroborates the findings in Tables 3 and 4. However, the tipping points decrease immediately upon allowing movement. The optimal rate of mobility appears to be somewhere around 20% for both network topologies, with a gradual increase for values beyond that. This increase can be attributed to the event where swapping with an influenced ordinary agent can actually de-influence them, causing a regressive effect.

4.3 Memory

Since it was difficult to discern the effects of memory, we decided to select a baseline case for both topologies and vary only M in our simulations. The lattice representative featured a mobility rate of .25, von Neumann neighborhoods, and no toral connections, while the standard small world set $m = .25$ and $k = 4$.

Figure 5 shows this relationship. It is now clear that increasing memory also increases the tipping point for the given simulation. This makes intuitive sense, since agents with a historically unwavering experience should be more difficult to persuade than those who simply do not remember their experiences.

The two baseline models performed quite similarly, though the small-world model does consistently obtain slightly lower thresholds than its lattice counterpart. This is likely do to the ease of

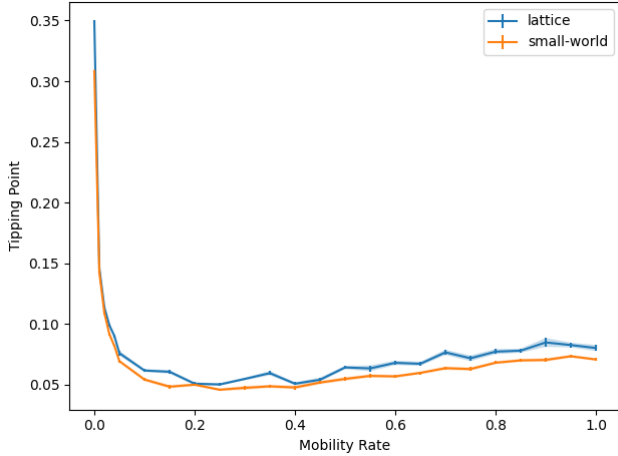


Figure 4: Tipping point vs. mobility rate

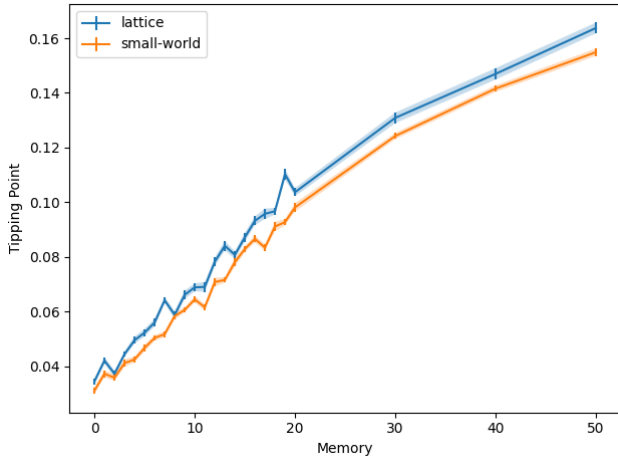


Figure 5: Tipping point vs. memory

traversal that both topologies have to offer. Additionally, both models look almost identical on the node-level, as each agent has 4 neighbors on average with which to interact. The small world’s ring-like structure could contribute to this slight advantage, as mobile agents are more likely to stay within a localized area. Lattice-based influencers can move in and out of any given area more freely, which is less helpful when attempting to convert agents with large memories.

5 DISCUSSION

The vast majority of the tipping-point thresholds we observed are significantly and consistently lower than those obtained from theoretical and empirical studies [6, 21]. The key difference between these two experiments is the interaction mechanism employed. Instead of randomly pairing individuals at each time step, the agents interact with only their neighbors, who are determined by the topology of their network or community. Thus, individuals of a

committed minority can collaborate more effectively to influence the others, even while their movements are random and uncoordinated.

The influencers’ frequency of movement is perhaps the most critical factor in lowering the tipping point. If there is no mobility, the "critical mass" of committed minority needs to represent nearly 33% of the total population to have a chance of succeeding. Even minimal movement causes the threshold to plummet nearly tenfold.

The number edges in the network is also a major contributor to the reduced tipping point thresholds. Generally speaking, mobile influencers prefer fewer interactions per individual, since fewer influencers are needed to successfully convert. Furthermore, topologies with edges, like the non-toral lattices, can greatly assist influencers. Not only do they reduce the total number of interactions in the grid, they offer more stability for communities of influenced agents than those positioned internally. These phenomena can even be seen in the example simulation in Figure 1.

However, we did not observe drastic threshold differences between lattices and small world networks. Tipping points were nearly identical given a similar number of interactions per agent in the network.

In our results, tipping points and memory capacity appear to be linearly correlated. This observation differs from that of Centola *et al.*, who presented logarithmic-like relationship between these variables [6]. However, the two cannot possibly follow a linear correlation forever, as the tipping point would be driven beyond 50%, which is the logical ceiling of the variable. In that case, however, large memory capacities ($M > 50$) would be needed to show significant asymptotic behavior, whereas the relationship was evident by $M = 20$ for Centola *et al.* [6].

6 CONCLUSION

Small-world and lattice topologies both lend themselves to significantly decreased tipping points for a committed minority to convert the whole population. If the average number of connections per agent is low and the mobility rate of the influencers is moderate (in the range of 15-25%), then the threshold needed to achieve a tipping point can be reduced by almost an entire order of magnitude.

Influencing agents significantly prefer communities where the memory capacity of their neighbors are low. Past experiences can significantly delay the conversion process and necessitate that the mobile influencers remain in one place, something which is not guaranteed with random movements.

The fact that these tipping points can be so drastically diminished without any planning or coordination between the influencers is quite exceptional. Should the agents learn to strategize their collective movements, or simply follow a greedy heuristic of their own, these thresholds could be brought down to only a handful of influencers per hundreds or even thousands of ordinary counterparts.

These discoveries can be utilized greatly for real-world communities. Political figures, elected officials, and civic leaders who have committed followers can take advantage of their community topology to influence public behavior. Additionally, these ideas can be used to model the spread and containment of transmittable diseases, similar to susceptible-infectious-recovered (SIR) models [19], which have become quite popular due to recent global health crises.

7 FUTURE WORK

The study of tipping points is only a couple of decades old and there is relatively work in this area in the field of multiagent systems. We envisage a number of fruitful dimensions to investigate to better understand and characterize what factors are influential and to what extent in facilitating or hindering tipping points. The following are some interesting and likely fruitful research directions that we envisage pursuing in the near future:

- Individuals in real-life are likely to have some vested interests and inertia that resists change. In particular, an individual is less likely to switch conventions immediately after changing to a new one. It would be interesting to see if we have some entrenched holdout of the existing conventions that can influence the population back after a new norm is reached among the non-entrenched population. The relative sizes of the tipping points of initial change and the reversal process may show hysteresis patterns.
- The mobility of influencer agents in the current implementation is random. We can evaluate more deliberate and intentional movements by the influencers, including (a) coordinating movements with other influencers, (b) adapt movement rate to interaction experience with current neighbors, (c) use movement trajectory memory to decide where to move next, etc.
- Placement of initial influencers can impact the rate and likelihood of adoption of the new convention. Strategic injection of influencers into the population topology can be evaluated.
- In addition to the grid and small-world networks used in this paper, it would be instructive to experiment with other common networks (scale-free, hub-and-spoke, etc.).

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