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A Individual Time Trial Strategy Model Based on Improved OmTTE and SCPO Model

Summary

Bicycle road races require cyclists to ride along specific routes throughout the race, with the shortest time to win. In this paper, the mathematical physics method is used, combined with human biology knowledge, to establish a description of the rider power profile model, and further put forward the SFAT model, in order to provide guidance for the participants.

Therefore, we need to establish **a full-time, full-terrain rider force-power-velocity modeltext**. Based on the previous TTE model describing power profile, we introduce the attenuation function to simulate the physical decline caused by lactic acid accumulation. The curve residual analysis R^2 between the newly established OmTTE model and the OmPD fitting the actual situation is 0.9902 ; based on the CPO model established by predecessors, the **angle variable** is introduced to expand the applicable section to the whole terrain. Based on the above models, we propose the SFAP model for the physical management **strategies of different sections of the riders**, so that the riders can improve the competition results by more than 5% through more reasonable physical management without improving their physical quality.

The highlights of this paper are as follows. Firstly, unlike the MMP curve based on average power used to predict competitions in recent years, our model applies a more reasonable TTE curve based on **instantaneous power** to analyze short-term decisions of competitions. Secondly, our model can reflect the physical differences of different riders, so as to put forward **reasonable suggestions for different groups**. Finally, this paper extends the application of the model and discusses the cooperation strategy of group timing competition.

For the first question, we fit the **power profiles** of various riders, bring them into our OmTTE model and derive their power curves.

For the second problem, we use our model to fit the data from the first problem into our SFAP model, and calculate the **performance of different types of riders in different courses**. The results are consistent with reality.

For the third problem, we introduce wind speed, humidity and altitude into the SCPO model to **verify the sensitivity of our results to these factors**. The conclusion is that humidity and altitude have little influence, but gale weather has great influence on our model.

For the fourth question, we explored **the possible rider factors and verified their impact on the SFAP model**.

For the last problem, we discuss the expansion of SFAP model for **teamwork**, and the results of collaboration model are much better than other models.

Keywords: ITT; TTE; OmPD; CP; Cycling power output

Contents

1 Introduction

1.1 Restatement of the Problem

The Individual Time Trial is a competition in the Road Cycling Race, which is about 40 km long for men and 20 km long for women. Each rider departure time interval between1-2minutes. No mutual help between riders. The rider with the least competition time wins. The riders' physical distribution strategy plays a vital role in the competition results. In addition, contestants' handling of undulating sections and sharp bends also affects the results.Team Time Trial is another competition in road cycling. Each group includes six riders, and the team time is determined by the fourth riders as he crosses the finish line.

Our mathematical model will accomplish the following tasks:

- Establish power time curve and power-velocity relationship in different terrains. And on this basis to analyze the reasonable game strategy.
- Define power profiles for riders of different types and genders and apply these power profiles to multiple time trial course.
- The sensitivity of the analysis model to environmental impact and the sensitivity of riders to deviate from the target power.
- Discuss the expansion of model in team collaboration. Improve the original model by taking team collaboration factors into consideration.

1.2 Literature Review

To analyze the relationship between human power output and time, Jeukendrup proposed time to exhaustion (TTE) in 1996 [1]. In fact, it refers to 'time to file', that the rider cannot maintain the power at a given rate. To describe the power output of individual efforts, vogt et al. proposed mean maximum power output (MMP) in 2007. MMP value represents the maximum average power recorded during the event in a given (arbitrary) duration [2]. MMP value plays an important role in participating riders and coaches [3]. Model OmPD of Michael J. Puchowicz et al. fitting MMP curve. Compared with other models, such as CP model, OmPD provides better goodness of fit and better theoretical characteristics [4].

In 1995, GERT et al. studied the effects of air friction and rolling resistance on cycling [5]. In order to better propose the ITT competition strategy, Scott et al. proposed a physical consumption model considering the influence of slope on the competition, and used the Lagrange multiplier method to solve the minimum problem [6]. In 2020, Andrea et al. proposed the mathematical models of lateral and longitudinal dynamics, and studied the influence of technical sections such as sharp bends on competition decisions.

1.3 Our work

Based on previous studies, we found that the existing output power-physical consumption model-TTE model-short-term prediction accuracy, while its long-term prediction performs not so well.

We believe that the reason for the bias is that the TTE model does not consider the long-term lactic acid accumulation. Therefore, we introduce the attenuation factor ϕ to simulate the lactic acid accumulation, and TCP_{max} to simulate the starting time of lactic acid accumulation, that is, the maximum time that aerobic exercise can maintain, so as to establish the OmTTE model to accurately describe the power image of athletes in a longer time range.

After analysis and calculation, we established the SFAT model, proposed several optimized physical distribution strategies, and brought them into the OmTTE and SCPO models. The rationality of these strategies was verified by numerical calculation.

2 Assumptions and Justifications

- **Riders have instantaneous maximum power, and the output power within a period of time will affect the speed of physical attenuation. The physical condition will affect the power output as well.** The variability from exercise time to fatigue is variable [7]. In practice, high-power cycling will increase the speed of physical attenuation within a certain period of time. When the strength is insufficient, the rider cannot maintain the power-time relationship as when the strength is abundant.

- **Riders exercise can be divided into aerobic exercise and anaerobic exercise, with aerobic power CP as the threshold. And the time limit for aerobic exercise is TCP_{max}.** performance is closely related to aerobic function [8]. When the exercise time exceeds TCP_{max} or the exercise power exceeds CP, the exercise will cause lactic acid accumulation and have a negative impact on the human body. The performance analysis of the rider will also be different from the aerobic exercise stage.

- **Consider the slope as a flat section with a fixed angle.** In a long slope, the slight slope change does not affect the output power and speed, which will not affect the final result of the trial.

- **Riders will choose the best strategy to win the trial.** In order to complete the competition in the shortest possible time, riders will reasonably allocate their physical strength to obtain greater time benefits.

3 Notations

Table 1: Notations

Symbole	Definition	Unit
E	Eatigue value	-
R	Change rate of fatigue value	-
ϕ	Physical attenuation function	-
P_0	Cruse power	W
P_{max}	Instantaneous maximum power output	W
TCP_{max}	Upper limit of aerobic time	s
a_r	Coefficient of friction	-
k_m	coefficient of air drag	-

4 Model 1: Omni Time to Exhaustion Model

4.1 Problem Analysis and Model Establishment

The power profile is also refered as TTE(Time to exhaustion) model, which is proposed to describe the maximum time that a rider can maintain the output of a given power [6]. Moreover, beyond this maximum time, the rider can still produce other power below the original given power and does not completely exhaust his strength.

However, according to previous studies, the most effective time for TTE curve to predict cycling competition is 3-15 minutes, which is because over a certain period of time, lactic acid will accumulate in the human body, thereby affecting the next play of the rider.

In order to define the rider 's force curve, the reasonable method is based on the concept of the existing TTE model, and the OmTTE model is proposed. The improvement of the model for TTE is to introduce a coefficient ϕ which increases with time. The coefficient is proposed based on TCP_{max}, which is used to explain the decrease of output power caused by human exceeding the time limit of lactic acid accumulation in actual experiments.

Assuming that the human body strength is E, E is a number between 0 and 1, the larger the value is, the more tired the human is. When $E = 1$, people will be unable to maintain the existing output power.

Assuming $R = \frac{dE}{dt}$, it represents the change of physical strength value E with time.

In the previous TTE model :

$$R = \frac{(P_m - P)(P - P_0)}{A(P_m - P)} \tag{1}$$

where P_m represents the maximum power that athletes can produce and P_0 represents the power that athletes can maintain forever (in other words, when athletes output P_0 power, it can last longer than the game time we care about), the visualization image of Equation 1 is Figure 1.

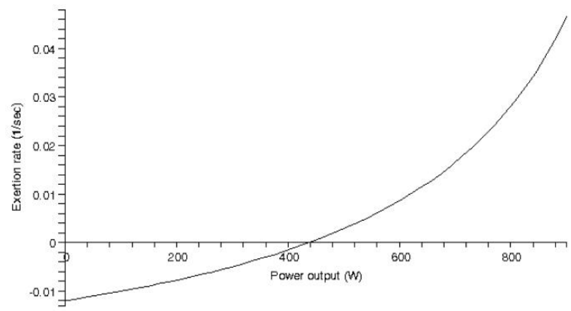


Figure 1: Exertion rate R versus power output P [6]

In the OmTTE model, considering the existence of TC_{max}, the ability of output power will decrease due to the accumulation of lactic acid after long-term exercise. Therefore, for the original TTE model, a reasonable improvement method is to introduce a bias term related to time. When $T > TCP_{max}$, the value of R should increase with T. In other words, it is common sense that when an athlete enters an anaerobic phase, he will consume faster physical strength, and he will be more likely to feel unable to maintain the original output power. Finally,

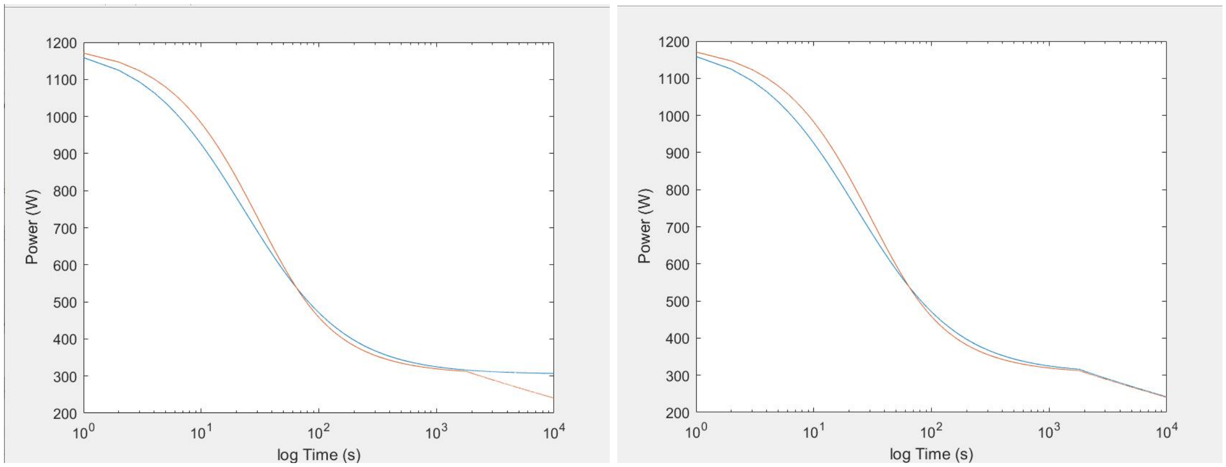
$$R = \begin{cases} \frac{(P_m - P)(P - P_0)}{A(P_m - P)} & \text{if } T <= TCP_{max}, \\ \frac{(P_m - (P+\phi))((P+\phi) - P_0)}{A(P_m - (P+\phi))} & \text{if } T > TCP_{max}. \end{cases}$$

$$\phi = B \ln \left(\frac{T}{\text{TCP max}} \right)$$

4.2 Discussion of Rationality

In order to verify the rationality of OmTTE model, OmTTE model is compared with OmPD model based on MMP. MMP is the relationship between the maximum average power of the rider and time measured by experiment. MMP is often used to estimate the maximum power that can be endured for each duration of the rider. [4] The OmPD model is a model proposed by Michael J. in 2020 that can well fit the MMP relationship within 0 10^4 s. The OmTTE model is a fitting model that the rider can maintain the limit time of the output of a given power, in other words, the power in the model is the instantaneous power at each moment. Macroscopically, OmTTE curves should be similar to OmPD curves.

Figure 2 (a) shows a comparison between the OmTTE model and the OmPD model without introducing the physical attenuation coefficient ϕ. The image shows that the difference between the two models becomes larger after a certain time. The concept of TCP_{max} is introduced in the study of OmPD [4]. When TCP_{max} is overused, the rider will be more prone to fatigue due to physical overuse. This human change is essentially a time-dependent physical decay function ϕ. According to this theory, the physical attenuation function ϕ is introduced into the OmTTE model. Figgure 2 (b) shows the comparison between the OmTTE model and the OmPD model with the introduction of the physical attenuation function ϕ. It can be seen that the values of the two models after introducing the physical attenuation function ϕ are relatively close. According to the data



(a) (b)

Figure 2: Comparison of OmPD Curves of OmTTE Curves

in Table 2, the differences of the two models are basically below 1%, $R^2 = 0.9902$. That is, the OmTTE model is realistic.

Table 2: Residual analysis table of OmTTE

Time (s)	OmTTF(W)	OmPD(W)	Residual	Residual Percentage
1	1158.7	1170.8	1.034	0.0883%
3	1092.8	1123.3	2.712	0.2414%
9	944.7	1000.3	5.560	0.5559%
27	713.9	756.8	5.662	0.7482%
81	501.4	493.6	1.578	0.3198%
243	381.7	367.8	3.790	1.0303%
729	332.1	325.3	2.113	0.6495%
2187	306.8	303.7	1.036	0.3410%

5 Model 2: Strategies for All Terrain Model

5.1 Strategies for Slope: Slope Cycling Power Output Model

5.1.1 Problem Analysis and Model Establishment

CPO Model is a formula proposed by Groot to describe the relationship between the output power of a cyclist on a flat road and the total weight of a bicycle and the steady speed of a bicycle. According to Groot' s modeling, the wind resistance on a bicycle is [5]

$$F_a = 1/2 * \rho C_d A_p v^2 = kv^2$$

where C_d stands for drag coefficient, A_p is the projected frontal area, ρ is the density of the air, this eqn indicates that the air friction force F_a has a quadratic relation with the speed.

Bicycle energy includes translational kinetic energy and wheel rotational kinetic energy. The rate at which bicycle energy changes equals the resultant power of the external force.

$$\frac{d}{dt}\left(\frac{1}{2}\left(m_1+\frac{2l}{r^2}\right)V^2\right)=(F_p-Fr-Fa)\,v$$

where m' is the total weight of bicycle, I is the moment of inertia of the tires and r is the radius of the tires. For convenience, the energy term can be replaced by a extra mass term m where

$$m=m'+2I/r^2$$

and the equation becomes

$$mv'v=(F_p-F_r-F_a)v \tag{2}$$

in equation (2) F_r is the resistance from the ground, expressed by the equation

$$F_r=a_r*m$$

in equation (2) F_a is the air resistance, expressed by the equation

$$F_a=m*k_m*v^2$$

in equation (2) a_r and k_m are the friction coefficient and air resistance coefficient of bicycles. Some data obtained by Groot and others are as Figure 3.

	A_0 (m^2)	m (kg)	k_m ($10^{-3}m^{-1}$)	a_r ($10^{-2}ms^{-2}$)
AD	0.44	103.7	1.75	4.01
FW	0.39	95.2	2.26	3.57
FP	0.40	90.1	2.00	3.85
SS	0.37	97.5	1.86	3.85
RS	0.36	81.1	2.08	3.64
AO	0.36	76.3	2.41	3.54
PA	0.38	84.3	2.69	3.90

Figure 3: Partial k_m and a_r data [5]

The modeling of Groot well explained the movement of bicycles on flat roads, and the parameters they gave were consistent with the actual situation. But because they do not take into account the form of bicycles on the slope, they need to introduce extra gravity to work.

We analyze the force on the bicycle (see Figure 4). On the slope, the positive pressure of the bicycle on the ground becomes the original $\cos\theta$ times, and the gravity will provide a downward force along the slope

$$F_g=mg\sin\theta$$

In summary, we propose an improved SCPO model. In the absence of stability:

$$v'=\frac{F_p}{m}-(arcos(\theta)+gsin(\theta))-k_m*v^2 \tag{3}$$

Once stabilized, $v'=0, P=F_p*v$, calculated:

$$P=m\left(arcos\left(\theta\right)+gsin\left(\theta\right)\right)v+mk_m*v^3 \tag{4}$$

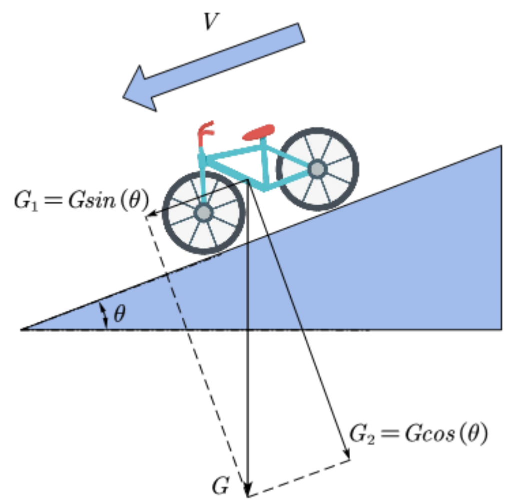


Figure 4: Slope Force Analysis

We selected the power, speed and slope data of the stable cycling section of the riders in the UCI professional challenge video, and compared with our model. The results show that the SCPO model can well explain the power of riders, and compared with the data of GROOT, after more than 20 years of development and progress, the bicycle manufacturing process has made great progress, especially the application of streamlined helmets, which makes the air resistance coefficient k_m of most riders less than that of GROOT 20 years ago.

After calculation, from equation (3), after deformation:

$$v' = \frac{P}{m} - (arcos(\theta) + gsin(\theta)) - k_m * v^2 \tag{5}$$

This differential equation has no analytical solution. Considering that its image is non-rigid, Fourth-order fifth-order Runge-Kutta algorithm is used to obtain its numerical solution. The initial value is assumed that the cyclist rushes up a 10% slope $(tan(\ theta) = 0.1)$ at a speed of 13 m/s (46.8 km/h) and different powers, so his speed-time curve is as Figure 5:

It can be seen that the speed of the cyclist has stabilized for about 20 seconds. In the bicycle race, a long slope often takes 10 minutes or more to pass. Therefore, in the next calculation, we ignore the time when the speed of the cyclist reaches a steady state. For equation (4) we use matlab to visualize the equation, which shows the power changes at a certain angle and speed(see Figure 6).

5.1.2 Strategies and Validation

According to the SCPO model, we can put forward better strategies for ITT competitions. In the following, we will divide the terrain into uphill, flat and downhill, and formulate SFAT. It is worth noting that the flat can essentially be seen as a slope of 0%.

In the case of flat ground, two possible reasonable situations are considered. The first is riding

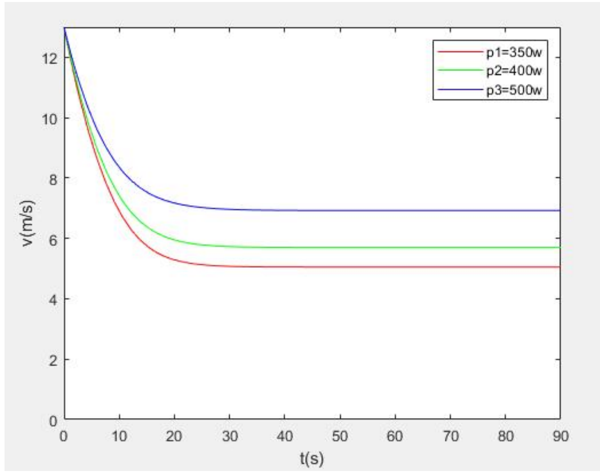


Figure 5: Speed-Time Curve

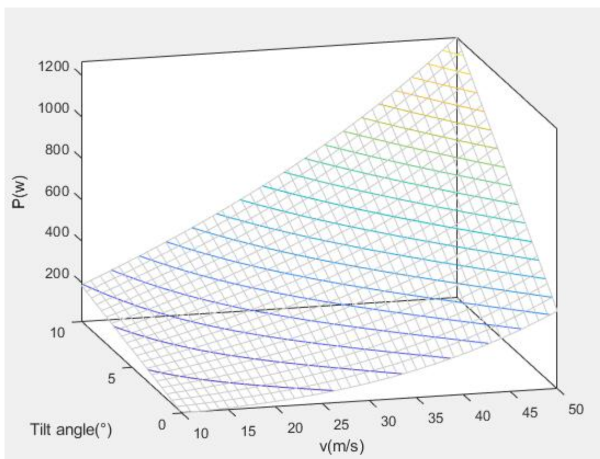


Figure 6: SCPO Power Visualization Image

with stable power output, which consumes physical consumption slowly and steadily until the end of the flat road. The second case is the high-power cycling in the previous section of the road, which increase E to a certain value, and the low-power cycling in the latter section of the road, which makes the physical strength restore to the initial state and cycle. By calculation we find that the first case can be completed in a shorter time, which is consistent with the literature. This also confirms the correctness of our model.

In the uphill situation, two possible reasonable situations are considered as well. The first case is to maintain the cruise power of the flat during the uphill process.

Although this situation does not lead to a decrease in the speed in the uphill, it does not cause excessive physical consumption, ensuring a stable power output in the subsequent competition. Although this situation does not lead to a decrease in the speed in the uphill, it does not cause excessive physical consumption, ensuring a stable power output in the subsequent competition. The second scenario is to increase power output at uphill to maintain speed advantage. In order to figure out which of the two cases is more reasonable, an experimental course of 500 m uphill, 5%

slope and 2000 m behind the slope was established (see Figure 7). Table 3 shows the simulation data of the two cases in the course.

Table 3: Differences Between the Two Climbing Slope Strategies

Quantity	Case 1	Case 2
Uphill power	572W	864W
Flat power	572W	376W
Uphill speed	12.72m/s	20.00m/s
Flat speed	17.22m/s	16.54m/s
Time	139.25s	146.38s

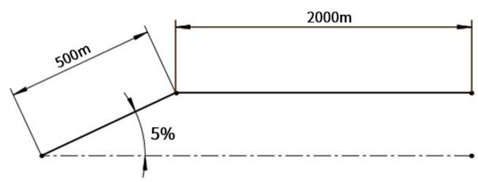


Figure 7: course schematics

It can be seen from the table that the first situation is more excellent. It is not difficult to analyze that in the second case, although the speed advantage is maintained in the uphill, it will lead to insufficient physical strength in the subsequent flat making it difficult to maintain the original cruise speed, resulting in 5.1% more final use than in the first case. In downhill cases, whether to increase power to increase speed is a main factor to be considered. In the study, we found that wind resistance is an important factor affecting the relationship between power and velocity of downhill. Similar to the previous case, a test course was established consisting of a 500 m downhill section with a slope of 5% first and a 2000 m horizontal section followed. Suppose the rider ' s fatigue value $E = 0.4$ due to riding on the previous section. Combined with the SCPO model, the calculation results of the downhill acceleration strategy show that the strategy of increasing power in the downhill will consume physical strength at a higher speed and bring greater wind resistance consumption, which leads to an increase of 0.7% in the total riding time and a higher fatigue value after the end of the road section. In order to obtain the maximum time benefit, it is necessary to increase the power acceleration in the section before the downhill and consume the physical strength to the exactly recoverable value in the downhill section.

5.1.3 Conclusion

Based on the above analysis, SFAT model has the following conclusions:

- Flat riding should adopt the strategy of maintaining stable cruise power and slowly consuming physical strength.

- The uphill riding should adopt the strategy of maintaining the cruise power close to the level and reducing the speed.
- Downhill riding should take the strategy of consuming physical strength before downhill and restoring physical strength to the original state in downhill.

5.2 Strategies for Sharp Turn

5.2.1 Problem Analysis

Generally speaking, the turning with radius less than15m and degree more than 90 degrees becomes a sharp turn, which is also called technical section in previous studies. Studies have shown that even if the rapid bending technology section is used to restore physical strength, the time lost in the rapid bending technology section cannot be regained in the straight line section [10]. Therefore, the rider cannot completely abandon the complete deceleration of the rapid bending technology section, but should adopt strategies to control the timing of increasing and decreasing power and increasing and decreasing speed more accurately. In a fixed course, the friction coefficient of a rider is stable, and the centripetal force is mainly provided by friction during the turning process, and the centripetal force provided by friction is limited. As shown in the formula

$$F_n = m\frac{v^2}{R},$$

where F_n is centripetal force, m is the total mass of people and cars, v is the riding speed, R is the turning radius. During the turning we need to ensure that the radial friction f is always greater than the centripetal force F_n. The following is based on the formula for the analysis of the problem of sharp bend technical sections :

- When turning at the same radius, the greater the speed is, the greater the centripetal force is needed. Otherwise, the centrifugal movement will be carried out, leading to sideslip and other dangerous situations. Therefore, high-speed riding is not allowed in the technical section of sharp bend.
- In order to shorten the riding time as much as possible, the rider needs to choose the riding path with small radius and short distance as much as possible, that is, to choose the lateral approach, and then to pass through the sharp-bend technical section along the path tangent to the arc of the track.
- In order to shorten the riding time as much as possible, the rider needs to choose the riding path with small radius and short distance as much as possible, that is, to choose the lateral approach, and then to pass through the sharp-bend technical section along the path tangent to the arc of the track. In reality, in the process of processing technical sections, the power transmission capacity is limited by the inclination angle, which means that the power output cannot be transmitted during the turning. That is to say, the output power should be reduced, the speed should be reduced, and the physical strength should be saved when the inclination angle is generated. After turning, the roll angle quickly recovers to 0% to generate power output. That is, at this point to increase power, restore to the original speed [10]. The cycling trajectory and power output of the sharp bend section are shown as Figure 8.

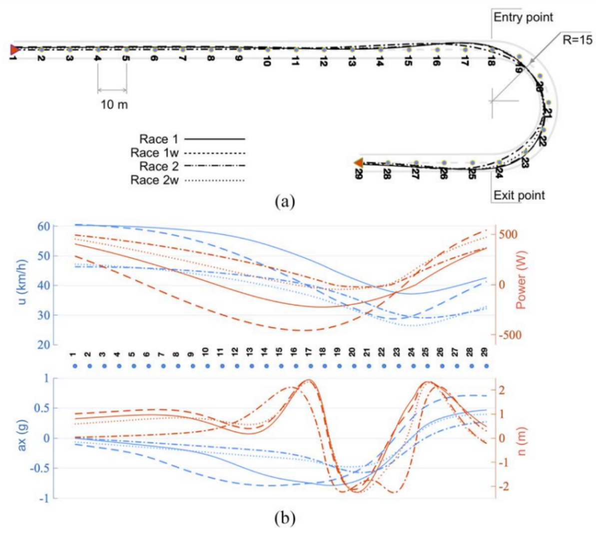


Figure 8: Riding Trajectory and Power Output of Rapid Curved Technical Section [10]

5.2.2 Conclusion

The rider shall ride to the outside road before entering the technical section, reduce the output power and speed and produce a turning angle. After restoring the turning angle, the rider should increase power and return to the original cruise speed.

In fact, after the analysis and collation of the game course, the data shows that the proportion of technical sections in the whole track of the individual timer is small. The impact of sharp bends on final race duration is small.

6 SFAT Application

6.1 OmTTE Application for Different Types of Rider

We apply the SFAT model to different types of riders, including male timer experts, female timer experts and male climbers. Their important parameter TCP_{max}, m, P_0, P_m is shown as Figure 9. Their OmTTE curve is shown as Figure 10.

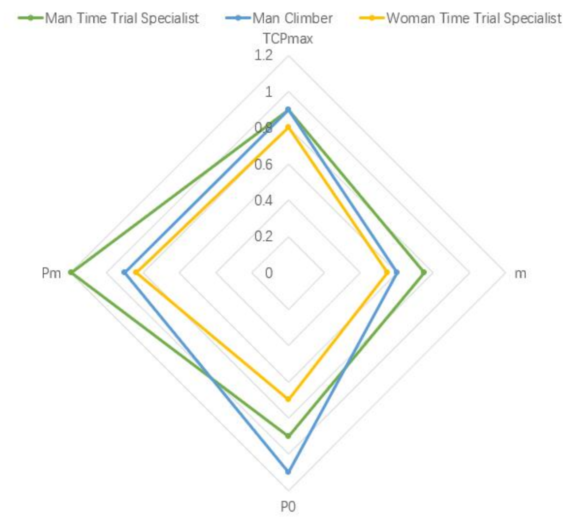


Figure 9: Ability Radar Chart of Three Types of Riders

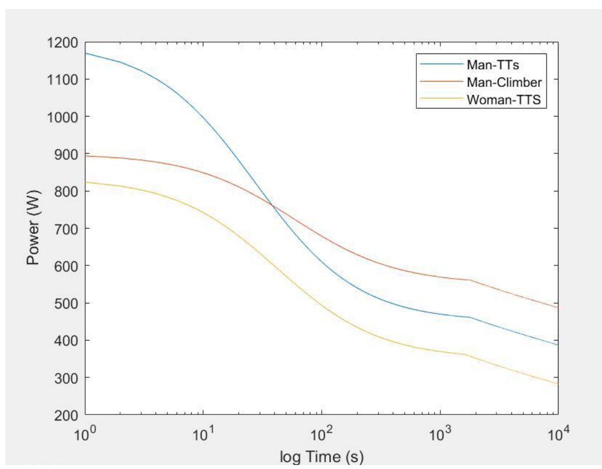


Figure 10: OmTTE curve of Three Types of Riders

6.2 SFAT Application in Different courses

We apply the SFAT model to the 2021 Tokyo Olympic Games, the UCI Urban Cycling World Championships and a bicycle race course we customize, and calculate the results.UCI course male track is a 43.3 km long road, female track is a 30.3 km long road. The difference between the highest altitude and the lowest altitude is less than 15 m, the slope is always less than 1.5%, and there is no obvious long slope. In addition, the rapid turning distance is less than one thousandth of the middle distance. Therefore, we believe that course can be approximately considered as a long straight road without fluctuation.

The course composition of Tokyo Olympic Games is complex, which can be described in Figure 11.

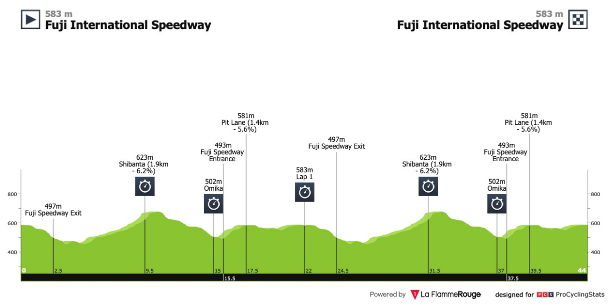


Figure 11: Fuji Individual time trial course [11]

We also designed a course consisting of two 6.4Km straight lines, a 3.6Km continuous uphill, a 3.6Km continuous downhill and four sharp turns. On this track, women need to ride a cycle, men need to ride two cycles. The 3D model of this course is as Figure 12.

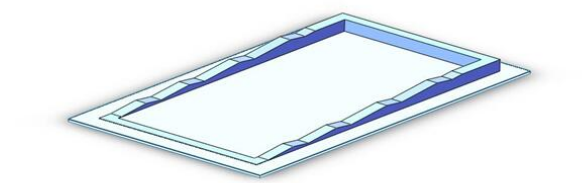


Figure 12: 3D Model of Customized Course

On three tracks, our model yields Table 4.

Table 4: The Results of Three Riders on Three Time Trail Courses

	Olympic	UCI	Custom Course
Man-TTS	59.9055	51.6468	51.0158
Woman-TTS	39.731505	45.35865	36.77364
Man-Climber	54.7667	55.8723	51.2384

6.3 SFAT Application in Different Types of Rider

The results of calculating the time spent on the formal track by the riders applying the above parameters show that the time is within the actual time-consuming range of the game, which indicates that our parameter design for three types of riders is reasonable.

Table 5: Time difference of different people in flat and uphill sections

Terrain	Slope	Flat
Man-Time-Trial-Specialist	139.2509s	124.0076s
Man-Climber	129.3273s	125.8743s
Woman-Time-Trial-Specialist	142.456s	126.1666s

In order to highlight the differences in the capabilities of the three riders, we designed two test roads, one with several long straight roads and the other with a 500m, 5% uphill segment (pictured), in which they performed as Table 5.

7 Sensitivity Analysis

7.1 Environment Factor

7.1.1 Wind Resistance

In order to explore the influence of wind resistance on the model, the angle variables in SCPO model and the values of a_r, k_m, m are fixed, and the wind speed V_w is introduced into V to simulate the influence of wind resistance on the required output power. It can be seen from the results of the Figure 13 that the wind speed has little effect on the output power under the condition of downwind. 10 km/h (secondary wind) can reduce the power output by 37% at 40 km/h cruise speed. Correspondingly, in the case of reverse wind, the influence of wind speed on the power output maintaining a certain cruise speed is not linear, but as the wind speed increases, the influence of wind resistance on SCPO is more obvious. Wind speed of 10 km/h (secondary wind) increases power output by 49% at cruise speed of 40 km/h, while it increases power output by 107% at 20 km/h (secondary wind). In other words, the influence of headwind on ITT performance is an important factor to be considered in the time trial.

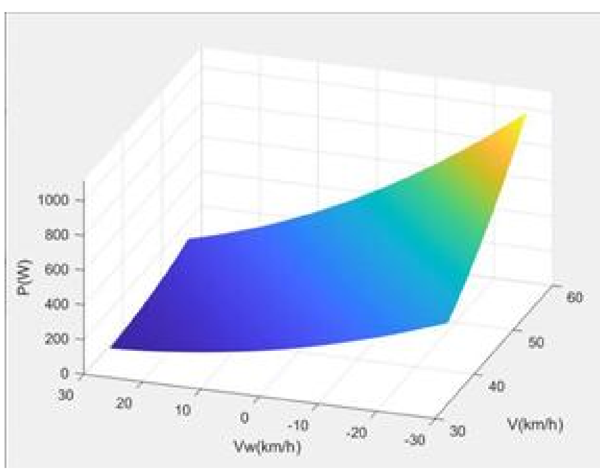


Figure 13: Influence of Wind Speed on SCPO Model

7.1.2 Humidity

The friction resistance coefficient a_r is affected by humidity in the personal timing competition. a_r changes caused by ambient humidity are the same for all riders along the road. However, the strategy changes made by different riders against the changes in a_r will result in differences in performance between different riders. Studies have shown that, compared with the dry environment, the rider should decelerate earlier in the humid environment, so as to be close to the optimal turning path. And for the 40 km ITT competition, only the sharp bend technical road no less than 25%, humidity will have no less than 1% impact on the final results [10].

7.1.3 Other Environmental Factors

Temperature is also one of the influencing factors of ITT competition results. Hot conditions can cause the rider output power to decline. Studies have shown that, after two weeks of adaptation, compared with cool conditions, professional riders can complete ITT basically unchanged time under hot conditions. On the contrary, they will have a significant decline in power output during ITT without adaptation [12].

Altitude can indirectly affect ITT results by influencing riders'CP values. In which the CP describes the relationship between sustainable power output and duration for severe-intensity exercise [13]. Studies have shown that the CP value of riders will decrease significantly with the increase of altitude(see Figure 14). In this paper, it can be obtained that the CP value is positively correlated with the cruise power P_0, that is, the higher the altitude is, the lower the cruise power and speed will be, and ultimately the competition will be time-varying [14].

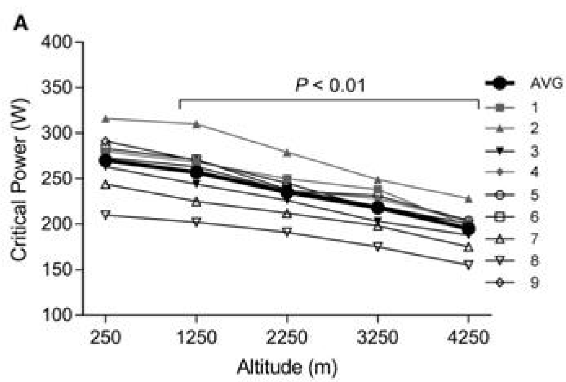


Figure 14: Effect of altitude on CP

7.2 Rider Factor

Our SFAT model is verified to improve about 5% of the competition results through more reasonable physical distribution without improving the physical quality of the riders. However, SFAT may not be accurately implemented due to rider factors. In order to explore the impact of rider factors on SFAT, we used multiple test courses to explore the time fluctuations caused by rider factors through calculation.

There are two factors leading to the rider' s misallocation of physical strength : the location of the power increase and the size of the power increase. After calculation, the influence of these two factors on the trial time is as Figure 15.

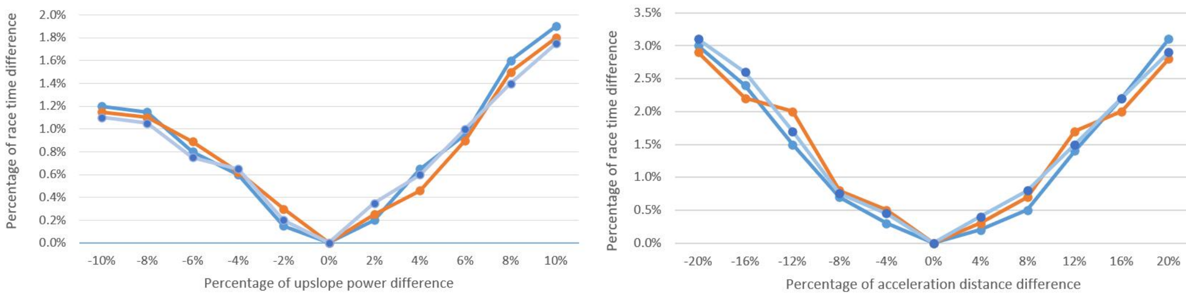


Figure 15: Increased Power or Position Error Resulting in Increased Duration

In the above two figures, each curve in each figure represents the impact of the position of the riders increasing power and the size of the increasing power on the competition time in a test section where the power needs to be increased at an appropriate position. In the figure, the point with zero abscissa indicates that the physical fitness is fully managed according to the strategy given by the SFAT. The ordinate indicates the percentage of the prolongation of the test section due to the rider factor compared with the standard SFAT strategy.

Besides, it can be seen that the score of the rider increases with the increase of his deviation from the SFAT model. When the difference of the position judgment of the rider' s acceleration reaches 20%, the game time of the SFAT will be extended by nearly 3%, which means that the SFAT model will be nearly invalid. At the same time, the trend shows that the impact of acceleration power increase on performance is more obvious than that of acceleration power decrease. We believe that this is because the increase in power will lead to a rapid increase in the fatigue value E, which will lead to the failure of riders to maintain power in the subsequent sections. Therefore, the reasonable approach is to increase the output power cautiously so as to avoid premature fatigue. To sum up, if the rider training properly, relatively accurate implementation of the strategy given by the SFAT model, in a slight error he will also get satisfactory results.After our calculations, the cooperative group performed 15% faster than the non-cooperative group on UCI course

8 Model Evaluation and Further Discussion

8.1 Model Evaluation for team game

- Since the performance of the fourth person in the team is the final performance of the team, it can be considered that the physical strength of one or two people in the team can be consumed in the later period to the value that cannot maintain the cruise speed. That is, these two people can consume more physical strength in the fight against wind resistance.

- Considering the factors of team cooperation, the team members can take turns to fight against the wind resistance at the head of the team. At the same time, the team members behind the team do not need to fight against the wind resistance while maintaining the cruise speed, and can slowly recover their physical strength.

- By measuring the impact of wind resistance on physical strength to determine the team leader rider windbreak time. The correlation functions of wind resistance, velocity and body force are established.

8.2 Futher Discussion

8.2.1 Advantages of the Model

- Different from the MMP curve based on the average power used to predict the competition in recent years, our model applies a more reasonable TTE curve based on instantaneous power to analyze the short-term cycle decision in the trial.
- Considering the limited physical strength of the rider, the physical attenuation function ϕ is introduced into the TTE model, and the OmTTE model is established. The TTE model is extended to the whole region to make it suitable for highway competitions (long-term competitions).
- The slope is introduced into the CPO model to establish the SCPO model, so that the SCPO model can be used for the analysis of the SFAT model.
- Our model can reflect the physical differences of various riders, so as to put forward reasonable suggestions for different groups.

8.2.2 Shortcomings of the Model

- Cannot explain the physiological meaning of the physical attenuation function ϕ introduced by OmTTE model after TCP_{max} time.
- Our SFAT model is only suitable for long and slow slopes, but not for steep and short slopes.

8.2.3 Promotion of the Model

- Extend the ITT-based SFAT model to TTT.
- Explain the physiological meaning of physical attenuation function ϕ.

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Appendices

Appendix A First appendix: Guidance

Dear Mr. Directeur Sportif,

Through our in-depth study of the problem of bicycle driving, we establish OmTTE(Omni Time To Exhaustion)and SCPO(Slope Cycling Power Output)mathematical models to reproduce the cycling state of athletes. Through the calculation of these two models, we obtain the SFAT(Strategy For All Terrine)model. SFAT model can guide your cyclist to allocate physical strength in the most reasonable way, so as to improve his athletic performance. In our model, the rider can use our physical distribution strategy to improve his individual time trial result by 5% without improving his physical ability.

A complex track may have a variety of factors affecting the physical management strategy. We select the most likely factors affecting the performance of the rider and give our suggestions respectively.

1:Slope strategy

- **Flat riding should adopt the strategy of maintaining stable cruise power and slowly consuming physical strength.**
- **The uphill riding should adopt the strategy of maintaining the cruise power close to the level and reducing the speed.**
- **Downhill riding should take the strategy of consuming physical strength before downhill and restoring physical strength in downhill.**

The reason explanation needs to use our athlete 's physical consumption model and the riding power model.

Physical consumption model points out that riders can maintain a certain cruise power. When the power output by the rider is higher than the cruise power, his strength will be consumed; when it is lower than the cruise power, his physical strength will recover slowly.

When riding on the ground, if the rider chooses to increase his physical strength to the cruise power first, he can improve the speed in a short time, but in the long run, because the rider needs additional time to restore physical strength, his average speed will decline.

When riding on the uphill, if the rider chooses to increase the power to maintain the speed as much as possible, he will use his strength too fast, resulting in the speed unable to be maintained, and reducing the overall average speed.

When riding downhill, if the rider chooses to use a higher output power, it will increase the speed of riding greatly, leading to a significant increase in air resistance. Thus it exerts a negative impact on his results.

It is worth noting that in view of the third proposal, our study finds that if the power of the rider increases too much in the road section before the downhill, it will lead to his premature exhaustion of energy before the slope, which will significantly exert a negative impact on the results. So we recommend that riders accumulate experience in physical fitness management to avoid this. If your rider needs quantitative research to increase the power value, you can use the OmTTE and SCPO models in our paper to calculate.

2:Sharp turn strategy

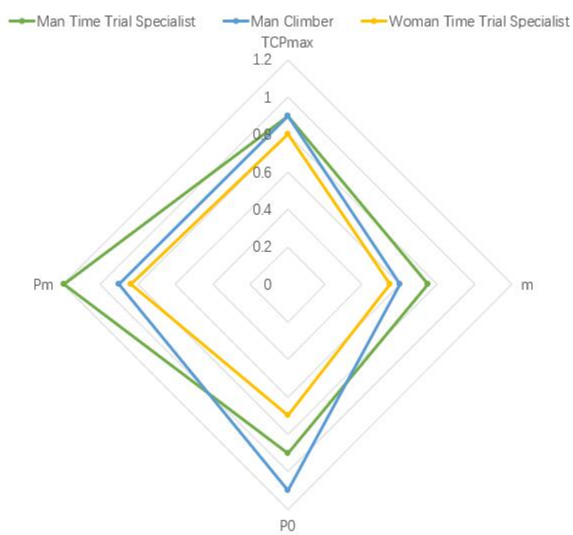
The rider shall ride to the outside road before entering the technical section, reduce the output power and speed and produce a turning angle. After restoring the turning angle, the rider should increase power and return to the original cruise speed.

We propose this suggestion because if the rider enters the technical section of the sharp turning at a high rate, it is likely to occur sideslip. At the same time, the power transmission capacity of the bicycle is limited by the inclination angle, and it is difficult for the rider to maintain high power output in the sharp turning section.

In fact, after the analysis and collation of the game course, the data shows that the proportion of technical sections in the whole track of the individual timing is small. The impact of sharp turns on final race duration is limited.

3:Individualized suggestions

Individualized suggestions Considering that the model needs to customize personalized schemes for different riders, we need the following important parameters measured by riders. To make the parameters simple and intuitive, we fit three typical indicators for your reference :



The most important indicator in the model is $P0$, which is a rider 's cruise power. It represents that a rider can continuously output cruise power without feeling tired. Our strategy is proposed around the cruise power. Therefore, it is necessary for you to equip your riders with a power meter so that they can find their own cruise power during training, and perform slope riding suggestions on the uphill and downhill slopes, believing that your riders can achieve more excellent results in the competition.

Appendix B Second appendix: Code

```
clear;clc
syms P T t A Pm P0 P R E TCPmax B;
P0 = 350;Pm = 835;A = 20408;TCPmax = 1600;B = 38;
T = 1 : 1 : 10000;
P = P0 + (A*(Pm-P0)) ./ (T .* (Pm-P0) + A) - B .* log(T ./ TCPmax) .* (T>=TCPmax);
% R=(Pm-P0).*(P-P0)./A./(Pm-P); % from P to R
semilogx(T,P);
xlabel ('log Time (s)')
ylabel('Power (W)')
legend('Man-TTS', 'Man-Climber', 'Woman-TTS')
hold on;

clear;clc;
syms W1 t Pmax  CP TCPmax A t;
W1=15506;
TCPmax=1800;
Pmax=1196;
CP=304;
A=38;
t=1:1:10000;
P2= (W1./t.*(1-exp(-t.*(Pmax-CP)/W1))+CP).*(t<=TCPmax)+ ...,
(W1./t.*(1-exp(-t.*(Pmax-CP)/W1))+CP-A*log(t./TCPmax)).*(t>TCPmax);
figure(1);
semilogx(t,P2);
xlabel ('log Time (s)')
ylabel('Power (W)')
hold on;

X = ones(1, 8);
Y = ones(1, 8);
Z = ones(1, 8);
T = ones(1, 8);
j = 1;i = 1;
while j <= 8
X(j) = P(i);
Y(j) = P2(i);
Z(j) = abs(X(j) - Y(j)) / Y(j) * 100;
T(j) = i;
j = j + 1;
i = i * 3;
end

F = [T,
X,
Y,
Z];

clear;clc
R2 = 1 - (sum((P - P2).^2) ./ sum((P2 - mean(P2)).^2));
```

```
clear;clc
syms m ar km p v g;
```

```
m=65;
ar=0.04;
km=0.001;
g=9.8;
p = m * km * v ^ 3 + ar * m * v * g;


syms a b c d A B C;
a = m * km;
b = 0;
c = ar * m * g;
d = -400;
A = b ^ 2 - 3 * a * c;
B = b * c - 9 * a * d;
C = c ^ 2 - 3 * b * d;
DET = B ^ 2 - 4 * A * C;
Y1 = A*b + 1.5*a*(-B + sqrt(DET));
Y2 = A*b + 1.5*a*(-B - sqrt(DET));
y1 = nthroot(Y1,3);  y2 = nthroot(Y2,3);
v = (-b-y1-y2)/(3*a);

clear;clc
l = 43.3e3;
P0 = 350;Pm = 835;A = 20408;TCPmax = 1200;B = 38;
syms dp ansP anst;
ansp = 0;
for p = 100 : 0.5 : 700
v = p_to_v(p);
T = l / v;
P = P0 + (A*(Pm-P0)) ./ (T .* (Pm-P0) + A) - B .* log(T ./ TCPmax) .* (T>=TCPmax);
if ((~ansp) || (dp > abs(P - p)))
ansp = p;
dp = abs(P - p);
anst = T;
end
end
disp(anst / 60);

clear;clc
P0 = 550;Pm = 906;A = 20408;TCPmax = 1800;B = 38;
syms dp ansP anst;
ansp = 0;
for p = 100 : 0.5 : 700
T = 0;
v5 = p_to_v(p);T = 21.4e3 / v5 + T;
v4 = p_theta_to_v(p, 5.6);T = T + 2.8e3 / v4;
v3 = p_theta_to_v(p, 6.2);T = T + 3.8e3 / v3;
v2 = p_theta_to_v(p, 2.2);T = T + 11e3 / v2;
v1 = p_theta_to_v(p, 4);T = T + 5e3 / v1;
P = P0 + (A*(Pm-P0)) ./ (T .* (Pm-P0)+A) - B .* log(T ./ TCPmax) .* (T >= TCPmax);
if ((~ansp) || (dp > abs(P - p)))
ansp = p;
dp = abs(P - p);
anst = T;
```

```
end
end
disp(anst / 60);

clear;clc
P0 = 450;Pm = 1196;A = 20408;TCPmax = 1800;B = 38;
syms dp ansP anst;
ansp = 0;
for p = 100 : 0.5 : 700
T = 0;
v5 = p_to_v(p);T = 32.8e3 / v5 + T;
v4 = p_theta_to_v(p, 5.6);T = T + 7.2e3 / v4;
P = P0+(A*(Pm-P0)) ./ (T .* (Pm-P0) + A) - B .* log(T ./ TCPmax) .* (T>=TCPmax);
if ((~ansp) || (dp > abs(P - p)))
ansp = p;
dp = abs(P - p);
anst = T;
end
end
disp(anst / 60);
```