CS 70 Discrete Mathematics and Probability Theory
Summer 2023 Huang, Suzani, and Tausik

HW 0

1 Administrivia

(a) Make sure you are on the course Ed (for Q&A) and Gradescope (for submitting homeworks, including this one). Find and familiarize yourself with the course website. What is its homepage's URL?

(b) Read the policies page on the course website.

- (i) What is the breakdown of how your grade is calculated?
- (ii) What is the attendance policy for discussions?
- (iii) When are homeworks released and when are they due?
- (iv) How many "drops" do you get for vitamins? For homework?
- (v) When are the quizzes? When is the final?
- (vi) What percentage score is needed to earn full credit on a homework?

Solution:

(a) The course website is located at `https://www.eecs70.org/`.

(b)
- (i) Discussion Attendance: 5%, Vitamins: 5%, Homework: 20%, Quizzes: 30%, Final: 40%.
- (ii) You will receive 1 attendance point for every discussion, and will need at least 13 points in order to receive full credit for discussion attendance. You are welcome to attend other discussion sections, but your attendance will only be counted for the section you are actually assigned.
- (iii) The homework for the current week is released on Gradescope on Sunday. The homework is due on Gradescope the following Thursday at 11:59 pm (grace period until Friday 11:59pm); the solutions for that homework will be released on Sunday along with the new homework.
- (iv) You can drop the lowest 2 vitamins and lowest 2 homeworks the entire semester. However, please save these drops for emergencies. We do not have the bandwidth to make personalized exceptions to this rule.
- (v) Quiz 1 Date: 6/28/23 Wednesday 7-8pm, Quiz 2 Date: 7/12/23 Wednesday 7-8pm, Quiz 3 Date: 7/26/23 Wednesday 7-8pm, Final Date: 8/11/23 Friday 6-9pm
- (vi) 73%.

2 Course Policies

Go to the course website and read the course policies carefully. Leave a followup on Ed if you have any questions. Are the following situations violations of course policy? Write "Yes" or "No", and a short explanation for each.

(a) Alice and Bob work on a problem in a study group. They write up a solution together and submit it, noting on their submissions that they wrote up their homework answers together.

(b) Carol goes to a homework party and listens to Dan describe his approach to a problem on the board, taking notes in the process. She writes up her homework submission from her notes, crediting Dan.

(c) Erin comes across a proof that is part of a homework problem while studying course material. She reads it and then, after she has understood it, writes her own solution using the same approach. She submits the homework with a citation to the website.

(d) Frank is having trouble with his homework and asks Grace for help. Grace lets Frank look at her written solution. Frank copies it onto his notebook and uses the copy to write and submit his homework, crediting Grace.

(e) Heidi has completed her homework using LaTeX. Her friend Irene has been working on a homework problem for hours, and asks Heidi for help. Heidi sends Irene her PDF solution, and Irene uses it to write her own solution with a citation to Heidi.

(f) Joe found homework solutions before they were officially released, and every time he got stuck, he looked at the solutions for a hint. He then cited the solutions as part of his submission.

Solution:

(a) Yes, this is a violation of course policy. All solutions must be written entirely by the student submitting the homework. Even if students collaborate, each student must write a unique, individual solution. In this case, both Alice and Bob would be culpable.

(b) No, this is not a violation of course policy. While sharing *written solutions* is not allowed, sharing *approaches* to problems is allowed and encouraged. Because Carol only copied down *notes*, not *Dan's solution*, and properly cited Dan's contribution, this is an actively encouraged form of collaboration.

(c) No, this is not a violation of course policy. Using external sources to help with homework problems, while less encouraged than peer collaboration, is fine as long as (i) the student makes sure to understand the solution; (ii) the student uses understanding to write a new solution, and does not copy from the external source; and (iii) the student credits the external source. However, looking up a homework problem online is a violation of course policies; the correct course of action upon finding homework solutions online is to close the tab.

(d) Yes, this is a violation of course policy, and both Frank and Grace would be culpable. Even though Frank credits Grace, written solutions should never be shared in the first place, and certainly not copied down. This is to ensure that each student learns how to write and present clear and convincing arguments. To be safe, try not to let anybody see your written solutions at any point in the course—restrict your collaboration to *approaches* and *verbal communication*.

(e) Yes, this is a violation of course policy. Once again, a citation does not make up for the fact that written solutions should never be shared, in written or typed form. In this case, both Heidi and Irene would be culpable.

(f) Yes, this is a violation of course policy. Joe should not be reading solutions before they are officially released. Instead, Joe should ask for help when he is stuck through Ed or Office Hours.

3 Use of Ed

Ed is incredibly useful for Q&A in such a large-scale class. We will use Ed for all important announcements. You should check it frequently. We also highly encourage you to use Ed to ask questions and answer questions from your fellow students.

(a) Read the Ed Etiquette section of the course policies and explain what is wrong with the following hypothetical student question: "Can someone explain the proof of Theorem XYZ to me?" (Assume Theorem XYZ is a complicated concept.)

(b) When are the weekly posts released? Are they required reading?

(c) If you have a question or concern not directly related to the course content, where should you direct it?

Solution:

(a) There are two things wrong with this question. First, this question does not pass the 5-minute test. This concept takes longer than 5 minutes to explain, and therefore is better suited to Office Hours. Second, this question does not hone in on a particular concept with which the student is struggling. Questions on Ed should be narrow, and should include every step of reasoning that led up to the question. A better question in this case might be: "I understood every step of the proof of Theorem XYZ in Note 2, except for the very last step. I tried to reason it like this, but I didn't see how it yielded the result. Can someone explain where I went wrong?"

(b) The weekly posts are released every Monday. They're required reading.

(c) Please send an email to `cs70-staff@berkeley.edu`.

4 Remote Course Option

Although this class is designated to be in-person and we strongly recommend taking the class in-person, we are facilitating remote options for those who are not physically at Berkeley. We ask everyone to fill out this form.

(a) What is the secret phrase?

Solution: "staff recommends taking the class in-person"

5 Implication

Note 0
Note 1

Which of the following implications are always true, regardless of P? Give a counterexample for each false assertion (i.e. come up with a statement $P(x,y)$ that would make the implication false).

(a) $\forall x \forall y P(x,y) \implies \forall y \forall x P(x,y)$.

(b) $\forall x \exists y P(x,y) \implies \exists y \forall x P(x,y)$.

(c) $\exists x \forall y P(x,y) \implies \forall y \exists x P(x,y)$.

Solution:

(a) True. For all can be switched if they are adjacent; since $\forall x, \forall y$ and $\forall y, \forall x$ means for all x and y in our universe.

(b) False. Let $P(x,y)$ be $x < y$, and the universe for x and y be the integers. Or let $P(x,y)$ be $x = y$ and the universe be any set with at least two elements. In both cases, the antecedent is true and the consequence is false, thus the entire implication statement is false.

(c) True. The first statement says that there is an x, say x' where for every y, $P(x,y)$ is true. Thus, one can choose $x = x'$ for the second statement and that statement will be true again for every y.

6 Propositional Practice

Note 1

In parts (a)-(c), convert the English sentences into propositional logic. In parts (d)-(f), convert the propositions into English. In part (f), let $P(a)$ represent the proposition that a is prime.

(a) There is one and only one real solution to the equation $x^2 = 0$.

(b) Between any two distinct rational numbers, there is another rational number.

(c) If the square of an integer is greater than 4, that integer is greater than 2 or it is less than -2.

(d) $(\forall x \in \mathbb{R})\ (x \in \mathbb{C})$

(e) $(\forall x, y \in \mathbb{Z})(x^2 - y^2 \neq 10)$

(f) $(\forall x \in \mathbb{N})\ [\ (x > 1) \implies (\exists a, b \in \mathbb{N})\ ((a + b = 2x) \wedge P(a) \wedge P(b))\ ]$

Solution:

(a) Let $p(x) = x^2$. The sentence can be read: "There is a solution x to the equation $p(x) = 0$, and any other solution y is equal to x". Or,

$$(\exists x \in \mathbb{R})\,((p(x) = 0) \wedge ((\forall y \in \mathbb{R})(p(y) = 0) \implies (x = y)))\,.$$

(b) The sentence can be read "If x and y are distinct rational numbers, then there is a rational number z between x and y." Or,

$$(\forall x, y \in \mathbb{Q})((x \neq y) \implies ((\exists z \in \mathbb{Q})(x < z < y \vee y < z < x)).$$

Equivalently,

$$(\forall x, y \in \mathbb{Q})((x = y) \vee (\exists z \in \mathbb{Q})(x < z < y \vee y < z < x)).$$

Note that $x < z < y$ is mathematical shorthand for $(x < z) \wedge (z < y)$, so the above statement is equivalent to

$$(\forall x, y \in \mathbb{Q})(x = y) \vee ((\exists z \in \mathbb{Q})\,((x < z) \wedge (z < y)) \vee ((y < z) \wedge (z < x))).$$

(c) $(\forall x \in \mathbb{Z})\ ((x^2 > 4) \implies ((x > 2) \vee (x < -2)))$

(d) All real numbers are complex numbers.

(e) There are no integer solutions to the equation $x^2 - y^2 = 10$.

(f) For any natural number x greater than 1, there are some prime numbers a and b such that $2x = a + b$.

In other words: Any even integer larger than 2 can be written as the sum of two primes.

Aside: This statement is known as Goldbach's Conjecture, and it is a famous unsolved problem in number theory (`https://xkcd.com/1310/`).

7 Riddles

Note 1

Shreyas claims the following. "If I'm not studying, I'm watching TV; and if I'm not watching TV, I'm sleeping." Assuming that Shreyas can only do one of these things at a time, is it possible to know what he is doing? Explain your reasoning.

Solution: Define the following propositions: p as "Shreyas is studying", q as "Shreyas is watching TV", and r as "Shreyas is sleeping".

Shreyas's claim can be written equivalently as $(\neg p \implies q) \wedge (\neg q \implies r)$.

p	q	r	$\neg p \implies q$	$\neg q \implies r$	$(\neg p \implies q) \wedge (\neg q \implies r)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	T	F	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	F	T	F
F	F	F	F	F	F

Since it's impossible to do more than one activity at once, we look at which row has only one of p,q,r being true and the far right column being true. The only row that matches this is $p = F, q = T, r = F$. Hence, Shreyas is watching TV.

8 Divisibility! Rules?

Note 2

(a) Prove that if n is an integer divisible by 6, then n^2 is divisible by 9.

(b) Prove that if m has a remainder of 1 when divided by 3, then m^3 also has a remainder of 1 when divided by 3. (Hint: you may find the expression $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ helpful)

(c) Prove that the sum of any three consecutive integers is divisible by 3.

Solution:

(a) If n is divisible by 6, then there exists some integer k such that $n = 6k$. Hence,

$$n^2 = (6k)^2 = 36k^2 = 9(4k^2) = 9k'$$

where $k' = 4k^2$. The form $n^2 = 9k'$ implies that n^2 is divisible by 9.

(b) If m has a remainder of 1 when divided by 3, then there exists some integer k such that $m = 3k+1$. Using the Hint,

$$m^3 = (3k+1)^3 = 27k^3 + 9k^2 + 3k + 1 = 3(9k^3 + 3k^2 + k) + 1 = 3k' + 1$$

where $k' = 9k^3 + 3k^2 + k$. The form $m^3 = 3k' + 1$ implies that m^3 leaves a remainder of 1 when divided by 3.

(c) Denote the arbitrary three consecutive integers as $n, n+1$, and $n+2$ and their sum as S. We see that

$$S = n + (n+1) + (n+2) = 3n + 3 = 3(n+1) = 3k'$$

where $k' = n+1$. The form $S = 3k'$ implies that the sum of any three arbitrary consecutive integers is divisible by 3.