Design proposal for a Mach-Zehnder Interferometer

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1. INTRODUCTION

A Mach-Zehnder Interferometer (MZI) is a device that exploits the interference between optical fields by changing the relative phase shift between the two arms of the design. It is a fundamental building block in silicon photonic design, and can be used in building switches, splitters and many other components.

We are to propose a design of our own MZI and simulate with our parameters.

2. THEORY

In the context of this course, we can look at a MZI as consisting of a Y-branch splitter which divides your input light into 2 arms, followed by a Y-branch combiner which then combines your optical fields. In between these 2 components, we have waveguides that determine our respective path lengths. Consider this outline of an MZI with the description above.
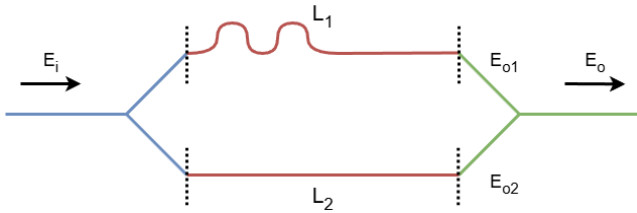

Figure 1. Basic outline of an imbalanced Mach-Zehnder interferometer.

Suppose we have an input electric field $E = E_i e^{-i(\beta z - \omega t)}$, with the propagation $\beta = 2\pi/(n_{eff}\lambda)$. This field is passed through the Y-branch splitter (blue) and has propagated its respective path length (red) to arrive at the Y-branch combiner (green), given that an ideal 50/50 splitter divides light equally, and the $I = |E|^2$ relationship, the resulting electric fields is then

$$E_{o1} = \frac{E_i}{\sqrt{2}} e^{-i\beta_1 L_1 - \frac{\alpha_1}{2}L_1} \tag{1}$$

$$E_{o2} = \frac{E_i}{\sqrt{2}} e^{-i\beta_2 L_2 - \frac{\alpha_2}{2}L_2} \tag{2}$$

Notes
- we are omitting the time dependent phase term ωt.
- α is real and represents the power attenuated per unit length.

The output of the Y-branch combiner is then the sum of the electric fields divided by $\sqrt{2}$.

$$E_o = \frac{1}{\sqrt{2}}(E_{o1} + E_{o2}) \tag{3}$$

which then leads us to this expression

$$E_o = \frac{E_i}{2}\left(e^{-i\beta_1 L_1 - \frac{\alpha_1}{2}L_1} + e^{-i\beta_2 L_2 - \frac{\alpha_2}{2}L_2}\right) \tag{4}$$

with intensity being expressed as

$$I_o = \frac{I_i}{4}\left|e^{-i\beta_1 L_1 - \frac{\alpha_1}{2}L_1} + e^{-i\beta_2 L_2 - \frac{\alpha_2}{2}L_2}\right|^2 \tag{5}$$

From this, we consider lossless propagation, with identical waveguides leading to equivalent propagation constants $\beta = \beta_1 = \beta_2$ and expression for intensity as

$$I_o = \frac{I_i}{2}[1 + \cos(\beta \Delta L)] \tag{6}$$

Equation (6) represents the transfer function for an **imbalanced interferometer**. A balanced interferometer would have both path lengths equal, where the resulting constructive and destructive interference would need to come from a variation in propagation constants β i.e. $\Delta\beta$ instead of ΔL.

Additionally, from the imbalanced MZI transfer function, we can determine the spacing between adjacent peaks. It is an indication at which wavelengths show constructive and destructive interference. This measurement is termed the free spectral range (FSR) and shows dependence on both the group index and path length. For an imbalanced MZI, as light propagates the length of its respective arm once, this gives us

$$FSR = \frac{\lambda^2}{|L_1 - L_2|n_g} \tag{7}$$

and group index

$$n_g = n_{eff} - \lambda \frac{dn_{eff}}{d_\lambda} \tag{8}$$

3. MODELLING AND SIMULATIONS

We will be using a strip waveguide of width $500nm$ and height $220nm$ in this course. Our simulation parameters are centered at $1550nm$ and we will be just considering the first quasi-TE polarized light. The waveguide mode profile is presented below.
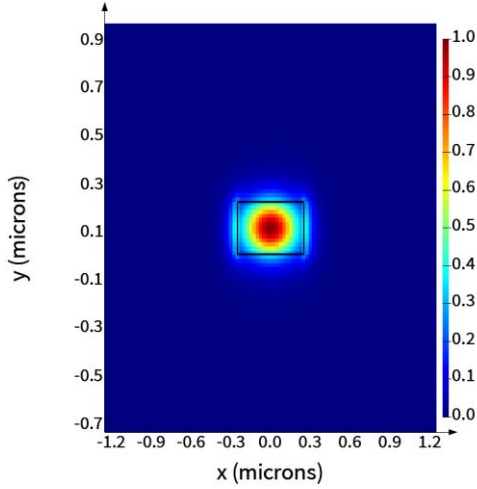

Figure 2. Waveguide mode profile showing electric field intensity of first quasi-TE mode.

Performing a wavelength sweep from $1550nm$ to $1650nm$ on our first quasi-TE mode, we find that the effective index decreases with wavelength while our group index increases.
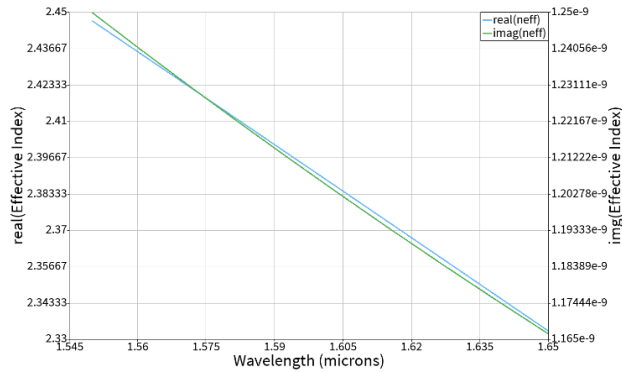

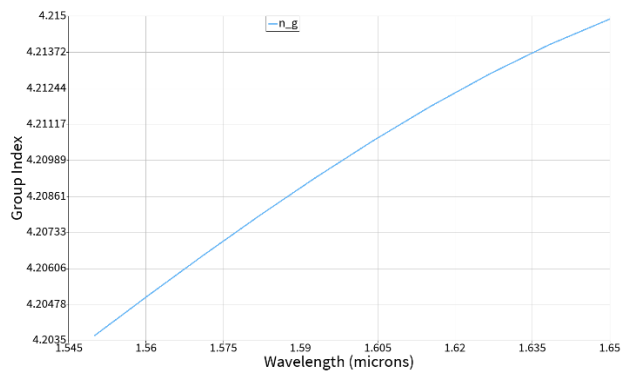
Figure 3. Effective index vs Wavelength

Figure 4. Group index vs Wavelength

The compact model of this waveguide can then be calculated in MATLAB and expressed as

$$n_{eff}(\lambda) = 2.4468 - 1.1338(\lambda - 1.55) - 0.0368(\lambda - 1.55) \quad (9)$$

where wavelength λ is in microns (μm).

4. IMBALANCED INTERFEROMETER

We are looking at testing different path lengths and quantifying our expectations by both calculating our FSR and simulating the FSR from INTERCONNECT (note that these simulated values are approximate values taken from the plot of FSR vs Wavelength).

For $1550nm$, and a group index $n_g = 4.204$ we get

$\Delta L \ [um]$	calc. FSR [nm]	sim. FSR [nm]
80	7.143	7.00
100	5.715	5.72
130	4.396	4.40
140	4.082	4.07
220	2.598	2.58

We are using the EBeam library in our INTERCONNECT simulations. The components in this library are modelled after real experimental data. Attached are the transfer functions of the Y-Branch and fibre grating coupler that were used.
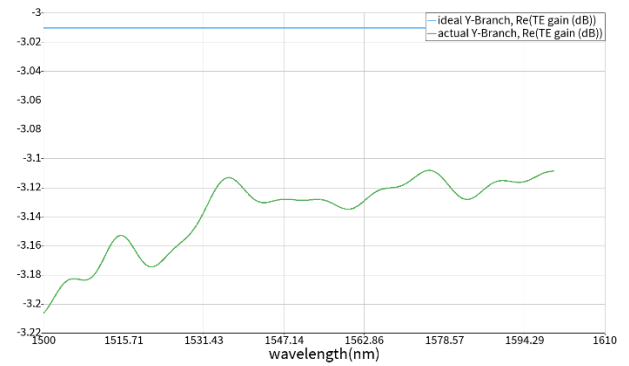

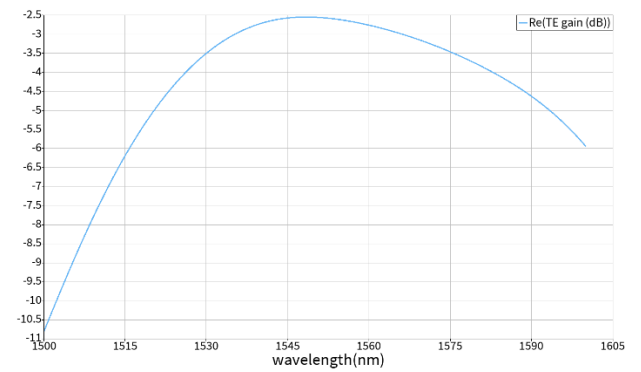*Figure 5. Transfer function of an ideal and typical Y-branch.*

Figure 6. Transfer function of fibre grating coupler.

As an example, we look at the MZI configuration with $\Delta L=140um$. Here we show the transfer function (or gain) of the MZI as well as the transmission spectrum.
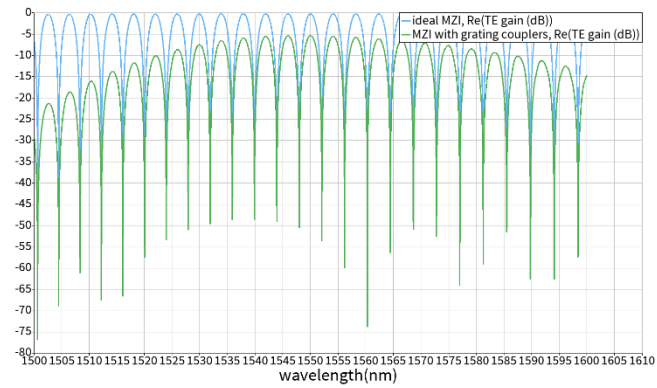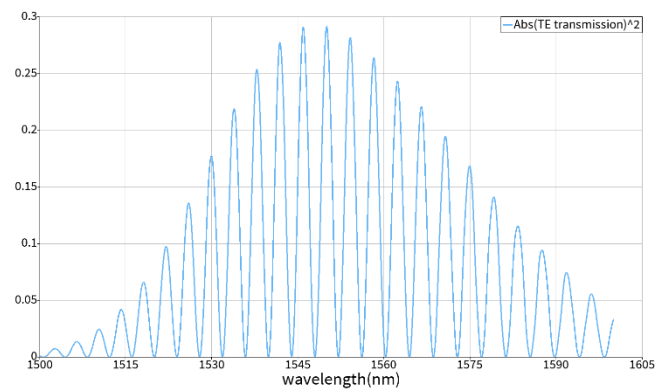

Figure 7. MZI output spectrum.

Figure 8. MZI transmission spectrum.