

# 自动控制原理

## CH5. 线性系统的频域分析法.

### 5.2 基本概念:

$$\cdot T(t) = X \sin \omega t, \quad Y_s(t) = A(\omega) X \sin [\omega t + \varphi(\omega)]$$

$$\begin{cases} G(j\omega) = A(\omega) e^{j\varphi(\omega)} \\ G(j\omega) = P(\omega) + jQ(\omega) \end{cases} \quad \begin{cases} A(\omega) = \sqrt{P^2(\omega) + Q^2(\omega)} \\ \varphi(\omega) = \arctan \frac{P(\omega)}{Q(\omega)} \end{cases} \quad \begin{cases} P(\omega) = A(\omega) \cos \varphi(\omega) \\ Q(\omega) = A(\omega) \sin \varphi(\omega) \end{cases}$$

### 5.3 对数坐标图.

$$\cdot 比例环节: G(j\omega) = k$$

$$L(\omega) = 20 \log |k|$$

$$\varphi(\omega) = \angle \{k\} = \begin{cases} 0 & k \geq 0 \\ -180^\circ & k < 0 \end{cases}$$

$$\cdot 延迟环节: G(j\omega) = e^{-j\omega \tau}$$

$$L(\omega) = 0$$

$$\varphi(\omega) = -57.37\omega^\circ$$

$$\cdot 积分环节: G(j\omega) = \frac{1}{j\omega}$$

$$\cdot 微分环节: G(j\omega) = j\omega$$

$$L(\omega) = -20 \log \omega$$

$$L(\omega) = 20 \log \omega$$

$$\varphi(\omega) = -90^\circ$$

$$\varphi(\omega) = 90^\circ$$

$$\cdot 滞后环节: G(j\omega) = \frac{1}{1+jT\omega}$$

$$\cdot 一阶微分: G(j\omega) = 1+jT\omega$$

$$L(\omega) \approx \begin{cases} 0 & \omega \ll \frac{1}{T} \\ -20 \log T\omega & \omega \gg \frac{1}{T} \end{cases}$$

$$L(\omega) \approx \begin{cases} 0 & \omega \ll \frac{1}{T} \\ 20 \log T\omega & \omega \gg \frac{1}{T} \end{cases}$$

$$\varphi(\omega) = -\arctan(T\omega)$$

$$\varphi(\omega) = \arctan(T\omega)$$

$$\cdot 振荡环节: G(j\omega) = \frac{1}{(1-T\omega^2)^2 + j2\zeta T\omega}$$

$$\cdot 二阶微分: G(j\omega) = 1-T^2\omega^2 + j2\zeta T\omega$$

$$L(\omega) \approx \begin{cases} 0 & \omega \ll \frac{1}{T} \\ -40 \log T\omega & \omega \gg \frac{1}{T} \end{cases}$$

$$L(\omega) \approx \begin{cases} 0 & \omega \ll \frac{1}{T} \\ 40 \log T\omega & \omega \gg \frac{1}{T} \end{cases}$$

$$\varphi(\omega) = -\arctan \frac{2\zeta T\omega}{1-T^2\omega^2}$$

$$\varphi(\omega) = \arctan \frac{2\zeta T\omega}{1-T^2\omega^2}$$

#### 5.4 极坐标图

· 绘制:  $w \rightarrow 0$ :  $G(jw) = \frac{K}{(jw)^n}$

$$A(w) = \frac{K}{w^n} \quad \varphi(w) = -n \cdot \frac{\pi}{2} \quad (w \rightarrow 0)$$

$w \rightarrow \infty$ :  $A(w) \rightarrow 0 \quad \varphi(w) = -(n-m) \frac{\pi}{2}$

$(n > m) \uparrow \quad \lim_{w \rightarrow \infty} \angle G(jw) = -(n-m) \frac{\pi}{2}$

· 增加零极点的影响: 见上.

#### 5.5 奈奎斯特稳定性判据

·  $F(s) = 1 + G(s)$   $\rightarrow$   $F(s)$  的零点是闭环的极点, 极点是开环的极点.

$N = Z - P$ .  $Z$ :  $F(s)$  零点在右半的个数: 闭环特征根在右半

$P$ :  $F(s)$  极点在右半的个数: 开环特征根在右半

闭环稳定:  $Z = 0$ . 开环稳定:  $P = 0$

$N$ :  $F(s)$  包围原点( $+\infty$ ),  $G(s)$  包围  $1 + j\omega$  ( $-\infty$ )

#### 5.6 稳定裕度:

$$A(w_c) = 1. \quad \gamma = 180^\circ + \varphi(w_c)$$

$$\varphi(w_c) = -180^\circ. \quad K_g = \frac{1}{A(w_c)} \rightarrow \text{闭环系统稳定的最大开环增益}$$

$$L_g = 20 \lg K_g$$

#### 5.7 闭环系统的频率特性

$$\varphi(jw) = \frac{G(jw)}{1 + G(jw)}$$

$$M(w) = | \varphi(jw) | \quad \alpha(w) = \angle \varphi(jw)$$

$$M_p \quad w_p. \quad w_b: M(w_b) = \frac{\sqrt{2}}{2} M(w)$$

$$M(w_p) = M_p = \max M(w)$$

### 5.8. 闭环系统性能分析:

1. 开环频率特性分析与系统稳定性:

$$L(\omega)|_{\omega=1} = 20 \lg K, \quad K = w_0^\nu$$

2. 闭环幅频特性分析:

单位阶跃:  $C(\omega) = M(\omega), \quad e_{ssr} = 1 - C(\omega) = 1 - M(\omega)$

$\nu = 0$  时,  $M(\omega) = \frac{K}{1+K}, \quad e_{ssr} = \frac{1}{1+K}$

$\nu \geq 1$  时,  $M(\omega) = 1, \quad e_{ssr} = 0$

3. 频率与时间的反比:

$$\underline{h}_1(j\omega) = \underline{h}_2(j\frac{\omega}{\alpha}) (\alpha > 0) \Rightarrow h_1(t) = h_2(\alpha t)$$

4. 典型二阶系统开环频域性能与瞬态性能:

$$\zeta = 0.01\beta, \quad T_s = \begin{cases} \frac{4}{3\omega_n} & \Delta = 2 \\ \frac{3}{3\omega_n} & \Delta = 5 \end{cases} = \begin{cases} \frac{8}{w_c} \cdot \frac{1}{\tan \beta} \\ \frac{8b}{w_c} \cdot \frac{1}{\tan \beta} \end{cases}$$

5. 欠阻尼闭环频域与瞬态性能:

$$w_p = \omega_n \sqrt{1 - 2\zeta^2}, \quad M_p = \frac{1}{2\zeta\sqrt{1 - 2\zeta^2}} \quad \star$$

$$w_b = \omega_n \sqrt{1 - 2\zeta^2 + \sqrt{2 - 4\zeta^2 + 4\zeta^4}}$$

## CH7. 线性系统的设计方法

### 7.2. 植生装置 (基于伯德图)

超前校正:

$$G_C(s) = \frac{1 + \alpha T s}{\alpha(1 + T s)} = \frac{s + \frac{1}{\alpha T}}{s + \frac{1}{T}} \quad (\alpha > 1)$$

$$w_m = \frac{1}{T \sqrt{\alpha}}$$

$$\varphi_m = \arcsin \frac{\alpha - 1}{\alpha + 1}$$

高通滤波器.

$$L(w_m) = 20 \log \alpha$$

$$\varphi_m = \varphi^* - \varphi + 5 \sim 12^\circ$$

解得  $\alpha$

$$w_c'' = w_m \cdot \text{斜率 } T$$

$$L(w_c'') + 20 \log \alpha = 0$$

解得  $w_c''$

滞后校正:

$$G_C(s) = \frac{1 + \beta T s}{1 + T s} = \beta \frac{s + \frac{1}{\beta T}}{s + \frac{1}{T}} \quad (\beta < 1)$$

$$w_m = \frac{1}{\sqrt{\beta} \cdot T} \quad \frac{1}{\beta T} = \frac{w_c''}{10}$$

$$\varphi_m = \arcsin \frac{1 - \beta}{1 + \beta}$$

$$\varphi = \varphi^* + 5 \sim 12^\circ$$

$$\text{使 } \varphi(w_c'') = -180^\circ + \varphi^* + 5 \sim 12^\circ$$

解  $w_c''$

$$L(w_c'') + 20 \log \beta = 0$$

$$\text{解 } \beta \rightarrow \frac{1}{\beta T} = \frac{w_c''}{10} \rightarrow \text{斜率 } T$$

超前滞后校正

$$G_C(s) = \frac{(1 + T_a s)(1 + T_b s)}{\left(1 + \frac{T_a s}{\alpha}\right)(1 + \alpha T_b s)}$$

$$1. \text{令 } w_c'' = w_g, \quad \frac{1}{T_b} = \frac{w_c''}{10}$$

$$2. \text{令 } \varphi_m = \varphi^*. \quad \alpha \Rightarrow \varphi_m = \arcsin \frac{\alpha - 1}{\alpha + 1}$$

3. 过  $(w_c'', L(w_c''))$

以  $+20 \text{ dB/dec}$  为斜率的直线.

与  $0 \text{ dB}$  及  $-20 \log \alpha \text{ dB}$  的交点 (端点)  
转角频率

$$\frac{1}{T_a}, \quad \frac{\alpha}{T_a} \text{ 分别为转角频率}$$

#### 7.4 基于根轨迹的小流校正.

· 增加开环极点: 根轨迹向右; 增加开环零点, 根轨迹向左

· 增加偶极子: 对稳定性无影响.

· 超前:  $G_C(s) = \frac{s+z}{s+p}, |z| < |p|$

1. 由  $s, w_n$  确定主导极点:

$$Re = -\Im w_n \quad Im = w_n \sqrt{1-\Re^2}$$

2. 取  $z = -Re$

3. 由相角条件求  $\theta_p$ , 进而求  $p$ .

滞后:  $G_C(s) = \frac{s+z}{s+p}, |z| > |p| \rightarrow z \text{ 和 } K_v, \gamma'$

1. 作原小流根轨迹.

2. 由  $\gamma'$ , 作  $\beta = \arccos \gamma'$ , 与根轨迹交点为主导极点.

3. 求出在主导极点处的增益  $K$ .  $K_v = \frac{K}{2}$

3.  $\frac{|z|}{|p|} = \frac{K_v'}{K_v}$ . 取  $z, p$ .

### CH3. 时域分析法

#### · 一阶系统

$$GK = \frac{K}{s}, \quad \Psi(s) = \frac{1}{Ts + 1}$$

$$\cdot t_d = 0.693T; \quad t_r = 2.197T; \quad t_s = \begin{cases} 4T, & \Delta=2 \\ 3T, & \Delta=3 \end{cases}$$

· 减小 T: (1) 负反馈 (2) 放大 K

#### · 二阶系统

$$GK(s) = \frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2}, \quad \Psi(s) = \frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2}$$

性能指标:

欠阻尼

$$t_r = \frac{\pi - \beta}{w_n \sqrt{1-\beta^2}}$$

$$t_p = \frac{\pi}{w_n \sqrt{1-\beta^2}}$$

$$t_s = \begin{cases} \frac{4}{3w_n}, & \Delta=2 \\ \frac{3}{2w_n}, & \Delta=3 \end{cases}$$

$$t_f = \frac{2\pi}{w_n \sqrt{1-\beta^2}}, \quad N = \frac{t_s}{t_f}$$

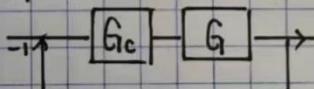
$$\delta\% = e^{\frac{-\pi\beta}{\sqrt{1-\beta^2}}}$$

过阻尼

$$t_r = \frac{1+1.5\zeta+\zeta^2}{w_n^2}$$

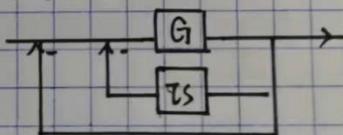
$$t_s = \begin{cases} \frac{8.4}{w_n}, & \Delta=2 \\ \frac{11.6}{w_n}, & \Delta=3 \end{cases}$$

性能改善: (1). 比例放大:  $G_{c(s)} = k_p + k_d s$



(2). 速度反馈: 反馈 Ts

不改变  $w_n$   
改变  $\zeta$



· 高阶系统. 稳.

· 稳定性:

1. 充要条件: 没有右半平面的特征根

2. 代数判据: 劳斯判据: 1) 列:  $\varepsilon \rightarrow$  全行: 临界 / 不稳

· 稳态误差:

$$E(s) = R(s) - Y(s), \quad e_{ss} = \lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s E(s)$$

$$R(s) = \frac{A}{s}, \quad K_p = \lim_{s \rightarrow 0} G_K(s), \quad e_{ss} = \begin{cases} \frac{A}{1+K_p} & K, v=0 \\ \frac{A}{1+K}, v=0 \\ 0, v \geq 1 \end{cases}$$

$$R(s) = \frac{B}{s^2}, \quad K_v = \lim_{s \rightarrow 0} s G_K(s), \quad e_{ss} = \begin{cases} \frac{B}{K_v} & 0, v=0 \\ \infty, v=0 \\ \frac{B}{K}, v=1 \\ 0, v \geq 2 \end{cases}$$

$$R(s) = \frac{C}{s^3}, \quad K_a = \lim_{s \rightarrow 0} s^2 G_K(s), \quad e_{ss} = \begin{cases} \frac{C}{K_a} & 0, v \leq 1 \\ \frac{C}{K}, v=2 \\ 0, v \geq 3 \end{cases}$$

# 现代控制理论

## CH1. 状态空间描述方法.

### 1.1. 状态变量模型

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

### 1.2 建立方法.

1. 从物理机理: 有n个储能元件  $\rightarrow$  n个状态变量.

2. 从微分方程:

1) 不含输入的导数:  $y^{(n)} + a_{n-1}y^{(n-1)} + \dots + a_1y' + a_0y = b_0u$

$$x_n = y^{(n-1)}$$
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_0 \end{bmatrix} u$$

$$y = [1 \ 0 \ 0 \ \dots \ 0]^T [x_1 \ x_2 \ \dots \ x_n]$$

2) 含输入的导数:  $y^{(n)} + a_{n-1}y^{(n-1)} + \dots + a_1y' + a_0y = b_nu + b_{n-1}u^{(n-1)} + \dots + b_1u' + b_0u$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 0 & \dots & 0 & -a_0 \\ 1 & 0 & \dots & 0 & -a_1 \\ 0 & 1 & \dots & 0 & -a_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} b_0 - a_0b_n \\ b_1 - a_1b_n \\ \vdots \\ b_{n-2} - a_{n-2}b_n \\ b_{n-1} - a_{n-1}b_n \end{bmatrix} u$$

$$y = [0 \ 0 \ \dots \ 0 \ 1]^T [x_1 \ \dots \ x_n]^T + b_n u$$

### 3. 从传递函数

1) 直接分解:  $G(s) = \frac{Y(s)}{U(s)} = \frac{b_n s^n + b_{n-1} s^{n-1} + \dots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \dots + a_1 s + a_0}$

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_0 - a_1 - a_2 - \dots - a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [b_0 - a_0 b_n, b_1 - a_1 b_n, \dots, b_{n-1} - a_{n-1} b_n]^T [x_1 \dots x_n] + b_n u.$$

2) 串联、并联分解: 略.

### 1.3. 传递函数矩阵:

$$G(s) = C(sI - A)^{-1}B + D$$

并联:  $G_1 = G_1 + G_2$

串联:  $G_1 = G_1 \cdot G_2$

反馈:  $G_1 = G_1 - G_1 \cdot G_2 \cdot G_1$

### 1.4. 线性非奇异变换

1.  $\bar{A} = P^{-1}AP, \bar{B} = P^{-1}B, \bar{C} = CP, \bar{D} = D$

义

2. 性质: 1) 不变特征值: 极点

2) 不变  $G(s)$

3) 不变能观能控.

### 3. 对角化型:

1) 由  $|sI - A| = 0$  解  $\lambda_i$

2) 由  $(\lambda_i I - A)v_i = 0$  解  $v_i$

3).  $P = [v_1 \ v_2 \ \dots \ v_n]$

## CH2. 线性控制系统的运动分析

### 2.1 线性连续常系数方程的解:

$$\begin{cases} x(t) = e^{At}x(0) \\ x(t) \underset{t=t_0}{=} x(t) : x(t) = e^{A(t-t_0)}x(t_0) \end{cases} \quad e^{At} \triangleq \sum_{k=0}^{\infty} \frac{1}{k!} A^k t^k$$

$$x(t) = \mathcal{L}^{-1}[(sI - A)^{-1}]x(0).$$

### 2.2 矩阵指数函数:

1. 性质: 1)  $A: n \times n : e^{A(t_1+t_2)} = e^{At_1}e^{At_2}$

$$2) e^{A(t-t)} = I$$

3)  $e^{At}$  非奇异

4) 若  $A \times B = B \times A : e^{(A+B)t} = e^{At}e^{Bt}$

$$5) \frac{d}{dt}(e^{At}) = e^{At} \cdot A$$

$$6) e^{P^T A P t} = P^T e^{At} P, \quad e^{At} = P e^{P^T A P t} P^{-1}$$

$$7) \star A = \text{diag}[\lambda_1, \lambda_2, \dots, \lambda_n], \quad e^{At} = \text{diag}[e^{\lambda_1 t}, e^{\lambda_2 t}, \dots, e^{\lambda_n t}]$$

### 2. 末解:

1) 直接法:  $e^{At} = \sum_{k=0}^{\infty} \frac{1}{k!} A^k t^k$

2) 拉氏变换:  $e^{At} = \mathcal{L}^{-1}[(sI - A)^{-1}]$

3) 标准型法:  $e^{At} = P e^{\bar{A}t} P^{-1}$

4) 特定示数: 路.  $P_{S_0 \sim S_2}$

## 2.3. 状态转移矩阵

### 1. 线性定常系统: (齐次)

$$\dot{x} = Ax : \begin{cases} x(t) = \bar{\Psi}(t) x(t_0) \\ x(t) = \bar{\Psi}(t-t_0) x(t_0) \end{cases} \quad \begin{cases} \bar{\Psi}(t) = e^{At} \\ \bar{\Psi}(t-t_0) = e^{A(t-t_0)} \end{cases}$$

条件: 1)  $\bar{\Psi}(t_0-t_0) = I$

$$2) \bar{\Psi}(t-t_0) = A \bar{\Psi}(t-t_0)$$

### 2. 线性时变系统:

$$x(t) = \bar{\Psi}(t, t_0) x(t_0)$$

### 3. 性质:

$$1) \bar{\Psi}(t_0-t_0) = I$$

$$5) \bar{\Psi}(t_1+t_2) = \bar{\Psi}(t_1) \bar{\Psi}(t_2) = \bar{\Psi}(t_2) \bar{\Psi}(t_1)$$

$$\bar{\Psi}(t_0, t_0) = I$$

$$2) \bar{\Psi}(t-t_0) = A \bar{\Psi}(t-t_0)$$

$$6) [\bar{\Psi}(t)]^n = \bar{\Psi}(nt)$$

$$\bar{\Psi}(t, t_0) = A(t) \bar{\Psi}(t, t_0)$$

$$3) \bar{\Psi}(t_2-t_1) \bar{\Psi}(t_1-t_0) = \bar{\Psi}(t_2-t_0)$$

4. 计算:

$$\bar{\Psi}(t_2, t_1) \bar{\Psi}(t_1, t_0) = \bar{\Psi}(t_2, t_0)$$

$$1) \bar{\Psi}(t) = e^{At}$$

$$4) \bar{\Psi}^{-1}(t-t_0) = \bar{\Psi}(t_0-t)$$

$$2) \bar{\Psi}(t-t_0) = A \bar{\Psi}(t-t_0), \text{ 且 } \bar{\Psi}(t-t) = I$$

$$\bar{\Psi}^{-1}(t, t_0) = \bar{\Psi}(t_0, t)$$

$$\therefore A = \bar{\Psi}(t, t_0) \Big|_{t=t_0}$$

### 2.4. 非齐次:

$$\dot{x} = Ax + Bu$$

$$\begin{aligned} x(t) &= \bar{\Psi}(t-t_0) x(t_0) + \int_{t_0}^t \bar{\Psi}(t-\tau) B u(\tau) d\tau & x(t) &= L \left[ [(sI-A)^{-1} x(0) + (sI-A)^{-1} B u(s)] \right] \\ &= e^{At-t_0} x(t_0) + \int_{t_0}^t e^{A(t-\tau)} B u(\tau) d\tau \end{aligned}$$

### CH3. 能控性和能观性

能控性

能观性

一. 判别:

$$1. Q_C = [B \ AB \ A^2B \ \dots \ A^{n-1}B]$$

$$\text{rank}(Q_C) = n$$

2. A化为对角阵  $\bar{A}$  后

$\bar{B}$  中无全0行

3. A化为约当阵  $\bar{A}$

$\bar{B}$  中每个若当块最后行不全0

$$Q_0 = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

$$\text{rank}(Q_0) = n$$

A化为  $\bar{A}$  后

$\bar{C}$  中无全0列

A化为  $\bar{A}'$  后

$\bar{C}$  每类第一列不全0

4. S平面判据:

若对消可能  $\bar{C}, \bar{C}'$

$$G_{1(S)} = C(SI - A)^{-1}b \text{ 三个子方程无零极点对消} \rightarrow 0 + C$$

二. 标量型

$$P_{C_1} = [b, Ab, \dots, A^{n-1}b] = Q_C$$

$$P_{C_1}^{-1} = Q_0 = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

$$P_{C_2} = [A^{n-1}b, A^{n-2}b, \dots, b]$$

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ 0_{n-1} & 1 & & 0 \\ \vdots & & \ddots & \\ 0_2 & 0 & & 0 \\ 0_1 & 0_2 & \dots & 1 \end{bmatrix}$$

$$P_{C_2}^{-1} = \begin{bmatrix} 1 & 0_{n-1} & \dots & 0_2 & 0_1 \\ 0 & 1 & \dots & 0 & 0_2 \\ \vdots & & \ddots & & \\ 0 & 0 & \dots & 1 & 0_{n-1} \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix} \begin{bmatrix} CA^{n-1} \\ CA^{n-2} \\ \vdots \\ CA \\ Cf \end{bmatrix}$$

三对偶原理:

$$\text{定义 } \left\{ \begin{array}{l} A_1 = A_2^T \\ B_1 = C_2^T \\ C_1 = B_2^T \end{array} \right.$$

$$\text{性质 } \left\{ \begin{array}{l} G_{1(1S)} = G_{2(1S)}^T \\ f_1(\lambda) = f_2(\lambda) \end{array} \right.$$

$$B_1 = C_2^T$$

$$\sum_i \alpha_i \Rightarrow \sum_i \alpha_i c \quad ; \quad \sum_i \alpha_i c \Leftrightarrow \sum_i \alpha_i 0$$

$$C_1 = B_2^T$$

## CH5. 状态反馈与观测器

### 5.1 状态反馈与极点配置

#### 5.1.1. 系统构成:

$$u = -Kx + v$$

$$G(s) = C[sI - (A - BK)]^{-1}B \rightarrow \text{闭环传递函数}$$

$$f(\lambda) = [\lambda I - (A - BK)] \rightarrow \text{特征多项式}$$

5.1.2. 极点配置: 系统完全能控  $\Leftrightarrow$  可任意配置极点

算法:  $f(\lambda) = f^*(\lambda) = \prod(\lambda - \lambda_i^*)$

5.1.3. O.C.:  $\Rightarrow$  状态反馈不影响  $\Rightarrow$  可能改变。

### 5.3 状态观测器设计

#### 5.3.1. 原理:

$$g(\lambda) = [\lambda I - (A - GC)] \rightarrow \text{将维数由 } n \text{ 增加至 } 2n.$$

5.3.2. 存在条件:  $\Sigma$  不能观了. 系是渐近稳定的 极点决定了观测器的收敛  
极点配置定理: 若可观  $\Rightarrow$  极点可任意配置  $\rightarrow$

5.3.2. 算法:  $g(\lambda) = g^*(\lambda) = \prod(\lambda - \lambda^*)$

#### 5.4. 带观测器的状态反馈:

分离性: 观测器极点比期望极点距虚轴远3~5倍。

传递函数不变性。