Effects of Network Topology, Mobility, Interaction, and Memory on Tipping Points

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Abstract. Understanding the conditions under which social norms or conventions reach tipping points is critical for modeling large-scale behavioral change. A tipping point is reached when a committed minority influences the rest of the population to switch from an existing norm. In this paper, we explore how network topology, influencer mobility, interactivity, and adaptive learning affect the acceleration and emergence of tipping points in agent-based coordination games. We investigate two network topologies (grids and small worlds), neighborhood interactivity, varied mobility rates of influencer nodes, and agent memory capacity. We use extensive simulations to identify favorable conditions as well as the size and distribution of influencer groups needed for norm change. We analyze the norm change process to show the interplay between network structure and influencer mobility on the rate of norm change in the tipping process. Our findings contribute to a deeper understanding of norm change in multiagent systems, offering insights for the design of more effective social interventions and mechanisms of norm enforcement.

Keywords: Tipping points · Norm change · Network topology · Mobility · Influencers.

1 Introduction

Social norms and conventions play a crucial role in shaping collective human behavior. Understanding how these norms emerge, spread, and undergo change is essential for various fields, including sociology, economics, and artificial intelligence [2, 13, 21, 25]. In particular, sudden and significant shifts in established behavioral norms, known as *tipping points*, where a previous minority behavior is adopted by the majority in the population, has received increasing attention because of the disruptive forces they are often associated with [1, 6, 11, 12, 20]. Of particular interest is the size of the minority group of like-minded influencers needed to effect a change in the established convention followed by the population. The success of such minority groups in effecting norm change, i.e., the presence of tipping points, is consistent with the theory of *critical mass* discussed in evolutionary game theory literature [16, 25]. In the context of multi-agent systems (MAS), the study of norm adoption and tipping points has garnered significant attention, as it provides insight into collective behaviors and how they can be influenced or accelerated under various conditions [3, 15, 17, 21].

In this paper, we explore the impact of several factors on the emergence and acceleration of tipping points in social norms using agent-based simulations. Specifically, we examine how different network topologies (such as lattice and small-world [23] networks), mobility patterns (stationary vs. migratory agents [7]), interaction frameworks (Moore vs. von Neumann neighborhoods on a grid [26], or small-world networks), and learning modes affect the rate and likelihood of tipping points. Our approach introduces systematic variations in these dimensions to better understand how specific configurations can either accelerate or delay the critical mass required for tipping points involving a change of established norms.

While much of the previous work on tipping points in social norms has focused on static networks or singular norm options, we extend this research by allowing for dynamic mobility of influencer agents and the presence of multiple competing norms with varying levels of similarity. Additionally, we incorporate memory into agent decision-making, where agents base their norm adoption not only on immediate observations but also on a history of past interactions [6]. This adds complexity and realism to the model, providing a richer understanding of the conditions that lead to tipping points.

Our contributions are twofold:

- We develop a simulation framework to perform extensive experimentation with a range of influential factors, including network topology, mobility, and memory history.
- We provide empirical results that show how varying these parameters can accelerate tipping points, offering new strategies for inducing rapid norm change in MAS.

This work builds upon existing literature on norm emergence, tipping points, and network science, but it moves beyond prior approaches by integrating dynamic mobility, memory-based decision making, and multiple norm options into a unified framework.

The rest of this paper is organized as follows: Section 2 explores related work, Section 3 outlines the methodology of our agent-based model, Section 4 presents our results, Section 5 discusses our findings, Section 6 concludes our study with a number of takeaways, and Section 7 looks ahead to the future of this research.

2 Related Works

The distinction between conventions and norms is not always clear. Typically, norms are viewed as having more regulation around them with punishment for a lack of adherence to a norm. Additionally, there are several classes of norms, such as legal, moral, and social [19]. In this paper, the terms convention and social norm may be viewed interchangeably, as we are more concerned with the emergence and prevalence of a minority state over a majority one.

2.1 Foundational Theories of Tipping Points & Norm Adoption

Our work is heavily influenced by the work of Centola *et al.* [6] who presents results, both from empirical study with human subjects and from simulation based experiments to validate formal predictions about the "critical mass" or proportion of minority influencers needed to effect a change in established norms. Centola *et al.* found that in contrast to no significant effect of varying population size, varying memory size, i.e., the number of recent interactions influencing agent decisions, have a significant effect on tipping point. They experimented with 10 independent groups where members of each group were randomly paired to select a name for a picture. Choosing the same name gave them higher rewards. Once a convention was established in each group, a minority of 10–30% individuals, using a different naming convention for pictures, was introduced. Data was collected over successive interactions to see if the naming conventions adopted by the population changed to that used by the introduced minority groups.

Under explicit incentives that reward social coordination among peers, the authors observed a tipping-point threshold percentage of influencers, in the mid-20s, is needed to change the established social convention to a newly introduced convention. This contrasts with prior theoretical models which hypothesized critical mass of as low as 10% to as high as 40% of the population size. Whereas Centola *et al.* examined norm shifts within isolated random pairings, our study extends this by placing agents in structured network topologies, allowing us to explore the spatial effects of minority influence in a more realistic setting.

Granovetter's "Threshold Models of Collective Behavior" provided an influential framework for understanding how individual decision-making, which is contingent on social thresholds, combines to create outcomes [14]. Granovetter's model explains why small initial shifts in individual behavior can eventually lead to large-scale social changes, especially when a critical mass is reached. His work offers an essential theoretical lens for studying tipping points in social networks, complementing Centola's empirical findings.

While Granovetter's model primarily focused on binary choices, later research by Young explored how social learning influences the establishment of conventions, emphasizing that small initial groups of adopters can play a disproportionately large role in norm-setting under the right conditions [25]. Young also emphasized the significance of coordination games in social learning, where repeated interactions lead to the emergence of stable outcomes. This line of work aligns closely with Centola's focus on clustered-lattice networks, where local reinforcement among agents can establish enduring behavioral patterns. Both Centola and Young's findings underscore the idea that tipping points are not solely a function of network size or randomness but depend critically on how agents interact and learn from one another over time.

Researchers, like Gelfand *et al.*, have also made extensive work of surveying the tipping-point literature and examining it for consensus opinions as well as novel findings [13]. Such work is imperative given the ease of exploration in social network dynamics in the modern day.

2.2 Role of Network Topology in Norm Change

Centola showed that the topology of networks influences how social norms spread, especially in the case of complex contagions, where adoption requires reinforcement from multiple neighbors [4]. Centola's work demonstrated that clustered-lattice networks are more conducive to norm adoption compared to random networks because they facilitate repeated interactions among individuals with similar thresholds. These findings highlight the crucial role that network structure plays in shaping the adoption process.

While Centola has looked at norm dynamics in a handful of topological settings where interactions are localized, each agent only sees one of its neighbors at a time during the interaction [5]. In a way, our paper seeks to combine the topological constraints and local interactions found in [5] with the memory capacity and committed minority of [6].

More recent research has focused on the role of minority influence in tipping dynamics. Baccino and Villata investigate how small, committed groups can influence the majority even when their views are initially unpopular [3]. Their findings suggest that the placement of such groups in network structures can greatly amplify their influence, particularly when they demonstrate high levels of commitment or "loudness." This research highlights an important aspect of norm adoption: the qualitative characteristics of influencers, rather than their sheer number, can be decisive in tipping points. This echoes earlier insights from Centola, who argued that network structure, when combined with strong behavioral reinforcement, plays a pivotal role in norm propagation. They suggest this committed minority thrives especially in small-world or scale-free networks.

Other studies have investigated the effects of topology and memory, such as Villatoro *et al.* [22]. However, these researchers initialized their networks with unbiased agents, each with a myriad of conventions to choose from. They do not study the necessary parameters for flipping a majority class by introducing a minority of stubborn influencers.

2.3 Influencer Characteristics & Mobility

The mobility of the influencer agents in our simulations is similar to that of a simple random walk [18]. Though, there is a probability that the influencers stay put at each time step, and they do not swap positions with other influencers, so these mathematical models based on random walks are not directly applicable to our experiments.

Mobility is a crucial parameter for conventions to spread in a multi-agent network. Many researchers will utilize a topological framework, but keep all agent positions fixed from the start [15]. While we do analyze the impact of complete immobility of all agents in the network, it is the presence of localized movement which brings about the most interesting effects on norm emergence.

Choi *et al.* observe different network effects resulting from the diffusion of innovations in a market setting [8]. However, the initial "enthusiasts" in the network are immobile, and there is no concept of memory introduced in their simulations.

Crawford *et al.* investigated the repulsion effect seen so often in real-world interactions [9]. The researchers proposed two models, one for repulsion by social judgment, and another for repulsion of by categorization. They also theorized about the formation of extremist groups and their implications under these models.

De *et al.* utilize an evolutionary model with a prisoner's dilemma payoff matrix at each pairwise interaction in their networks. They even introduce the idea of mobility [10]. However, the rate of mobility is kept relatively low (below 8%) and works by moving the agent to a randomly selected open node of the network, which exists due to the birth and death rates of their model. Unlike our study, mobility is not localized in their model.

Together, these studies provide a comprehensive view of how individual decision-making, network structure, and influencer strategies interact to shape tipping points in social systems. The current research builds on this foundation by investigating how different network topologies, agent mobility, and interaction frameworks can alter tipping dynamics, with a specific focus on the role of influencers in driving collective behavior toward critical thresholds.

3 Methodology

3.1 Topological Generation

These experiments tested two different topologies: square lattices and small-world networks. They were chosen for their applicability to real-world social networks. Physical crowds of individuals can resemble lattices due to the dense packing dynamics and inability to reach beyond one's immediate neighbors. Small-world networks largely retain this localization of connections, but introduce occasional jumps across the network. This can resemble certain online social networks, or even loosely packed physical ones where good friends tend to stand near one another, though some occasionally act as bridges between social groups. This localization will be important for the movement of our influencer agents, which we will discuss later.

Lattice. This topology featured $N = n^2$ agents that were placed in an $n \times n$ grid. The connectivity of each node was determined by two boolean parameters: $torus$ and $Moore$. The former indicated whether the lattice should be treated like a torus (i.e. the left and right edges of the network were connected as well as the top and bottom). The latter specified whether each agent was surrounded by a von Neumann neighborhood (4 adjacencies) or a Moore neighborhood (8 adjacencies).

Small World. These networks consisted of N agents connected within a small-world graph obtained using the Watts–Strogatz model. They were built by specifying two parameters: k (the number of nearest neighbors each node was initially connected to in a ring topology) and p (the probability of rewiring an edge). For all of our simulations, $p = .1$.

3.2 Agent Types

The agents in these simulations came in two varieties: ordinary and influencer. While an agent's type stayed the same throughout a simulation, its internal state value, representing its current norm, could change.

Ordinary. Most agents at the beginning of each simulation were ordinary. They represented the dominant majority all following the same norm, but could be convinced to switch. To represent the dominant norm, their initial state was set to 0. These agents did not move on their own, but they did interact with their direct neighbors at each time step. The result of this interaction determined the state they assumed for that time step. To reach that result, the agent observed the states of its current neighbors along with the previous M steps of neighbor states in its memory. The agent would then take on the majority norm or remain unchanged if there was a tie.

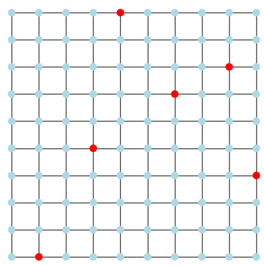
Influencer. A fraction f of the agents, known as influencers, were given an alternative norm, represented as state 1. The members of this committed minority never changed their norm, but they did move throughout the network in hopes of converting other agents. At each time step, an influencer had a chance—denoted by the mobility rate m—to swap places with one of its ordinary neighbors, chosen at random. This swap movement retains the network structure, ensures each node represents a single agent at all times, and keeps mobility localized, allowing for the gradual diffusion of conventions.

3.3 Simulation

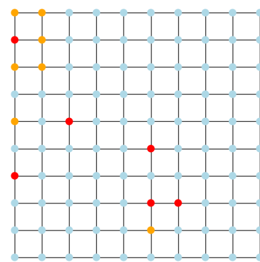
After generating the topology, the influencers were placed evenly throughout the network according to a Halton sequence with Owen scrambling for sufficiently different placements between simulations. For lattices, a two-dimensional sample was calculated, then each coordinate was rounded to the nearest integer and mapped to the corresponding node on the grid in which to place an influencer. For small worlds, a one-dimensional sample was chosen and directly mapped to the node IDs in the network.

Full simulations using a both network topologies are displayed in Figure 1. Red dots represent the influencers, blue dots are ordinary agents that still adhere to the initial norm of the majority, and orange dots are those that have switched to the minority norm.

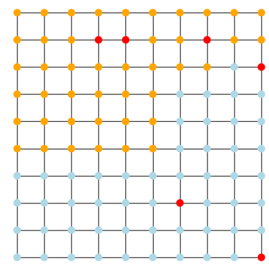
At each time step, influencers moved according to their mobility rate m, and ordinary agents interacted with their neighborhoods. The minority norm's adoption rate of the entire population was then recorded, and the cycle would



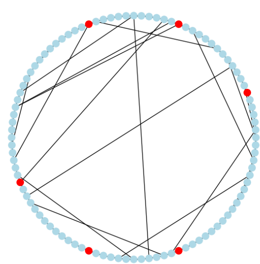
(a) Step 0



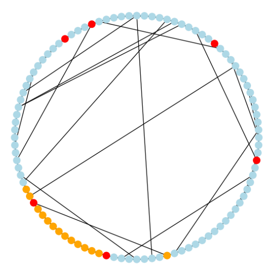
(b) Step 500



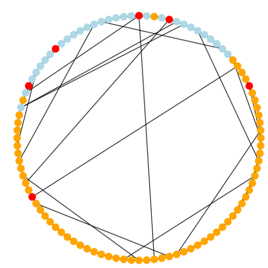
(c) Step 1000



(d) Step 0

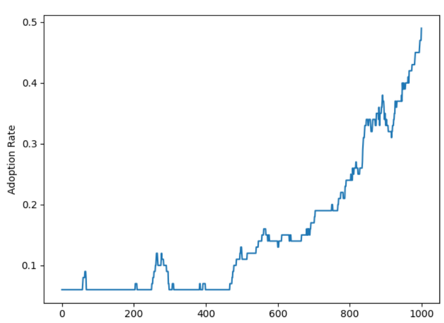


(e) Step 500

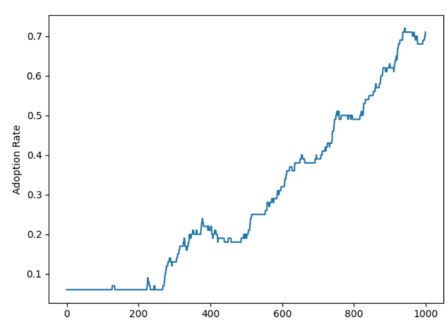


(f) Step 1000

Fig. 1: Simulations using lattice (a–c) and small-world (d–f) networks. Blue nodes are ordinary agents in state 0, orange nodes are ordinary agents who have been influenced to take on state 1, and red nodes are influencers.



(a) Lattice.



(b) Small world.

Fig. 2: Adoption rates over time for the simulations found in Figure 1.

repeat. If the alternative norm (state 1) ever reached total ubiquity, the simulation would cease. Figure 2 shows the adoption rates over time of the simulations in Figure 1. It is important to note that the adoption rate is not increasing monotonically, and there are several times when the influencers' progress is lost due to their movements.

3.4 Tipping Point Estimation

In order to determine an estimate for the true tipping point f^* of a single configuration, 50 evenly spaced f values were used to run 50 simulations. The final adoption rate A of the minority norm was recorded after each simulation. The 50 (f, A) samples would be used to fit a curve, where the midpoint $(f^*, .5)$ of that curve would contain the estimate for the tipping point of the configuration.

The first and simplest curve that appeared to fit the data was a logistic curve, given by Equation 1.

$$A_{ord}(f) = \frac{1}{1 + e^{-a(f-b)}} \tag{1}$$

Here, b is the f value at the curve's midpoint, while a describes the steepness of the curve near its midpoint. This curve fits the behavior of our data as it is bounded between 0 and 1, with a steep but smooth increase in the middle.

However, this curve only describes the final adoption rate of the ordinary agents given f, since it stays very close to 0 at low values of f, hence the syntax $A_{ord}(f)$. The overall curve would combine this effect with the final adoption rate of the influencers given by $A_{inf}(f)$. This function was always 1 for our simulations, as the influencers never changed their norm. Thus, we arrive at Equation 2, which describes the final adoption rate of the whole network, and is the objective function for our curve-fitting problem.

$$A(f) = fA_{inf}(f) + (1-f)A_{ord}(f) = f + \frac{1-f}{1 + e^{-a(x-b)}} \tag{2}$$

This is the statistically superior model. Over 65% of the data exhibited a lower sum of squared residuals (SSR) from the logistic model, improving both the mean and median SSR. This is because it encapsulates the linear trend in A at low values of f, as the influencers grow in size while not realizing any change in their ordinary neighbors. However, once f is large enough to flip ordinary agents for good, then the logistic behavior begins. It is important to note that a and b are still the only parameters for this function. The biggest difference is that the midpoint no longer occurs at b.

Now that we had our model function, we used non-linear least squares to fit the simulation results of each configuration to Equation 2. This provided us with the optimized curve parameters and their uncertainties. We then utilized root-finding algorithms on the resulting curve to determine f^*. Figure 3 demonstrates all of these results for a single configuration.

Table 1 describes all of the variables present in the simulations and provides any default values they may take when not explicitly varied.

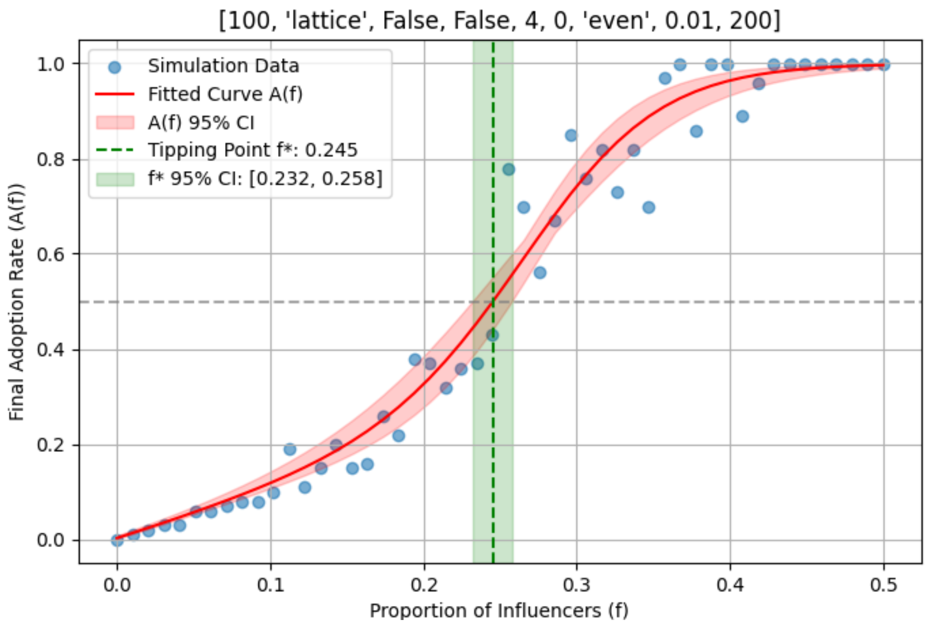


Fig. 3: An example $A(f)$ curve fitted to simulation data. The title contains the values of the configuration parameters, which were [N, *topology*, *torus*, $Moore$, k, M, *dist*, m, *steps*].

Table 1: Variable descriptions, possible values, and default values.

Variable	Description	Possible Values	Default
A	Final adoption rate	[0, 1]	-
dist	Initial influencer distribution	{even}	even
f	Fraction of influencers	[0, 1]	-
f^*	Tipping point	[0, 1]	-
k	Small world: Average node degree	$\mathbb{N}$	4
M	Agent memory	$\mathbb{N}$	5
m	Influencer mobility rate	[0, 1]	.05
$Moore$	Lattice: Moore neighborhoods?	{true, false}	false
N	Number of agents/network nodes	$\mathbb{N}$	100
p	Small world: Edge-rewiring probability	[0, 1]	.1
steps	Maximum number of timesteps	$\mathbb{N}$	200
topology	Lattice or small-world network	{lattice, small-world}	-
torus	Lattice: Torus?	{true, false}	false

4 Results

4.1 Sensitivity Analysis

We ran an extensive sensitivity analysis over five principal configuration variables: influencer mobility rate (m), agent memory (M), the number of agents in the network (N), the maximum number of simulation steps (*steps*), and the network structure (*topology*). This involved simulating 50 different f values on 432 different configurations, resulting in 21,600 total (f, A) data points. Each configuration was fitted to a model curve from which an estimate of f^* could be extracted. It is important to note that the lattice and small-world networks were built using their default values from Table 1.

Figure 4 displays pairwise heatmaps of the aggregated tipping points. These heatmaps reveal a number of general trends. First, it appears that f^* decreases significantly as mobility rate first increases, then it flattens out for large values of m. This effect is the most drastic of all the trends found from this analysis. Second, the tipping point decreases with increasing *steps*. Third, increasing the agents' memory inflates the value of f^*. Lastly, the default lattice network consistently has a slightly higher tipping point than its small-world counterpart. Using Spearman's rank correlation, we can confirm that mobility rate ($\rho = -.69$), maximum timesteps ($\rho = -.53$), and memory ($\rho = .15$) correlate the most with f^*, while the network topology ($\rho = -.08$) and the number of agents ($|\rho| < .01$) appear uncorrelated.

Since the first three of these trends are the most significant, we investigate their combined effects further in Figure 5. In most scenarios, the tipping point seems to decay exponentially with increasing influencer mobility. The value of f^* also clearly decreases with more *steps*. However, at large values of m, these two behaviors change according to M. With no memory at all, f^* seems to decrease monotonically. However, as the agents' memory capacity increases, so too does the tipping point. This effect is so great that, with a mobility rate of 50%, a 200-step, no-memory simulation exhibits roughly the same f^* as one with 5000 steps and memory capacity of 10.

4.2 Memory

We ran a number of supplementary simulations to determine the true relationship between memory and the tipping point. These extra samples came from configurations which fixed three additional variables to their default values in Table 1: m, N, and *steps*. After recording the data and fitting the curves to several different linear, logarithmic, and exponential functions, the clear winner was a shifted logarithmic function given by Equation 3, obtaining an R^2 value of about .97.

$$f^*(M) = a \ln(M - b) \tag{3}$$

It should be noted that, while this function does not have a horizontal asymptote, the tipping point cannot logically surpass .5. However, M would have to be impractically large for this to occur with the typical optimized parameters. Figure 6 displays the supplemental data along with this fitted curve.

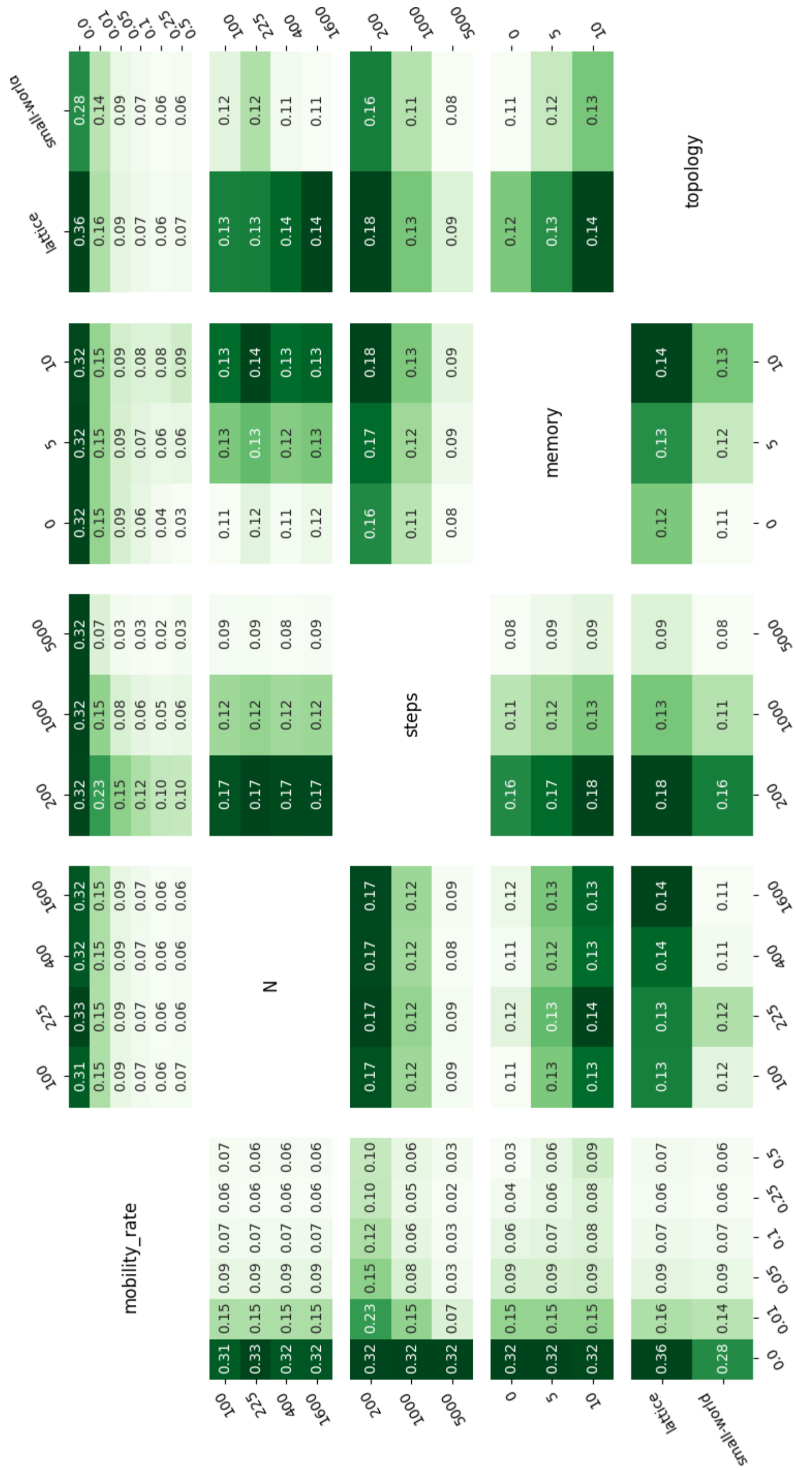


Fig. 4: Pairwise tipping point heatmaps over the following configuration parameters: m, N, *steps*, M, and *topology*. All cells represent averaged f^* values. The shade of a cell corresponds to the relative size of f^* within its own subplot, independent of the values in other subplots.

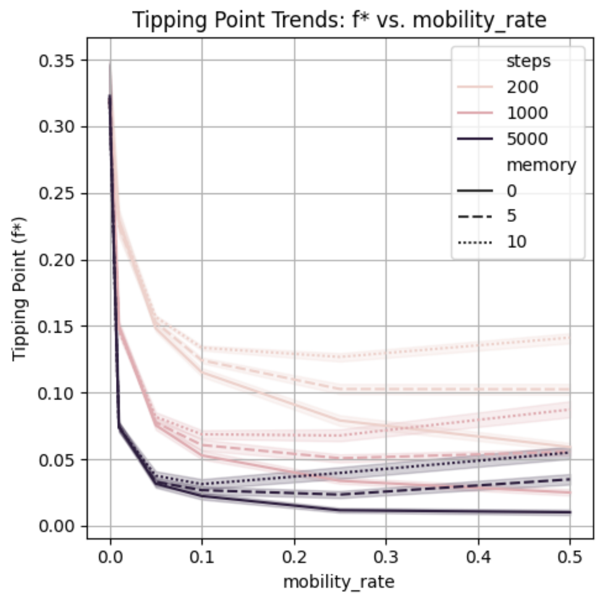


Fig. 5: Trends in f^* vs. mobility rate, memory, and maximum timesteps.

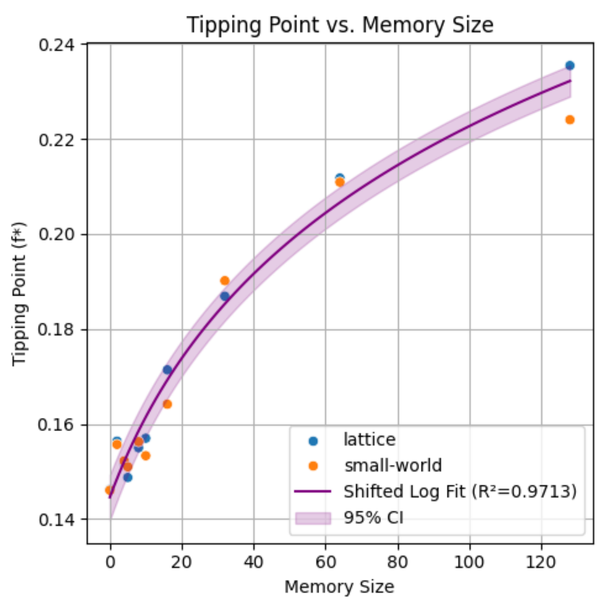


Fig. 6: Tipping point vs. memory, featuring supplementary f^* samples after fixing m, N, and *steps* to their default values.

4.3 Steps

Similar to above, we simulated some more data to further characterize the effects of *steps* on f^*. In doing so, the configurations fixed m, N, and *memory* to their defaults. This time, a reciprocal function was the best fit with an R^2 value of about .98. Equation 4 gives the parameterized form.

$$f^*(steps) = \frac{a}{steps^b} \tag{4}$$

This function features a horizontal asymptote at $f^* = 0$, which might imply that any configuration could drive its tipping point down near zero given enough timesteps. Figure 7 displays the supplemental data along with this fitted curve.

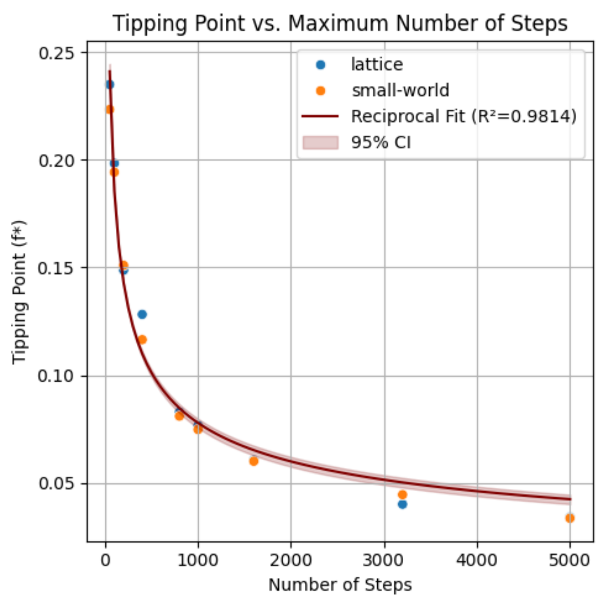


Fig. 7: Tipping point vs. *steps*, featuring supplementary f^* samples after fixing m, N, and M to their default values.

4.4 Topology

Table 2 shows the results for yet another subset of our simulations. In it, we varied the topological parameters of both network types, along with mobility rate, due to its significant interactions with f^*. For a given value of m, the number of network connections increases down the table. We can see that, given

Table 2: The effects of topological changes and mobility rate in lattices (left) and small worlds (right), holding all other parameters constant.

m	$torus$	$Moore$	f^*	95% CI
0			.354	[.322, .381]
0	✓		.399	[.363, .428]
0		✓	.317	[.293, .335]
0	✓	✓	.338	[.307, .358]
.10			.121	[.113, .127]
.10	✓		.134	[.129, .139]
.10		✓	.152	[.145, .158]
.10	✓	✓	.178	[.162, .189]
.25			.110	[.105, .116]
.25	✓		.127	[.124, .129]
.25		✓	.151	[.138, .162]
.25	✓	✓	.194	[.180, .203]

m	k	f^*	95% CI
0	2	.379	[.364, .394]
0	4	.280	[.268, .291]
0	6	.347	[.333, .350]
0	8	.319	[.307, .331]
.10	2	.121	[.114, .129]
.10	4	.122	[.117, .127]
.10	6	.144	[.141, .147]
.10	8	.164	[.161, .166]
.25	2	.084	[.079, .089]
.25	4	.098	[.093, .102]
.25	6	.127	[.125, .130]
.25	8	.168	[.167, .168]

a positive mobility rate, the tipping point is directly related to the number of network connections in both topologies.

Toral lattices ensure that edge cells have the same number of connections as interior cells. Moore neighborhoods are twice as large as von Neumann neighborhoods. Given $n = 15$, a non-toral grid with von Neumann neighborhoods would only have $2n(n-1) = 420$ edges, whereas a torus with Moore neighborhoods would have over double with $4n^2 = 900$. Moore neighborhoods asymptotically double the number of edges in a lattice as n grows. This increases the number of influencers needed around an ordinary agent to convert its norm. Additionally, non-toruses require fewer influencers to convert those near the edge. Persuaded agents on the edge form a much more stable block than a coalition in the center of the grid.

For small worlds, k directly controls the number of adjacencies in the network. This network type behaves similarly to lattices in that an increase in k—and thus the total number of connections—seems to increase f^* when influencers are mobile. The pattern is less clear for stationary influencers in both topologies, where their immobility results in staggeringly high tipping points for any configuration.

5 Discussion

The vast majority of the tipping points we observed are significantly and consistently lower than those obtained from theoretical and empirical studies [6, 25]. The key difference between these two experiments is the interaction mechanism employed. Instead of randomly pairing individuals at each time step, the agents interact with only their neighbors, who are determined by the topology of their network or community. Thus, individuals of a committed minority can collaborate more effectively to influence the others, even while their movements are random and uncoordinated.

The influencers' frequency of movement is by far the most critical factor in lowering the tipping point. If there is no mobility, the "critical mass" that committed minority must exceed is 25% of the total population to have a chance of succeeding. Even the slightest chance of movement causes this threshold to plummet.

The time horizon of the overall interaction is also quite an important factor. Our model suggests that any configuration's norm can switch given enough time and given the bare minimum number of influencers to flip the first ordinary agent. This occurs despite the fact that the adoption rate over time is not guaranteed to increase monotonically.

In our results, tipping points and memory capacity appear to be logarithmically correlated. This observation agrees with that of Centola *et al.*, who presented a logarithmic-like relationship between these variables [6]. This confirms that increasing memory reduces susceptibility to norm shifts but at a diminishing rate. Additionally, even though our model did not include an explicit asymptote at $f^* = .5$, agents would need an unfeasibly large memory capacity ($M \gg 50$) to approach this value.

The number edges in the network is also a major contributor to the reduced tipping points. Generally speaking, mobile influencers prefer fewer interactions per individual, since fewer influencers are needed to successfully convert. Furthermore, topologies with edges, like the non-toral lattices, can greatly assist influencers. Not only do they reduce the total number of interactions in the grid, they offer more stability for communities of influenced agents than those positioned internally. These phenomena can even be seen in the example simulation in Figure 1.

However, we did not observe drastic threshold differences between lattices and small-world networks. The tipping points were quite comparable given a similar number of interactions per agent in the network. We also noted that the number of agents in the network had little to no effect on f^*. Thus, the 100-agent networks we simulated were just as representative as one with millions of agents.

6 Conclusion

Our findings highlight that mobility plays the dominant role in reducing the tipping point threshold, far outweighing the effects of network topology, agent memory, or interaction radius. Even a modest mobility rate dramatically lowers f^* by an order of magnitude.

Influencing agents significantly prefer communities where the memory capacity of their neighbors are low. Past experiences can significantly delay the conversion process and necessitate that the mobile influencers remain in one place, something which is not guaranteed with random movements.

Finally, small-world and lattice topologies can lend themselves to significantly decreased tipping points for a committed minority to convert the whole population. If the average number of connections per agent is low, mobile influencers can easily succeed.

The fact that these tipping points can be so drastically diminished without any planning or coordination between the influencers is quite exceptional. If the agents learn to strategize their collective movements, or simply follow a greedy heuristic of their own, these thresholds could be brought down to only a handful of influencers per hundreds or even thousands of ordinary counterparts.

These discoveries can be utilized greatly for real-world communities. Political figures, elected officials, and civic leaders who have committed followers can take advantage of their community topology to influence public behavior. Additionally, these ideas can be used to model the spread and containment of transmittable diseases, similar to susceptible-infectious-recovered (SIR) models [24], which have become quite popular due to recent global health crises.

7 Future Work

The study of tipping points is only a couple of decades old and there is relatively little work in this area in the field of multiagent systems. We envisage a number of fruitful dimensions to investigate to better understand and characterize what factors are influential and to what extent in facilitating or hindering tipping points. The following are some interesting and likely fruitful research directions that we envisage pursuing in the near future:

- Individuals in real-life are likely to have some vested interests and inertia that resists change. In particular, an individual is less likely to switch conventions immediately after changing to a new one. It would be interesting to see if we have some entrenched holdout of the existing conventions that can influence the population back after a new norm is reached among the non-entrenched population. The relative sizes of the tipping points of intitial change and the reversal process may show hysteresis patterns.
- The mobility of influencer agents in the current implementation is random. We can evaluate more deliberate and intentional movements by the influencers, including (a) coordinating movements with other influencers, (b) adapt movement rate to interaction experience with current neighbors, (c) use movement trajectory memory to decide where to move next, etc.
- Placement of initial influencers can impact the rate and likelihood of adoption of the new convention. Strategic injection of influencers into the population topology can be evaluated.
- Experimenting with additional network structures (scale-free, hup-and-spoke, etc.) could be instructive.

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