2nd Project Statement

Course: MMB

Choose one of the references from the following list:

- http://dx.doi.org/10.1051/proc/2010002
- https://doi.org/10.1371/journal.pone.0090497
- https://doi.org/10.1007/s10439-015-1299-2
- https://doi.org/10.1016/j.jtbi.2020.110229

1. Describe the process of atherosclerosis according to the chosen paper, including:
   - the modelling assumptions and the purpose of the model in the paper;
   - identification of the main variables;
   - the system of mathematical equations determining these variables.
2. Implement an algorithm for the numerical approximation of the variables involved in the initiation of the inflammatory process, including the accumulation of foam cells, but excluding plaque growth. For this purpose:

   (a) Domain and modelling assumptions
   - Consider a 2D idealized domain with length L_x and height L_{y1} representing a 2D slice of the lumen;
   - an adjacent 2D domain with the same length and height L_{y2} representing the intima layer.

   The values L_x, L_{y1}, L_{y2} should be defined according to the chosen reference or sources cited therein.

   Enumerate the model assumptions, the variables, parameters, governing equations, and boundary conditions to be solved in each domain. The velocity to be used in domain 1 should have the form

   $$v = (v1(x, y), v2(x, y)) = (vmax * P(y), 0)$$

   where $P(y)$ is a quadratic positive funtion, whitin the domain 1, with zero values at the bottom and top of the domain. Whenever the Wall Shear Stress will be required, it should be computed using this velocity field.

   (b) Numerical implementation

   Implement an explicit finite difference scheme to solve the coupled system. It is suggested to use:
   - FD for time derivative (backward difference),

- BD for transport (upwind),
- CD for diffusion.

Depict results at suitable time instants to illustrate the relevant biological dynamics.

(c) Convergence analysis

Verify convergence of the numerical solution with respect to:

- time step refinement,
- spatial mesh refinement.

Interpret the convergence behaviour in terms of expected stability and accuracy of the scheme.

(d) Interpretation of results

Discuss the observed dynamics in light of the modelling assumptions and the biological processes being represented.

(e) Alternative numerical method

Choose an alternative numerical scheme that presents advantages with respect to the explicit FD scheme and justify your choice.

3. Final deliverables:

Prepare a LaTeX Beamer presentation including:

- description of the model;
- statement of assumptions;
- governing equations;
- numerical methods and convergence;
- results and interpretation;
- references.

The project must be presented orally with participation of all group members. Evaluation will be for the group, but individual penalisation may occur.

After the presentation, submit:

- slides in PDF format,
- runnable code scripts,
- a declaration of each member's contribution,
- a statement describing any use of AI tools.

Evaluation will strongly emphasize:

- deep understanding of the numerical implementation,
- correct formulation of the mathematical models,
- justification and consistency of model assumptions.