0.1 MIMO-OFDM Property

Definition 0.1

A MIMO-OFDM channel with N sub-carriers can be viewed as N independent MIMO channels over a flat fading channel.

Remark

A M by K (M transmit and K receiver antennas) MIMO channel with cyclic prefix at transmitter and guard removal at receiver has the following equivalent expression:

$$\tilde{y} = \tilde{D}\tilde{x} + \tilde{n} \tag{1}$$

where $\tilde{D}$ is the K*N by M*N equivalent channel matrix given by

$$\tilde{D} = \begin{pmatrix} FH^{1,1}F^H & \dots & FH^{1,M}F^H \\ \vdots & \ddots & \vdots \\ FH^{K,1}F^H & \dots & FH^{K,M}F^H \end{pmatrix} \tag{2}$$

Remark

$\tilde{x}$ is an aggregated input vector with size M*N, which in other words can be viewed as M transmit antennas each with N sub-carriers, since each element of the $\tilde{D}$ matrix is in fact a diagonal matrix, which means each sub-carrier is independent of other sub-carrier from the same antenna. The only data streams that will cause interference for a sub-carrier are all the sub-carriers with the same index from other antennas.

Definition 0.2

We have for the nth receiver sub-carrier

$$y_n^k = \sum_{m=1}^{M} D_n^{(n,m)} u_n^m \tag{3}$$

Remark

This means we can now apply pre-equalization to sub-carriers independently. However, it may be surprising that the overall pre-equalization matrix is still in the form of an aggregated channel inversion matrix.

In fact, the overall equalization matrix takes the form

$$\tilde{S} = \begin{pmatrix} S^{(1,1)} & \dots & S^{(K,1)} \\ \vdots & \ddots & \vdots \\ S^{(1,M)} & \dots & S^{(K,M)} \end{pmatrix} \tag{4}$$

Each element is a N by N diagonal matrix, The overall matrix expression $\tilde{S}$ is given by (For ZF)

$$\tilde{S} = \tilde{D}^H(\tilde{D} * \tilde{D}^H)^{-1} \tag{5}$$

Definition 0.3

Note, however, this does not mean each element of $\tilde{S}$ is an equivalent inversion of the diagonal matrix in $\tilde{D}$.

☞ **Remark**

Then what is the relation between aggregated channel matrix inversion and code independently across each sub-carriers? We investigate by looking at a 2 by 2 MIMO with 4 sub-carriers. The aggregated channel matrix and overall equalization matrix are of the following:

$$D = \begin{pmatrix} D_1^{1,1} & 0 & 0 & 0 & D_1^{1,1} & 0 & 0 & 0 \\ 0 & D_2^{1,1} & 0 & 0 & 0 & D_2^{2,1} & 0 & 0 \\ 0 & 0 & D_3^{1,1} & 0 & 0 & 0 & D_3^{2,1} & 0 \\ 0 & 0 & 0 & D_4^{1,1} & 0 & 0 & 0 & D_4^{2,1} \\ D_1^{1,2} & 0 & 0 & 0 & D_1^{2,2} & 0 & 0 & 0 \\ 0 & D_2^{1,2} & 0 & 0 & 0 & D_2^{2,2} & 0 & 0 \\ 0 & 0 & D_3^{1,2} & 0 & 0 & 0 & D_3^{2,2} & 0 \\ 0 & 0 & 0 & D_4^{1,2} & 0 & 0 & 0 & D_4^{2,2} \end{pmatrix} \tag{6}$$

$$S = \begin{pmatrix} S_1^{1,1} & 0 & 0 & 0 & S_1^{2,1} & 0 & 0 & 0 \\ 0 & S_2^{1,1} & 0 & 0 & 0 & S_2^{2,1} & 0 & 0 \\ 0 & 0 & S_3^{1,1} & 0 & 0 & 0 & S_3^{2,1} & 0 \\ 0 & 0 & 0 & S_4^{1,1} & 0 & 0 & 0 & S_4^{2,1} \\ S_1^{1,2} & 0 & 0 & 0 & S_1^{2,2} & 0 & 0 & 0 \\ 0 & S_2^{1,2} & 0 & 0 & 0 & S_2^{2,2} & 0 & 0 \\ 0 & 0 & S_3^{1,2} & 0 & 0 & 0 & S_3^{2,2} & 0 \\ 0 & 0 & 0 & S_4^{1,2} & 0 & 0 & 0 & S_4^{2,2} \end{pmatrix} \tag{7}$$

Where subscript means the sub-carrier, so for instance, for sub-carrier one, we can find the relationship between received signal and transmitted signal as (Assuming without pre-coding matrix):

$$\begin{pmatrix} y_1^1 \\ y_1^2 \end{pmatrix} = \begin{pmatrix} D_1^{1,1} & D_1^{1,1} \\ D_1^{1,2} & D_1^{2,2} \end{pmatrix} \begin{pmatrix} x_1^1 \\ x_1^2 \end{pmatrix} \tag{8}$$

This means the received sub-carrier one on both receive antennas only depends on sub-carrier one from both transmit antenna.

Definition 0.4

The inversion of

$$\begin{pmatrix} D_1^{1,1} & D_1^{1,1} \\ D_1^{1,2} & D_1^{2,2} \end{pmatrix} \tag{9}$$

is in fact

$$\begin{pmatrix} S_1^{1,1} & S_1^{1,1} \\ S_1^{1,2} & S_1^{2,2} \end{pmatrix} \tag{10}$$

☞ **Remark**

This means that the sub-carriers can be in fact independently equalized and the result will be an aggregated channel inversion matrix.