Mach-Zehnder Interferometer – Final Report

Yaron Kodriano

I. Introduction

A Mach-Zehnder Interferometer (MZI) is an optical device used for the measurement of phase shifts between two light beams. The incoming optical field is split between two arms and recombined at the output. The phase difference of the two arms is determined by the optical path difference which is affected by path length difference and effective index difference.

The MZI is a versatile device that can be use for splitters, modulators, switches and more. This report proposes a layout design to measure waveguide properties from varying MZI parameters, in this the path difference, ΔL.

II. Theory

The unbalanced Mach–Zehnder interferometer is shown in *Figure 1*, consisting of two Y-branch splitters to split and combine two waveguides path at different length.

Figure 1: Mach–Zehnder interferometer, layout example.[1]

Considering an input electrical field to the first Y-branch, E_i, the electrical fields at the entrance of the second is described by:

$$(1)\ E_{o1} = \frac{E_i}{\sqrt{2}} e^{-i\beta_1 L_1 - \frac{\alpha_1}{2} L_1},$$

$$(2)\ E_{o2} = \frac{E_i}{\sqrt{2}} e^{-i\beta_2 L_2 - \frac{\alpha_2}{2} L_2},$$

where, $\beta_{1,2}$ is the propagation constant of each arm, $L_{1,2}$ is the length and $\alpha_{1,2}$ is the propagation loss. For simplicity we will neglect losses, i.e., $\alpha_1 = \alpha_2 = 0$. Also, the waveguides in the proposed layout have similar properties, thus $\beta_1 = \beta_2 = \beta$.

The output field at the second Y-branch is given by:

$$(3)\ E_0 = \frac{E_{o1} + E_{o2}}{\sqrt{2}} = \frac{E_i}{2}\left(e^{-i\beta L_1} + e^{-i\beta L_2}\right),$$

and the output intensity is given by:

$$(4)\ I_0 = \frac{I_i}{4}\left|e^{-i\beta L_1} + e^{-i\beta L_2}\right|^2 = \frac{I_i}{2}[1 + \cos(\beta \Delta L)],$$

where $\Delta L = L_1 - L_2$ is the path length difference of the two waveguides.

III. Modeling and Simulation

A. *Waveguide Design*

The wave used in the proposed layout is a strip waveguide of 500 nm width and 220 nm height. The waveguide is designed for light at wavelength of 1550 nm and support only two guided modes at this wavelength. We will use the first guided mode which is quasi-TE polarized. *Figure 2* displays the electrical field of the TE mode using Lumerical MODE simulation.

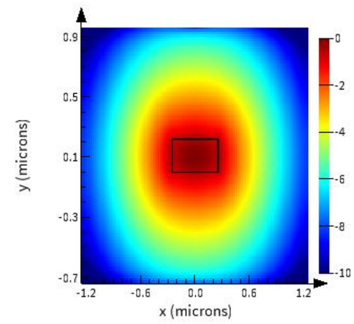

Figure 2:Electrical field intensity for the TE mode, logarithmic scale.

Figure 3 displays the calculated effective index versus wavelength for the TE mode. The effective index can be approximated using a second order Taylor expansion around a central wavelength, λ_0, as:

[1] Chrostowski L, Hochberg M. *Silicon Photonics Design: From Devices to Systems*. Cambridge University Press; 2015

$$(5)\ n_{eff}(\lambda) \approx n_1 + n_2(\lambda - \lambda_0) + n_2(\lambda - \lambda_0)^2,$$

with

$$n_{eff} = n_1$$
$$n_g = n_1 - n_2 \cdot \lambda_0$$
$$D = -2 \cdot \lambda_0 \cdot \frac{n_3}{c} [s/m^2]$$

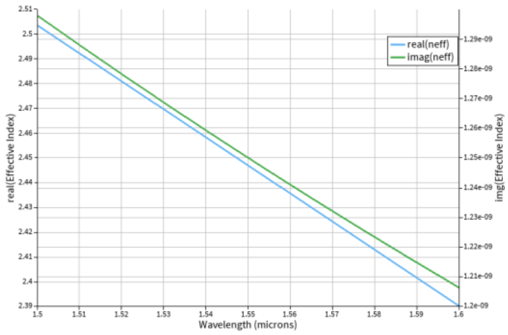

Figure 3: Effective index vs. Wavelength of the strip waveguide discussed.

The approximation around λ_0=*1.55* µm for the data in *Figure 3* is given by:

$$(6)\ n_{eff} \approx 2.442 \ - 1.13 \cdot (\lambda - \lambda_0) - 0.0416 \cdot (\lambda - \lambda_0)^2$$

Figure 4 displays the group index versus wavelength.

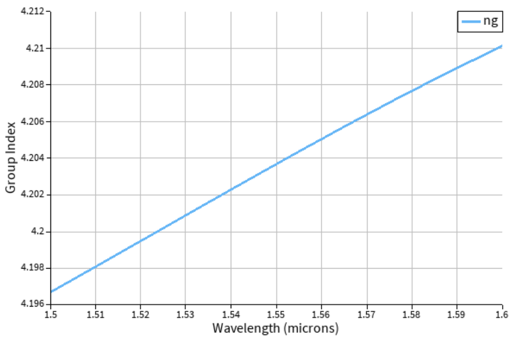

Figure 4: Group index vs. Wavelength of the strip waveguide discussed.

There is a good agreement between the extracted data and the approximation.

B. Mach-Zehnder Interferometer

The optical transfer function of a Mach-Zehnder interferometer is given by

$$(7)\ T(\lambda) = \frac{I_o}{I_i} = \frac{1}{2}[1 + \cos(\beta \Delta L)] = \frac{1}{2}\left[1 + \cos\left(\frac{2\pi n_g}{\lambda} \Delta L\right)\right]$$

Where I_o, I_i are respectively the input and output light intensities, and we assume constant and uniform temperature.

The free spectral range (FSR) is defined as the distance between two adjacent peaks in the electrical field intensity, hence:

$$(8)\ FSR = \frac{\lambda^2}{n_g \Delta L}$$

IV. Layout Proposal

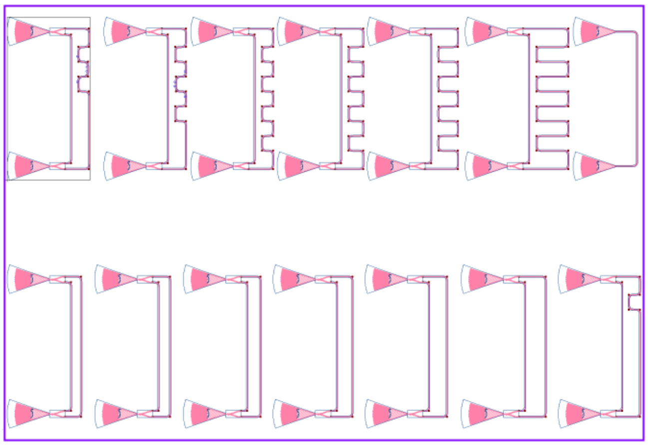

Figure 5: Layout proposal

The layout proposal in *Figure 5* is constructed from 14 un-balanced Mach-Zehnder interferometers with different path difference and loopback to de-embed the grating couplers spectral response.

Table 1 lists the path difference of the MZIs proposed and their calculated FSRs.

Table 1: path differences of the proposed MZIs and their corresponding FSR

	ΔL [µm]	FSR [nm]	ΔFSR [nm]
MZI1	30	19.1	0
MZI2	33	17.3	1.7
MZI3	36	15.9	1.4
MZI4	40	14.3	1.6
MZI5	45	12.7	1.6
MZI6	50	11.4	1.3
MZI7	60	9.5	1.9
MZI8	70	8.2	1.4
MZI9	85	6.7	1.4
MZI10	105	5.4	1.3
MZI11	140	4.1	1.4
MZI12	185	3.1	1.0
MZI13	300	1.9	1.2

The path differences were chosen such that the FSR difference from path to path would be on approximately 1-2 nm. The minimal path difference was chosen to be approximately 20 nm to have several FSRs in a single MZI transmission spectrum. The maximal was chosen to give an FSR of 2 nm, that will not impose high resolution requirement for the measurement.

Figure 6 display the FSR versus the path length. The circles denote the values of the proposed layout.

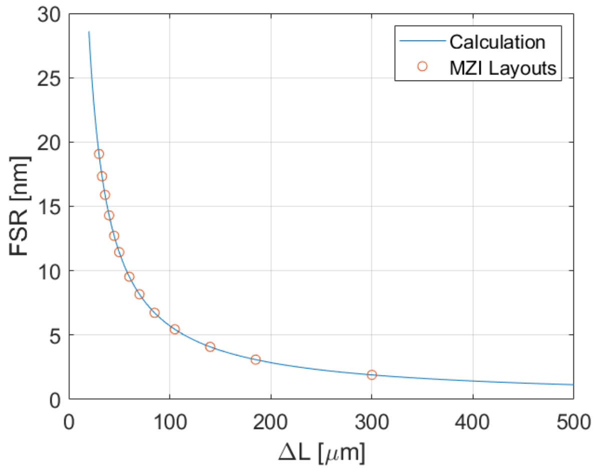

Figure 6: FSR vs. Path difference of an unbalanced MZI. The circles denote the path differences for the MZIs in the proposed layout and their corresponding FSR

V. MEASUREMENTS & ANALYSIS

The spectrum of each of the devices described in the previous section was measured. The spectrum of the loopback enables to de-embed the grating couplers spectral response. *Figure 7* display the transmission spectrum and a 4-degree polynomial fit used to de-embed grating couplers response.

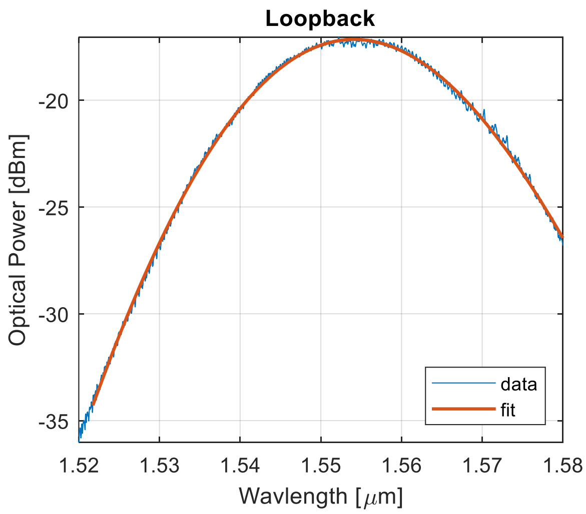

Figure 7: Loopback device spectrum (blue) and fitted spectrum used for de-embedding (red)

For each measurement the fitted loopback response was subtracted and the resonance detected by MATLAB findpeaks. *Figure 8* display the de-embedded spectrum of MZI10 device along with the observed resonances.

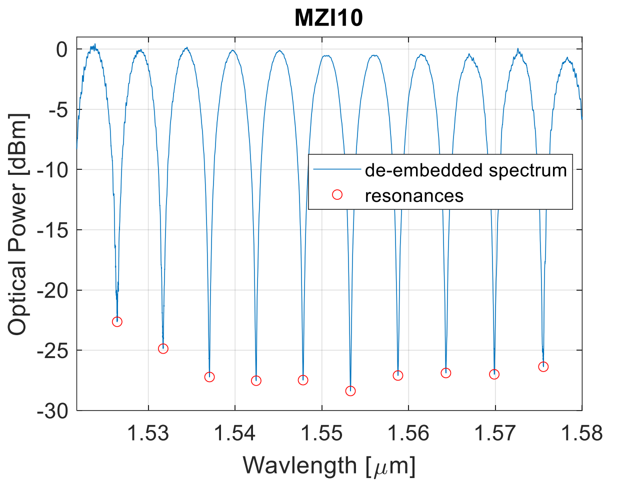

Figure 8; De-embedded spectrum of MZI10 device and observed resonances

The FSR was calculated for each pair of resonances and the group index was calculated according to Eq. (8). The results for MZI10 device are displayed in *Figure 9*.

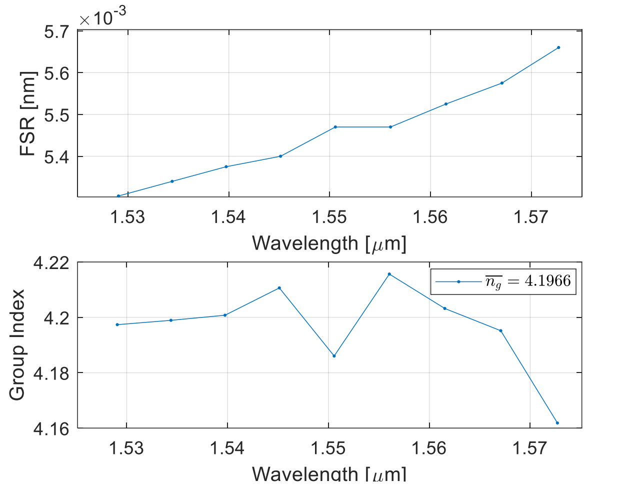

Figure 9: Calculated FSR (top) and group index (bottom) for the data presented in Figure 8

For each measurement the de-embedded data was fitted to:

$$F = 10\log_{10}\left(\frac{1}{4}\left|1+\exp\left[-i\frac{2\pi n_{eff}}{\lambda}\Delta L-\frac{\alpha\Delta L}{2}\right]\right|^{2}\right)+b$$

Where α represents absorption and b represents the excess insertion loss. The results for MZI10 are displayed in Figure 10.

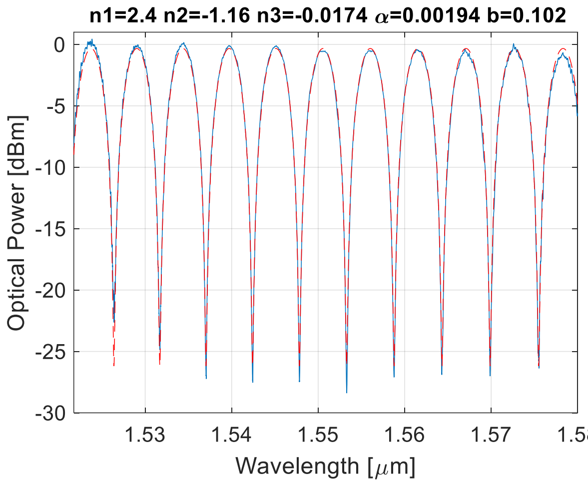

Figure 10

Figure 11 displays box plots for the fitted values of n_1, n_2, & n_3.

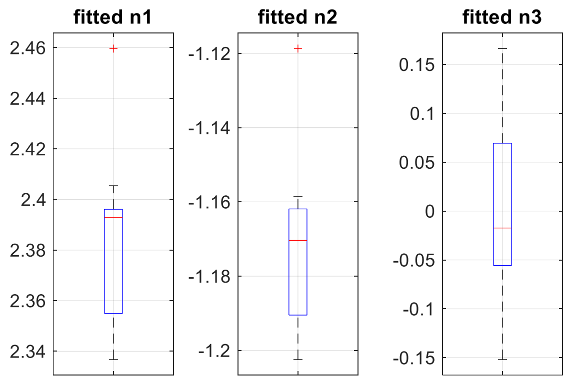

Figure 11

Taking the median for all devices measurement gives an empirical value for n_{eff}

$$n_{eff} = 2.392 - 1.17 \cdot (\lambda - \lambda_0) - 0.0173 \cdot (\lambda - \lambda_0)^2$$

VI. Conclusions

In this report I presented devices layouts of several MZIs used to evaluate empirically the effective index of the fabricated waveguide. The model and simulations were displayed, and the measured transmission spectrum was used to give an empirical value.