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# A comparison of chance-constrained DEA and stochastic frontier analysis: bank efficiency in Taiwan

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We employed both chance-constrained data envelopment analysis (CCDEA) and stochastic frontier analysis (SFA) to measure the technical efficiency of 39 banks in Taiwan. Estimated results show that there are significant differences in efficiency scores between chance-constrained DEA and stochastic frontier production function. The advanced setting of the chance-constrained mechanism of DEA does not change the instinctive differences between DEA and SFA approaches. We further find that the ownership variable is still a significant variable to explain the technical efficiency in Taiwan, irrespective of whether a DEA, CCDEA or SFA approach is used.

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## Introduction

In this paper, we compare chance-constrained data envelopment analysis (CCDEA) with stochastic frontier analysis (SFA) to estimate technical efficiency indices, and to explore the effect of ownership on the technical efficiency for policy purposes in the banking sector. We focus on the efficiency assessment since we believe that efficiency and/or performance will become strategic variables in tackling the increasing competitive pressure and structural changes within this industry. We incorporate a stochastic mechanism and employ both CCDEA and SFA approaches to our analysis since we consider that the unpredictability of customer demand makes the banks' input–output relationship stochastic. There are two reasons: firstly, bank service often consists of one-off activities and the results of bank service processes are commonly of a stochastic nature. Both the nature of deposit/loans risk and the attitude toward deposit/loan risk are often different in different customers. Thus, the outputs of bank services provide various indicators of performance and they may be the result of inexperience, added to the intrinsic difficulty of prediction and the unpredictability of customer demand. Secondly, the turbulent Asian financial crisis period made market conditions more unpredictable and stochastic. The CCDEA and SFA results can suggest the presence of theoretical output variability components in efficiency levels, meaning that the best practice efficiency indices of each bank are relatively stable. We also examine the relationship between technical efficiency and bank ownership in order to provide some recommendations for bank management.

Deregulation in the financial market is now perceived as a worldwide phenomenon and Taiwan is no exception to this international trend. Taiwan began to follow the trend in the early 1990s in order to increase operating efficiency and to attract funds into the loanable fund supply market. The policy of deregulation involves two distinct streams: privatization of ownership and the establishment of new banks in the market. The former includes the well-known examples of the First Bank, Hua-nan Bank and Chang-hua Bank, each of which were privatized and came to be owned jointly by both the private and public sectors. The Government shareholding in banks in Taiwan has been reduced. The latter comprises the newly established banks in private ownership in Taiwan. However, the question remains as to whether or not the privatization of mixed banks really improves the performance of the enterprise. In 1991, the Government of Taiwan announced the Commercial Bank Establishment Promotion Decree as a means of opening up the bank market further, by inviting private investors to participate in Taiwan's banking industry. The investors set up more than 30 new commercial banks, pushing the total number of domestic commercial banks in Taiwan in 1999 to 44. The Bank of Taiwan provides the authority to open a new bank or a new branch, if certain prerequisites on capital adequacy are met. The implementation of such recent policy clearly signals Taiwan's desire to make banking more competitive and to level the playing field for public-owned and private-owned banks. After the Asian currency crisis began, no bankruptcy of banks occurred in Taiwan, revealing that the impacts of the Asian currency crisis on Taiwan's banking industry has been relatively slight. However, this does not

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mean that Taiwan's banking structure will be problem-free in the near future. This is an appropriate time to quantify, and also to explain, the anticipated different types of efficiency among banks, since we believe that efficiency will become a strategic variable in tackling the increasing competitive pressure and structural changes within the banking industry.

## The two study approaches

In this section, we first introduce the non-parametric programming approach of CCDEA to evaluate efficiency and then propose the alternative parametric programming approach of SFA.

The DEA approach uses a mathematical programming technique to construct a piecewise linear frontier and it can be referred as a non-parametric programming approach.[1,2] DEA allows researchers to avoid specification of a given functional form or error structure, and many researchers have focused on estimating the technical efficiency and scale efficiency of banks by utilizing this technique.[3–6] Amongst these, Schaffnit presents a best practice analysis of bank branches based on a DEA assurance region (DEA-AR) model containing output multiplier constraints, with standard transaction and maintenance times, in order to evaluate allocative efficiency. In addition, there have been a number of papers, which have compared parametric and non-parametric approach to efficiency analysis such as Bjurek *et al*,[7] Giokas,[8] Resti,[9,10] Ferrier and Lovell[11] and Cooper and Tone.[12] Among them, Bjurek *et al* and Giokas argue that the deterministic DEA and loglinear model do not exhibit significant difference. Resti also argues that various kinds of DEA and stochastic cost functions do not differ dramatically. However, Ferrier and Lovell argues that deterministic DEA and stochastic cost functions differ both in structure and in implementation and the disagreements between the results of the two approaches are still substantial, although the related model error of the deterministic setting has been mitigated. Cooper and Tone discuss DEA and stochastic cost functions and identify some serious problems of bias in SFA approaches.[12] This paper then extends the basic deterministic DEA method to incorporate the chance-constrained DEA mechanism in order that we can obtain a more similar comparison base between stochastic parametric programming and CCDEA approaches.

As to the CCDEA approach, Charnes *et al* first propose chance-constrained programming to measure efficiency in the case of uncertainty and that analyse the cases of the possibility of violated constraints.[13] Land *et al* introduces the basic CCDEA model to measure productive efficiency to the case of stochastic inputs and outputs in 49 school sites.[14] Land *et al* further apply the technique to assess the relative economic performance of capitalist and state socialist systems.[15]

In the DEA approach, we aim to identify the most efficient decision-making unit (DMU) among all DMUs, and to estimate the relative efficiency of DMUs. Consider a set of $R$ banks, each consuming different amounts of $j$ inputs to produce $i$ outputs. The basic Charnes–Cooper–Rhodes efficiency of an individual bank, $r_0$, can be calculated through the following linear program (CCR model).[1]

$$\begin{aligned}&\text{Min}\,\theta\\ &\text{subject to}\quad \sum_r Y_{ir}\lambda_r \geqslant Y_{ir_0} \qquad \lambda_r \geqslant 0,\ r=1,2,\ldots,R\\ &\qquad\qquad\quad \theta X_{jr_0} - \sum_r X_{jr}\lambda_r \geqslant 0\end{aligned}$$

where $Y_{ir}$ is the amount of the $i$th output produced by the $r$th DMU, $X_{jr}$ is the amount of the $j$th input used by the $r$th DMU, $\lambda_r$ is the input weight and $i$ runs from 1 to $m$, $j$ runs from 1 to $n$, and $r = 1, 2, \ldots, R$. The contraction factor $\theta$ cannot exceed unity, $\theta \leqslant 1$. In deterministic DEA, we can find observations that fall on the efficiency frontier. Note that the efficiency here is an *ex ante* concept, which is based on Pareto and Koopman's efficiency (see Charnes *et al*), and can be smoothly extended to the chance-constrained setting (see Land *et al*).[14]

Here, we then permit some stochastic variation around the efficiency frontier and we can incorporate the stochastic considerations into the model to accommodate the measurement and specification errors. We consider that the setting of the outputs is stochastic whilst the inputs are predetermined. This simplifies the presentation without loss of generalization. To do this, we modify our constraint equation and add the mechanism of the chance-constraint introduced by Land *et al*. Thus, the corresponding chance-constrained efficiency measure is calculated as

$$\begin{aligned}&\text{Min}\,\theta\\ &\text{subject to}\quad \text{Prob}\left[\sum_r Y_{ir}\lambda_r \geqslant Y_{ir_0}\right] \geqslant \alpha \qquad i=1,2,\ldots,I\\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad j=1,2,\ldots,J\\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad r=1,2,\ldots,R\\ &\qquad\qquad\quad \theta X_{jr_0} - \sum_r X_{jr}\lambda_r \geqslant 0\end{aligned}$$

We can find that the best practice output and best practice input are denoted as $\sum_r Y_{ir}$ and $\sum_r X_{jr}$, respectively. Best practice represents the top performers. The chance constraints: Prob $[\sum_r Y_{ir}\lambda_r \geqslant Y_{iro}] \geqslant \alpha$ indicates that the probability of the best-practice output exceeding observed output shall be at least level $\alpha$. We assume $\alpha = 0.95$ so that most DMUs (say, 5%) will be set as best performers. Following this, we can transform the chance constraint as $\mathrm{E}(\sum_r Y_{ir}\lambda_r - Y_{iro}) - 1.645\sigma \geqslant 0$ where E is the sign for mathematical expected value and 1.645 is in the one-tailed standard normal distribution statistic, $\sigma$ is the standard deviation of $\sum_r Y_{ir}\lambda_r - Y_{iro}$, which is equal to $m[\sum_{r=1}^{r=R}\sum_{s=1}^{s=S}\lambda_r\lambda_s\,\mathrm{Cov}\,(Y_{ir}, Y_{is})]^{1/2}$ where Cov is the covariance operator and $m$ is within-bank standard deviation for each output. Note that different projected points can be obtained when the primary objective is to maximize either the proportional increase in outputs or the proportional decrease in inputs. Here we use input oriented to identify a projected point that minimizes the value of $\theta$. That is, of all projections possible, one that maximizes the proportional reduction of inputs is identified.

To simplify the presentation, here we also assume: (i) all outputs are stochastically independent; the performance at one bank is independent of that at another bank, that is, $\mathrm{Cov}(Y_{ir}, Y_{is}) = 0$ for all $i$ for $r \neq s$; (ii) individual bank variability of each output (as measured by the variance) is the same for all outputs and at all banks, that is, $\mathrm{Var}(Y_{ir}) = m = 1$; and (iii) all observed outputs match their mathematical expectations, that is, the observed performance at each banks serves as an unbiased estimate of the true performance at that site, that is, $\mathrm{E}(Y_{ir}) = Y_{ir}$ for all $i$ and $r$. Please note that we can add another chance constraint form, Prob $[\theta X_{jro} - \sum_r X_{jr}\lambda_r \geqslant \alpha]$, to replace the current form, $\theta X_{jro} - \sum_r X_{jr}\lambda_r \geqslant 0$, if we want to consider the case that the inputs are to be stochastic.

Alternatively, the parametric programming approach is concerned with the production or cost function base and focuses on an estimation of the characteristics of the functions under the assumption that all firms operate under rational economic behaviour. From Farrell's introduction of the production function model to measure the efficiency of production, a number of researchers, particularly Sealey and Lindley, have further developed the concept of the stochastic production function employing a stochastic frontier model.[16,17] The advanced models developed by Battese and Coelli allow us to estimate time-varying inefficiency levels.[18,19] Coelli further proposes an algorithm to estimate the maximum likelihood estimator (MLE) of the SFA proposed by Battese and Coelli.[20]

In the SFA, our concept of efficiency is productive efficiency, as introduced by Farrell. Consider a set of $R$ banks, each consuming different amounts of $j$ inputs to produce $i$ outputs. We also assume that each bank has at least one positive input and one positive output and satisfies the free disposability of inputs and outputs. These disposability assumptions imply that an increase in inputs never results in a decrease in outputs, and that any reduction in outputs remains possible with the same amount of inputs.

Note that the inefficiency effect is defined as how far the firm operates below the frontier production function. Coelli argues that the inefficiency effects, which cause the firm to operate below the frontier production (or above the cost function), can be referred to as the technical inefficiency.[20] In order to investigate the inefficiency index, we employ a maximum likelihood estimate approach, as proposed by Battese and Coelli in 1995, to estimate the parameters of the stochastic frontier and the inefficiency indices.[19] This includes the specification and the estimation of the stochastic frontier function, and the prediction of the predicted technical inefficiency. The stochastic frontier of the familiar translog production function can be defined as follows:

$$\begin{aligned}\ln(Y) = {} & \hat{\alpha}_0 + \hat{\alpha}_1 \ln K + \hat{\alpha}_2 \ln L + \hat{\alpha}_3 \ln D + \tfrac{1}{2}\hat{\alpha}_{11}(\ln K)^2 \\ & + \tfrac{1}{2}\hat{\alpha}_{12}(\ln L)^2 + \tfrac{1}{2}\hat{\alpha}_{13}(\ln D)^2 + \hat{\rho}_{11}\ln(K)\ \ln(L) \\ & + \hat{\rho}_{12}\ln(k)\ln(D) + \hat{\rho}_{13}\ln(L)\ln(D) \\ & + V_{rt} + U_{rt} \quad \text{(translog form)}\end{aligned}$$

where $Y$ is the output, $K$ is the input of asset, $L$ is the input of labour, $D$ is the input of deposit, $V_{rt}$ is the random error term of firm at time $t, r = 1, 2, \ldots, 44, t = 1, 2, 3,$ where the random disturbances $\hat{\varepsilon}_{rt}$ are assumed to be independently distributed as $N(0, \sigma_U^2)$. The technical efficiencies are derived from the stochastic frontier function using an error components model.

## The evidence from Taiwan's banks

In choosing outputs and inputs, we use the intermediation approach, which views banks as financial intermediaries where deposits are treated as inputs, assuming that a bank's main business is to borrow funds from depositors to lend to others (see Oral and Yolalan[21]). This approach can effectively benefit banking operations and improve efficiency in Taiwan's competitive environment. We utilize sensitivity analysis to determine the output and input items. We calculate the technical efficiency scores for each bank for eight cases (see Table 1). The basic case is case A, and the remaining seven cases are used in our sensitivity analysis in order to capture various aspects of technical efficiency. The data consists of 273 samples (39 banks within seven years) in the model and the rejection criteria appears to be (i) high correlation with the base case A and/or (ii) a greater number of efficient DMUs than base case A. Case B is included to determine the impact of adding the number of bank branches to the model, and it has a less important effect on the results, as indicated by the correlation coefficient of 0.9679 and the 17 efficient banks. Therefore, we do not need to consider case B. Similar results were obtained in case C by adding the interest revenue (correlation coefficient is 0.8596) and the 20 efficient banks to the model given in case C. Case D is the worst case when we included the number of bank branches and the interest revenue into the model at the same time. The number of efficient banks increases to 33. In addition, Case E indicates that there are no significant effects when we omit the non-interest revenue from the model. The identical number of efficient banks (10) and the high correlation coefficient supports our conclusion. Again, the number (10 *vs* 15) of efficient banks in case A is smaller than that in case F, suggesting that case A is superior to case F. Similar results were obtained in case G and H by directly substituting bank loans and investments with an alternative definition (interest revenue).

**Table 1** Result of sensitivity analysis on the chosen DEA model

| *Items* | *Case A* | *Case B* | *Case C* | *Case D* | *Case E* | *Case F* | *Case G* | *Case H* |
|---|---|---|---|---|---|---|---|---|
| Outputs | | | | | | | | |
| loans | T | T | T | T | T | T | | |
| investments | T | T | T | T | T | T | | |
| non-interest revenue | T | T | T | T | | | T | T |
| interest revenue | | | T | T | | | T | T |
| Inputs | | | | | | | | |
| labour | T | T | T | T | T | T | T | T |
| assets | T | T | T | T | T | T | T | T |
| deposits | T | T | T | T | T | T | T | T |
| branches | | T | | T | | T | | T |
| Estimated results | | | | | | | | |
| SCC with Case A | — | 0.9679 | 0.8596 | 0.7483 | 0.9092 | 0.8923 | 0.5861 | 0.5276 |
| number of efficient banks | 10 | 17 | 20 | 33 | 10 | 15 | 12 | 18 |
| mean efficiency score | 0.702 | 0.729 | 0.763 | 0.799 | 0.674 | 0.694 | 0.679 | 0.728 |
| standard deviation | 0.152 | 0.157 | 0.133 | 0.138 | 0.163 | 0.166 | 0.148 | 0.161 |
| minimum efficiency score | 0.339 | 0.401 | 0.432 | 0.507 | 0.325 | 0.362 | 0.406 | 0.528 |

Notes: SCC means Spearman correlation coefficient; all correlation coefficients are significant at a 5% level of significance. We use DEA to obtain the technicial efficiency scores and conduct this senstivity analysis. The data consists of 273 samples (39 banks within seven years) in the model. As to the rejection criteria, it appears to be (i) high correlation with the base case A and/or (ii) a greater number of efficient DMUs than base case A.

Based on these findings, our DEA model has the following three output variables: loans (including business and individual loans), investments (mainly government securities and shares, along with public and private enterprise securities), and non-interest revenue (including transaction fees, the revenue on securities investment and other business revenues). The first two types of output constitute the main activities of banks, whilst the third type is an extra source of revenue for banks. These three types of output use operating resources through three types of input, namely bank staff, assets and deposits. We refer to the bank deposits accounted for by current deposits, time deposits and savings deposits, and since interest revenue is the main return of loans and investment, we exclude interest revenue output and view it as being equivalent to loans and investments. Bank assets include mainly net fixed assets, which can be estimated as a proxy by the total domestic assets except bank loans and investments. The official report from the Department of Finance, Central Bank, and the ROC Commission on National Corporations of the Ministry of Economic Affairs provides a rich source of data on the operations of all banks in Taiwan. We have gathered the requisite data for 39 banks, which represents most of the domestic banks in Taiwan, covering the period 1994 to 2000. Table 2 lists the descriptive statistics of the input data. The seven years' data includes the ante-crisis (Asian Financial Crisis) period (1994–1996) and the post-crisis period (1997–2000) that it can effectively avoid the one-off shock effect in the overall efficiency evaluation. We employ GAMS (general algebraic modelling system) software in this paper to solve the nonlinear chance-constrained programming problem and we use Frontier-41 software to run the stochastic frontier analysis.

The mean ordinary DEA technical efficiency score (CCR model) in Table 3 is 0.920, implying that banks could have produced the same level of output using 92% of the input actually used. Using a chance-constrained DEA approach, we find that the chance-constrained DEA efficiency scores (0.931) are, on average, higher than that of deterministic DEA scores (0.920). The chance-constrained frontier of banks is naturally a 'soft' frontier, at which the output

**Table 2** Summary of the descriptive statistics of the input data

| *Years* | *Staff employed (person)* | *Bank assets (billion)* | *Bank deposit (billion)* | *Bank loans (billion)* | *Bank investment (billion)* | *Non-interest revenue (billion)* |
|---|---|---|---|---|---|---|
| 1994–1996 | 1915 | 193.10 | 206.45 | 182.61 | 32.19 | 3.02 |
| 1997–2000 | 2279 | 288.09 | 496.90 | 237.88 | 43.72 | 9.30 |
| 1994–2000 | 2123 | 247.38 | 372.42 | 214.19 | 38.77 | 6.61 |
| Public | 4451 | 558.04 | 770.10 | 442.64 | 79.06 | 12.62 |
| Private | 1250 | 130.88 | 223.31 | 128.52 | 23.67 | 4.36 |

Note: The dollars value is in billions of New Taiwan dollars, a billion $\Rightarrow 10^6$ American dollars. The data consists of 39 domestic commercial banks and seven years' data. The data consists of seven public-owned banks and 32 private-owned banks.

**Table 3** Efficiency estimated among DEA, CCDEA and SFA methods

| *Bank name* | *DEA* | *CCDEA* | *SFA* |
|---|---|---|---|
| Aetna | 1.000 | 1.000 | 0.866 |
| Agriculture | 0.991 | 1.000 | 0.885 |
| Asia Pacific | 0.858 | 0.868 | 0.821 |
| Baodao | 0.973 | 0.987 | 0.845 |
| Center Trust | 0.558 | 0.569 | 0.594 |
| Chang-hua | 0.868 | 0.878 | 0.766 |
| Chiao-tung | 0.808 | 0.809 | 0.785 |
| China International | 0.953 | 0.975 | 0.638 |
| China Trust | 0.996 | 1.000 | 0.737 |
| Chinese | 1.000 | 0.871 | 0.875 |
| Chinfon | 0.996 | 1.000 | 0.750 |
| Chung-shing | 0.832 | 0.838 | 0.756 |
| Cosmos | 0.780 | 0.800 | 0.749 |
| Dah-an | 0.957 | 0.969 | 0.851 |
| E. Sun | 0.921 | 0.930 | 0.826 |
| Far Eastern | 1.000 | 1.000 | 0.868 |
| First | 0.957 | 1.000 | 0.805 |
| Fu-bon | 0.723 | 0.745 | 0.711 |
| Grand | 0.715 | 0.720 | 0.738 |
| Hsinchu City | 0.970 | 0.988 | 0.817 |
| Hualien City | 0.937 | 1.000 | 0.623 |
| Hun-nan | 0.831 | 0.842 | 0.762 |
| Kaohsiung | 1.000 | 1.000 | 0.775 |
| Kaohsiung City | 0.956 | 0.969 | 0.673 |
| Land | 0.992 | 0.996 | 0.790 |
| Our Corp. | 0.856 | 0.877 | 0.807 |
| Pan-Asia | 1.000 | 1.000 | 0.854 |
| Shanghai | 0.903 | 0.944 | 0.740 |
| Sinopac | 0.881 | 0.906 | 0.825 |
| Taishin | 0.955 | 1.000 | 0.795 |
| Taichung City | 1.000 | 1.000 | 0.812 |
| Tainan City | 0.983 | 1.000 | 0.786 |
| Taipei | 0.977 | 0.980 | 0.833 |
| Taipei Business | 1.000 | 1.000 | 0.837 |
| Taitung City | 1.000 | 1.000 | 0.647 |
| Taiwan | 1.000 | 1.000 | 0.776 |
| Taiwan Provincial | 1.000 | 1.000 | 0.851 |
| Union | 0.782 | 0.798 | 0.772 |
| United World | 1.000 | 1.000 | 0.841 |
| Average | 0.920 | 0.932 | 0.782 |
| Standard deviation | 0.1022 | 0.1008 | 0.0718 |

observations of banks are in some cases allowed to cross the envelope. Then they can move closer to do any observation. However, the deterministic frontier is a 'hard' frontier for any given figures, and the envelope is located far from the chance-constrained one. Hence, the chance-constrained DEA frontier may be crossed by a few efficient banks, but most of the banks (95% or more) are still assumed to fall on or beneath the frontier. That is, the efficiency scores of CCDEA are higher than ordinary DEA.[14]

We further find that the average technical efficiency is 0.782 from the stochastic frontier analysis (SFA), which is significantly lower than the result gained from the chance-constrained DEA approach (0.931) (Table 3). Table 4 shows the estimated coefficients of the stochastic translog production function and the statistics for noise (sigma-squared) and the inefficiency component (LR test of the one-sided error).[22] Both noise and inefficiency are significant, indicating the noise component is also present, and that the stochastic frontier analysis model should be stochastic. We observe that the log-likelihood function for the final maximum likelihood estimate of the stochastic frontier model is $-6.94$ and the figure for the ordinary least squares fit of the production function is $-15.03$. Hence, we can compute the one-sided generalised likelihood-ratio statistics as 16.18 by $-2\times(-6.94-(-15.03))$, which is larger than the critical level of $\chi^2(5, 0.95) = 10.37$. The null hypothesis of no technical inefficiency effects in the model is therefore rejected. The stochastic frontier is significantly different from the deterministic frontier model with no random error included.

Two banker's asymptotic DEA efficiency tests have been used to test for inefficiency differences between two different efficiency scores.[23] First, we assume that the two inefficiencies ($1-\theta_a$ and $1-\theta_b$) follow the exponential distribution. The test statistic is $(\sum_r(1-\theta_{ar})/N_a)/(\sum_r(1-\theta_{br})/N_b)$, evaluated relative to the $F$ distribution with $(2N_a, 2N_b)$ degrees of freedom. Secondly, we assume that the two inefficiencies ($1-\theta_a$ and $1-\theta_b$) follow the half-normal distribution. The test statistic is $(\sum_r(1-\theta_{ar})^2/N_a)/\ (\sum_r(1-\theta_{br})^2/N_b)$, evaluated relative to the $F$ distribution with $(N_a, N_b)$ degrees of freedom. Another two traditional test procedures, Welch's mean test and the Mann–Whitney test, have also been used to test for comparison inefficiency differences between the different efficiency scores.[24] For Welch's mean test, the test statistic, under the assumption of unequal variances, is given by $\overline{X}_a-\overline{X}_b/\sqrt{(\sigma_a^2/N_a)+(\sigma_b^2/N_b)}$, which follows the $t$ distribution of freedoms calculated as $(\sigma_a^2/N_a)+(\sigma_b^2/N_b)/(\sigma_a^2/N_a)^2/N_a-1+(\sigma_b^2/N_b)^2/N_b-1$, where $\overline{X}_a$ and $\overline{X}_b$ and $\sigma_a^2$ and $\sigma_b^2$ are the sample means and variances of the inefficiencies. In the Mann—Whitney test, the test statistic $Z$ value is calculated by $Z=u-E(u)/\sqrt{V(u)}$, and $u$ is the lower figure between the calculated magnitude of $U_a$ and $U_b$:

$$U_a = N_aN_b + \frac{N_a(N_a+1)}{2} - W_a$$

$$U_b = N_aN_b + \frac{N_b(N_b+1)}{2} - W_b$$

$$E(\mu) = N_aN_b/2 \quad V(\mu) = N_aN_b(N_a+N_b+1)/12$$

where $W_a$ and $W_b$ are the rank sums of each selected sample. In our case, one of $N$ has large sample sizes ($N > 15$), we can generate a $Z$ value and refer to the standardized normal distribution to test the null hypothesis.

We use four tests: banker's two asymptotic DEA tests, Welch's mean test and the Mann–Whitney test in our study. All tests show that there is a significant difference among the average efficiency scores of ordinary DEA *vs* SFA

**Table 4** Estimated results of the stochastic translog production function

| *Items* | *Estimated parameters* | *Standard error* | *T value* |
|---|---|---|---|
| Constant | 6.0517 | 2.0542 | 2.9459** |
| ln ($L$) | −0.0439 | 0.6179 | −0.0711 |
| ln ($K$) | −0.5751 | 0.1893 | 3.0380** |
| ln ($M$) | 0.2844 | 0.5119 | 0.5556 |
| $(\ln K)^2$ | 0.0146 | 0.1095 | 0.1337 |
| $(\ln L)^2$ | 0.02762 | 0.0776 | 3.5584** |
| $(\ln M)^2$ | 0.2164 | 0.0376 | 5.7513** |
| ln $L$ ln$K$ | −0.0195 | 0.0707 | −0.2763 |
| ln $L$ ln$M$ | −0.0374 | 0.0771 | −0.4859 |
| ln $K$ ln$M$ | −0.2342 | 0.0708 | −3.3086** |
| Sigma-squared | 0.1368 | 0.0178 | 7.6912** |
| Gamma | 0.8799 | 0.0467 | 18.8251** |
| Log likelihood (OLS) | −15.03 | | |
| Log likelihood (MLE) | −6.94 | | |
| LR test of the one-sided error | 16.18** | | |

Note: ** represents significant at 0.05 level.

methods (see the third column, Table 5), and CCDEA *vs* SFA methods (see the fourth column, Table 5). Ferrier and Lovell[11] compared the deterministic DEA and parametric programming approaches and they presented a similar conclusion. This is consistent with the presentation of Farrell and Lovell, who argued that different techniques do lead to dramatically different results when they are used inside a similar methodological framework. The advanced setting of the chance-constrained mechanism of DEA does not change the instinctive differences between DEA and SFA approaches. Moreover, derived results can be obtained when DEA and CCDEA are compared and technical efficiencies obtained within these two approaches are not different as confirmed in four kinds of tests (see the fifth column, Table 5).

Note that the calculated individual bank variability is set to unity in the CCDEA approach ($\mathrm{Var}(Y_{ir}) = m = 1$). The smaller the stochastic variability of outputs (the smaller the moderator; $m$), the lower the efficiency scores. The frontier becomes more rigid and the band of bank output domain has a lower possibility to go outside the envelope, and approaches the case of deterministic DEA. Here we choose four scenarios ($m = 0.1$; $m = 0.2$; $m = 0.4$; $m = 0.6$) to highlight and compare our findings (see Table 6). Note that we do not extend to the scenarios of $m = 0.8$ and/or $m = 1.0$ because there are too many efficient banks in the range $m = 0.6$. Obviously, the greater the degree of uncertainty, the greater the requirement of the contingency margin, and banks need more slack to reach the performing scores. Also, when the chance-constrained efficiency score equals one, the optimal buffers of each bank's output will slide down to zero and collapse. For any banks, the magnitude of output buffers is computed by $1.645\sigma$. Assuming that three outputs have the same variance, the numerical value of the buffer of banks is identical for three outputs. The buffers of outputs can be interpreted as assessing how a given bank is performing compared to its reference banks, the required assignments needing to be larger than the

**Table 5** Summary of efficiency difference test results

| *Classification* | *Test procedure*[1] | *DEA* vs *SFA* | *CCDEA* vs *SFA* | *DEA* vs *CCDEA* |
|---|---|---|---|---|
| Banker's asymptotic DEA tests[2] | Exponential type | 2.682** | 3.182** | 1.187 |
| | Half-normal type | 3.137** | 3.610** | 1.151 |
| Traditional efficiency tests | Welch test[3] | 6.866** | 7.525** | 0.480 |
| | Mann–Whitney test[4] | −5.340** | 5.589** | 1.037 |

Notes: 1. ** represents significance at the 0.05 level.
2. As to Banker's asymptotic DEA tests, there are six tests performed: the exponential type, (1) DEA *vs* SFA (=0.2183/0.0814=2.682), (2) CCDEA *vs* SFA (=0.2183/0.0686=3.182) and (3) DEA *vs* CCDEA (=0.0814/0.0686=1.187); the half-normal type, (4) DEA *vs* SFA (=0.0527/0.0168=3.137), (5) CCDEA *vs* SFA (=0.0527/0.0146=3.610) and (6) DEA *vs* CCDEA (=0.0168/0.0146=1.151).
3. As to Welch efficiency tests, there are three tests performed: (1) DEA *vs* SFA (=(0.920 − 0.782)/0.0201=6.866), (2) CCDEA *vs* SFA (=(0.931 − 0.782)/0.0198=7.525) and (3) DEA vs CCDEA (=(0.931 − 0.920)/0.0229=0.480).
4. As to Mann–Whitney efficiency tests, there are also three tests performed, that is, (1) DEA *vs* SFA of technical efficiency ((226 − 760.5)/100.1=−5.340), (2) CCDEA *vs* SFA of technical efficiency ((201 − 760.5)/100.1=−5.589) and (3) DEA *vs* CCDEA of technical efficiency ((656.5 − 760.5)/100.1=−1.039). Note that 760.5 and 100.1 are the calculated average and the standard deviation of the selected sample.

**Table 6** Various output variation degrees of the chance-constrained efficiency score and buffers of inefficient banks

| | | *0.1* | | *0.2* | | *0.4* | | *0.6* | |
|---|---|---|---|---|---|---|---|---|---|
| *Scenarios (m) bank name* | *Basic DEA* | *Eff.* | *Buffers* | *Eff.* | *Buffers* | *Eff.* | *Buffers* | *Eff.* | *Buffers* |
| Aetna | 1.000 | 1.000 | — | 1.000 | — | 1.000 | — | 1.000 | — |
| Agriculture | 0.991 | 0.995 | 0.49 | 1.000 | — | 1.000 | — | 1.000 | — |
| Asia Pacific | 0.858 | 0.862 | 0.17 | 0.868 | 0.34 | 0.906 | 0.68 | 0.962 | 1.01 |
| Baodao | 0.973 | 0.978 | 0.18 | 0.987 | 0.38 | 1.000 | — | 1.000 | — |
| Center Trust | 0.558 | 0.563 | 0.23 | 0.569 | 0.47 | 0.582 | 0.78 | 0.594 | 1.12 |
| Chang-hua | 0.868 | 0.871 | 0.55 | 0.878 | 1.11 | 0.889 | 1.34 | 0.896 | 1.50 |
| Chiao-tung | 0.808 | 0.809 | 0.28 | 0.809 | 0.57 | 0.811 | 1.13 | 0.817 | 1.64 |
| China International | 0.953 | 0.966 | 0.21 | 0.975 | 0.43 | 0.994 | 0.86 | 1.000 | — |
| China Trust | 0.996 | 0.991 | 1.21 | 1.000 | — | 1.000 | — | 1.000 | — |
| Chinese | 1.000 | 0.857 | 0.82 | 0.871 | 1.10 | 0.892 | 1.39 | 0.914 | 1.72 |
| Chinfon | 0.996 | 1.000 | — | 1.000 | — | 1.000 | — | 1.000 | — |
| Chung-shing | 0.832 | 0.835 | 0.17 | 0.838 | 0.35 | 0.844 | 0.68 | 0.850 | 1.02 |
| Cosmos | 0.780 | 0.790 | 0.20 | 0.800 | 0.39 | 0.823 | 0.79 | 0.846 | 1.06 |
| Dah-an | 0.957 | 0.960 | 0.19 | 0.969 | 0.39 | 0.994 | 0.80 | 1.000 | — |
| E. Sun | 0.921 | 0.924 | 0.17 | 0.930 | 0.34 | 0.946 | 0.74 | 0.969 | 1.08 |
| Far Eastern | 1.000 | 1.000 | — | 1.000 | — | 1.000 | — | 1.000 | — |
| First | 0.957 | 0.970 | 0.79 | 1.000 | — | 1.000 | — | 1.000 | — |
| Fu-bon | 0.723 | 0.733 | 0.21 | 0.745 | 0.41 | 0.769 | 0.75 | 0.794 | 1.06 |
| Grand | 0.715 | 0.718 | 0.17 | 0.720 | 0.34 | 0.726 | 0.68 | 0.731 | 1.03 |
| Hsinchu City | 0.970 | 0.978 | 0.21 | 0.988 | 0.44 | 1.000 | — | 1.000 | — |
| Hualien City | 0.937 | 0.965 | 0.17 | 1.000 | — | 1.000 | — | 1.000 | — |
| Hun-nan | 0.831 | 0.836 | 0.75 | 0.842 | 1.62 | 0.857 | 1.91 | 0.865 | 1.97 |
| Kaohsiung | 1.000 | 1.000 | — | 1.000 | — | 1.000 | — | 1.000 | — |
| Kaohsiung City | 0.956 | 0.958 | 0.16 | 0.969 | 0.33 | 1.000 | — | 1.000 | — |
| Land | 0.992 | 0.994 | 1.01 | 0.996 | 2.01 | 0.999 | 3.58 | 1.000 | — |
| Our Corp. | 0.856 | 0.865 | 0.20 | 0.877 | 0.43 | 0.905 | 0.75 | 0.931 | 1.07 |
| Pan-Asia | 1.000 | 1.000 | — | 1.000 | — | 1.000 | — | 1.000 | — |
| Shanghai | 0.903 | 0.925 | 0.20 | 0.944 | 0.38 | 1.000 | — | 1.000 | — |
| Sinopac | 0.881 | 0.890 | 0.23 | 0.906 | 0.49 | 0.939 | 0.81 | 1.000 | — |
| Taishin | 0.955 | 0.967 | 0.21 | 1.000 | — | 1.000 | — | 1.000 | — |
| Taichung City | 1.000 | 1.000 | — | 1.000 | — | 1.000 | — | 1.000 | — |
| Tainan City | 0.983 | 1.000 | — | 1.000 | — | 1.000 | — | 1.000 | — |
| Taipei | 0.977 | 0.978 | 0.24 | 0.980 | 0.48 | 0.982 | 1.02 | 0.985 | 1.44 |
| Taipei Business | 1.000 | 1.000 | — | 1.000 | — | 1.000 | — | 1.000 | — |
| Taitung City | 1.000 | 1.000 | — | 1.000 | — | 1.000 | — | 1.000 | — |
| Taiwan | 1.000 | 1.000 | — | 1.000 | — | 1.000 | — | 1.000 | — |
| Taiwan Provincial | 1.000 | 1.000 | — | 1.000 | — | 1.000 | — | 1.000 | — |
| Union | 0.782 | 0.789 | 0.18 | 0.798 | 0.40 | 0.823 | 0.75 | 0.850 | 1.07 |
| United World | 1.000 | 1.000 | — | 1.000 | — | 1.000 | — | 1.000 | — |
| Average | 0.920 | 0.924 | | 0.932 | | 0.943 | | 0.951 | |

Note: Four scenarios ($m=0.1$; $m=0.2$; $m=0.4$; $m=0.6$) of stochastic variability of various degrees within-bank standard deviation are chosen. Eff. = efficiency.

observed outputs. For example, assessing the bank loans obtained at a specific bank (say Asia Pacific Bank), the reference bank output (say, loans) with which Asia Pacific Bank is compared should be at least 95% larger than or equal to the actual loans. In order to reach this condition, Asia Pacific Bank needs a lot of slack through achieved bank loans. Moreover, the bank investment and/or non-interest revenue of Asia Pacific Bank also needs some slack.

We further conduct a regression analysis to determine whether the efficiency scores are related to the Asian financial crisis, ownership or size of bank characteristics. We transform the basic regression form to a logistic probability function since the efficiency ranges from zero to one. The transformed regression is expressed as $\ln(Y/(1-Y)) = \alpha + \beta X$ which is derived from the logistic probability function of $Y = F(\alpha + \beta X) = 1/(1 + \exp(-\alpha - \beta X))$ where $Y$ and $X$ represent a dependent variable and independent variable, respectively. Using the transformed regression, the issue of probabilistic prediction range 0 to 1 has been formulated to the issue of the occurrence prediction of specific events in the whole real set. Bank size is measured by staff employed or bank assets. The status of ownership is measured by government shareholding. The results of the regression analysis are shown in Table 7,

**Table 7** Estimated results of the regression analysis

| *Model Dep. var.* | *M1 DEA* | *M2 DEA* | *M3 DEA* | *M4 CCDEA* | *M5 CCDEA* | *M6 CCDEA* | *M7 SFA* | *M8 SFA* | *M9 SFA* |
|---|---|---|---|---|---|---|---|---|---|
| Indep. var. | 0.874 | 0.862 | 0.867 | 0.891 | 0.918 | 0.896 | 0.850 | 0.849 | 0.848 |
| intercept | 62.83** | 59.17** | 62.22** | 36.33** | 24.10** | 24.31** | 77.22** | 72.46** | 75.50** |
| | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) |
| Ownership | 0.062 | 0.053 | 0.098 | 0.063 | 0.042 | 0.058 | 0.054 | 0.039 | 0.053 |
| | 2.63** | 2.25** | 1.72** | 2.44** | 1.25 | 1.67* | 3.20** | 1.99** | 2.80** |
| | (0.009) | (0.025) | (0.093) | (0.017) | (0.216) | (0.098) | (0.002) | (0.048) | (0.005) |
| Staff | | $1.61 \times 10^{-5}$ | | | $6.63 \times 10^{-6}$ | | | $3.80 \times 10^{-6}$ | |
| | | 3.18** | | | 0.93 | | | 0.94 | |
| | | (0.003) | | | (0.358) | | | (0.350) | |
| Assets | | | $1.36 \times 10^{-7}$ | | | $1.07 \times 10^{-8}$ | | | $5.97 \times 10^{-8}$ |
| | | | 3.91** | | | 0.19 | | | 2.13** |
| | | | (0.002) | | | (0.847) | | | (0.034) |
| Asian financial | 0.210 | 0.199 | 0.205 | 0.180 | 0.177 | 0.1791 | 0.126 | 0.114 | 0.118 |
| crisis effect | 11.92** | 11.96** | 12.28** | 7.58** | 7.31** | 7.24** | 9.08** | 8.51** | 8.78** |
| | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) |
| Adj. $R^2$ | 0.577 | 0.579 | 0.588 | 0.443 | 0.424 | 0.436 | 0.228 | 0.207 | 0.218 |
| *F* value | 47.88 | 48.41 | 50.95 | 32.69 | 21.37 | 20.87 | 28.37 | 25.02 | 26.54 |
| *P* value | 0.001** | 0.001** | 0.001** | 0.001** | 0.001** | 0.001** | 0.001** | 0.001** | 0.001** |

Notes: 1. Indep. Var. = independent variables; Dep. Var. = dependent variable; here are the efficiency scores derived from DEA, CCDEA, or SFA model. $N = 273$.
2. ** represents significant at 0.05 level.
3. The ownership is switched to the dummy variables 0 and 1 as a proxy. The cut-off point is 33.3% in government shareholding.

demonstrating that ownership, size of bank and the Asian financial crisis effect variables are significant variables in explaining technical efficiency. Similar results were also obtained in the CCDEA efficiency and SFA efficiency scores, and these support the results listed above in this section. We find that the ownership variable still provides a significant variable to explain the technical efficiency in the efficiency model, irrespective of whether a DEA, CCDEA or SFA approach is considered. We can reasonably infer that public-owned banks often undertake lower technical efficiency and there is a disparity in efficiency. Also Taiwan having a basis in bank ownership may be substantial when the Asian financial crisis effect has been considered.

## Concluding remarks

In this study, ordinary DEA, chance-constrained DEA, and stochastic frontier analysis approaches are employed to compare the technical efficiency of 39 banks in Taiwan. The seven-year data set (1994–2000), which includes the ante-crisis (Asian financial crisis) period of 1994–1996, along with the post-crisis period of 1997–2000, is used to effectively avoid the one-off shock effect in the overall efficiency evaluation. Estimated results show that there are significant differences in efficiency scores between chance-constrained DEA and stochastic frontier production function. Similar to the results of Ferrell and Lovell, we find that different approaches (DEA *vs* SFA) will result in different results when they are employed in the similar methodological framework. The advanced setting of the chance-constrained mechanism of DEA does not change the instinctive differences between DEA and SFA approaches. We further find that the ownership variable still provides a significant variable to explain the technical efficiency in the efficiency model, irrespective of whether DEA, CCDEA or a SFA approach is employed.

## References


1 Charnes A, Cooper WW and Rhodes E (1978). Measuring the efficiency of decision making units. *Eur J Opl Res* **2**: 429–444.
2 Banker RD, Charnes A and Cooper WW (1984). Some models for estimation of technical and scale inefficiencies in data envelopment analysis. *Mngt Sci* **30**: 1078–1092.
3 Oral M and Yolalan R (1990). An empirical study on measuring operating efficiency and profitability of bank branches. *Eur J Opl Res* **46**: 282–294.
4 Favero CA and Papi L (1995). Technical efficiency and scale efficiency in the Italian banking sector. *Appl Econ* **27**: 385–395.
5 Schaffnit C, Rosen D and Paradi JC (1997). Best practice analysis of bank branches: an application of DEA in a large Canadian bank. *Eur J Opl Res* **98**: 269–289.
6 Fukuyama H, Guerra R and Weber WL (1999). Efficiency and ownership: evidence from Japanese credit cooperatives. *J Econ Bus* **51**: 473–487.
7 Bjurek H, Hjalmarsson L and Forsund FR (1990). Deterministic parametric and nonparametric estimating of efficiency in service production. *J Economet* **46**: 213–227.
8 Giokas DI (1991). Bank branch operating efficiency: a comparative application of DEA and the loglinear model. *Omega* **19**: 549–557.
9 Resti A (1997). Evaluating the cost efficiency of the Italian banking system: what can be learned from the joint application of parametric and non-parametric techniques. *J Banking Fin* **21**: 221–250.

10 Resti A (2000). Efficiency measurement for multi-product industries: a comparison of classic and recent techniques based on simulated data. *Eur J Opl Res* **121**: 559–578.
11 Ferrier GD and Lovell CAK (1990). Measuring cost efficiency in banking: econometric and linear programming evidence. *J Economet* **46**: 229–245.
12 Cooper WW and Tone K (1997). Measures of inefficiency in data envelopment analysis and stochastic frontier estimation. *Eur J Opl Res* **99**: 72–88.
13 Charnes A and Cooper WW (1959). Chanced-constrained programming. *Mngt Sci* **15**: 73–79.
14 Land KC, Lovell CAK and Thore S (1993). Chance-constrained data envelopment analysis. *Mngt Decis Econ* **14**: 541–554.
15 Land KC, Lovell CAK and Thore S (1994). Productive efficiency under capitalism and state socialism: an empirical inquiry using chance-constrained data envelopment analysis. *Technol Forecast Social Change* **46**: 139–152.
16 Farrell M (1957). The measurement of productive efficiency. *J R Statist Soc A* **120**: 253–281.
17 Sealey CW and Lindley JT (1997). Inputs, outputs and a theory of production and cost at depository financial institutions. *J Fin* **32**: 1251–1266.
18 Battese GE and Coelli TJ (1992). Frontier production function, technical efficiency and panel data with application to paddy farmers in India. *J Prod Anal* **3**: 153–169.
19 Battese GE and Coelli TJ (1995). A model for technical inefficiency effects in a stochastic frontier production function for panel data. *Empirical Econ* **20**: 325–332.
20 Coelli TJ (1996). Simulators and hypothesis tests for a stochastic model: a Monte Carlo analysis. *J Prod Anal* **6**: 247–268.
21 Oral M and Yolalan R (1990). An empirical study on measuring operating efficiency and profitability of bank branches. *Eur J Opl Res* **46**: 282–294.
22 Coelli T, Rao DSP and Battese GE (1998). *An Introduction to Efficiency and Productivity Analysis*. Kluwer: Norwell, MA, pp 190–198.
23 Banker RD (1996). Hypothesis tests using data envelopment analysis. *J Prod Anal* **7**: 139–159.
24 Banker RD and Chang H (1995). A simulation study of hypothesis tests for differences in efficiencies. *Int J Prod Econ* **39**: 37–54.

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