0.1 MU-Communication

Definition 0.1

The up-link received signal is:

$$y[m] = \sum_{k=1}^{K} x_k[m] + w[m] \tag{1}$$

With user k having power constraint P_k

Definition 0.2

The general K-user up-link capacity is:

$$\sum_{k \in S} R_k < log(1 + \frac{\sum_{k \in S} P_k}{N_0}) \quad for \quad all \quad S \subset 1, ..., K \tag{2}$$

☞ Remark

The right hand of the equation is a capacity region with $2^k - 1$ constraints. i.e. The sum rate of all individual users cannot exceed the sum capacity of the system. The K! corner points are achieved by SIC, one corner point for each cancellation order. At receiver the cancellation order is always to decode weaker users before decoding its own data.

Definition 0.3

The down-link received signal for each user is

$$y_k[m] = h_k x[m] + w_k[m] \tag{3}$$

with $|h1| \leq |h2| \leq \cdots \leq |h_K|$

Definition 0.4

The boundary of the capacity region is given by rate tuples:

$$R_k = log(1 + \frac{P_k |h_k|^2}{N_0 + (\sum_{j=k+1}^{K})|h_k|^2}) \tag{4}$$

☞ Remark

For all possible splits $P = \sum_k P_k$ is the total power at the base-station and is achieved with super-position coding at the transmitter

Definition 0.5

The complex base-band representation of the up-link flat fading channel with K users:

$$y[m] = \sum_{k=1}^{K} h_k[m] x_k[m] + w[m] \tag{5}$$

where $h_k[m]_m$ is the fading process of user k.

Consider the slow fading up-link, where the time-scale of communication is short relative to the coherence time interval for all users. i.e. $h_k[m] = h_k$ for all m. The standard up-link AWGN channel with received SNR of user k equal to $|h_k|^2P/N_0$.

Definition 0.6

The probability of the outage even can be written as

$$p_{out}^{ul} = P(log(1 + SNR\sum_{k\in S}|h_k|^2) < |S|R, \quad for \quad some \quad S \subset 1, \dots, K) \tag{6}$$

☞ Remark

For the non-fading case, there is no channel gain due to fading (At least I hope that's what is going on).

Now turn to fast fading scenario where each $h_k[m]_m$ is modelled as a time-varying ergodic process.

Definition 0.7

The sum capacity of the up-link fast fading channel can be expressed as:

$$C_{sum} = E[log(1 + \frac{\sum_{k=1}^{K}|h_k|^2P}{N_0})] \tag{7}$$

Definition 0.8

The sum capacity with water-filling is

$$C_{sum} = E[log(1 + \frac{P_{k*}(h)|h_{k*}|^2}{N_0}] \tag{8}$$

☞ Remark

$k^*(h)$ is the index of the user with strongest channel at joint channel state h.

Definition 0.9

The down-link fading channel with K users has the following expression:

$$y_k[m] = h_k[m]x[m] + w_k[m], \quad k = 1, \dots, K \tag{9}$$

where $h_k[m]_m$ is the channel fading process of user k

Definition 0.10

With Channel state information at receiver only, analogous to AWGN down-link analysis, we can obtain

$$\sum_{k=1}^{K} R_k < E[log(1 + \frac{|h|^2 P}{N_0})] \tag{10}$$

Definition 0.11

With full channel side information, and with water-filling solution, the sum capacity of down link is:

$$E[log(1 + \frac{P^*(h)(max_{k=1,\ldots,K}|h_k^2|)}{N_0})] \tag{11}$$

0.2 MU-MIMO

We begin by investigating the narrow-band time-invariant up-link with each user having a single transmit antenna and BS equipped with an array of antennas.

Definition 0.12

The base-band model is

$$y[m] = \sum_{k=1}^{K} h_k x_k[m] + w[m] \tag{12}$$

The SDMA capacity region, for the multiple receive antenna case, is the natural extension:

$$R_1 < log(1 + \frac{||h_1||^2 P_1}{N_0}) \tag{13}$$

$$R_2 < log(1 + \frac{||h_2||^2 P_2}{N_0}) \tag{14}$$

$$R_1 + R_2 < logdet(I_{n_\tau} + \frac{1}{N_0} H K_x H^*) \tag{15}$$

Where $K_x = diag(P1, P2)$.

Definition 0.13

The capacity region is now a K-dimensional polyhedron: the set of rates (R1, ..., R_k) such that

$$\sum_{k \subset S} R_k < logdet(I_{n_r} + \frac{1}{N_0} \sum_{k \subset S} P_k h_k h_k^*), \quad for \quad each \quad S \in 1, \ldots, K \tag{16}$$

The slow fading model for every user k, $h_k[m] = h_k$ for all time m.

Definition 0.14

The probability of the outage even is:

$$p_{out}^{ul-mimo} = P(logdet(I_{n_r} + SNR\sum_{k\subset S} h_k h_k^* < |S|R, \quad for \quad some \quad S \in 1,\dots,K) \tag{17}$$

Assuming receiver CSI, the sum capacity is

Definition 0.15

$$C_{sum} = \max_{p_k(h1,h2),k=1,2} E[logdet(I_{n_r} + \frac{1}{N_0} H K_x H^*] \tag{18}$$

Now we move on to MU-MIMO up-link, where both the BS and MS are equipped with multiple antennas.

Definition 0.16

The MIMO up-link time-invariant channel is an extension of equation (12):

$$y[m] = \sum_{k=1}^{K} H_k x_k[m] + w[m] \tag{19}$$

where H_k is now a fixed n_r by n_{tk} matrix.

Take and simplest case and assume 2 users, the rates R_1, R_2 achieved by the transceiver architecture must now satisfy the constraints as:

$$R_k \le logdet(I_{n_r} + \frac{1}{N_0} H_k K_{xk} H_k^*) \tag{20}$$

$$R_1 + R_2 \le logdet(I_{n_r} + \frac{1}{N_0}\sum_{k=1}^{2} H_k K_{xk} H_k^*) \tag{21}$$

We then include the fast-fading channel, what is an extension of equation 19 as:

Definition 0.17

The fast-fading channel MU-MIMO is the following:

$$y[m] = \sum_{k=1}^{K} H_k[m] x_k[m] + w[m] \tag{22}$$

The channel variation $H_k[m]_m$ are independent across users k and stationary and ergodic in time m.

The rate tuples for two users can be updated as:

$$R_k \le E[logdet(I_{n_r} + \frac{1}{N_0} H_k K_{xk} H_k^*)] \tag{23}$$

$$R_1 + R_2 \leq E[logdet(I_{n_r} + \frac{1}{N_0}\sum_{k=1}^{2} H_k K_{xk} H_k^*)] \tag{24}$$

We now turn to the down-link MU-MIMO channel, assuming the users only has, for now, a single antenna and transmitter is equipped with multiple antennas.

Definition 0.18

The base-band model of the narrow-band down-link with base-station having n_t antennas and K users with each user having a single receive antenna is

$$y_k[m] = h_k^* x[m] + w_k[m], \quad k = 1, \ldots, K \tag{25}$$

where h_k^* is an n_t dimensional row vector representing the channel from the base-station to user k. The transmitted signal is:

$$x[m] = \sum_{k=1}^{K} \tilde{x_k}[m] u_k \tag{26}$$

The form follows from linear pre-coding.

With transmit beam-forming vector u_k, the received signal of user k is given by:

$$y_k[m] = (h_k^* u_k)\tilde{x}_k[m] + \sum_{j<k}(h_k^* u_j)\tilde{x}_j[m] + \sum_{j>k}(h_k^* u_j)\tilde{x}_j[m] + w_k[m] \tag{27}$$

Whereas, we apply Costa pre-coding for user k and treat the interference from users 1 to k-1 as known and from users k+1 to K as Gaussian Noise, the rate that user k gets is:

$$R_k = log(1 + SINR_k) \tag{28}$$