

# 自动控制原理

## CH5. 线性系统的频域分析法

### 5.2 基本概念:

$$x(t) = X \sin \omega t$$

$$y_s(t) = A(\omega) X \sin[\omega t + \varphi(\omega)]$$

$$\begin{cases} G(j\omega) = A(\omega) e^{j\varphi(\omega)} \\ G(j\omega) = P(\omega) + jQ(\omega) \end{cases} \begin{cases} A(\omega) = \sqrt{P(\omega)^2 + Q(\omega)^2} \\ \varphi(\omega) = \arctan \frac{Q(\omega)}{P(\omega)} \end{cases} \begin{cases} P(\omega) = A(\omega) \cdot \cos \varphi(\omega) \\ Q(\omega) = A(\omega) \cdot \sin \varphi(\omega) \end{cases}$$

### 5.3 对数坐标图

比例环节:  $G(j\omega) = K$

$$L(\omega) = 20 \lg |K|$$

$$\varphi(\omega) = \angle |K| = \begin{cases} 0 & K > 0 \\ -180^\circ & K < 0 \end{cases}$$

延迟环节:  $G(j\omega) = e^{-j\omega T}$

$$L(\omega) = 0$$

$$\varphi(\omega) = -57.3 \omega T^\circ$$

积分环节:  $G(j\omega) = \frac{1}{j\omega}$

$$L(\omega) = -20 \lg \omega$$

$$\varphi(\omega) = -90^\circ$$

微分环节:  $G(j\omega) = j\omega$

$$L(\omega) = 20 \lg \omega$$

$$\varphi(\omega) = 90^\circ$$

惯性环节:  $G(j\omega) = \frac{1}{1+j\omega T}$

$$L(\omega) \approx \begin{cases} 0 & \omega \ll \frac{1}{T} \\ -20 \lg \omega T & \omega \gg \frac{1}{T} \end{cases}$$

$$\varphi(\omega) = -\arctan(\omega T)$$

一阶微分:  $G(j\omega) = 1+j\omega T$

$$L(\omega) \approx \begin{cases} 0 & \omega \ll \frac{1}{T} \\ 20 \lg \omega T & \omega \gg \frac{1}{T} \end{cases}$$

$$\varphi(\omega) = \arctan(\omega T)$$

振荡环节:  $G(j\omega) = \frac{1}{(1-T^2\omega^2) + j2\zeta T\omega}$

$$L(\omega) \approx \begin{cases} 0 & \omega \ll \frac{1}{T} \\ -40 \lg \omega T & \omega \gg \frac{1}{T} \end{cases}$$

$$\varphi(\omega) = -\arctan \frac{2\zeta T\omega}{1-T^2\omega^2}$$

二阶微分:  $G(j\omega) = 1-T^2\omega^2 + j2\zeta T\omega$

$$L(\omega) \approx \begin{cases} 0 & \omega \ll \frac{1}{T} \\ 40 \lg \omega T & \omega \gg \frac{1}{T} \end{cases}$$

$$\varphi(\omega) = \arctan \frac{2\zeta T\omega}{1-T^2\omega^2}$$

#### 5.4 极坐标图

· 绘制:  $\omega \rightarrow 0: G(j\omega) = \frac{K}{(j\omega)^v}$

$$A(\omega) = \frac{K}{\omega^v}$$

$$\varphi(\omega) = -v \cdot \frac{\pi}{2} \quad (\omega \rightarrow 0)$$

$$\omega \rightarrow \infty: A(\omega) \rightarrow 0$$

$$\varphi(\omega) = -(n-m) \frac{\pi}{2}$$

$(n > m) \uparrow$

$$\lim_{\omega \rightarrow \infty} \angle G(j\omega) = -(n-m) \frac{\pi}{2}$$

· 增加零极点的影响: 见上.

#### 5.5 奈奎斯特稳定判据

·  $F(s) = 1 + G_K(s) \rightarrow F(s)$  的零点是闭环的极点, 极点是开环的极点.

$N = Z - P$   $Z$ :  $F(s)$  零点在右半的个数: 闭环特征根在右半

$P$ :  $F(s)$  极点 (开环极点) 在右半的个数: 开环特征根在右半

闭环稳定:  $Z = 0$  开环稳定:  $P = 0$

$N$ :  $F(s)$  包围原点 (顺),  $G_K(s)$  包围  $(-1, j0)$  (顺)

#### 5.6 稳定裕度:

$$A(\omega_c) = 1, \quad \varphi = 180^\circ + \varphi(\omega_c)$$

$$\varphi(\omega_g) = -180^\circ, \quad K_g = \frac{1}{A(\omega_g)} \rightarrow \text{闭环系统不稳定时的最大开环增益}$$

$$L_g = 20 \lg K_g$$

#### 5.7 闭环系统的频率特性.

$$\Phi(j\omega) = \frac{G(j\omega)}{1 + G(j\omega)}$$

$$M(\omega) = |\Phi(j\omega)| \quad \alpha(\omega) = \angle \Phi(j\omega)$$

$$M_p, \omega_p, \omega_b: M(\omega_b) = \frac{\sqrt{2}}{2} M(\omega)$$

$$M(\omega_p) = M_p = \max M(\omega)$$



### 5.8. 闭环系统性能分析:

#### 1. 开环频率特性分析系统稳态:

$$L(j\omega)|_{\omega=1} = 20 \lg K, \quad K = \omega_0^v$$

#### 2. 闭环幅频特性分析:

单位阶跃:  $C(\infty) = M(0), \quad e_{ssr} = 1 - C(\infty) = 1 - M(0)$

$$v=0 \text{ 时, } M(0) = \frac{K}{1+K}, \quad e_{ssr} = \frac{1}{1+K}$$

$$v \geq 1 \text{ 时, } M(0) = 1, \quad e_{ssr} = 0$$

#### 3. 频率与时间的反比:

$$\Phi_1(j\omega) = \Phi_2(j\frac{\omega}{\alpha}) \quad (\alpha > 0) \Rightarrow h_1(t) = h_2(\alpha t)$$

#### 4. 典型二阶系统开环频域性能与瞬态性能:

$$\zeta = 0.01 \text{ 时, } t_s = \begin{cases} \frac{4}{\zeta \omega_n} & \Delta = 2 \\ \frac{3}{\zeta \omega_n} & \Delta = 5 \end{cases} = \begin{cases} \frac{8}{\omega_c} \cdot \frac{1}{\tan \varphi} \\ \frac{8.6}{\omega_c} \cdot \frac{1}{\tan \varphi} \end{cases}$$

#### 5. 欠阻尼 闭环频域与瞬态性能:

$$\omega_p = \omega_n \sqrt{1 - \zeta^2}, \quad M_p = \frac{1}{2\zeta \sqrt{1 - \zeta^2}} \quad \star$$

$$\omega_b = \omega_n \sqrt{1 - 2\zeta^2 + \sqrt{2 - 4\zeta^2 + 4\zeta^4}}$$

## CH7. 线性系统设计方法

### 7.2. 校正装置 (基于伯德图)

超前校正:

$$G_C(s) = \frac{1 + \alpha Ts}{1 + Ts} = \frac{s + \frac{1}{\alpha T}}{s + \frac{1}{T}} \quad (\alpha > 1)$$

$$\omega_m = \frac{1}{T\sqrt{\alpha}}$$

$$\varphi_m = \arcsin \frac{\alpha - 1}{\alpha + 1}$$

高通滤波器

$$L(\omega_m) = 10 \lg \alpha$$

$$\varphi_m = \varphi^* - \varphi + 5 \sim 12^\circ$$

解得  $\alpha$

$$\hookrightarrow \omega_c'' = \omega_m \text{ 解得 } T$$

$$L'(\omega_c'') + 10 \lg \alpha = 0$$

解得  $\omega_c''$

滞后校正:

$$G_C(s) = \frac{1 + \beta Ts}{1 + Ts} = \beta \frac{s + \frac{1}{\beta T}}{s + \frac{1}{T}} \quad (\beta < 1)$$

$$\omega_m = \frac{1}{\sqrt{\beta} \cdot T} \quad \frac{1}{\beta T} = \frac{\omega_c''}{10}$$

$$\varphi_m = \arcsin \frac{1 - \beta}{1 + \beta}$$

$$\varphi = \varphi + 5 \sim 12^\circ$$

$$\text{使 } \varphi(\omega_c'') = -180^\circ + \varphi + 5 \sim 12^\circ$$

解得  $\omega_c''$

$$\hookrightarrow L'(\omega_c'') + 20 \lg \beta = 0$$

$$\hookrightarrow \text{解 } \beta \rightarrow \frac{1}{\beta T} = \frac{\omega_c''}{10} \rightarrow \text{解 } T$$

超前滞后超前

$$G_C(s) = \frac{(1 + T_\alpha s)(1 + T_\beta s)}{(1 + \frac{T_\alpha s}{\alpha})(1 + \alpha T_\beta s)}$$

$$1. \text{ 令 } \omega_c'' = \omega_g, \quad \frac{1}{T_\beta} = \frac{\omega_c''}{10}$$

$$2. \text{ 令 } \varphi_m = \varphi^*, \quad \alpha \Rightarrow \varphi_m = \arcsin \frac{\alpha - 1}{\alpha + 1}$$

$$3. \text{ 过 } (\omega_c'', L(\omega_c''))$$

以  $+20 \text{ dB/dec}$  为斜率作直线.

与  $0 \text{ dB}$  及  $-20 \lg \alpha \text{ dB}$  交线 (点) 为转折频率

$\frac{1}{T_\alpha}, \frac{\alpha}{T_\alpha}$  分别为转折频率



#### 7.4. 基于根轨迹的系统校正.

- 增加开环极点: 根轨迹向右; 增加开环零点, 根轨迹向左
- 增加偶极子: 对稳定性无影响.

超前:  $G_c(s) = \frac{s+z}{s+p}$ ,  $|z| < |p|$

1. 由  $\zeta, \omega_n$  确定主导极点:

$$\text{Re} = -\zeta\omega_n \quad \text{Im} = \omega_n\sqrt{1-\zeta^2}$$

2. 取  $z = -\text{Re}$

3. 由相位条件求  $\theta_p$ , 进而求  $p$ .

滞后:  $G_c(s) = \frac{s+z}{s+p}$ ,  $|z| > |p| \rightarrow \zeta'$  和  $K_v', \zeta'$

1. 作原系统根轨迹.

2. 由  $\zeta'$  作  $\beta = \arccos \zeta'$ , 与根轨迹交点为主极点.

3. 求出在主极点处增益  $K$ ,  $K_v = \frac{K}{z}$

3.  $\frac{|z|}{|p|} = \frac{K_v'}{K_v}$ , 取  $z, p$ .

### CH3. 时域分析法

#### 一阶系统

$$G(s) = \frac{K}{s}, \quad \Phi(s) = \frac{1}{Ts+1}$$

$$t_d = 0.693T; \quad t_r = 2.197T; \quad t_s = \begin{cases} 4T, & \Delta=2 \\ 3T, & \Delta=3 \end{cases}$$

减小T: (1) 负反馈 (2) 放大K

#### 二阶系统

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \quad \Phi(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

性能指标:

欠阻尼

$$t_r = \frac{\pi - \beta}{\omega_n \sqrt{1 - \zeta^2}}$$

$$t_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}}$$

$$t_s = \begin{cases} \frac{4}{\zeta\omega_n}, & \Delta=2 \\ \frac{3}{\zeta\omega_n}, & \Delta=3 \end{cases}$$

$$t_f = \frac{2\pi}{\omega_n \sqrt{1 - \zeta^2}}, \quad N = \frac{t_s}{t_f}$$

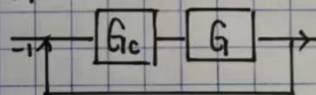
$$\delta\% = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

过阻尼

$$t_r = \frac{1 + 1.5\zeta + \zeta^2}{\omega_n}$$

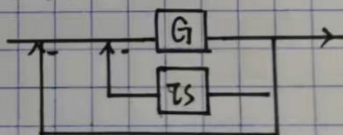
$$t_s = \begin{cases} \frac{8.4}{\omega_n}, & \Delta=2 \\ \frac{6.6}{\omega_n}, & \Delta=3 \end{cases}$$

性能改善: 1). 比例微分:  $G_c(s) = K_p + K_d s$



2). 速度反馈: 反馈  $\tau s$

不改变  $\omega_n$   
改变  $\zeta$





· 高阶系统. 略.

· 稳定性:

1. 充要条件: 没有右半平面特征根

2. 代数判据: 劳斯判据: 1) 0列:  $\varepsilon$  2) 全0行: 临界/非稳

· 稳态误差:

$$E(s) = R(s) - Y(s), \quad e_{ss} = \lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s E(s)$$

$$R(s) = \frac{A}{s}, \quad K_p = \lim_{s \rightarrow 0} G_K(s), \quad e_{ss} = \frac{A}{1+K_p}$$
$$\begin{cases} K, & v=0 \\ \infty, & v \geq 1 \end{cases} \quad \begin{cases} \frac{A}{1+K}, & v=0 \\ 0, & v \geq 1 \end{cases}$$

$$R(s) = \frac{B}{s^2}, \quad K_v = \lim_{s \rightarrow 0} s G_K(s), \quad e_{ss} = \frac{B}{K_v}$$
$$\begin{cases} 0, & v=0 \\ K, & v=1 \\ \infty, & v \geq 2 \end{cases} \quad \begin{cases} \infty, & v=0 \\ \frac{B}{K}, & v=1 \\ 0, & v \geq 2 \end{cases}$$

$$R(s) = \frac{C}{s^3}, \quad K_a = \lim_{s \rightarrow 0} s^2 G_K(s), \quad e_{ss} = \frac{C}{K_a}$$
$$\begin{cases} 0, & v \leq 1 \\ K, & v=2 \\ \infty, & v \geq 3 \end{cases} \quad \begin{cases} \infty, & v \leq 1 \\ \frac{C}{K}, & v=2 \\ 0, & v \geq 3 \end{cases}$$

# 现代控制理论

## CH1. 状态空间描述方法.

### 1.1. 状态变量模型

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du. \end{cases}$$

### 1.2. 建立方法.

1. 从物理机理: 有  $n$  个储能元件  $\rightarrow$  几个状态变量.

2. 从微分方程:

① 不含输入项子数:  $y^{(n)} + a_{n-1}y^{(n-1)} + \dots + a_1y' + a_0y = bu$

$$\begin{aligned} x_n &= y^{(n-1)} \\ \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b \end{bmatrix} u \end{aligned}$$

$$y = [1 \ 0 \ 0 \ \dots \ 0] [x_1 \ x_2 \ \dots \ x_n]^T$$

② 含输入项子数:  $y^{(n)} + a_{n-1}y^{(n-1)} + \dots + a_1y' + a_0y = b_nu + b_{n-1}u^{(1)} + \dots + b_1u' + b_0u$

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & \dots & 0 & -a_0 \\ 0 & 0 & 1 & \dots & 0 & -a_1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} b_0 - a_0b_n \\ b_1 - a_1b_n \\ \vdots \\ b_{n-1} - a_{n-1}b_n \end{bmatrix} u \end{aligned}$$

$$y = [0 \ 0 \ \dots \ 0 \ 1] [x_1 \ \dots \ x_n]^T + b_nu$$



### 3. 从传递函数

1) 直接分解:  $G(s) = \frac{Y(s)}{U(s)} = \frac{b_n s^n + b_{n-1} s^{n-1} + \dots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \dots + a_1 s + a_0}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [b_0 - a_0 b_n, b_1 - a_1 b_n, \dots, b_{n-1} - a_{n-1} b_n] [x_1 \dots x_n]^T + b_n u.$$

2) 串联、并联分解: 略.

1.3. 传递函数分解:

$$G(s) = C(sI - A)^{-1}B + D$$

$$\begin{cases} \text{串联: } G = G_1 + G_2 \\ \text{并联: } G = G_1 G_2 \\ \text{反馈: } G = G_1 - G_1 G_2 G_1 \end{cases}$$

1.4. 线性非奇异变换

1.  $\bar{A} = P^{-1}AP$ ,  $\bar{B} = P^{-1}B$ ,  $\bar{C} = CP$ ,  $\bar{D} = D$

2. 性质: 1) 不变特征值, 极点

2) 不变  $G(s)$

3) 不变能观可控.

3. 对角线型:

1) 由  $|\lambda I - A| = 0$  解  $\lambda_i$

2) 由  $(\lambda_i I - A)v_i = 0$  解  $v_i$

3)  $P = [v_1 \ v_2 \ \dots \ v_n]$

设

## CH2. 线性控制系统的运动分析

### 2.1 线性连续定常系统方程的解:

$$\begin{cases} \dot{x}(t) = A x(t) \\ x(0) = x(0) \end{cases}$$

$$e^{At} \triangleq \sum_{k=0}^{\infty} \frac{1}{k!} A^k t^k$$

$$x(t) = x(t_0) \big|_{t=t_0} = x(t_0) = e^{A(t-t_0)} x(t_0)$$

$$x(t) = \mathcal{L}^{-1}[(sI - A)^{-1}] x(0).$$

### 2.2 矩阵指数函数:

1. 性质: 1)  $A: n \times n : e^{A(t_1+t_2)} = e^{At_1} e^{At_2}$

2)  $e^{A(t-t)} = I$

3)  $e^{At}$  非奇异

4) 若  $A \times B = B \times A : e^{(A+B)t} = e^{At} e^{Bt}$

5)  $\frac{d}{dt}(e^{At}) = e^{At} A$

6)  $e^{P^{-1}APt} = P^{-1} e^{At} P, e^{At} = P e^{P^{-1}APt} P^{-1}$

7) 若  $A = \text{diag}[\lambda_1, \lambda_2, \dots, \lambda_n], e^{At} = \text{diag}[e^{\lambda_1 t}, e^{\lambda_2 t}, \dots, e^{\lambda_n t}]$

### 2. 求解:

1) 直接法:  $e^{At} = \sum_{k=0}^{\infty} \frac{1}{k!} A^k t^k$

2) 拉氏变换:  $e^{At} = \mathcal{L}^{-1}[(sI - A)^{-1}]$

3) 标准型法:  $e^{At} = P e^{\bar{A}t} P^{-1}$

4) 待定系数: 略.  $P_{50 \sim 52}$



## 2.3. 状态转移矩阵

### 1. 线性定常系统: (齐次)

$$\dot{x} = Ax : \begin{cases} x(t) = \Phi(t) x(0) \\ x(t) = \Phi(t-t_0) x(t_0) \end{cases}$$

$$\begin{cases} \Phi(t) = e^{At} \\ \Phi(t-t_0) = e^{A(t-t_0)} \end{cases}$$

条件: 1)  $\Phi(t_0-t_0) = I$

2)  $\dot{\Phi}(t-t_0) = A \Phi(t-t_0)$

### 2. 线性时变系统:

$$x(t) = \Phi(t, t_0) x(t_0)$$

### 3. 性质:

1)  $\Phi(t_0-t_0) = I$

$$\Phi(t_0, t_0) = I$$

2)  $\dot{\Phi}(t-t_0) = A \Phi(t-t_0)$

$$\dot{\Phi}(t, t_0) = A(t) \Phi(t, t_0)$$

3)  $\Phi(t_0-t_0) \Phi(t_0-t_0) = \Phi(t_0-t_0)$

$$\Phi(t_0, t_0) \Phi(t_0, t_0) = \Phi(t_0, t_0)$$

4)  $\Phi^{-1}(t-t_0) = \Phi(t_0-t)$

$$\Phi^{-1}(t, t_0) = \Phi(t_0, t)$$

5)  $\Phi(t_1+t_0) = \Phi(t_0) \Phi(t_1) = \Phi(t_1) \Phi(t_0)$

6)  $[\Phi(t)]^n = \Phi(nt)$

### 4. 计算:

1)  $\Phi(t) = e^{At}$

2)  $\dot{\Phi}(t-t_0) = A \Phi(t-t_0)$  且  $\Phi(t-t_0) = I$

$$\therefore A = \dot{\Phi}(t-t_0) \big|_{t=t_0}$$

### 2.4. 非齐次: $\dot{x} = Ax + Bu$

$$\begin{aligned} x(t) &= \Phi(t-t_0) x(t_0) + \int_{t_0}^t \Phi(t-\tau) B u(\tau) d\tau \\ &= e^{A(t-t_0)} x(t_0) + \int_{t_0}^t e^{A(t-\tau)} B u(\tau) d\tau \end{aligned}$$

$$x(s) = \mathcal{L}[(sI-A)^{-1} x(0) + (sI-A)^{-1} B U(s)]$$

### CH3. 能控性和能观性

#### 能控性

#### 能观性

一. 判别:

$$1. Q_c = [B \ AB \ A^2B \ \dots \ A^{n-1}B]$$

$$\text{rank}(Q_c) = n$$

2. A化为对角阵 $\bar{A}$ 后

$\bar{B}$ 中无全0行

3. A化为约当阵 $\bar{A}$

$\bar{B}$ 中每个若当块最后一列不全0

$$Q_o = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

$$\text{rank}(Q_o) = n$$

A化为 $\bar{A}$ 后

$\bar{C}$ 中无全0列

A化为 $\bar{A}$ 后

$\bar{C}$ 中每块第一列不全0

4. s平面判据:

$$G(s) = C(sI - A)^{-1}B \quad \text{若对消, 可能 } \bar{0}, \bar{C}, \text{ or } \bar{0}$$

分子分母无零极点抵消  $\rightarrow 0 + C$

二. 标准型

$$P_c = [b, Ab, \dots, A^{n-1}b] = Q_c$$

$$P_o^{-1} = Q_o = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

$$P_c = [A^{n-1}b, A^{n-2}b, \dots, b]$$

$$P_o^{-1} = \begin{bmatrix} 1 & a_{n-1} & \dots & a_2 & a_1 \\ 0 & 1 & \dots & 0 & a_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & a_{n-1} \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix} \begin{bmatrix} CA^{n-1} \\ CA^{n-2} \\ \vdots \\ CA \\ C \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ a_{n-1} & 1 & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_2 & 0 & & 0 \\ a_1 & a_2 & \dots & 1 \end{bmatrix}$$

三. 对偶原理:

$$\text{定义} \begin{cases} A_1 = A_2^T \\ B_1 = C_2^T \\ C_1 = B_2^T \end{cases}$$

$$\text{性质} \begin{cases} G_1(s) = G_2(s)^T \\ f_1(u) = f_2(u) \\ \sum_1 \delta \delta_0 \Leftrightarrow \sum_2 \delta \delta_c ; \sum_1 \delta \delta_c \Leftrightarrow \sum_2 \delta \delta_0 \end{cases}$$



## CH5. 状态反馈与观测器

### 5.1 状态反馈与极点配置

#### 5.1.1. 系统构成:

$$u = -Kx + v$$

$$G(s) = C[sI - (A - BK)]^{-1}B \rightarrow \text{闭环传递函数}$$

$$f(\lambda) = |\lambda I - (A - BK)| \rightarrow \text{特征多项式}$$

#### 5.1.2. 极点配置: 系统完全能控 $\Leftrightarrow$ 可任意配置极点

$$\text{算法: } f(\lambda) = f^*(\lambda) = \prod (\lambda - \lambda_i^*)$$

#### 5.1.3. O.C: 1) 状态反馈不影响 $\zeta$ $\rightarrow$ 可能改变 $\sigma$

### 5.2 状态观测器设计

#### 5.2.1. 原理:

$$g(\lambda) = |\lambda I - (A - GC)| \rightarrow \text{将维数由 } n \text{ 增加至 } 2n.$$

#### 5.2.2. 存在条件: $\Sigma$ 不能观测系统是渐近稳定的 极点决定了观测器的收敛

极点配置定理: 若可观  $\Rightarrow$  极点可任意配置  $\rightarrow$

$$\text{5.2.2. 算法: } g(\lambda) = g^*(\lambda) = \prod (\lambda - \lambda_i^*)$$

#### 5.4. 带观测器的状态反馈:

分离性: 观测器极点比期望极点距虚轴远3~5倍.

传递函数不变性.