Carrier Frequency Offset Estimation for OCDM with Null Subchirps

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***Abstract*—In this paper, we investigate the carrier frequency offset (CFO) estimation problem in orthogonal chirp division multiplexing (OCDM) systems. We propose a transmission scheme by inserting consecutive null subchirps. A CFO estimator is developed to achieve a full acquisition range. We further demonstrate that the proposed transmission scheme not only helps to resolve CFO identifiability issues but also enables multipath diversity for OCDM systems. Simulation results corroborate our theoretical findings.**

***Index Terms*—OCDM, CFO, identifiability, multipath diversity**

I. Introduction

ORTHOGONAL chirp division multiplexing (OCDM) is a multi-carrier modulation scheme that uses orthogonal chirp waveforms to carry information symbols. Compared to the currently dominant modulation scheme, i.e., orthogonal frequency division multiplexing (OFDM), OCDM [1] [2] is more robust against frequency-selective channels, enjoys lower bit error rate (BER) under various equalization methods, and performs better against burst interferences. Gaining more attentions recently, OCDM also sees increased interests in the research of the multi-user communications, channel and CFO estimation [3]–[7].

In an OCDM system, the inverse discrete Fresnel transform (IDFnT) is used as the transform kernel to spread each information symbol over the entire bandwidth and the whole symbol block. Thus, OCDM has the double-spreading feature. A non-sinusoidal transform basis may enjoy certain advantages under the influence of CFO. For instance, simulation results in [8] indicate that OCDM with a minimum mean square error (MMSE) equalizer or decision feedback equalizer (DFE) performs better in terms of BER than OFDM over frequency-selective channels with CFOs at medium signal-to-noise ratios (SNRs). It is also observed in [9] that chirp-based OFDM systems outperform traditional OFDM with the presence of CFO.

The double-spreading property of chirp basis suggests that frequency-selective channels may have different impacts on the receiver's ability to correctly identify a CFO in OCDM compared to OFDM. The identifiability issue is well documented in the case of OFDM [10]. Owing to the sinusoidal kernel, OFDM converts a frequency-selective channel into a set of frequency-flat channels. However, channel nulls in the frequency domain can create multiple indistinguishable candidate CFOs in OFDM systems. Thus, it was proposed that judicially placed null subcarriers can restore the CFO identifiability in OFDM [10], [11]. The optimality and identifiability of data-aided CFO estimation approaches for OFDM systems with training sequences are explored in [12], [13].

On the other hand, some preliminary efforts are directed towards handling the CFO in OCDM. In [5], an excessive cyclic prefix (CP) is used to cross correlate with its counterpart in an OCDM block to estimate the CFO. In [6], a preamble-type synchronization and channel estimation method is applied to an OCDM system. Although both of them show decent accuracy, they require redundant CPs or symbols. In addition, they still face the identifiability issues for large CFO [5], [6]. For instance, the CP-based approach [5] has a CFO acquisition range as half of the subchirp frequency spacing.

This paper resolves the CFO identifiability issue in OCDM with the help of the proposed transmission scheme, which inserts consecutive null subchirps. A CFO estimator is designed accordingly to achieve a full acquisition range of the CFO. We further prove that the proposed transmission scheme not only helps to address the CFO identifiability issue, but also enables multipath diversity for OCDM systems. Simulation results verify the identifiability of the proposed CFO estimator, and indicate that the BER performances of both the linear equalizers (LEs) and maximum likelihood estimator (MLE) benefit from the enabled multipath diversity.

Notations: Upper (lower) case bold letters are used to denote matrices (vectors). The notation $[\mathbf{A}]_{m,n}$ indicates the entry at the m-th row and n-th column of the matrix $\mathbf{A}$, whereas $[\mathbf{A}]_{m,:}$ and $[\mathbf{A}]_{:,n}$ denote the m-th row and n-th column of the matrix $\mathbf{A}$, respectively. The identity matrix of size $P\times P$ is denoted by $\mathbf{I}_P$ and the zero matrix of size $M\times N$ is $\mathbf{0}_{M\times N}$. The notations $(\cdot)^T$, $(\cdot)^{\mathcal{H}}$, $(\cdot)^{-1}$, $(\cdot)^{\dagger}$, and $(\cdot)^*$ represent transpose, conjugate transpose, inverse, Moore-Penrose inverse, and element-wise conjugate, respectively. We use $E[\cdot]$ to denote the expectation with respect to (w.r.t.) all random variables within the brackets.

II. System Model

In this section, we derive a baseband equivalent OCDM system model with a CFO and time-invariant frequency-selective channel is assumed with an impulse response $h(l)$ of order L, where $l = 0, 1, \ldots, L$.

Consider the i-th symbol block that is composed of the information symbol block $\mathbf{s}(i)$ and null symbols. The null symbols in the Fresnel domain correspond to null subchirps in the time domain, which are added consecutively. Thus, the i-th symbol block of length N is assembled as $\mathbf{T}_{\mathrm{zp}}\mathbf{s}(i)$, where $\mathbf{s}(i)$ is of length K and $\mathbf{T}_{\mathrm{zp}} = [\mathbf{I}_K \quad \mathbf{0}_{K\times(N-K)}]^T$.

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We further assume that the covariance matrix of $\mathbf{s}(i)$ is given by $\mathbf{R}_{ss} = E[\mathbf{s}(i)\mathbf{s}^{\mathcal{H}}(i)] = E_s\mathbf{I}_K$, where E_s is the symbol energy. The i-th symbol block is modulated with the IDFnT. The resulting time-domain symbol block can be written as $\mathbf{x}(i) = \mathbf{\Phi}^{\mathcal{H}}\mathbf{T}_{\text{zp}}\mathbf{s}(i)$, where the DFnT matrix $\mathbf{\Phi}$ of size $N \times N$ is defined as:

$$[\mathbf{\Phi}]_{m,n} = \frac{1}{\sqrt{N}} e^{-j\frac{\pi}{4}} \times \begin{cases} e^{j\frac{\pi}{N}(m-n)^2}, & N = 0 \bmod 2, \\ e^{j\frac{\pi}{N}(m+\frac{1}{2}-n)^2}, & N = 1 \bmod 2, \end{cases} \quad m,n = 1,\dots,N. \tag{1}$$

The matrix $\mathbf{\Phi}$ is circulant and unitary, i.e., $\mathbf{\Phi}^{\mathcal{H}}\mathbf{\Phi} = \mathbf{\Phi}\mathbf{\Phi}^{\mathcal{H}} = \mathbf{I}_N$. Moreover, a cyclic prefix (CP) of length L is inserted in the time domain before $\mathbf{x}(i)$ to combat multipath delay effects.

The transmitted OCDM signal goes through a time-invariant frequency-selective channel. At the receiver side, we consider a normalized CFO $w_o = 2\pi f_o T_s$ in the full range $[-\pi, \pi)$, where f_o is the CFO in Hz, and T_s is the sampling period. After synchronization and removal of the CP, the received signal block $\mathbf{y}(i)$ of length N is given by

$$\mathbf{y}(i) = e^{jw_o(i(N+L)-N)}\mathbf{D}_N(w_o)\mathbf{H}\mathbf{\Phi}^{\mathcal{H}}\mathbf{T}_{\text{zp}}\mathbf{s}(i) + \mathbf{n}(i), \tag{2}$$

where $\mathbf{D}_N(w_o)$ of size $N \times N$ is the diagonal CFO matrix with the n-th element in the diagonal given by $e^{jw_o(n-1)}$, the first column of the circulant channel matrix $\mathbf{H}$ is given by $\mathbf{h} = [h(0), \dots, h(L), \mathbf{0}_{N-L-1}^T]^T$, and $\mathbf{n}(i)$ is the independent and identically distributed (i.i.d.) zero-mean additive white Gaussian noise (AWGN) vector with the covariance matrix $\sigma^2\mathbf{I}_N$.

Once the CFO estimate $\hat{w}_o$ is obtained, we compensate the CFO effect on the received signal block and achieve $\mathbf{r}(i) = e^{-j\hat{w}_o(i(N+L)-N)}\mathbf{D}_N^{\mathcal{H}}(\hat{w}_o)\mathbf{y}(i)$. Consequently, LEs $\big($denoted by $\mathbf{G}_{(\cdot)}\big)$ or non-linear equalizers can be applied to $\mathbf{r}(i)$. For example, the data symbols can be recovered using LEs as $\hat{\mathbf{s}}(i) = \mathbf{G}_{(\cdot)}\mathbf{r}(i)$, where the zero-forcing (ZF) or MMSE equalizers are given by

$$\mathbf{G}_{\text{ZF}} = \mathbf{B}^{\dagger}, \tag{3}$$

$$\mathbf{G}_{\text{MMSE}} = \mathbf{B}^{\mathcal{H}}\left(\frac{\sigma^2}{E_s}\mathbf{I}_N + \mathbf{B}\mathbf{B}^{\mathcal{H}}\right)^{-1}, \tag{4}$$

with $\mathbf{B} = \mathbf{H}\mathbf{\Phi}^{\mathcal{H}}\mathbf{T}_{\text{zp}}$, respectively.

III. CFO Estimator and its Identifiability

In this section, we develop an estimator for the true CFO w_0 using the left null space (LNS) of the covariance matrix $\mathbf{R}_{yy} = E[\mathbf{y}(i)\mathbf{y}^{\mathcal{H}}(i)]$. We first analyze the LNS of $\mathbf{R}_{yy}$. Moreover, a CFO estimator based on the LNS is proposed, and its identifiability is proved.

A. The Proposed CFO Estimator

Based on (2), we derive the covariance matrix $\mathbf{R}_{yy}$ as follows,

$$\mathbf{R}_{yy} = \mathbf{D}_N(w_0)\mathbf{\Phi}^{\mathcal{H}}\mathbf{H}\mathbf{T}_{\text{zp}}\mathbf{R}_{ss}\mathbf{T}_{\text{zp}}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{\Phi}\mathbf{D}_N^{\mathcal{H}}(w_0) + \sigma^2\mathbf{I}_N, \tag{5}$$

where (5) is achieved using the property of circulant matrices $\mathbf{H}\mathbf{\Phi}^{\mathcal{H}} = \mathbf{\Phi}^{\mathcal{H}}\mathbf{H}$. Furthermore, the noiseless part of the covariance matrix is defined as

$$\bar{\mathbf{R}}_{yy} = \mathbf{\Phi}^{\mathcal{H}}\mathbf{H}\mathbf{T}_{\text{zp}}\mathbf{R}_{ss}\mathbf{T}_{\text{zp}}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{\Phi}, \tag{6}$$

where $\bar{\mathbf{R}}_{yy}$ is of size $N \times N$ and $\text{rank}(\mathbf{T}_{\text{zp}}) = K$. Thus, the rank of $\bar{\mathbf{R}}_{yy}$ is upper-bounded by $\text{rank}(\bar{\mathbf{R}}_{yy}) \leq \min(\text{rank}(\mathbf{H}), \text{rank}(\mathbf{T}_{\text{zp}})) < N$, according to Sylvester's inequality. Hence, there exists the LNS of $\bar{\mathbf{R}}_{yy}$.

The zero-padding (ZP) matrix $\mathbf{T}_{\text{zp}}$ makes $\mathbf{H}\mathbf{T}_{\text{zp}}$ a Toeplitz matrix of size $N \times K$. Recall that the multipath channel order is L. When $N - K > L$, the number of null subchirps is greater than L, and the last $N - K - L$ rows of $\mathbf{H}\mathbf{T}_{\text{zp}}$ only contain zeros. As a result, we arrive at

$$\mathbf{\Phi}^{\mathcal{H}}\mathbf{H}\mathbf{T}_{\text{zp}} = [\mathbf{\Phi}_{\mathcal{S}}^* \ \mathbf{\Phi}_{\mathcal{N}}^*]\begin{bmatrix} \bar{\mathbf{H}} \\ \mathbf{0}_{(N-K-L)\times K} \end{bmatrix}, \tag{7}$$

where $\mathbf{\Phi}_{\mathcal{S}}^* = \left[\mathbf{\Phi}^{\mathcal{H}}\right]_{:,1:K+L}$ of size $N \times (K+L)$, $\mathbf{\Phi}_{\mathcal{N}}^* = \left[\mathbf{\Phi}^{\mathcal{H}}\right]_{:,K+L+1:N}$ of size $N \times (N-K-L)$, and $\bar{\mathbf{H}} = [\mathbf{H}]_{1:K+L,1:K}$ of size $(K+L) \times K$. According to (7), we further prove that the null subchirps $\mathbf{\Phi}_{\mathcal{N}} = \left[\mathbf{\Phi}^T\right]_{:,K+L+1:N}$ is in the LNS of $\bar{\mathbf{R}}_{yy}$ as

$$\begin{aligned} &\mathbf{\Phi}_{\mathcal{N}}^T [\mathbf{\Phi}_{\mathcal{S}}^* \ \mathbf{\Phi}_{\mathcal{N}}^*]\begin{bmatrix} \bar{\mathbf{H}} \\ \mathbf{0}_{(N-K-L)\times K} \end{bmatrix} \\ = \ &\left[\mathbf{0}_{(N-K-L)\times(K+L)} \ \mathbf{I}_{N-K-L}\right]\begin{bmatrix} \bar{\mathbf{H}} \\ \mathbf{0}_{(N-K-L)\times K} \end{bmatrix} \\ = \ &\mathbf{0}_{(N-K-L)\times K}. \end{aligned} \tag{8}$$

Define a set of indices $\mathcal{N}_\phi = \{K+L+1, \dots, N\}$. With this notation, the CFO w_0 can be estimated using the LNS of $\bar{\mathbf{R}}_{yy}$ as

$$\hat{w}_0 = \arg\min_{w} J(w), \tag{9}$$

where the cost function $J(w)$ is defined as

$$J(w) = \sum_{k \in \mathcal{N}_\phi} \boldsymbol{\phi}_k^T \mathbf{D}_N^{-1}(w)\mathbf{R}_{yy}\mathbf{D}_N(w)\boldsymbol{\phi}_k^*, \tag{10}$$

with $\boldsymbol{\phi}_k = \left[\mathbf{\Phi}^T\right]_{:,k}$ and $\boldsymbol{\phi}_k^* = \left[\mathbf{\Phi}^{\mathcal{H}}\right]_{:,k}$. By leveraging $\bar{\mathbf{R}}_{yy}$, the cost function (10) can be rewritten as

$$J(w) = \sum_{k \in \mathcal{N}_\phi} \boldsymbol{\phi}_k^T \mathbf{D}_N(w_0 - w)\bar{\mathbf{R}}_{yy}\mathbf{D}_N(w - w_0)\boldsymbol{\phi}_k^* + \sigma^2(N-K-L), \tag{11}$$

where the equivalent noise term $\sigma^2(N-K-L)$ is the result of $\sigma^2\sum_{k\in\mathcal{N}_\phi}\boldsymbol{\phi}_k^T\mathbf{D}_N^{-1}(w)\mathbf{I}_N\mathbf{D}_N(w)\boldsymbol{\phi}_k^*$, independent of the candidate CFO w. When $w = w_0$, the result of $\mathbf{D}_N(w_0 - w)$ is an identity matrix, and the cost function is at a minimum, i.e., $J(w_0) = \sigma^2(N-K-L)$.

B. CFO Identifiability

The CFO identifiability issue arises for the CFO estimator (9), if the following necessary condition is fulfilled for some $w \neq w_0$

$$\left\{\boldsymbol{\phi}_{\tilde{k}}^T \mathbf{D}_N(w_0 - w)\right\}_{\tilde{k}\in\mathcal{N}_\phi} \subset \{\boldsymbol{\phi}_k^T\}_{k\in\mathcal{N}_\phi}. \tag{12}$$

The condition in (12) for some $w \neq w_0$ results in multiple minimums. Based on the definition of $\mathbf{\Phi}$ in (1), it is clear that $\boldsymbol{\phi}_k$ is a chirp sequence and $\boldsymbol{\phi}_{k+\Delta k}$ is a circularly shifted version of $\boldsymbol{\phi}_k$, where $\boldsymbol{\phi}_{k+\Delta k}$ is obtained by circularly shifted $\boldsymbol{\phi}_k$ down (up) by Δk samples, with Δk being a non-negative (non-positive) integer. Hence, supposing that $k = K+L+\Delta k$, the LNS of the results of $\bar{\mathbf{R}}_{yy}$ can be rewritten as

$$\{\boldsymbol{\phi}_k^T\}_{k\in\mathcal{N}_\phi} = \{\boldsymbol{\phi}_{K+L+\Delta k}^T\}_{\Delta k=1}^{N-K-L}. \tag{13}$$

Assume a block size of N with even value as an illustrative example. The m-th entry of $\boldsymbol{\phi}_{K+L+\Delta k}$ is given by:

$$\begin{aligned}
&\left[\boldsymbol{\phi}_{K+L+\Delta k}\right]_m \qquad (14)\\
=\ & \frac{e^{-\frac{j\pi}{4}}}{\sqrt{N}} e^{j\frac{\pi}{N}(K+L+\Delta k-m)^2},\\
=\ & \frac{e^{-\frac{j\pi}{4}}}{\sqrt{N}} e^{j\frac{\pi}{N}(K+L-m)^2 + j\frac{\pi}{N}(\Delta k^2+2\Delta k(K+L)-2m\Delta k)},\\
& m = 1,\ldots,N, \Delta k = 1,\ldots,N-K-L.
\end{aligned}$$

On the other hand, the sequence $\boldsymbol{\phi}_{\widetilde{k}}^T \mathbf{D}_N(w_0-w)$ is also a shifted $\boldsymbol{\phi}_{\widetilde{k}}^T$. Let us define $\widetilde{k} = K+L+\Delta\widetilde{k}$, where $\Delta\widetilde{k} = 1,\ldots,N-K-L$. Therefore, the m-th entry of $\boldsymbol{\phi}_{\widetilde{k}}^T\mathbf{D}_N(w_0-w) = \boldsymbol{\phi}_{K+L+\Delta\widetilde{k}}^T\mathbf{D}_N(w_0-w)$ is given by

$$\begin{aligned}
&\left[\boldsymbol{\phi}_{K+L+\Delta\widetilde{k}}^T\mathbf{D}_N(w_0-w)\right]_m \qquad (15)\\
=\ & \frac{e^{-\frac{j\pi}{4}}}{\sqrt{N}} e^{j(\frac{\pi}{N}(K+L+\Delta\widetilde{k}-m)^2+(w_0-w)m)},\\
=\ & \frac{e^{-\frac{j\pi}{4}}}{\sqrt{N}} e^{j\frac{\pi}{N}(K+L-m)^2}\\
& \times e^{j\frac{\pi}{N}(\Delta\widetilde{k}^2+2\Delta\widetilde{k}(K+L)-2m\Delta\widetilde{k}+\frac{N}{\pi}(w_0-w)m)},\\
& m=1,\ldots,N, \Delta\widetilde{k} = 1,\ldots,N-K-L.
\end{aligned}$$

Comparing (14) and (15), we observe that $\boldsymbol{\phi}_{K+L+\Delta\widetilde{k}}^T\mathbf{D}_N(w_0-w)$ would belong to the LNS of $\bar{\mathbf{R}}_{yy}$ if and only if the two following conditions are fulfilled at the same time

$$\begin{aligned}
&\frac{\pi}{N}(\Delta\widetilde{k}^2+2\Delta\widetilde{k}(K+L)) \bmod 2\pi\\
=\ & \frac{\pi}{N}(\Delta k^2+2\Delta k(K+L)) \bmod 2\pi,
\end{aligned} \tag{16}$$

and

$$\begin{aligned}
&\frac{2\pi}{N}m\Delta k \bmod 2\pi\\
=\ & \left(\frac{2\pi}{N}m\Delta\widetilde{k}+(w_0-w)m\right) \bmod 2\pi,
\end{aligned} \tag{17}$$

for Δk and $\Delta\widetilde{k} \in \{1,\ldots,N-K-L\}$. Recall that $(w_0-w) \in (-2\pi, 2\pi)$. It is easy to verify that the aforementioned two conditions can be satisfied simultaneously if and only if $\Delta k = \Delta\widetilde{k}$, and $(w_0-w) \bmod 2\pi = 0$, i.e., $w_0 = w$. Therefore, the CFO estimator employing the cost function (10) has a unique minimum and the CFO identifiability issue is resolved.

We conclude with the following proposition.

Proposition 1: For the cost function $J(w)$ in (10) to have a unique minimum, the insertion of at least $L+1$ consecutive null subchirps guarantees the CFO acquisition range $[-\pi,\pi)$ for an OCDM system under a multipath channel of order up to L.

C. Error Performance

Next, we explore the performance of the proposed OCDM transmission scheme with consecutive null subchirps (named OCDM-NSC) w.r.t. multipath diversity. A well known fact is that null subcarriers do not affect BER performance in OFDM w.r.t. multipath diversity. This is to say that the multipath diversity is always 1 for OFDM systems due to the inherent diagonal structure of the equivalent channel. On the other hand, null subchirps affect the OCDM performance w.r.t. multipath diversity differently. The following proposition is herein.

Proposition 2: Suppose that the number of consecutive null subchirps is greater than or equal to the channel order L, i.e., $N-K \geq L$, the OCDM-NSC scheme achieves full multipath diversity by LEs and MLE.

Proof: There is a link between an OCDM with null subchirps and a ZP-only single carrier (SC) transmission scheme. It turns out that the demodulated OCDM block with null subchirps also assumes the same mathematical form as the ZP-only SC block under multipath channels:

$$\begin{aligned}
\mathbf{z}(i) &= \mathbf{\Phi}\mathbf{y}(i)\\
&= \mathbf{\Phi}\mathbf{H}\mathbf{\Phi}^{\mathcal{H}}\mathbf{T}_{\text{zp}}\mathbf{s}(i)+\mathbf{n}(i)\\
&= \mathbf{H}\mathbf{T}_{\text{zp}}\mathbf{s}(i)+\mathbf{n}(i),
\end{aligned} \tag{18}$$

where $\mathbf{H} = \mathbf{\Phi}\mathbf{H}\mathbf{\Phi}^{\mathcal{H}}$ by the property of the Fresnel matrix and the circulant matrix. It is observed in [16] that ZP-only SC transmission with an equivalent tall Toeplitz channel matrix $\mathbf{H}\mathbf{T}_{zp}$ enables full multipath diversity with both LEs and MLE by providing a better channel matrix condition. OCDM-NSC after demodulation has the equivalent channel model. Therefore, this conclusion applies to OCDM-NSC. ■

It is worth noting that the spectral efficiency for the proposed OCDM-NSC is $K/(N+L)$. When the OCDM-NSC scheme achieves full diversity based on Proposition 2, its spectral efficiency is no greater than $(N-L)/(N+L)$.

IV. Simulations

In this section, we illustrate the performance results of the proposed method through simulations. In particular, the covariance matrix $\mathbf{R}_{yy}$ is calculated empirically across N_b blocks as:

$$\hat{\mathbf{R}}_{yy} = \frac{1}{N_b}\sum_{i=1}^{N_b}\mathbf{y}(i)\mathbf{y}^{\mathcal{H}}(i). \tag{19}$$

For all the simulations, we set that $N_b = 1000$ in (19). This is a sufficiently large empirical average to approximate $\mathbf{R}_{yy}$. The SNR is defined as E_s/σ^2. Multipath channels of order $L=2$ with Rayleigh fading coefficients are employed. The block size is $N=16$. We set $K=12$ such that $N-K$ (i.e., 4) null subchirps are inserted, and $N-K-L-1$ (i.e., 1) null subchirps are employed for the proposed CFO estimator. The QPSK modulation is adopted.

To evaluate the performance of the CFO estimation and verify its identifiability, we define the mean square error

(MSE) as $\sum_{q=1}^{Q}(w_0^{(q)} - \hat{w}_0^{(q)})^2/Q$, where Q is the number of Monte Carlo (MC) runs, and $w_0^{(q)}$ and $\hat{w}_0^{(q)}$ are the true and estimated CFO for the q-th MC run, respectively. As our proposed CFO estimator is pilot-free, we opt to use the CP-based CFO estimator [5] for baseline comparison instead of the pilot-aided method proposed in [6]. The performance of the CP-based CFO estimator depends on the amount of training data used for cross-correlation, and one can expect the performance to be related with the length of the excessive CP and the number of transmission blocks. Here we choose a CP of length $L_{cp} = 4 > (L+1)$ and 1000 transmission blocks for the CP-based CFO estimator for a fair comparison.

Although the CP-based CFO estimator [5] is in a closed form, it faces the identifiability issue when $|w_0| > \pi/N$. The comparison results are shown in Fig. 1, where three cases of $w_0 \in [-0.05\pi, 0.05\pi)$, $w_0 \in [-0.1\pi, 0.1\pi)$, and $w_0 \in [-\pi, \pi)$ are presented, respectively. Due to the data-aided nature of the cross-correlation based methods, the CP-based CFO estimator outperforms the proposed one, in the first case, where $-\pi/N \leq w_0 < \pi/N$. In contrast, when $|w_0| > \pi/N$, the CFO identifiability issue arises for the CP-based CFO estimator. The ambiguity error dominates its performance and results in an error floor. Since the identifiability issue is well addressed for the proposed CFO estimator, its MSE does not depend on the true CFO, and the proposed estimator remains robust for the full range of the CFO, i.e., $w_0 \in [-\pi, \pi)$.

We remark here that calculating the empirical covariance matrix in (18) requires a computational complexity of $\mathcal{O}(N_b N^2)$. To solve the minimization problem in (9), the complexity scales with $\mathcal{O}(N_c K N^2)$, where N_c is the number of candidate CFOs. Thus, the total complexity of the proposed CFO estimator is $\mathcal{O}((N_c K + N_b)N^2)$. In comparison, the method in [5] has a complexity of $\mathcal{O}(N_b(L_{cp} - L))$ for the cross-correlation. Moreover, our proposed CFO estimator can work well in complement with the CP-based CFO estimator. The proposed method addresses the identifiability issue and achieves a coarse CFO compensation. The CP-based method [5] can subsequently carries out a finer tuning. The resulting two-step CFO compensation scheme reaps both high accuracy and robustness against a full range CFO.

Next, we demonstrate the BER performance of the OCDM-NSC system with the proposed CFO estimator in Fig. 2. The LEs specified in (3) and (4) are applied, respectively. The true CFO w_0 is randomly generated following a uniform distribution in the range of $[-\pi, \pi)$ for each MC run. The performance of the OFDM system with $N-K$ null subcarriers using the CFO estimator in [10], and the one of the OCDM system with the CP-based CFO estimation method [5] are also shown in Fig. 2, respectively. Evidently from Fig. 2, the null subchirps enable the multipath diversity of the OCDM-NSC system, and all three equalizers for OCDM-NSC collect full diversity, verifying Proposition 2. With this setup, OFDM asymptotically has unit diversity, whereas the performance of OCDM with the CP-based CFO estimation method suffers from the identifiability ambiguity. In this regard, we have also testified that the proposed CFO estimator guarantees the CFO identifiability.

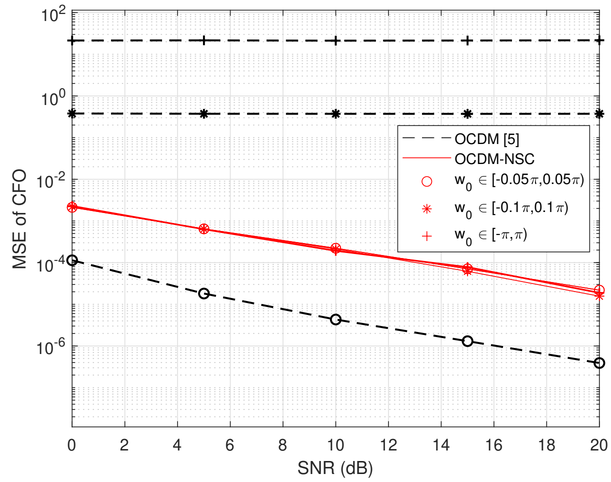

Fig. 1. MSEs of CFO estimators versus SNR.

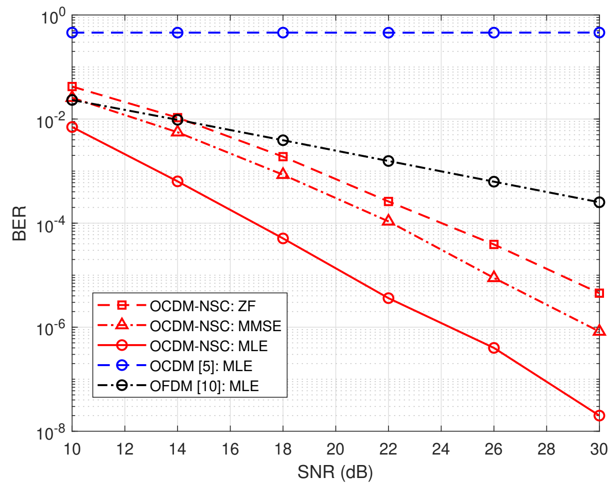

Fig. 2. BER performance with the proposed CFO estimator.

V. Conclusion

This paper investigates the CFO identifiability problem in OCDM systems. We propose to insert consecutive null subchirps to facilitate the CFO estimation. These null subchirps restore the CFO identifiability by creating channel independent null subspace to overcome adverse channel conditions. A CFO estimator is accordingly proposed to achieve a full acquisition range. It has also been demonstrated that the OCDM system with consecutive null subchirps enables better performance than its plain counterpart. Finally, the CFO identifiability for the proposed estimator is validated through simulations. Performance comparisons are made against other methods to show the advantages of the proposed OCDM-NSC.

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