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## PRODUCTION & MANUFACTURING | RESEARCH ARTICLE

# A novel decision-making method using fuzzy DEA credibility constrained and RC index

Rahmad Wisnu Wardana<sup>1</sup>, Ilyas Masudin<sup>1\*</sup>, Dian Palupi Restuputri<sup>1</sup> and Adhi Nugraha<sup>1</sup>

**Abstract:** Decision-making based on expert's opinion by data envelopment analysis (DEA) models requires crisp number. However, the assumption of expert might not always be accurate. This paper introduces fuzzy DEA credibility constrained and relative closeness (RC) index to cope with uncertainty data. The proposed approach transforms the traditional DEA models to be fuzzy events by using credibility measure. Furthermore, RC index was used to increase the discrimination power of traditional DEA. Two numerical examples with different credibility level are presented for demonstrate the proposed fuzzy DEA credibility constrained and RC index. The findings indicate that the credibility level impact on the RC index on both numerical examples. The increasing level of credibility value would drop the value of RC index.

**Subjects:** Operations Research; Operations Management; Operations Research

**Keywords:** decision-making; fuzzy DEA; credibility constrained; relative closeness

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### PUBLIC INTEREST STATEMENT

Decision-making based on expert's opinion by data envelopment analysis (DEA) model requires crisp number. Vagueness is the main source of uncertainty because the assumption of the experts might not always be accurate. Vagueness refers to fuzzy input and output data from experts. To solve this problem, the integration of fuzzy DEA credibility constrained and relative closeness (RC) index is introduced. This approach transforms the traditional DEA models to be fuzzy events by using credibility measure. RC index was used to increase the discrimination power of traditional DEA. The credibility level of the proposed approach provides the flexibility for decision-maker to set their own acceptable credibility level in making decision. Moreover, the efficiency score of each DMU is decreased by increasing the credibility level. The preserved output data applying by the integration of RC index and credibility level are the cause of the discrimination power improvement.

## 1. Introduction

Data envelopment analysis (DEA) was introduced by Charnes et al. (1978) to measure the relative efficiency of decision-making units (DMUs). The relative efficiency of any DMU is obtained by using a maximum of ratio of weighted outputs to weighted inputs. Nowadays, DEA still grows very fast, around 2000 more papers related with DEA have been published during 2010 to 2014 (Liu et al., 2016). The activities related to DEA grow rapidly because some advantages, such as DEA, does not need a set of weight either inputs or outputs (Mirhedayatian et al., 2013), DEA does not need to transform data to be same unit (Shabani et al., 2012), and DEA can handle quantitative and qualitative variables (Calik et al., 2018; Mousavi-Nasab & Sotoudeh-Anvari, 2017). However, the main issue of DEA is related with discrimination power that is still low. DEA is still possible to have same efficiency score in the several DMU, and this causes impossible to know the exact ranking of DMUs.

To increase the discrimination power of DEA, Wang and Jean (2006) proposed two virtual DMUs known as ideal DMU (IDMU) and anti-ideal DMU (ADMU) into the DEA. Furthermore, the two virtual DMUs are combined to relative closeness (RC) index, and the ranking of DMUs can be obtained. In their approach, all the data either inputs or outputs are assumed to be a crisp number. However, the assumption might not always be accurate or known as lack of precision of judgment of the expert. Hence, enhancing their model is necessary to make it useful in uncertainty data.

Recently, some researchers develop fuzzy set theory and apply in many problem situations. For example, a study using a type-2 fuzzy network DEA for fast-moving consuming goods (FMCG) performance evaluation (Olfat & Pishdar, 2017), a fuzzy DEA-Vikor used for supplier selection (Karami et al., 2020; Mohaghar et al., 2013) and an interval type-2 fuzzy dynamic network DEA used for airports' sustainability performance (Olfat et al., 2016). Furthermore, many methods that can make fuzzy DEA models. Lertworasirikul et al. (2003) provides for group of fuzzy DEA models, namely tolerance-based approach,  $\alpha$ -based approach, fuzzy ranking approach, and possibility approach, while Olfat and Pishdar (2017) modeled a type-2 fuzzy for dynamic network DEA considering double frontiers. Based on those approaches, the proper approach for solving the uncertainty data is possibility approach (Hatami-Marbini et al., 2013).

The main objective of this paper is to construct a new approach, namely fuzzy DEA credibility constrained and RC index. In the proposed approach, the models are modeled as fuzzy variable; hence, the constraints are considered as fuzzy events. In doing so, the credibility measure also will be used to handle the uncertainty data, it is more practical to determine either inputs or outputs variable. Finally, the numerical example also will be presented to illustrate the effectiveness of the proposed approach.

The paper is organized as follows. In section 2, recalling the basic concept of DEA and RC index model. After that, developing the proposed approach is presented in section 3. In section 4, focus on discussing the numerical example that is divided into two parts (simple and complex). Section 5 concludes the paper.

## 2. Basic concept

### 2.1. Data envelopment analysis (DEA) model

The main issue of conventional DEA cannot cope with uncertainty data. In the group, decision-making has two main uncertainty data, such as ambiguity and vagueness (Fasanghari et al., 2015). The ambiguity refers to multiple interpretations of the expert, and vagueness is due to the lack of expert opinion precision. In order to solve these problems, the DEA credibility constrained model is applied to solve ambiguity, and RC index technique is applied for solve vagueness. CCR (Charnes-Cooper-Rhodes) model is the most common DEA models (Charnes et al., 1978). Furthermore, suppose there are  $j$ th DMUs, and each of the DMUs has  $m$  inputs  $x_{ij}$  ( $i = 1, \dots, n$ ) and  $s$  output  $y_{rj}$  ( $r = 1, \dots, m$ ). In order to solve CCR model, the fractional programming (FP) can be used as follows:

$$E_j = \frac{\sum_{r=1}^m u_r y_{rj}}{\sum_{i=1}^n v_i x_{ij}}$$

Subject to:

(1)

$E_j$  is the efficiency of  $j$ th DMU with inputs ( $x_{ij}$ ) and outputs ( $y_{rj}$ ). The  $v_i$  is valued for input weight and the  $u_r$  is the output weight. Supposed DMU has efficiency score equals 1 ( $E_j = 1$ ), it means the DMU is efficient; otherwise, it is inefficient.

## 2.2. DEA and RC index model

DEA is an appropriate method, simple modeling, non-parametric solution, and an effective approach for multi criteria decision-making. Moreover, DEA does not require a decision-maker to set the weights of the inputs and outputs. However, on several occasions, it is still common to find the same DMU values so that it is difficult to decide the best criteria (Kim et al., 2019). In the conventional DEA, the efficiency score of DMUs is possible to have several same score, and this causes impossible to decide the exact ranking of DMUs. In other words, the discrimination power of DEA is not satisfactory. To improve the discrimination power of DEA, Wang and Jean (2006) proposed integrated DEA and RC index. In their paper, they compare two virtual DMUs, namely ideal DMU (IDMU) and anti-ideal DMU (ADMU), and it ranks them by the RC. In this proposed model, the efficiency score of IDMU ( $\theta_{IDMU}$ ) must be calculated first and the model as follows:

$$\theta_{IDMU} = \max \frac{\sum_{r=1}^m u_r y_r^{max}}{\sum_{i=1}^n v_i x_i^{min}}$$

Subject to:

$$\frac{\sum_{r=1}^m u_r y_{rj}}{\sum_{i=1}^n v_i x_{ij}} \leq 1, \quad j$$

$$v_i, u_r \geq 0 \quad i, r \quad (2)$$

$y_r^{max}$  and  $x_i^{min}$  are the input and output of IDMU which are determined by  $y_r^{max} = \max_j \{y_{rj}\}$ , and  $= \min_j \{x_{ij}\}$ , respectively. Furthermore, let  $\theta_{IDMU}^*$  is the optimum efficiency score of IDMU, and the next step is to determine the score of each DMU compared to the efficiency score of  $\theta_{IDMU}^*$ , and the model as follows:

$$\theta_j = \max \frac{\sum_{r=1}^m u_r y_{rj}}{\sum_{i=1}^n v_i x_{ij}}$$

Subject to:

$$\sum_{r=1}^m u_r y_r^{max} - \sum_{i=1}^n v_i (\theta_{IDMU}^* x_i^{min}) = 0$$

$$\frac{\sum_{r=1}^m u_r y_{rj}}{\sum_{i=1}^n v_i x_{ij}} \leq 1, \quad j$$

$$v_i, u_r \geq 0 \quad i, r \quad (3)$$

$\theta_j$  denotes the efficiency score of  $j$ th DMU compared to the efficiency score of . In the next step, determine the efficiency score of ADMU ( $\phi_{ADMU}$ ) by the following model:

$$\varphi_{ADMU} = \min \frac{\sum_{r=1}^m u_r y_r^{min}}{\sum_{i=1}^n v_i x_i^{max}}$$

Subject to:

$$\frac{\sum_{r=1}^m u_r y_{rj}}{\sum_{i=1}^n v_i x_{ij}} \geq 1, \quad j$$

$$v_i, u_r \geq 0 \quad i, r \quad (4)$$

$y_r^{min}$  and  $x_i^{max}$  are the input and output of IDMU which are determined by  $y_r^{min} = \min_y \{y_{rj}\}$ , and  $x_i^{max} = \max_x \{x_{ij}\}$ , respectively. Furthermore, let  $\varphi_{ADMU}^*$  is the optimum efficiency score of IDMU, and the next step is to determine the score of each DMU compared to the efficiency score of  $\varphi_{ADMU}^*$ , and the model as follows:

$$\varphi_j = \min \frac{\sum_{r=1}^m u_r y_{rj}}{\sum_{i=1}^n v_i x_{ij}}$$

Subject to:

$$\sum_{r=1}^m u_r y_r^{min} - \sum_{i=1}^n v_i (\varphi_{ADMU}^* x_i^{max}) = 0$$

$$\frac{\sum_{r=1}^m u_r y_{rj}}{\sum_{i=1}^n v_i x_{ij}} \geq 1, \quad j$$

$$v_i, u_r \geq 0 \quad i, r \quad (5)$$

$\varphi_j$  denote the efficiency score of  $j^{\text{th}}$  DMU compared to the efficiency score of  $\varphi_{ADMU}^*$ . After determining the score of  $\theta_{IDMU}^*$ ,  $\theta_j$ ,  $\varphi_{ADMU}^*$ , and  $\varphi_j$ , the RC of DMUs to IDMU as follows:

$$RC_j = \frac{\varphi_j^* - \varphi_{ADMU}^*}{(\varphi_j^* - \varphi_{ADMU}^*) + (\theta_{IDMU}^* - \theta_j^*)} \quad (6)$$

### 3. Proposed approach

The model (2–5) is traditional model of integrated DEA with RC index model. Moreover, all the inputs and the output assumed to be crisp number in the traditional model. However, in the real situation, the assumption is not always true. Vagueness, as the main sources of uncertainty, refers to fuzzy input and output data from expert. To cope with this problem, the fuzzy DEA credibility constrained and RC index model are developed as a linear programming model to be useful in real problem. Furthermore, Meng and Liu (2007) adopted the concept of fuzzy chance constraints programming (FCCP) on the DEA to cope with uncertainty data. Subsequently, we take the basic concept of credibility theory used to apply in the model (2–5), and it becomes a new model that is fuzzy DEA credibility constrained and RC index.

**Definition 1** Let  $\xi$  is a fuzzy variable with the distribution function  $\mu : \mathcal{R} \rightarrow [0, 1]$ . A fuzzy variable is said normal if there exists a real number  $r$  such that  $\mu(r) = 1$ .

**Definition 2** Let Pos and Nec be two particular fuzzy measure defined on  $(\mathcal{R}, U)$ , where  $U$  is power set of  $\mathcal{R}$ . Furthermore, Pos and Nec are a pair of dual fuzzy measures, and the model is  $\text{Pos}\{A\} = 1 - \text{Nec}\{A^c\}$  with  $A^c$  is the complement of  $A$ .

**Definition 3** The credibility measure model as follows:

$$Cr(A) = \frac{1}{2} (\text{Pos}\{A\} + \text{Nec}\{A\}) \quad (7)$$

For any  $A \in U$ .

Consider  $\xi$  as triangular fuzzy number  $(k^1, k^2, k^3)$  which is known so the membership function as follows:

$$\mu(r) = \begin{cases} \frac{r-k^1}{k^2-k^1} & \text{if } k^1 \leq r \leq k^2 \\ \frac{k^2-r}{k^2-k^3} & \text{if } k^2 \leq r \leq k^3 \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

Based on the definition, a general fuzzy chance constraints programming model can be formulated as follows (Xiang Li & Liu, 2006):

$$\text{Min } \sum_{j=1}^n C_j x_j$$

Subject to:

$$\text{Cr} \left\{ \sum_{j=1}^n a_{ij} x_j \leq \tilde{b}_i \right\} \geq \lambda_i, i = 1, \dots, m,$$

$$x_j \geq 0, i = 1, \dots, n. \quad (9)$$

Based on model (9), the first constraint  $(\sum_{j=1}^n a_{ij} x_j \leq \tilde{b}_i)$  should be greater than or equal to  $\lambda_i$ ,  $\lambda_i$  is a scalar of credibility level, and normally the credibility level should be greater than 0.5 (Meng & Liu, 2007; Y. Zhang et al., 2012). Furthermore, the fuzzy chance constraints programming should become deterministic model to simplify the optimization model. Theorem 1 is utilized to convert FCCP to their equivalent crisp ones.

**Theorem 1** Let  $\tilde{k}_i$  be and independent triangular fuzzy number  $(k_i^1, k_i^2, k_i^3)$  and be independent triangular fuzzy number  $(s_o^1, s_o^2, s_o^3)$ . Moreover, for any given credibility level  $\alpha \in [0.5, 1]$ .

$$\text{Cr} \left\{ \sum_j^n u_i \tilde{k}_i / \sum_o^n h_o \tilde{s}_o \leq c \right\} \geq \alpha \text{ if and only if}$$

$$\frac{(2\alpha - 1) \sum_j^n u_i k_i^3 + 2(1 - \alpha) \sum_j^n u_i k_i^2}{(2\alpha - 1) \sum_o^n h_o s_o^1 + 2(1 - \alpha) \sum_o^n h_o s_o^2} \leq c; \quad (10)$$

$$\text{Cr} \left\{ \sum_j^n u_i \tilde{k}_i / \sum_o^n h_o \tilde{s}_o \geq c \right\} \geq \alpha \text{ if and only if}$$

$$\frac{(2\alpha - 1) \sum_j^n u_i k_i^1 + 2(1 - \alpha) \sum_j^n u_i k_i^2}{(2\alpha - 1) \sum_o^n h_o s_o^3 + 2(1 - \alpha) \sum_o^n h_o s_o^2} \leq c; \quad (11)$$

According to FCCP concept, the models (2), (3), (4), and (5) can be transformed into the credibility constrained model as follows:

$$\text{max } \theta_{IDMU}$$

Subject to:

$$\left\{ \frac{\sum_{r=1}^m u_r \tilde{y}_r^{\max}}{\sum_{i=1}^n v_i \tilde{x}_i^{\min}} \geq \theta_{IDMU} \right\} \geq \alpha$$

$$\left\{ \frac{\sum_{r=1}^m u_r \tilde{y}_{rj}}{\sum_{i=1}^n v_i \tilde{x}_{ij}} \leq 1 \right\} \geq \beta_j, \quad j$$

$$v_i, u_r \geq 0 \quad i, r$$
(12)

$\max \theta_j$

Subject to:

$$\left\{ \frac{\sum_{r=1}^m u_r \tilde{y}_{rj}}{\sum_{i=1}^n v_i \tilde{x}_{ij}} \geq \theta_j \right\} \geq \alpha$$

$$\sum_{r=1}^m u_r y_r^{\max} - \sum_{i=1}^n v_i (\theta_{IDMU}^* x_i^{\min}) = 0$$

$$\left\{ \frac{\sum_{r=1}^m u_r \tilde{y}_{rj}}{\sum_{i=1}^n v_i \tilde{x}_{ij}} \leq 1 \right\} \geq \beta_j, \quad j$$

$$v_i, u_r \geq 0 \quad i, r$$
(13)

$\min \varphi_{ADMU}$

Subject to:

$$\left\{ \frac{\sum_{r=1}^m u_r \tilde{y}_{rj}}{\sum_{i=1}^n v_i \tilde{x}_{ij}} \geq 1 \right\} \geq \beta_j, \quad j$$

$$v_i, u_r \geq 0 \quad i, r$$
(14)

$\min \varphi_j$

Subject to:

$$\left\{ \frac{\sum_{r=1}^m u_r \tilde{y}_{rj}}{\sum_{i=1}^n v_i \tilde{x}_{ij}} \leq \varphi_j \right\} \geq \alpha$$

$$\sum_{r=1}^m u_r y_r^{\min} - \sum_{i=1}^n v_i (\varphi_{ADMU}^* x_i^{\max}) = 0$$

$$\left\{ \frac{\sum_{r=1}^m u_r \tilde{y}_{rj}}{\sum_{i=1}^n v_i \tilde{x}_{ij}} \geq 1 \right\} \geq \beta_j, \quad j$$

$$v_i, u_r \geq 0 \quad i, r$$
(15)

According to theorems 1, the model (14), (15), (16), and (17) are converted to equivalent crisp ones as follows:

$\max \theta_{IDMU}$

Subject to:

$$\frac{(2\alpha - 1) \sum_r^m u_r y_r^{1\max} + 2(1 - \alpha) \sum_r^m u_r y_r^{2\max}}{(2\alpha - 1) \sum_i^n v_i x_i^{3\min} + 2(1 - \alpha) \sum_i^n v_i x_i^{2\min}} \geq \theta_{IDMU}$$

$$v_i, u_r \geq 0 \quad i, r \quad (16)$$

$$\max \theta_j$$

Subject to:

$$\frac{(2\alpha - 1) \sum_r^m u_r y_{rj}^1 + 2(1 - \alpha) \sum_r^m u_r y_{rj}^2}{(2\alpha - 1) \sum_i^n v_i x_{ij}^3 + 2(1 - \alpha) \sum_i^n v_i x_{ij}^2} \geq \theta_j$$

$$\sum_{r=1}^m u_r y_r^{2\max} - \sum_{i=1}^n v_i (\theta_{IDMU}^* x_i^{2\min}) = 0$$

$$\frac{(2\beta_j - 1) \sum_r^m u_r y_{rj}^3 + 2(1 - \beta_j) \sum_r^m u_r y_{rj}^2}{(2\beta_j - 1) \sum_i^n v_i x_{ij}^1 + 2(1 - \beta_j) \sum_i^n v_i x_{ij}^2} \leq 1, \quad j$$

$$v_i, u_r \geq 0 \quad i, r \quad (17)$$

$$\min \varphi_{ADMU}$$

Subject to:

$$\frac{(2\alpha - 1) \sum_r^m u_r y_r^{3\min} + 2(1 - \alpha) \sum_r^m u_r x_r^{2\min}}{(2\alpha - 1) \sum_i^n v_i x_i^{1\max} + 2(1 - \alpha) \sum_i^n v_i x_i^{2\max}} \leq \varphi_{ADMU}$$

$$\frac{(2\beta_j - 1) \sum_r^m u_r y_{rj}^1 + 2(1 - \beta_j) \sum_r^m u_r y_{rj}^2}{(2\beta_j - 1) \sum_i^n v_i x_{ij}^3 + 2(1 - \beta_j) \sum_i^n v_i x_{ij}^2} \geq 1, \quad j$$

$$v_i, u_r \geq 0 \quad i, r \quad (18)$$

$$\min \varphi_j$$

Subject to:

$$\frac{(2\alpha - 1) \sum_r^m u_r y_{rj}^3 + 2(1 - \alpha) \sum_r^m u_r y_{rj}^2}{(2\alpha - 1) \sum_i^n v_i x_{ij}^1 + 2(1 - \alpha) \sum_i^n v_i x_{ij}^2} \leq \varphi_j$$

$$\sum_{r=1}^m u_r y_r^{2\min} - \sum_{i=1}^n v_i (\varphi_{ADMU}^* x_i^{2\max}) = 0$$

$$\frac{(2\beta_j - 1) \sum_r^m u_r y_{rj}^1 + 2(1 - \beta_j) \sum_r^m u_r y_{rj}^2}{(2\beta_j - 1) \sum_i^n v_i x_{ij}^3 + 2(1 - \beta_j) \sum_i^n v_i x_{ij}^2} \geq 1, \quad j$$

$$v_i, u_r \geq 0 \quad i, r \quad (19)$$

The model (16), (17), (18), and (19) are equivalent to the following model:



$$\theta_{DMU} = \max(2\alpha - 1) \sum_r^m u_r y_r^{1^{max}} + 2(1 - \alpha) \sum_r^m u_r y_r^{2^{max}}$$

Subject to:

$$(2\alpha - 1) \sum_i^n v_i x_i^{3^{min}} + 2(1 - \alpha) \sum_i^n v_i x_i^{2^{min}} = 1$$

$$(2\beta_j - 1) \left( \sum_r^m u_r y_{rj}^3 - \sum_i^n v_i x_{ij}^1 \right) + 2(1 - \beta_j) \left( \sum_r^m u_r y_{rj}^2 - \sum_i^n v_i x_{ij}^2 \right) \leq 0, \quad j$$

$$v_i, u_r \geq 0 \quad i, r \quad (20)$$

Table 2 is the results obtained by referring to equations 16 to 20. It shows that the efficiency score of each DMU decrease by increasing the credibility level. The results also indicated that the discrimination power is improved significantly in this step because all the DMUs are fully ranked in each credibility level. The real cause for improving the discrimination power is that in this proposed model, the output data are preserved by applying both the RC index and the credibility level parameters. Therefore, the highly discriminated results are generated without any loss of information.

$$\theta_j = \max(2\alpha - 1) \sum_r^m u_r y_{rj}^1 + 2(1 - \alpha) \sum_r^m u_r y_{rj}^2$$

Subject to:

$$(2\alpha - 1) \sum_i^n v_i x_{ij}^3 + 2(1 - \alpha) \sum_i^n v_i x_{ij}^2 = 1$$

$$\sum_{r=1}^m u_r y_r^{2^{max}} - \sum_{i=1}^n v_i (\theta_{DMU}^* x_i^{2^{min}}) = 0$$

$$(2\beta_j - 1) \left( \sum_r^m u_r y_{rj}^3 - \sum_i^n v_i x_{ij}^1 \right) + 2(1 - \beta_j) \left( \sum_r^m u_r y_{rj}^2 - \sum_i^n v_i x_{ij}^2 \right) \leq 0, \quad j$$

$$v_i, u_r \geq 0 \quad i, r \quad (21)$$

$$\varphi_{ADMU} = \min(2\alpha - 1) \sum_r^m u_r y_r^{3^{min}} + 2(1 - \alpha) \sum_r^m u_r x_r^{2^{min}}$$

Subject to:

$$(2\alpha - 1) \sum_i^n v_i x_i^{1^{max}} + 2(1 - \alpha) \sum_i^n v_i x_i^{2^{max}} = 1$$

$$(2\beta_j - 1) \left( \sum_r^m u_r y_{rj}^1 - \sum_i^n v_i x_{ij}^3 \right) + 2(1 - \beta_j) \left( \sum_r^m u_r y_{rj}^2 - \sum_i^n v_i x_{ij}^2 \right) \geq 0, \quad j \quad (22)$$

$$\varphi_j = \min(2\alpha - 1) \sum_r^m u_r y_{rj}^3 + 2(1 - \alpha) \sum_r^m u_r y_{rj}^2$$

Subject to:

$$\begin{aligned}
 (2\alpha - 1) \sum_i^n v_i x_{ij}^1 + 2(1 - \alpha) \sum_i^n v_i x_{ij}^2 &= 1 \\
 \sum_{r=1}^m u_r y_r^{2min} - \sum_{i=1}^n v_i (\varphi_{ADMU}^* x_i^{2max}) &= 0 \\
 (2\beta_j - 1) \left( \sum_r^m u_r y_{rj}^1 - \sum_i^n v_i x_{ij}^3 \right) + 2(1 - \beta_j) \left( \sum_r^m u_r y_{rj}^2 - \sum_i^n v_i x_{ij}^2 \right) &\geq 0, \quad j \\
 &\geq 0, \quad j \\
 v_i, u_r &\geq 0 \quad i, r
 \end{aligned} \tag{23}$$

According to the model above, a novel decision-making method using fuzzy DEA credibility constrained and RC index can be summarized in the following form.

Step 1: Determine the efficiency score of IDMU through solving model (16) or (20).

Step 2: Determine the efficiency score of DMUs, in comparison with the IDMU score through solving model (17) or (21).

Step 3: Determine the efficiency score of ADMU through solving model (18) or (22).

Step 4: Determine the efficiency score of DMUs, in comparison with the ADMU score through solving model (19) or (23).

Step 5: Rank the DMUs by RC through solving model (6).

#### 4. Numerical example and discussion

In this section, we present the illustration of the proposed model using two numerical examples. The first example is a simple problem and the second example is a complex problem. Moreover, the data set is taken from random number. Step 1 to step 4 were computed by the general algebraic modeling system (GAMS) software, and MS-Excel solved step 5. In the calculation, the credibility level should be satisfied with the same level ( $\beta_j = \alpha$ ).

**Example 1.** This situation has five DMUs that each DMU consists of two fuzzy inputs ( $\tilde{x}_{ij}$ ) and two fuzzy outputs ( $\tilde{y}_{rj}$ ). Furthermore, the data set is shown in [Table 1](#).

The result of proposed approach with different credibility level (0.6, 0.7, 0.8, 0.9, and 1) is presented in [Table 2](#). By set  $\beta_j = \alpha = 0.6$ , the following ranking order was obtained: DMU1  $\succ$  DMU4 DMU2  $\succ$  DMU3  $\succ$  DMU5, where “ $\succ$ ” is “better than”. Furthermore, the ranking of each DMU is same by increasing the credibility level to 0.7, 0.8, 0.9, and 1. Need to be noted that if the score of  $\theta_{IDMU}$  is decreased by increasing credibility level, whereas the score of  $\varphi_{ADMU}$  increase by increasing credibility level as shown in [Table 3](#).

**Example 2.** This situation has thirty-three DMUs that each DMU consists of five fuzzy inputs ( $\tilde{x}_{ij}$ ) and two fuzzy outputs ( $\tilde{y}_{rj}$ ). Furthermore, the data set is shown in [Table 3](#).

The result of proposed approach with different credibility level (0.6, 0.7, 0.8, 0.9, and 1) is presented in [Table 4](#). It is clearly seen in [Table 4](#) that the ranking of the DMU changes and it is caused by credibility level. However, the different ranking of DMU is not too significant and by this approach can recognize which DMU is either the best or the worst. Furthermore, same like simple

**Table 1. Represents an example of the simple data used in the method developed in this article. The data set used in this study consisted of 5 DMU candidates with two fuzzy inputs and two fuzzy outputs as the selection criteria.1 Data of 5 DMUs with 2 fuzzy inputs and 2 fuzzy outputs as follows**

DMU	$\tilde{x}_{1j}$			$\tilde{x}_{2j}$			$\tilde{y}_{1j}$			$\tilde{y}_{2j}$		
	$x_{1j}^1$	$x_{1j}^2$	$x_{1j}^3$	$x_{2j}^1$	$x_{2j}^2$	$x_{2j}^3$	$y_{1j}^1$	$y_{1j}^2$	$y_{1j}^3$	$y_{2j}^1$	$y_{2j}^2$	$y_{2j}^3$
1	2	3	4	2	3	4	9	10	11	9	10	11
2	2	3	4	4	5	6	7	8	9	7	8	9
3	4	5	6	2	3	4	5	6	7	7	8	9
4	2	3	4	2	3	4	9	10	11	7	8	9
5	4	5	6	4	5	6	5	6	7	5	6	7
IDMU	$x_1^{1min}$	$x_1^{2min}$	$x_1^{3min}$	$x_2^{1min}$	$x_2^{2min}$	$x_2^{3min}$	$y_1^{1max}$	$y_1^{2max}$	$y_1^{3max}$	$y_2^{1max}$	$y_2^{2max}$	$y_2^{3max}$
	2	3	4	2	3	4	9	10	11	9	10	11
ADMU	$x_1^{1max}$	$x_1^{2max}$	$x_1^{3max}$	$x_2^{1max}$	$x_2^{2max}$	$x_2^{3max}$	$y_1^{1min}$	$y_1^{2min}$	$y_1^{3min}$	$y_2^{1min}$	$y_2^{2min}$	$y_2^{3min}$
	4	5	6	4	5	6	5	6	7	5	6	7

example, the score of  $\theta_{IDMU}$  decrease by increasing credibility level, whereas the score of  $\varphi_{ADMU}$  increases by increasing credibility level. Figures 1 and 2 show a detailed the relationship between RC score and the credibility levels in different number of DMUs.

The results of the simulation with small size data set indicate that each DMU has the same linear pattern in RC score when the level of credibility increases by 0.1. This result is in line with a study by Zhang and Guo (2017) who found that there is a variation of results under different credibility levels on the flexible decision solutions. Other relevant study also shows that optimal decision-making could be obtained using the combination of the different level of credibility Li et al. (2015).

The results of the simulation using 33 DMUs in the different of credibility levels result in a variation of RC scores (Figure 2). The increasing value of credibility level from 0.6 to 1 produces a lower RC index, while the pattern of the graphs for each value of credibility level. In the case of the same value of RC score in DMUs such as in DMU 1 and DMU 2 which have the same RC score (0.765) in the credibility level of 0.6, the decision-making would be obtained by increasing the level of credibility. As seen in Figure 2, by increasing the credibility level to 0.7, the RC score of DMU 1 drops to 0.674 and DMU 2 has the value of 0.653. Thus, the decision-making process would be easier to make after the credibility level changes.

These findings are in line with previous relevant studies results that indicate the function credibility level in decision-making process for different cases of applications such as in manufacturing industry for welding selection process (Wardana et al., 2020) and in agriculture industry for water allocation (Zamani et al., 2020; C. Zhang & Guo, 2017). Moreover, Fasanghari et al. (2015) who incorporated the concept of Data Envelope Analysis (DEA) model into Enterprise Architecture (EA) with fuzzy credibility constrained program found that the model provide a useful solution for large-scale uncertainty decision-making problems. In the model of this article, the RC index would be the parameter to change the value of credibility level. As the value of RC scores for all the DMUs are different, it is not then necessary to change the credibility level to make decision.

Table 2. The RC score of 5 DMUs under a different credibility level

DMUs	$\alpha = \theta_j = 0.6$				$\alpha = \theta_j = 0.7$		$\alpha = \theta_j = 0.8$		$\alpha = \theta_j = 0.9$		$\alpha = \theta_j = 1$	
	$\theta_j$	$\varphi_j$	RC	Rank	RC	Rank	RC	Rank	RC	Rank	RC	Rank
1	0.068	4.057	0.9835	1	0.9783	1	0.9789	1	0.9820	1	0.9860	1
2	0.227	1.861	0.8913	3	0.8999	3	0.9141	3	0.9298	3	0.9449	3
3	0.229	1.126	0.8310	4	0.8449	4	0.8672	4	0.8917	4	0.9156	4
4	0.07	3.037	0.9775	2	0.9709	2	0.9721	2	0.9765	2	0.9818	2
5	0.56	1.122	0.6671	5	0.7250	5	0.7800	5	0.8305	5	0.8740	5
$\theta_{DMU}$	0.841				0.706		0.591		0.493		0.409	
	1.158				1.342		1.556		1.806		2.100	

Table 3. Data of 33 DMUs with five fuzzy inputs and two fuzzy outputs as follows

		DMU															
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$\tilde{x}_{1j}$	$x_{1j}^1$	5	0	5	6	5	8	9	6	3	2	6	3	5	9	0	7
	$x_{1j}^2$	6	1	6	7	6	9	10	7	4	3	7	4	6	10	1	8
	$x_{1j}^3$	7	2	7	8	7	10	11	8	5	4	8	5	7	11	2	9
$\tilde{x}_{2j}$	$x_{2j}^1$	0	9	8	8	5	3	9	8	3	1	4	7	4	7	3	3
	$x_{2j}^2$	1	10	9	9	6	4	10	9	4	2	5	8	5	8	4	4
	$x_{2j}^3$	2	11	10	10	7	5	11	10	5	3	6	9	6	9	5	5
$\tilde{x}_{3j}$	$x_{3j}^1$	8	7	7	0	9	8	7	1	7	9	0	0	1	7	7	5
	$x_{3j}^2$	9	8	8	1	10	9	8	2	8	10	1	1	2	8	8	6
	$x_{3j}^3$	10	9	9	2	11	10	9	3	9	11	2	2	3	9	9	7
$\tilde{x}_{4j}$	$x_{4j}^1$	0	1	2	2	2	3	0	5	3	9	9	3	8	6	9	1
	$x_{4j}^2$	1	2	3	3	3	4	1	6	4	10	10	4	9	7	10	2
	$x_{4j}^3$	2	3	4	4	4	5	2	7	5	11	11	5	10	8	11	3
$\tilde{x}_{5j}$	$x_{5j}^1$	1	5	3	9	6	3	6	5	4	3	4	8	8	9	3	4
	$x_{5j}^2$	2	6	4	10	7	4	7	6	5	4	5	9	9	10	4	5
	$x_{5j}^3$	3	7	5	11	8	5	8	7	6	5	6	10	10	11	5	6
$\tilde{y}_{1j}$	$y_{1j}^1$	22	45	20	48	44	28	47	21	21	44	42	20	30	43	23	37
	$y_{1j}^2$	23	46	21	49	45	29	48	22	22	45	43	21	31	44	24	38
	$y_{1j}^3$	24	47	22	50	46	30	49	23	23	46	44	22	32	45	25	39
$\tilde{y}_{2j}$	$y_{2j}^1$	23	40	31	22	40	34	31	36	49	24	26	32	30	41	39	19
	$y_{2j}^2$	24	41	32	23	41	35	32	37	50	25	27	33	31	42	40	20
	$y_{2j}^3$	25	42	33	24	42	36	33	38	51	26	28	34	32	43	41	21

(Continued)

Table 3. (Continued)

		DMU																				
		1		2	3	4	5		6	7	8		9	10	11		12	13	14	15	16	
		DMU																	IDMU		ADMU	
		17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33				
$\tilde{x}_{1j}$	$x^1_{1j}$	4	2	6	1	8	8	1	2	7	3	9	4	6	3	4	1	7	$x^{min}_1$	0	$x^{max}_1$	9
	$x^2_{1j}$	5	3	7	2	9	9	2	3	8	4	10	5	7	4	5	2	8	$x^{min}_1$	1	$x^{max}_1$	10
	$x^3_{1j}$	6	4	8	3	10	10	3	4	9	5	11	6	8	5	6	3	9	$x^{min}_1$	2	$x^{max}_1$	11
$\tilde{x}_{2j}$	$x^1_{2j}$	6	8	7	9	3	2	3	9	9	4	6	6	2	7	5	0	2	$x^{min}_2$	0	$x^{max}_2$	9
	$x^2_{2j}$	7	9	8	10	4	3	4	10	10	5	7	7	3	8	6	1	3	$x^{min}_2$	1	$x^{max}_2$	10
	$x^3_{2j}$	8	10	9	11	5	4	5	11	11	6	8	8	4	9	7	2	4	$x^{min}_2$	2	$x^{max}_2$	11
$\tilde{x}_{3j}$	$x^1_{3j}$	6	6	9	5	6	9	7	6	6	3	3	8	7	9	6	2	4	$x^{min}_3$	0	$x^{max}_3$	9
	$x^2_{3j}$	7	7	10	6	7	10	8	7	7	4	4	9	8	10	7	3	5	$x^{min}_3$	1	$x^{max}_3$	10
	$x^3_{3j}$	8	8	11	7	8	11	9	8	8	5	5	10	9	11	8	4	6	$x^{min}_3$	2	$x^{max}_3$	11
$\tilde{x}_{4j}$	$x^1_{4j}$	8	0	4	7	2	2	1	3	4	6	1	5	9	4	3	6	8	$x^{min}_4$	0	$x^{max}_4$	9
	$x^2_{4j}$	9	1	5	8	3	3	2	4	5	7	2	6	10	5	4	7	9	$x^{min}_4$	1	$x^{max}_4$	10
	$x^3_{4j}$	10	2	6	9	4	4	3	5	6	8	3	7	11	6	5	8	10	$x^{min}_4$	2	$x^{max}_4$	11
$\tilde{x}_{5j}$	$x^1_{5j}$	9	0	0	0	2	5	5	3	5	6	9	9	4	2	8	1	3	$x^{min}_5$	0	$x^{max}_5$	9
	$x^2_{5j}$	10	1	1	1	3	6	6	4	6	7	10	10	5	3	9	2	4	$x^{min}_5$	1	$x^{max}_5$	10
	$x^3_{5j}$	11	2	2	2	4	7	7	5	7	8	11	11	6	4	10	3	5	$x^{min}_5$	2	$x^{max}_5$	11
$\tilde{y}_{1j}$	$y^1_{1j}$	23	45	40	43	48	40	48	22	32	20	39	41	37	23	37	48	45	$y^{max}_1$	48	$y^{min}_1$	20
	$y^2_{1j}$	24	46	41	44	49	41	49	23	33	21	40	42	38	24	38	49	46	$y^{max}_1$	49	$y^{min}_1$	21
	$y^3_{1j}$	25	47	42	45	50	42	50	24	34	22	41	43	39	25	39	50	47	$y^{max}_1$	50	$y^{min}_1$	22
$\tilde{y}_{2j}$	$y^1_{2j}$	27	44	48	38	47	20	34	33	21	40	22	49	21	19	41	35	45	$y^{max}_2$	49	$y^{min}_2$	19
	$y^2_{2j}$	28	45	49	39	48	21	35	34	22	41	23	50	22	20	42	36	46	$y^{max}_2$	50	$y^{min}_2$	20

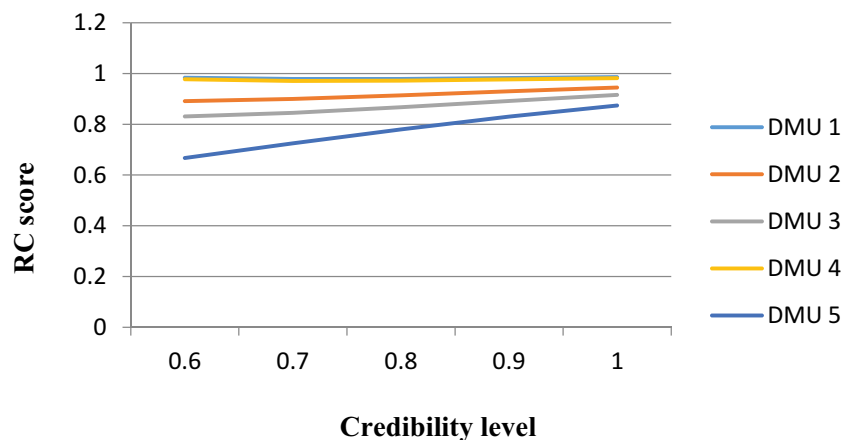
**Table 4. The RC score of 33 DMUS under a different credibility level**

DMUs	$\alpha = \theta_j = 0.6$				$\alpha = \theta_j = 0.7$		$\alpha = \theta_j = 0.8$		$\alpha = \theta_j = 0.9$		$\alpha = \theta_j = 1$	
	$\theta_j$	$\varphi_j$	RC	Rank	RC	Rank	RC	Rank	RC	Rank	RC	Rank
1	0.825	1.802	0.2665	20	0.3167	22	0.3658	23	0.4148	24	0.4635	24
2	0.872	3.224	0.4393	5	0.5076	5	0.5666	5	0.6192	5	0.6659	5
3	0.471	1.65	0.2248	32	0.2716	32	0.3194	32	0.3685	32	0.4184	32
4	0.899	1.837	0.2762	17	0.3326	17	0.3873	17	0.4398	17	0.4899	18
5	0.681	3.217	0.4253	6	0.4887	7	0.5460	7	0.5987	8	0.6467	8
6	0.578	2.264	0.3167	15	0.3736	16	0.4285	16	0.4816	16	0.5334	16
7	0.725	2.524	0.3570	13	0.4243	13	0.4856	13	0.5403	13	0.5904	13
8	0.868	1.732	0.2579	24	0.3095	24	0.3585	25	0.4075	25	0.4563	25
9	0.905	1.829	0.2753	18	0.3301	19	0.3806	19	0.4316	19	0.4830	20
10	0.445	1.988	0.2732	19	0.3305	18	0.3863	18	0.4398	18	0.4918	17
11	0.747	2.141	0.3108	16	0.3752	15	0.4331	15	0.4886	15	0.5386	15
12	0.865	1.654	0.2450	26	0.2951	26	0.3444	26	0.3934	26	0.4420	26
13	0.633	2.412	0.3381	14	0.4008	14	0.4589	14	0.5118	14	0.5621	14
14	0.486	3.273	0.4176	8	0.4774	10	0.5332	10	0.5856	10	0.6342	10
15	0.624	1.819	0.2582	23	0.3147	23	0.3704	22	0.4235	22	0.4738	22
16	0.753	1.737	0.2524	25	0.3090	25	0.3625	24	0.4155	23	0.4677	23
17	0.346	1.8	0.2417	28	0.2891	29	0.3372	29	0.3865	28	0.4361	28
18	0.902	3.584	0.4727	2	0.5400	2	0.5994	2	0.6514	2	0.6971	2
19	0.69	3.073	0.4125	10	0.4817	8	0.5445	8	0.5993	7	0.6476	7
20	0.681	3.068	0.4114	11	0.4789	9	0.5405	9	0.5956	9	0.6444	9
21	0.907	3.809	0.4908	1	0.5587	1	0.6167	1	0.6650	1	0.7081	1
22	0.603	1.674	0.2348	31	0.2894	28	0.3382	28	0.3852	29	0.4324	30
23	0.903	3.013	0.4214	7	0.4901	6	0.5522	6	0.6081	6	0.6579	6
24	0.578	1.739	0.2437	27	0.2926	27	0.3416	27	0.3907	27	0.4399	27
25	0.395	1.74	0.2352	30	0.2827	31	0.3304	31	0.3788	31	0.4280	31
26	0.704	1.839	0.2655	22	0.3195	21	0.3741	21	0.4290	20	0.4835	19
27	0.758	1.826	0.2665	21	0.3224	20	0.3764	20	0.4270	21	0.4761	21
28	0.652	3.141	0.4164	9	0.4770	11	0.5331	11	0.5853	11	0.6336	11
29	0.386	1.747	0.2358	29	0.2851	30	0.3341	30	0.3837	30	0.4333	29
30	0.296	1.581	0.2064	33	0.2504	33	0.2962	33	0.3434	33	0.3914	33
31	0.683	2.957	0.4007	12	0.4609	12	0.5176	12	0.5708	12	0.6206	12
32	0.903	3.287	0.4472	4	0.5180	4	0.5814	3	0.6375	3	0.6873	3
33	0.703	3.574	0.4577	3	0.5208	3	0.5780	4	0.6300	4	0.6770	4
$\theta_{DMU}$	4.298				3.475		2.862		2.387		2.01	
$\varphi_{ADMU}$	0.54				0.577		0.616		0.657		0.701	

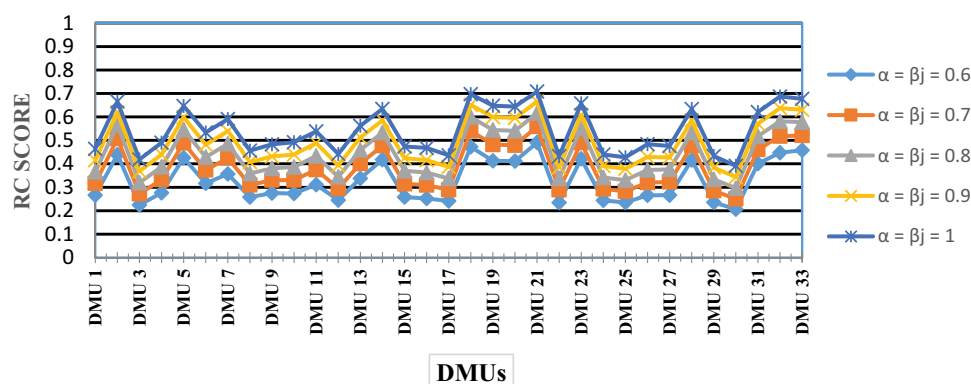
## 5. Conclusion

To deal with uncertainty data, fuzzy set theory was proposed to combine with integrated DEA and RC index. Furthermore, this paper uses credibility constraint programming for solve fuzzy event, and finally the method that used is fuzzy DEA credibility constrained and RC index. Two cases such as simple and complex, are presented to demonstrate the implementation and interpreted the result with different credibility level. The credibility level of the proposed approach provides the flexibility for decision-maker to set their own acceptable credibility level in making decision. Moreover, the preserved output data applying by the integration of RC index and credibility level

**Figure 1. Relationship between credibility level and RC score.**



**Figure 2. Pattern of the RC score in the different level of credibility.**



in this article are the cause of the discrimination power improvement. For further study, the number of experimental simulation would be beneficial to test the proposed approach for more robust results. In terms of the input and output data which are the conditions to be provided in this study, further experimental simulation might focus on the results of the conditions where input or output data is not provided.

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