0.1 Burst-Trapping Algorithm

Demonstration of Binary Cyclic Code's Ability in decoding burst-error.

Definition 0.1

Suppose then that C is a burst-b error correcting code, and that the syndrome S(x) satisfies the two conditions:

$$S(0) \neq 0 \tag{1}$$

$$deg(S(x)) \leq b-1 \tag{2}$$

☞ **Remark**
deg(S(x))is the highest degree of S(x)

Suppose we use a (7,3) Abramson Code with generator polynomial characterized by

$$g(x) = x^4 + x^3 + x^2 + 1 \tag{3}$$

with a burst-correcting ability b=2; the received bit-stream is characterised by

$$R(x) = x^6 + x^5 + x^2 + 1 \tag{4}$$

☞ **Remark**
In matrix form $R(x) = [1010011]$

Then the first syndrome is

$$S_0(x) = R(x) \mod g(x) = (x^6 + x^5 + x^2 + 1) \mod (x^4 + x^3 + x^2 + 1) = X^3 + x^2 \tag{5}$$

☞ **Remark**
Use polynomial long-division

Definition 0.2

Meggitt's lemma
For $j \geq 0$ define:

$$S_i(x) = [x^i R(x)]_n \mod g(x) \tag{6}$$

$$S_{j+1}(x) = [xS_j(x)] \mod g(x) \tag{7}$$

☞ **Remark**
$S_j(x)$ is the remainder syndrome of the jth cyclic shift of $R(x)$

We therefore have the following:

$$S_1(x) = [xS_0(x)] \mod g(x) = x^2 + 1 \tag{8}$$

$$S_2(x) = [xS_1(x)] \mod g(x) = x^3 + x \tag{9}$$

☞ **Remark**
$xS_1(x)$ is degree-insufficient and therefore remainder is itself

$$S_3(x) = [xS_2(x)] \mod g(x) = x^3 + 1 \tag{10}$$

$$S_4(x) = [xS_3(x)] \mod g(x) = x^3 + x^2 + x + 1 \tag{11}$$

$$S_5(x) = [xS_4(x)] \mod g(x) = x + 1 \tag{12}$$

5th Syndrome finally satisfies definition one with degree of 1. Therefore transforming into matrix form, the burst error is 11 of location $-5 \mod 7 = 2$.

The error vector is therefore $E = [0011000]$ and the corrected sequence is $R + E = [1001011]$