Convolutional Neural Networks

Most of this material is from Prof. Andrew Ng'and Chang's slides.

Computer Vision Problems

Image Classification

→ Cat? (0/1)

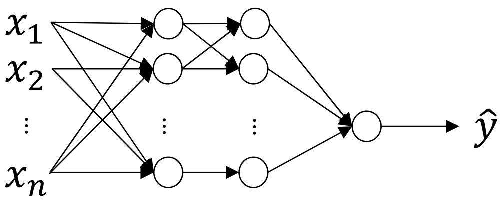
64x64

Object detection

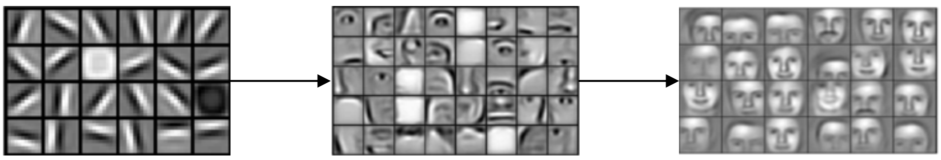
Neural Style Transfer

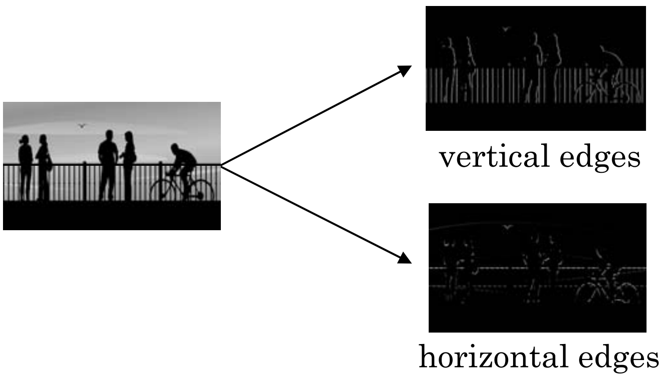
Deep Learning on large images

64x64



Computer Vision Problem





Vertical edge detection

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

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0	-2	-4	-7
-3	-2	-3	-16

Vertical edge detection

10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0

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1	0	-1
1	0	-1
1	0	-1

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0	30	30	0
0	30	30	0
0	30	30	0
0	30	30	0

Vertical edge detection examples

10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0

$*$

1	0	-1
1	0	-1
1	0	-1

$=$

0	30	30	0
0	30	30	0
0	30	30	0
0	30	30	0

0	0	0	10	10	10
0	0	0	10	10	10
0	0	0	10	10	10
0	0	0	10	10	10
0	0	0	10	10	10
0	0	0	10	10	10

$*$

1	0	-1
1	0	-1
1	0	-1

$=$

0	-30	-30	0
0	-30	-30	0
0	-30	-30	0
0	-30	-30	0

Vertical and Horizontal Edge Detection

1	0	-1
1	0	-1
1	0	-1

Vertical

1	1	1
0	0	0
-1	-1	-1

Horizontal

10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
0	0	0	10	10	10
0	0	0	10	10	10
0	0	0	10	10	10

$*$

1	1	1
0	0	0
-1	-1	-1

$=$

0	0	0	0
30	10	-10	-30
30	10	-10	-30
0	0	0	0

Learning to detect edges

1	0	-1
1	0	-1
1	0	-1

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

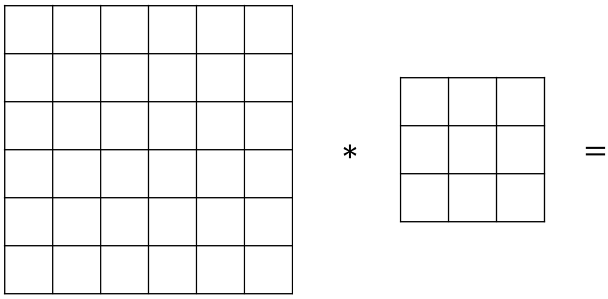
w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

Padding

$*$

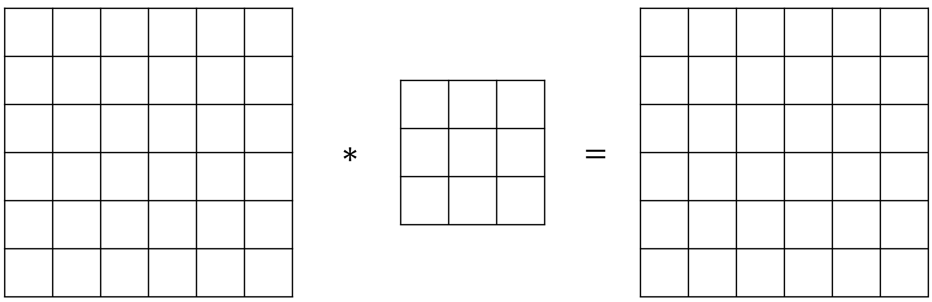
$=$

Padding



input: $n \times n$ filter: $f \times f$ output: $(n-f+1) \times (n-f+1)$

Padding



input: $n \times n$ filter: $f \times f$ output: $(n-f+1) \times (n-f+1)$

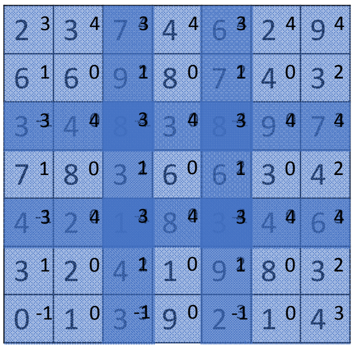
If we use padding p $\Rightarrow$ output: $(n+2p-f+1) \times (n+2p-f+1)$

Valid and Same convolutions

"Valid": No padding

"Same": Pad so that output size is the same as the input size.

Strided convolution

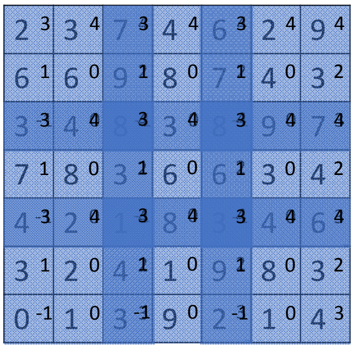


$*$

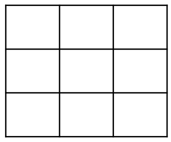
3	4	4
1	0	2
-1	0	3

$=$

Strided convolution



$$* \begin{bmatrix} 3 & 4 & 4 \\ 1 & 0 & 2 \\ -1 & 0 & 3 \end{bmatrix} = $$



$$\left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor \times \left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor$$

Summary of convolutions

$n \times n$ image $\qquad f \times f$ filter

padding p $\qquad$ stride s

$$\left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor \times \left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor$$

Technical note on cross-correlation vs. convolution

Convolution in math textbook:

2	3	7	4	6	2
6	6	9	8	7	4
3	4	8	3	8	9
7	8	3	6	6	3
4	2	1	8	3	4
3	2	4	1	9	8

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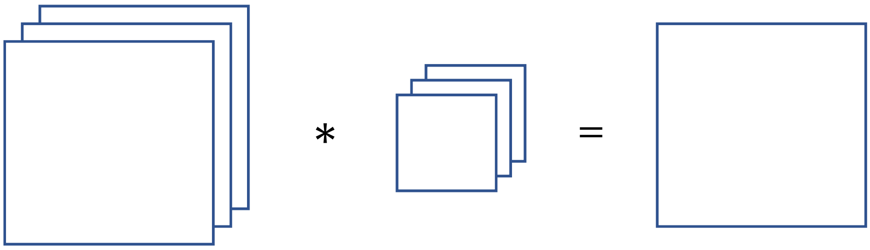
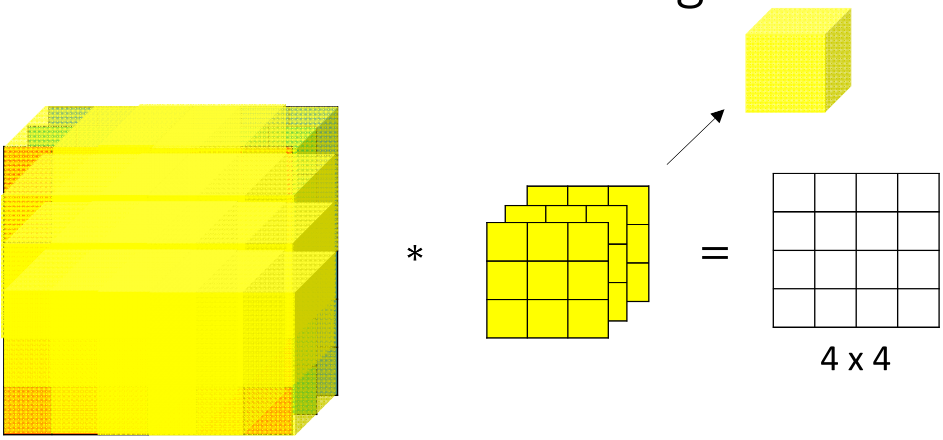
3	4	5
1	0	2
-1	9	7

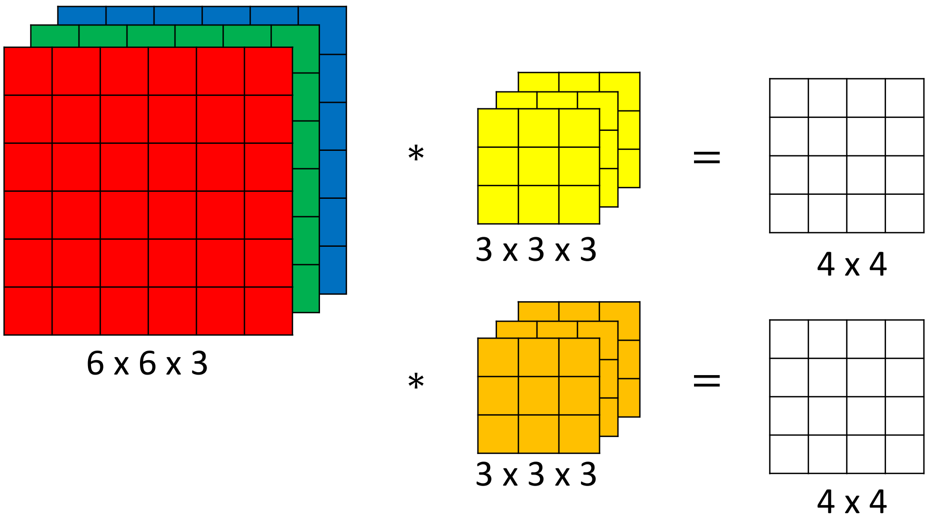
Convolutions on RGB images



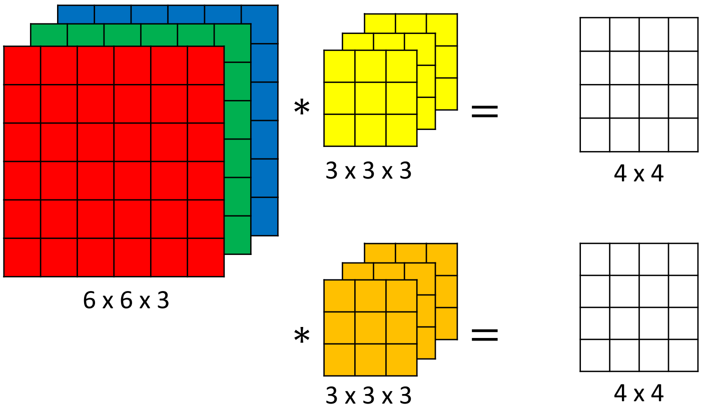
Convolutions on RGB images



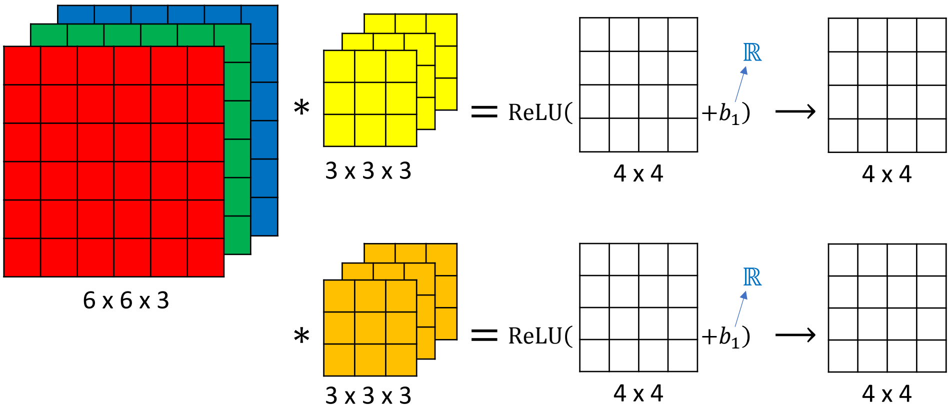
Multiple filters



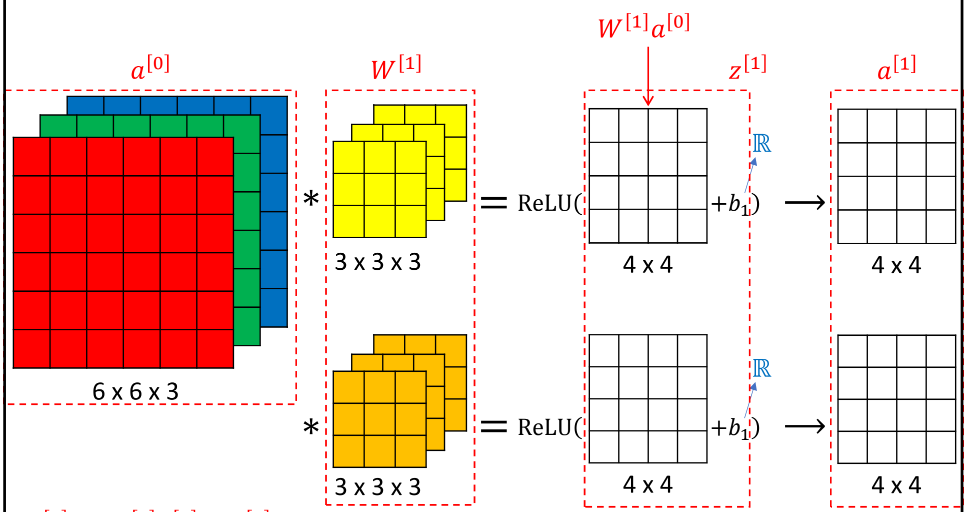
Example of a layer



Example of a layer



Example of a layer



$z^{[1]} = W^{[1]}a^{[0]} + b^{[1]}$

$a^{[1]} = g(z^{[1]})$

Number of parameters in one layer

- If you have 10 filters that are 3 x 3 x 3 in one layer of a neural network, how many parameters does that layer have?

Summay of notation

If layer l is a convolution layer:

$f^{[l]}$ = filter size

$p^{[l]}$ = padding

$s^{[l]}$ = stride

Input: $n_H^{[l-1]} \times n_W^{[l-1]} \times n_C^{[l-1]}$

Output: $n_H^{[l]} \times n_W^{[l]} \times n_C^{[l]}$

Summay of notation

If layer l is a convolution layer:

$f^{[l]}$ = filter size

$p^{[l]}$ = padding

$s^{[l]}$ = stride

$n_C^{[l]}$ = number of filters

Input: $n_H^{[l-1]} \times n_W^{[l-1]} \times n_C^{[l-1]}$

Output: $n_H^{[l]} \times n_W^{[l]} \times n_C^{[l]}$

$$n_H^{[l]} = \left\lfloor \frac{n_H^{[l-1]} + 2p^{[l]} - f^{[l]}}{s^{[l]}} + 1 \right\rfloor$$

$$n_W^{[l]} = \left\lfloor \frac{n_W^{[l-1]} + 2p^{[l]} - f^{[l]}}{s^{[l]}} + 1 \right\rfloor$$

Summay of notation

If layer l is a convolution layer:

$f^{[l]}$ = filter size

$p^{[l]}$ = padding

$s^{[l]}$ = stride

$n_C^{[l]}$ = number of filters

Each filter is $f^{[l]} \times f^{[l]} \times n_C^{[l-1]}$

Input: $n_H^{[l-1]} \times n_W^{[l-1]} \times n_C^{[l-1]}$

Output: $n_H^{[l]} \times n_W^{[l]} \times n_C^{[l]}$

$$n_H^{[l]} = \left\lfloor \frac{n_H^{[l-1]} + 2p^{[l]} - f^{[l]}}{s^{[l]}} + 1 \right\rfloor$$

$$n_W^{[l]} = \left\lfloor \frac{n_W^{[l-1]} + 2p^{[l]} - f^{[l]}}{s^{[l]}} + 1 \right\rfloor$$

Summay of notation

If layer l is a convolution layer:

$f^{[l]}$ = filter size

$p^{[l]}$ = padding

$s^{[l]}$ = stride

$n_C^{[l]}$ = number of filters

Each filter is $f^{[l]} \times f^{[l]} \times n_C^{[l-1]}$

Activations: $a^{[l]} \to n_H^{[l]} \times n_W^{[l]} \times n_C^{[l]}$

Input: $n_H^{[l-1]} \times n_W^{[l-1]} \times n_C^{[l-1]}$

Output: $n_H^{[l]} \times n_W^{[l]} \times n_C^{[l]}$

$$n_H^{[l]} = \left\lfloor \frac{n_H^{[l-1]} + 2p^{[l]} - f^{[l]}}{s^{[l]}} + 1 \right\rfloor$$

$$n_W^{[l]} = \left\lfloor \frac{n_W^{[l-1]} + 2p^{[l]} - f^{[l]}}{s^{[l]}} + 1 \right\rfloor$$

Summay of notation

If layer l is a convolution layer:

$f^{[l]}$ = filter size

$p^{[l]}$ = padding

$s^{[l]}$ = stride

$n_C^{[l]}$ = number of filters

Each filter is $f^{[l]} \times f^{[l]} \times n_C^{[l-1]}$

Activations: $a^{[l]} \rightarrow n_H^{[l]} \times n_W^{[l]} \times n_C^{[l]}$

$A^{[l]} \rightarrow m \times n_H^{[l]} \times n_W^{[l]} \times n_C^{[l]}$

Input: $n_H^{[l-1]} \times n_W^{[l-1]} \times n_C^{[l-1]}$

Output: $n_H^{[l]} \times n_W^{[l]} \times n_C^{[l]}$

$$n_H^{[l]} = \left\lfloor \frac{n_H^{[l-1]} + 2p^{[l]} - f^{[l]}}{s^{[l]}} + 1 \right\rfloor$$

$$n_W^{[l]} = \left\lfloor \frac{n_W^{[l-1]} + 2p^{[l]} - f^{[l]}}{s^{[l]}} + 1 \right\rfloor$$

Summay of notation

If layer l is a convolution layer:

$f^{[l]}$ = filter size

$p^{[l]}$ = padding

$s^{[l]}$ = stride

$n_C^{[l]}$ = number of filters

Each filter is $f^{[l]} \times f^{[l]} \times n_C^{[l-1]}$

Activations: $a^{[l]} \rightarrow n_H^{[l]} \times n_W^{[l]} \times n_C^{[l]}$

$A^{[l]} \rightarrow m \times n_H^{[l]} \times n_W^{[l]} \times n_C^{[l]}$

Weights: $f^{[l]} \times f^{[l]} \times n_C^{[l-1]} \times n_C^{[l]}$

Bias: $n_C^{[l]}$

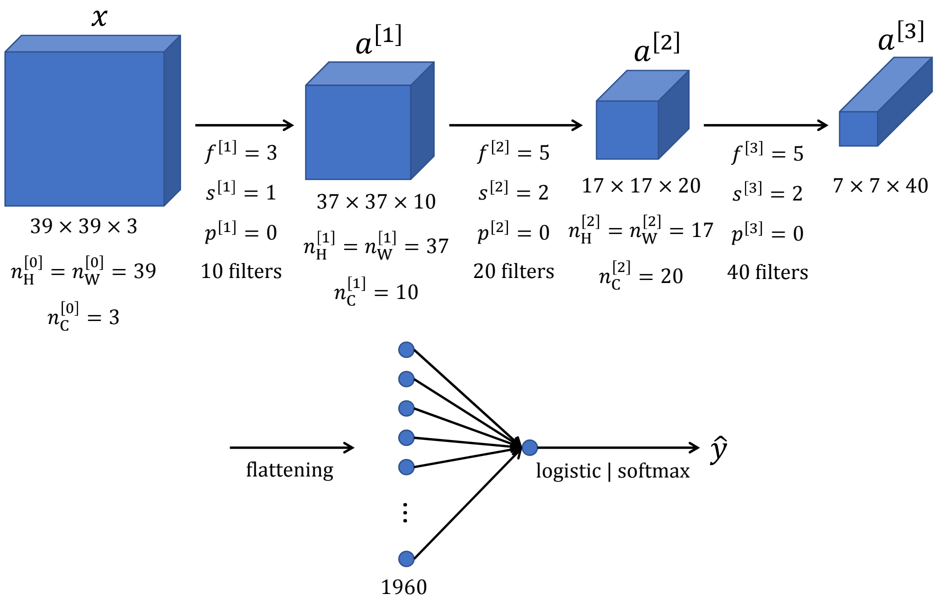
Input: $n_H^{[l-1]} \times n_W^{[l-1]} \times n_C^{[l-1]}$

Output: $n_H^{[l]} \times n_W^{[l]} \times n_C^{[l]}$

$$n_H^{[l]} = \left\lfloor \frac{n_H^{[l-1]} + 2p^{[l]} - f^{[l]}}{s^{[l]}} + 1 \right\rfloor$$

$$n_W^{[l]} = \left\lfloor \frac{n_W^{[l-1]} + 2p^{[l]} - f^{[l]}}{s^{[l]}} + 1 \right\rfloor$$

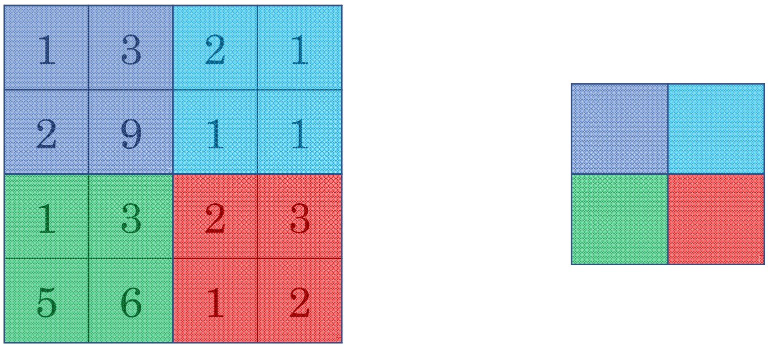
Example ConvNet



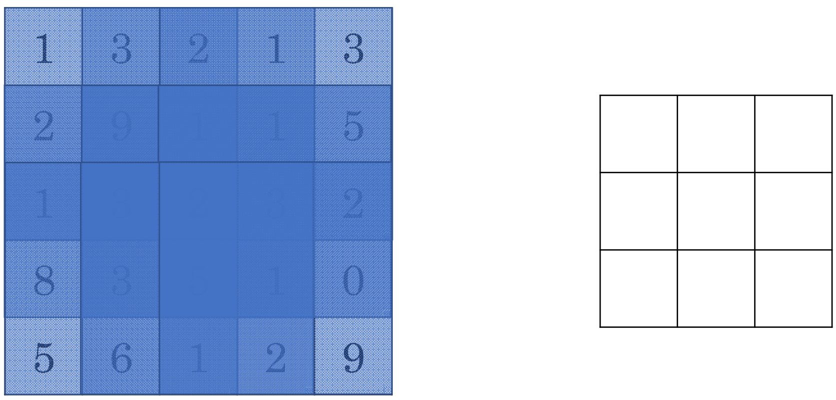
Types of layer in a convolutional network

- Convolution (CONV)
- Pooling (POOL)
- Fully connected (FC)

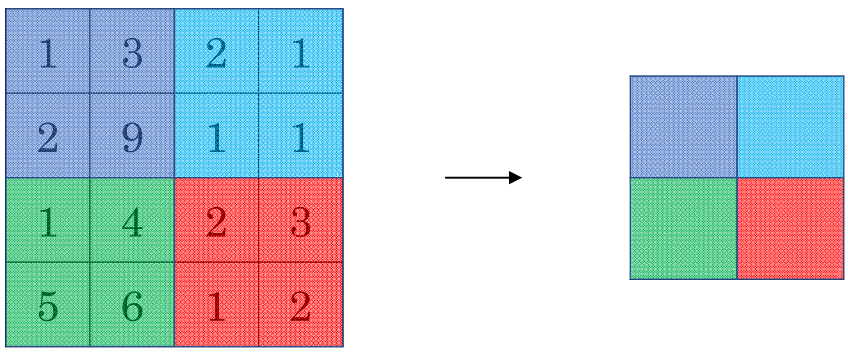
Pooling layer: Max pooling



Pooling layer: Max pooling



Pooling layer: Average pooling



Summary of pooling

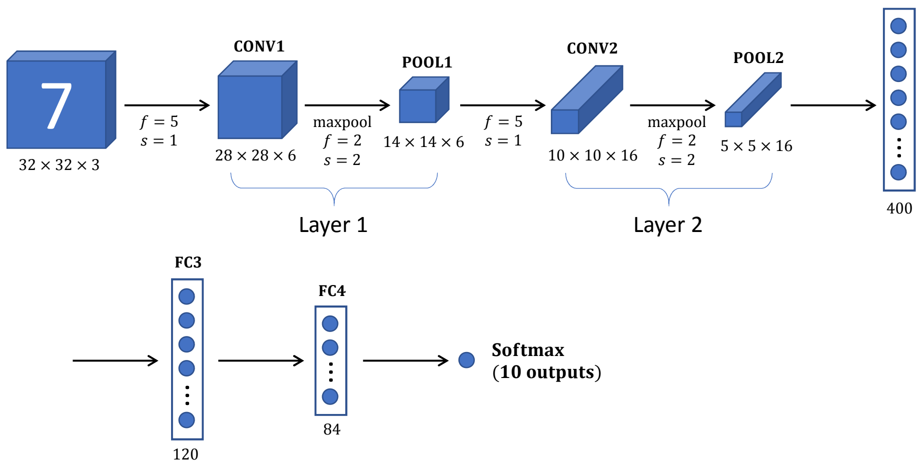
Hyperparameters:

f : filter size

s : stride

Max or average pooling

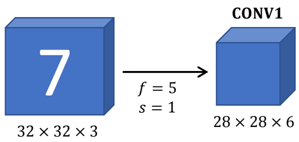
Neural network example (LeNet-5)



Neural network example (LeNet-5)

	Activation shape	Activation Size	# parameters
Input:	(32,32,3)	3,072	0
CONV1 (f=5, s=1)	(28,28,8)	6,272	208
POOL1	(14,14,8)	1,568	0
CONV2 (f=5, s=1)	(10,10,16)	1,600	416
POOL2	(5,5,16)	400	0
FC3	(120,1)	120	48,120
FC4	(84,1)	84	10,164
Softmax	(10,1)	10	850

Why convolutions?



Why convolutions?

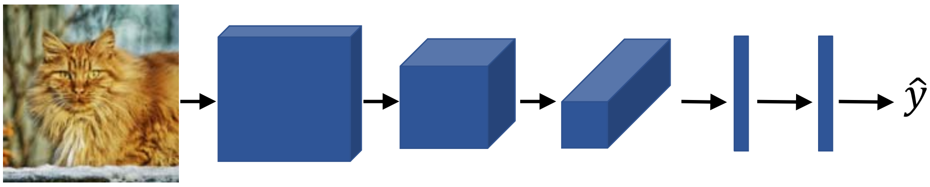
$$\begin{bmatrix} 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \\ 10 & 10 & 10 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \\ 0 & 30 & 30 & 0 \end{bmatrix}$$

Parameter sharing: A feature detector (such as a vertical edge detector) that's useful in one part of the image is probably useful in another part of the image.

Sparsity of connections: In each layer, each output value depends only on a small number of inputs.

Putting it together

Training set $(x^{(1)}, y^{(1)}) \ldots (x^{(m)}, y^{(m)})$.



Cost $J = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$

Use gradient descent to optimize parameters to reduce J