A solver based on normal modes for the resolution of 3D coupled poroelastic problems

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Context

$$[\widetilde{\mathbf{A}}(\omega)]\mathbf{X}(\mathbf{F},\omega) = \mathbf{F}$$

✓ Large size system (WL of Biot waves)
✓ Complex systems and frequency dependent

New representation

$$\mathbf{X}(\mathbf{F},\omega) = \sum_{i=1}^{n} \mathbf{\Phi}_i q_i(\mathbf{F},\omega) = [\mathbf{\Phi}]\mathbf{q}(\mathbf{F},\omega)$$

Recomposition Family

Contributions (unknowns)

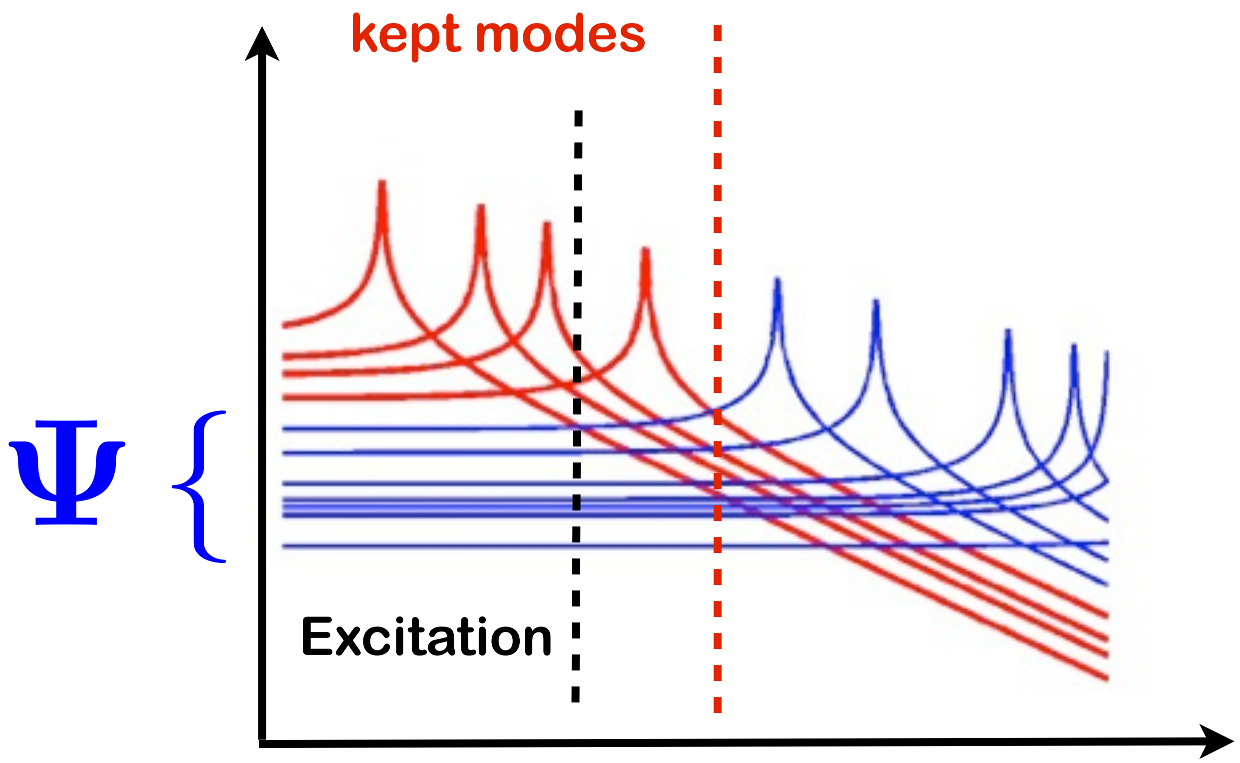
Objective: To propose a representation adapted to PEM

✓ Two phases
✓ Dispersion
✓ Usually combined with other structures (air, plates …)

Idea: Simple "modal" forms + "analytical-static" correction

Static correction (basic ideas)

$$\mathbf{X}(\mathbf{F},\omega) = \sum_{i=1}^{n} \mathbf{\Phi}_i q_i(\mathbf{F},\omega) = \sum_{i=1}^{m} \mathbf{\Phi}_i q_i(\mathbf{F},\omega) + \boxed{\mathbf{\Psi}(\mathbf{F})}$$



✓ Attachment modes can be determined without knowing higher modes

✓ Decoupled normal modes of the solid and fluid phase

✓ Corrections

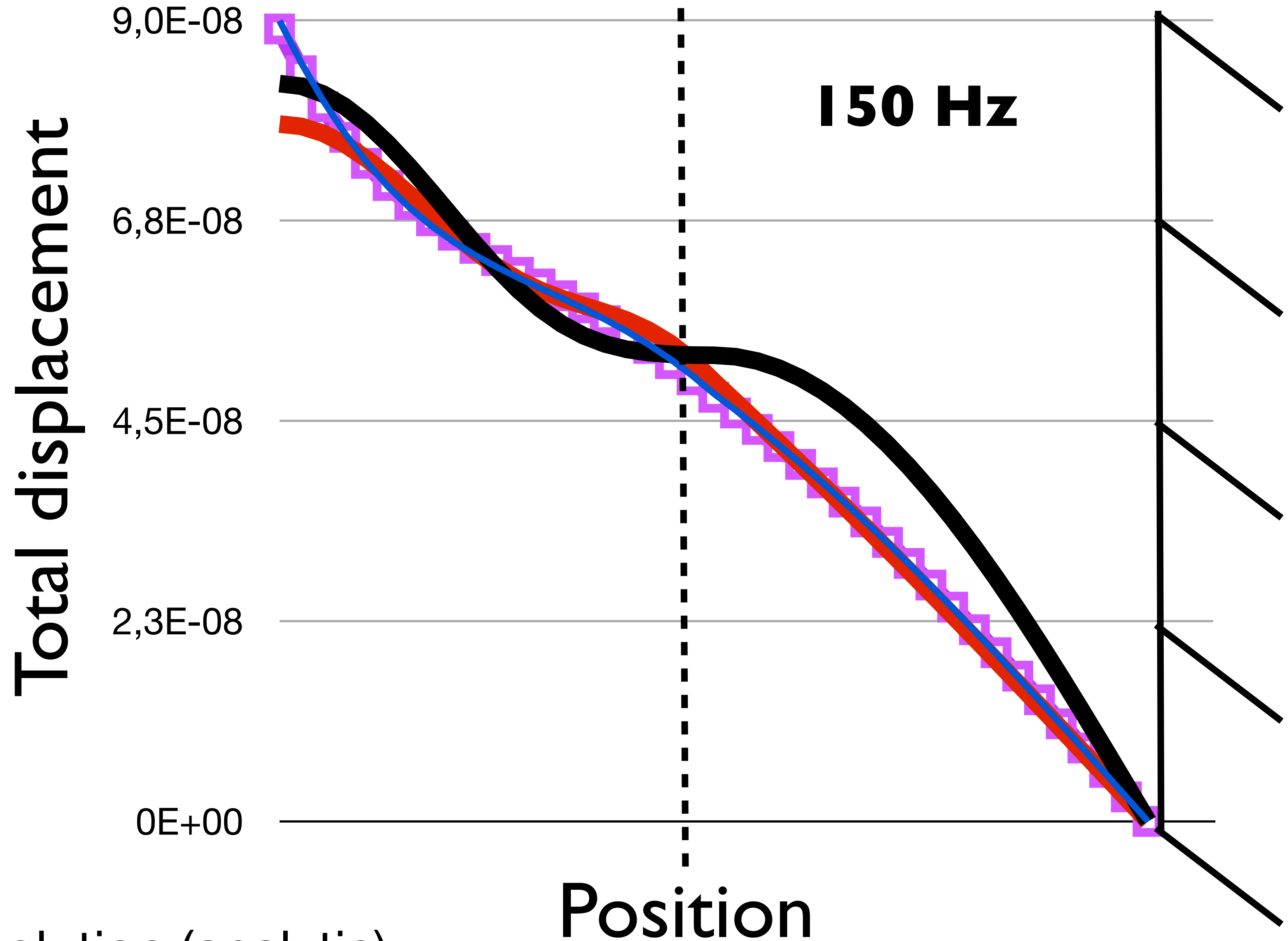
➡ Interfaces

➡ External forces

➡ Solid/fluid inside the PEM

A 1D problem as introduction

Foam
✓Rigid body
✓I elastic mode

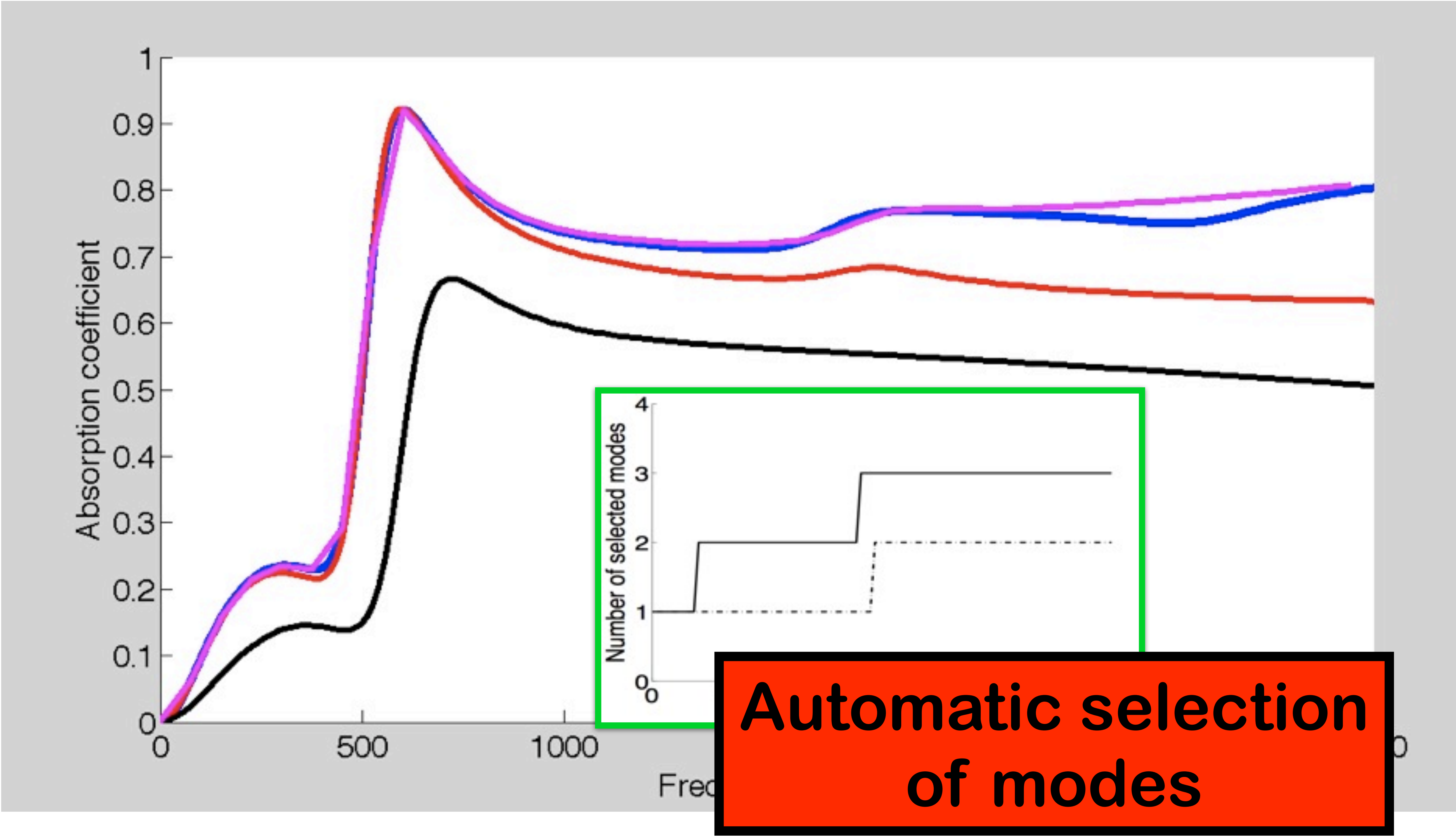


Fibrous
✓I elastic mode

— Reference solution (analytic)
— No correction (DMA)
— Craig & Chang (classical)
□ Proposed method

Dazel *et al.* AAA **96(2)** 2010

Absorption coefficient (2,1) modes



Analytical	No correction	Classical	Proposed

Fluid-poroelastic problem

$$\begin{bmatrix} \dfrac{[\mathbf{H}_a]}{\rho_a\omega^2} - \dfrac{[\mathbf{Q}_a]}{K_a} & [0] & [0] & [\mathbf{L}_1]^t \\ [0] & \hat{A}[\mathbf{K}] - \omega^2\widetilde{\rho}[\mathbf{M}] & -\widetilde{\gamma}[\mathbf{C}]^t & [0] \\ [0] & \widetilde{\gamma}[\mathbf{C}] & \dfrac{[\mathbf{H}]}{\widetilde{\rho}_{eq}\omega^2} - \dfrac{[\mathbf{Q}]}{\widetilde{K}_{eq}} & [\mathbf{L}_2]^t \\ [\mathbf{L}_1] & [0] & [\mathbf{L}_2] & [0] \end{bmatrix} \begin{Bmatrix} \mathbf{P}_a \\ \mathbf{u}^s \\ \mathbf{P} \\ \boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \mathbf{u}_a \\ \mathbf{F}_s \\ \mathbf{u}_P \\ \mathbf{0} \end{Bmatrix}$$

Decoupled normal modes

- **Air** $\quad [[\mathbf{H}_a], [\mathbf{Q}_a]] \Rightarrow \{[\mathbf{\Phi}_a], [\mathbf{k}_a^2]\}$
- **Solid** $\quad [[\mathbf{K}], [\mathbf{M}]] \Rightarrow \{[\mathbf{\Phi}_s], [\mathbf{k}_s^2]\}$
- **Pressure** $\quad [[\mathbf{H}], [\mathbf{Q}]] \Rightarrow \{[\mathbf{\Phi}_P], [\mathbf{k}_P^2]\}$

Projected system and simplifications

$$
\left[\begin{array}{cc|cccc|c}
\frac{[\mathbf{k}_a^2]}{\rho_a\omega^2}-\frac{[\mathbf{I}_a]}{K_a} & [\mathbf{0}] & [\mathbf{0}] & [\mathbf{0}] & [\mathbf{0}] & [\mathbf{0}] & [\mathbf{L}_1^k]^t \\
[\mathbf{0}] & \frac{[\mathbf{r}_a^2]}{\rho_a\omega^2}-\frac{[\mathbf{I}'_a]}{K_a} & [\mathbf{0}] & [\mathbf{0}] & [\mathbf{0}] & [\mathbf{0}] & [\mathbf{L}_1^h]^t \\
\hline
[\mathbf{0}] & [\mathbf{0}] & \hat{A}[\mathbf{k}_s^2]-\omega^2\widetilde{\rho}[\mathbf{I}_s] & [\mathbf{0}] & -\widetilde{\gamma}[\mathbf{\Gamma}_{kk}]^t & -\widetilde{\gamma}[\mathbf{\Gamma}_{hk}]^t & [\mathbf{0}] \\
[\mathbf{0}] & [\mathbf{0}] & [\mathbf{0}] & \hat{A}[\mathbf{r}_s^2]-\omega^2\widetilde{\rho}[\mathbf{I}'_s] & -\widetilde{\gamma}[\mathbf{\Gamma}_{kh}]^t & -\widetilde{\gamma}[\mathbf{\Gamma}_{hh}]^t & [\mathbf{0}] \\
[\mathbf{0}] & [\mathbf{0}] & -\widetilde{\gamma}[\mathbf{\Gamma}_{kk}] & -\widetilde{\gamma}[\mathbf{\Gamma}_{kh}] & \frac{[\mathbf{k}_P^2]}{\widetilde{\rho}_{eq}\omega^2}-\frac{[\mathbf{I}_p]}{\widetilde{K}_{eq}} & [\mathbf{0}] & [\mathbf{L}_2^k]^t \\
[\mathbf{0}] & [\mathbf{0}] & -\widetilde{\gamma}[\mathbf{\Gamma}_{hk}] & -\widetilde{\gamma}[\mathbf{\Gamma}_{hh}] & [\mathbf{0}] & \frac{[\mathbf{r}_P]}{\widetilde{\rho}_{eq}\omega^2}-\frac{[\mathbf{I}'_p]}{\widetilde{K}_{eq}} & [\mathbf{L}_2^h]^t \\
\hline
[\mathbf{L}_1^k] & [\mathbf{L}_1^h] & [\mathbf{0}] & [\mathbf{0}] & [\mathbf{L}_2^k] & [\mathbf{L}_2^h] & [\mathbf{0}]
\end{array}\right]
\left\{\begin{array}{c}\mathbf{q}_a\\ \mathbf{h}_a\\ \mathbf{q}_s\\ \mathbf{h}_s\\ \mathbf{q}_P\\ \mathbf{h}_P\\ \boldsymbol{\lambda}\end{array}\right\}
=
\left\{\begin{array}{c}\mathbf{\Phi}_s^t\mathbf{u}_a\\ \mathbf{\Phi}_s'^t\mathbf{u}_a\\ \mathbf{\Phi}_s^t\mathbf{F}_s\\ \mathbf{\Phi}_s'^t\mathbf{F}_s\\ \mathbf{\Phi}_P^t\mathbf{u}_P\\ \mathbf{\Phi}_P'^t\mathbf{u}_P\\ \mathbf{0}\end{array}\right\}.
$$

$$
\left[\begin{array}{cc}
\frac{[\mathbf{k}_a^2]}{\rho_a\omega^2}-\frac{[\mathbf{I}_a]}{K_a} & [\mathbf{0}] \\
[\mathbf{0}] & \frac{[\mathbf{r}_a^2]}{\rho_a\omega^2}-\cancel{\frac{[\mathbf{I}'_a]}{K_a}}
\end{array}\right]
$$

$$
\frac{K_a}{\rho_a} r_a^2 = \omega_a^2 >> \omega^2
$$

$$
\left[\begin{array}{cccc}
\hat{A}[\mathbf{k}_s^2]-\omega^2\widetilde{\rho}[\mathbf{I}_s] & [\mathbf{0}] & -\widetilde{\gamma}[\mathbf{\Gamma}_{kk}]^t & -\widetilde{\gamma}[\mathbf{\Gamma}_{hk}]^t \\
[\mathbf{0}] & \hat{A}[\mathbf{r}_s^2]-\cancel{\omega^2\widetilde{\rho}[\mathbf{I}'_s]} & -\widetilde{\gamma}[\mathbf{\Gamma}_{kh}]^t & -\widetilde{\gamma}[\mathbf{\Gamma}_{hh}]^t \\
-\widetilde{\gamma}[\mathbf{\Gamma}_{kk}] & -\widetilde{\gamma}[\mathbf{\Gamma}_{kh}] & \frac{[\mathbf{k}_P^2]}{\widetilde{\rho}_{eq}\omega^2}-\frac{[\mathbf{I}_p]}{\widetilde{K}_{eq}} & [\mathbf{0}] \\
-\widetilde{\gamma}[\mathbf{\Gamma}_{hk}] & \cancel{-\widetilde{\gamma}[\mathbf{\Gamma}_{hh}]} & [\mathbf{0}] & \frac{[\mathbf{r}_P]}{\widetilde{\rho}_{eq}\omega^2}-\cancel{\frac{[\mathbf{I}'_p]}{\widetilde{K}_{eq}}}
\end{array}\right]
$$

A little bit of algebra

$$\mathbf{P}_a = [\boldsymbol{\Phi}_a]\mathbf{q}_a + \underbrace{[\boldsymbol{\Phi}'_a]\mathbf{h}_a}_{\mathbf{P}_a^h}, \quad \mathbf{u}_s = [\boldsymbol{\Phi}_s]\mathbf{q}_s + \underbrace{[\boldsymbol{\Phi}'_s]\mathbf{h}_s}_{\mathbf{u}^h}, \quad \mathbf{P} = [\boldsymbol{\Phi}_P]\mathbf{q}_P + \underbrace{[\boldsymbol{\Phi}'_P]\mathbf{h}_P}_{\mathbf{P}^h}$$

$$\mathbf{P}^h = \widetilde{\rho}_{eq}\omega^2(\boldsymbol{\Psi}_P + \widetilde{\gamma}[\boldsymbol{\Xi}_s]\mathbf{q}_s - [\boldsymbol{\Lambda}_P]\boldsymbol{\lambda})$$

$\mathbb{R}$

Calculated once { **Force** — **Solid-pressure coupling** — **Air-porous coupling**

Similarily

$$\mathbf{P}_a^h = \rho_a\omega^2(\boldsymbol{\Psi}_a - [\boldsymbol{\Lambda}_a]\boldsymbol{\lambda})$$

$$\mathbf{u}^h = \frac{1}{\hat{A}}(\boldsymbol{\Psi}_s + \widetilde{\gamma}[\boldsymbol{\Xi}_P]\,\mathbf{q}_P + \widetilde{\rho}_{eq}\widetilde{\gamma}\omega^2\boldsymbol{\Psi}'_s + \widetilde{\rho}_{eq}\widetilde{\gamma}^2\omega^2[\boldsymbol{\Xi}'_s]\mathbf{q}_s - \widetilde{\rho}_{eq}\widetilde{\gamma}\omega^2[\boldsymbol{\Lambda}'_s]\boldsymbol{\lambda})$$

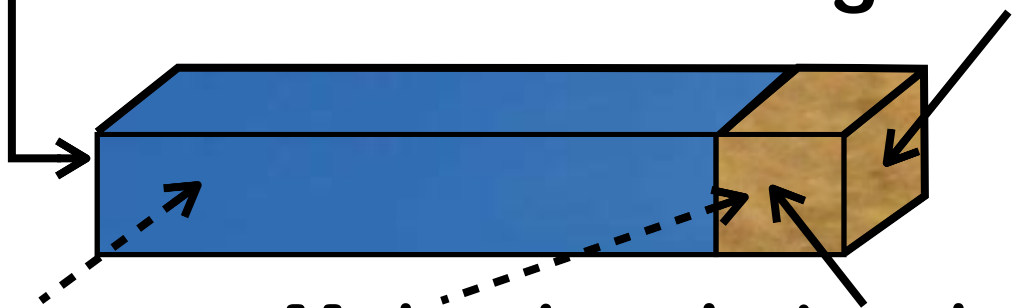
$$\left[\begin{array}{c|cc|c} [\mathbf{R}_{aa}] & [\mathbf{0}] & [\mathbf{0}] & [\mathbf{L}_1^k]^t \\ \hline [\mathbf{0}] & [\mathbf{R}_{ss}] & [\mathbf{R}_{sP}] & [\mathbf{R}_{s\lambda}] \\ [\mathbf{0}] & [\mathbf{R}_{Ps}] & [\mathbf{R}_{PP}] & [\mathbf{R}_{P\lambda}] \\ \hline [\mathbf{L}_1^k] & [\mathbf{R}_{\lambda s}] & [\mathbf{L}_2^k] & [\mathbf{0}] \end{array}\right] \left\{\begin{array}{c} \mathbf{q}_a \\ \mathbf{q}_s \\ \mathbf{q}_P \\ \boldsymbol{\lambda} \end{array}\right\} = \left\{\begin{array}{c} [\mathbf{F}_a] \\ [\mathbf{F}_s] \\ [\mathbf{F}_P] \\ [\mathbf{F}_\lambda] \end{array}\right\}$$

Laurel damped cavity (1D eq.)

Unitary piston excitation

Lx=1 cm; Ly=1 cm

Rigid backing



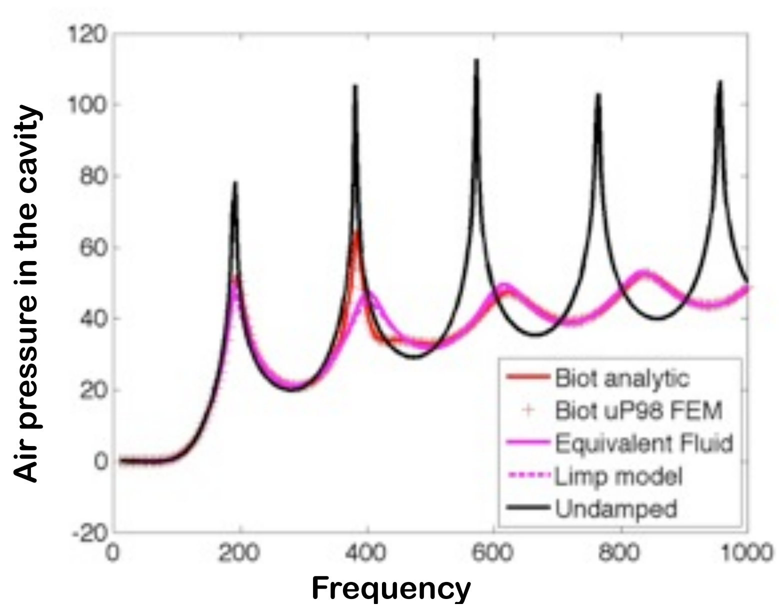
Air (80 cm)

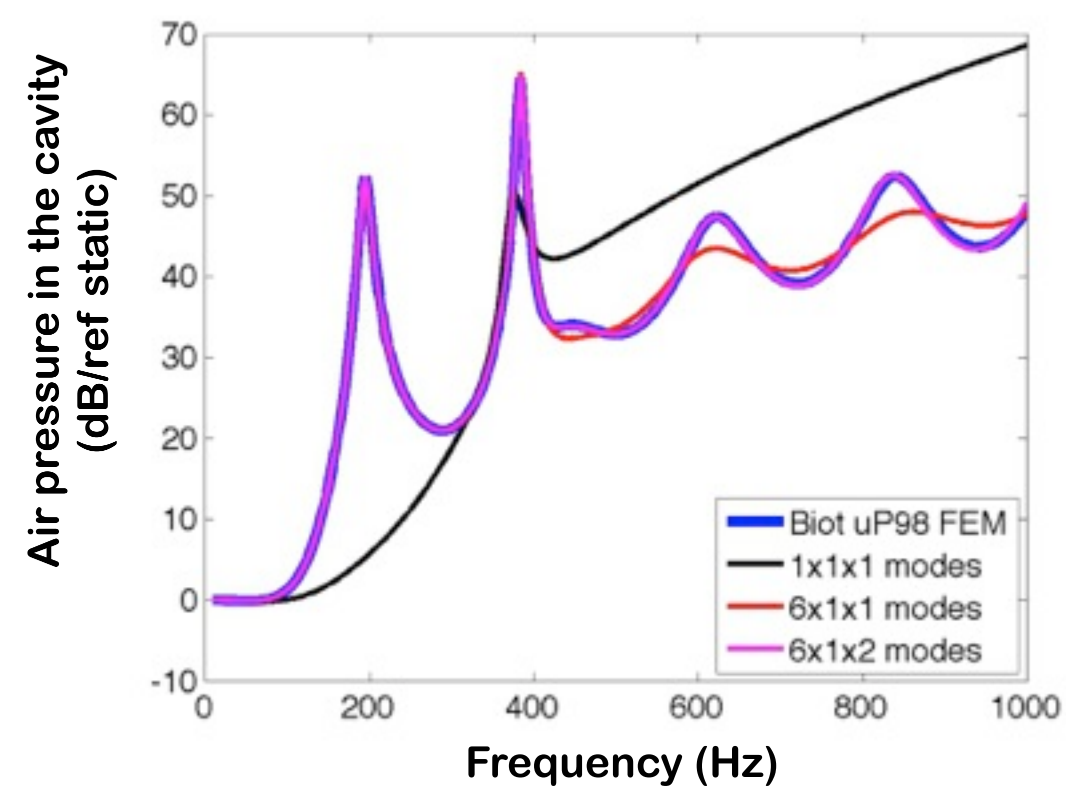
Melamine (10 cm)

Lateral sliding conditions

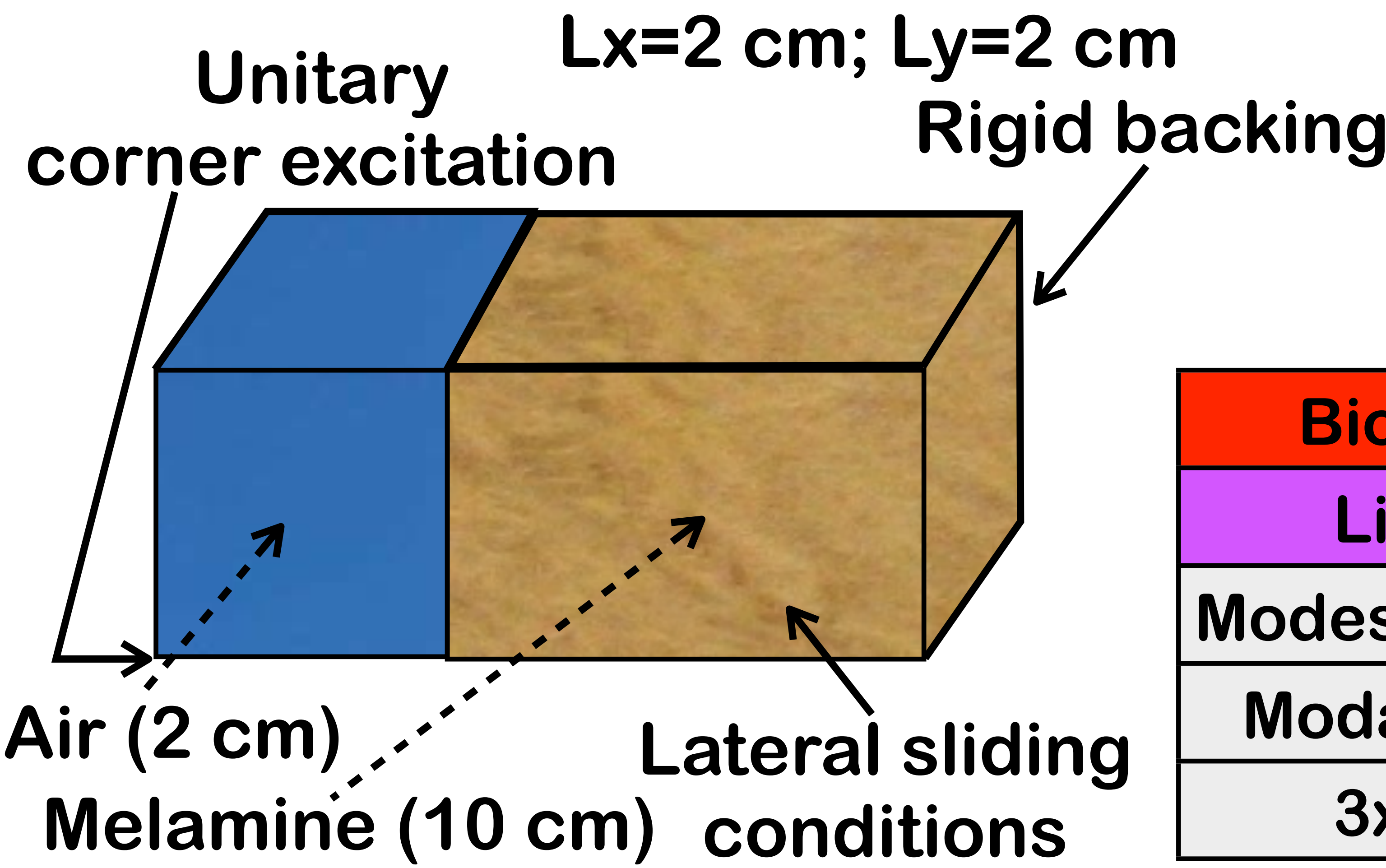
Home made Fortran code
Biot uP 1998
(20+10x1x1) HEXA27
300 Frequencies

Biot uP HSL ME57	16.89 s
Limp HSL ME57	3.16 s
Modes comp. time	1.42 s
Modal frequency loop	0.11 s
6x1x2 total time	1.53 s



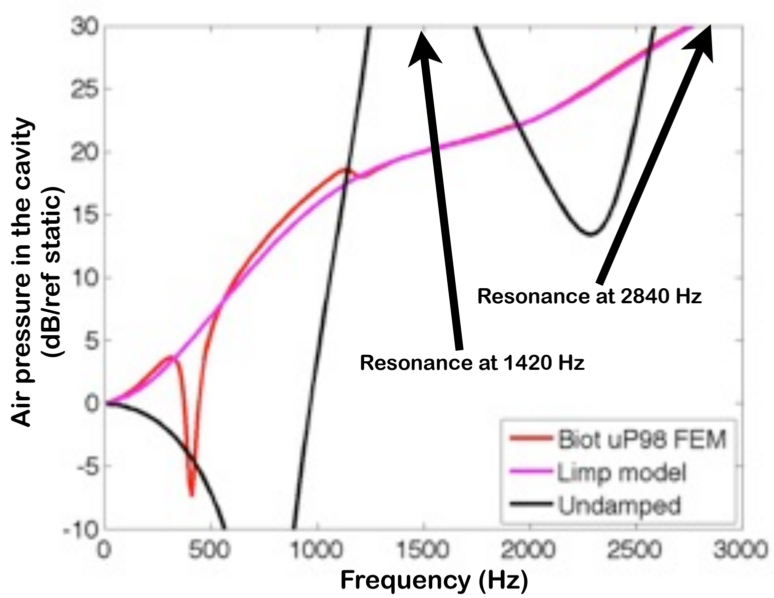


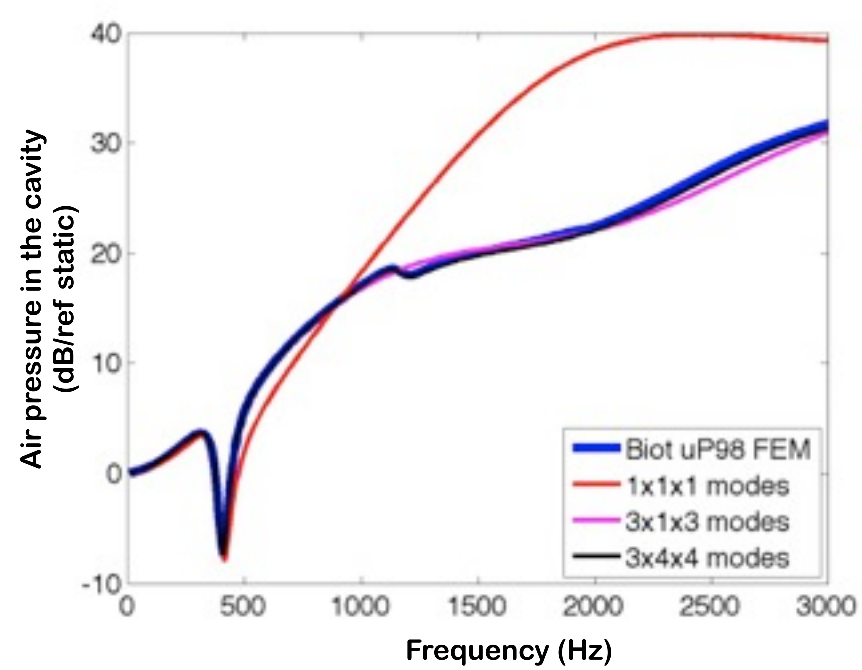
Hardy damped cavity (3D)



Home made Fortran code
Biot uP 1998
(4+10x6x6) HEXA27
300 Frequencies

Biot uP HSL ME57	4887 s
Limp HSL ME57	190 s
Modes computation time	32 s
Modal frequency loop	4 s
3x4x3 total time	36 s

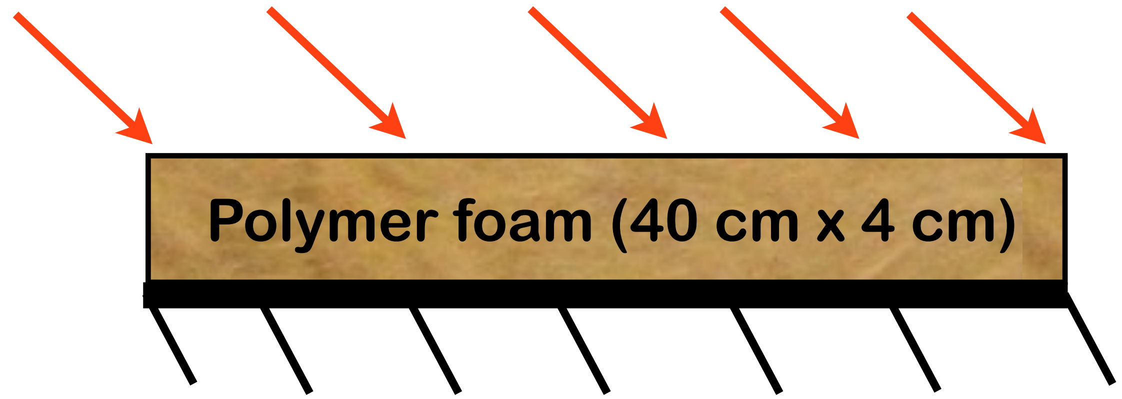


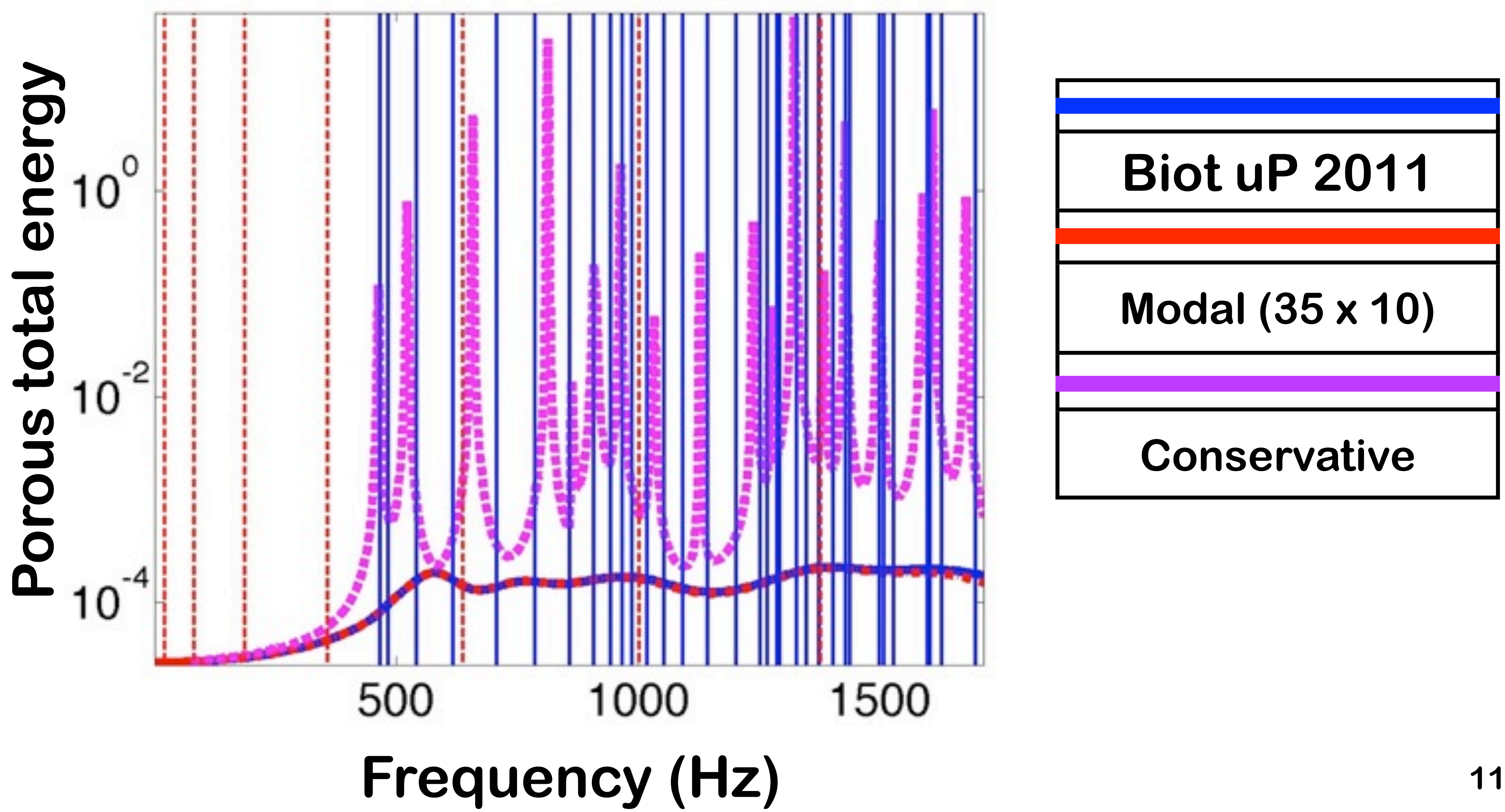


Resonances: a first approximation

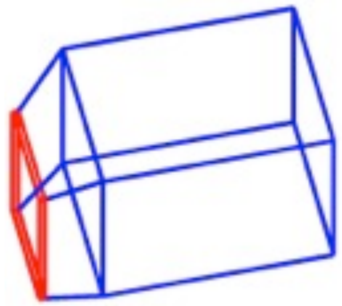
$$[\mathbf{R}_{ss}] = \hat{A}[\mathbf{k}_s^2] - \omega^2\widetilde{\rho}[\mathbf{I}] - \omega^2\widetilde{\rho}_{eq}\widetilde{\gamma}^2[\mathbf{\Phi}_s]^t[\mathbf{C}]^t[\mathbf{\Xi}_s]$$

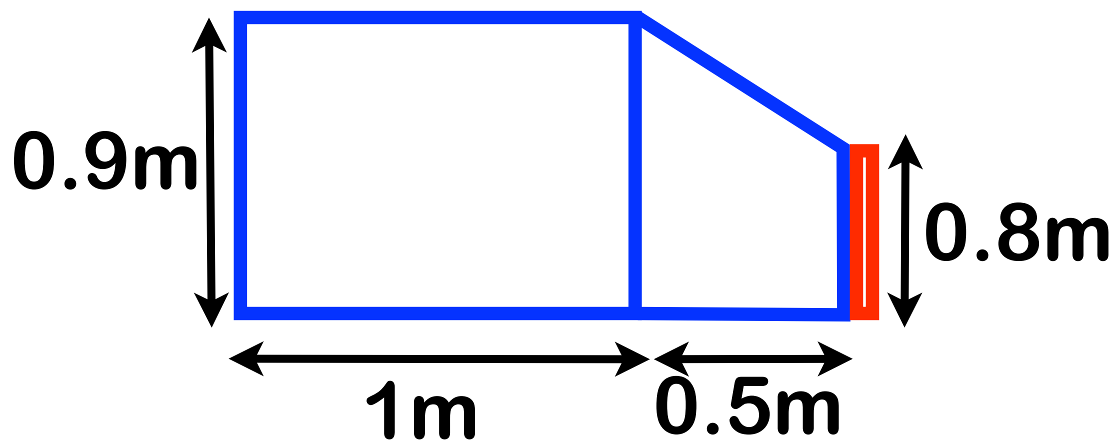
$$\omega_i \approx \sqrt{\mathcal{R}\left(\frac{\hat{A}[\mathbf{k}_s^2(i)]}{\widetilde{\rho} + \widetilde{\rho}_{eq}\widetilde{\gamma}^2[\mathbf{\Phi}_s(i)]^t[\mathbf{C}]^t[\mathbf{\Xi}_s(i)]}\right)}$$

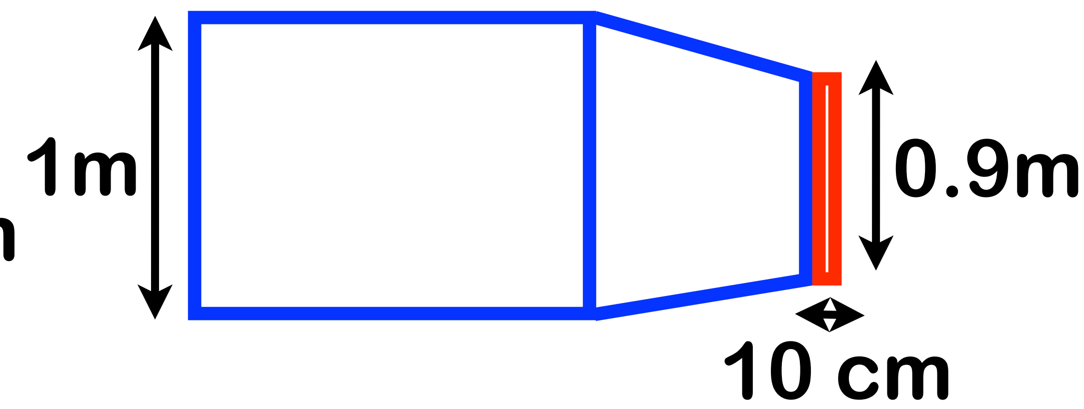


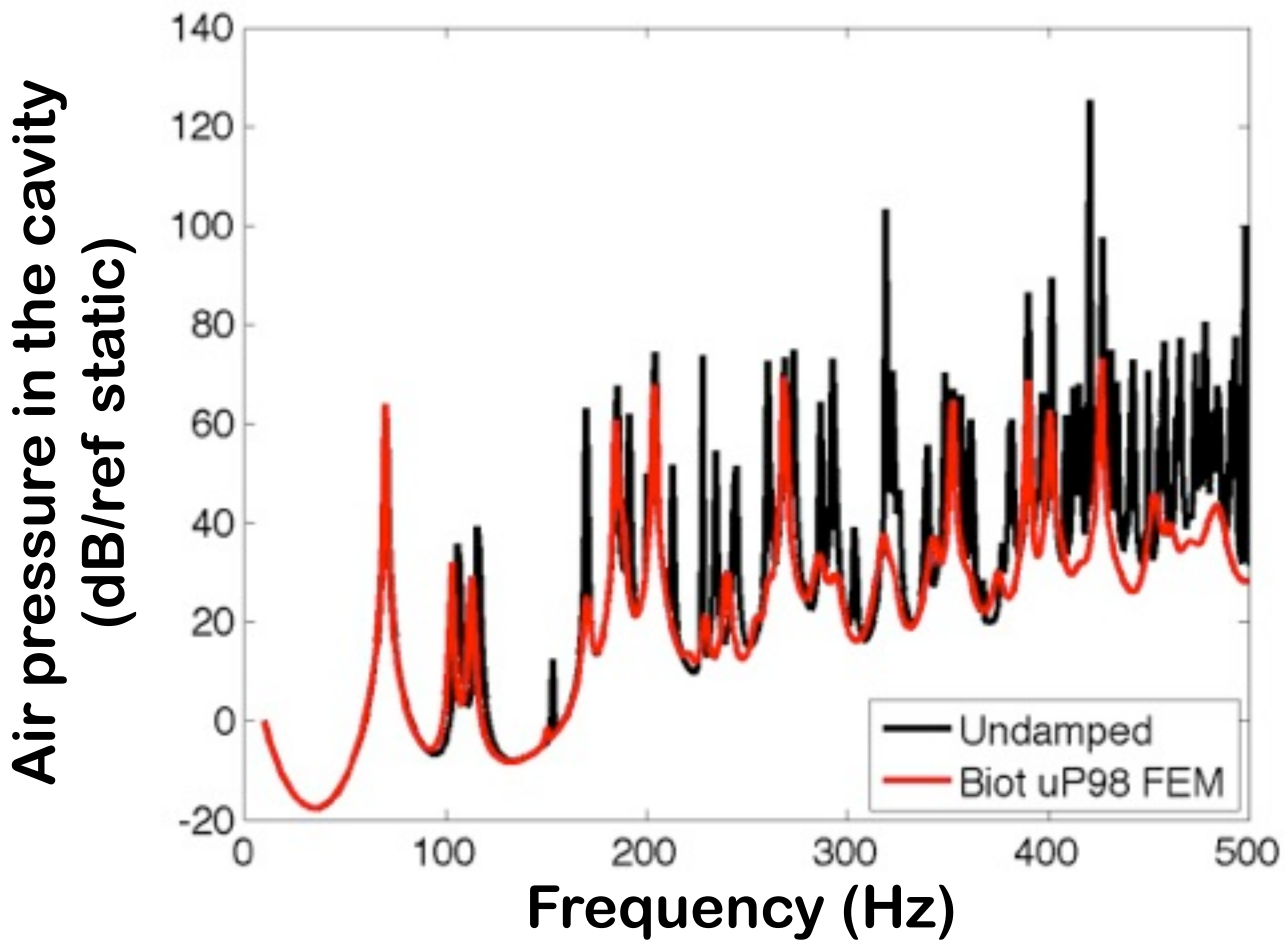


A non cartesian geometry





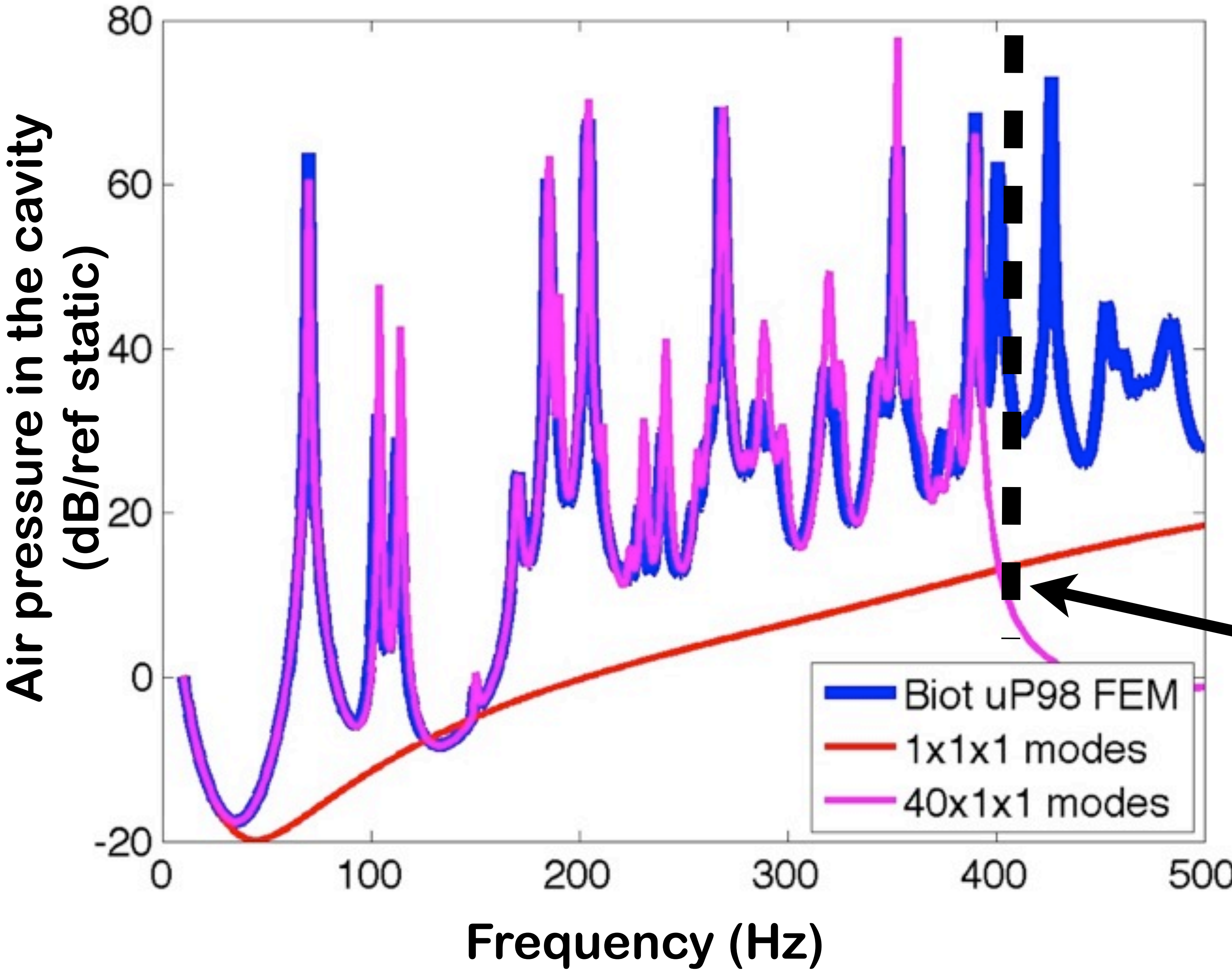




Home made Fortran code
Biot uP 1998
(35+5x8x7) HEXA27
500 Frequencies

Biot uP HSL ME57	18 802 s
Limp HSL ME57	3 012 s

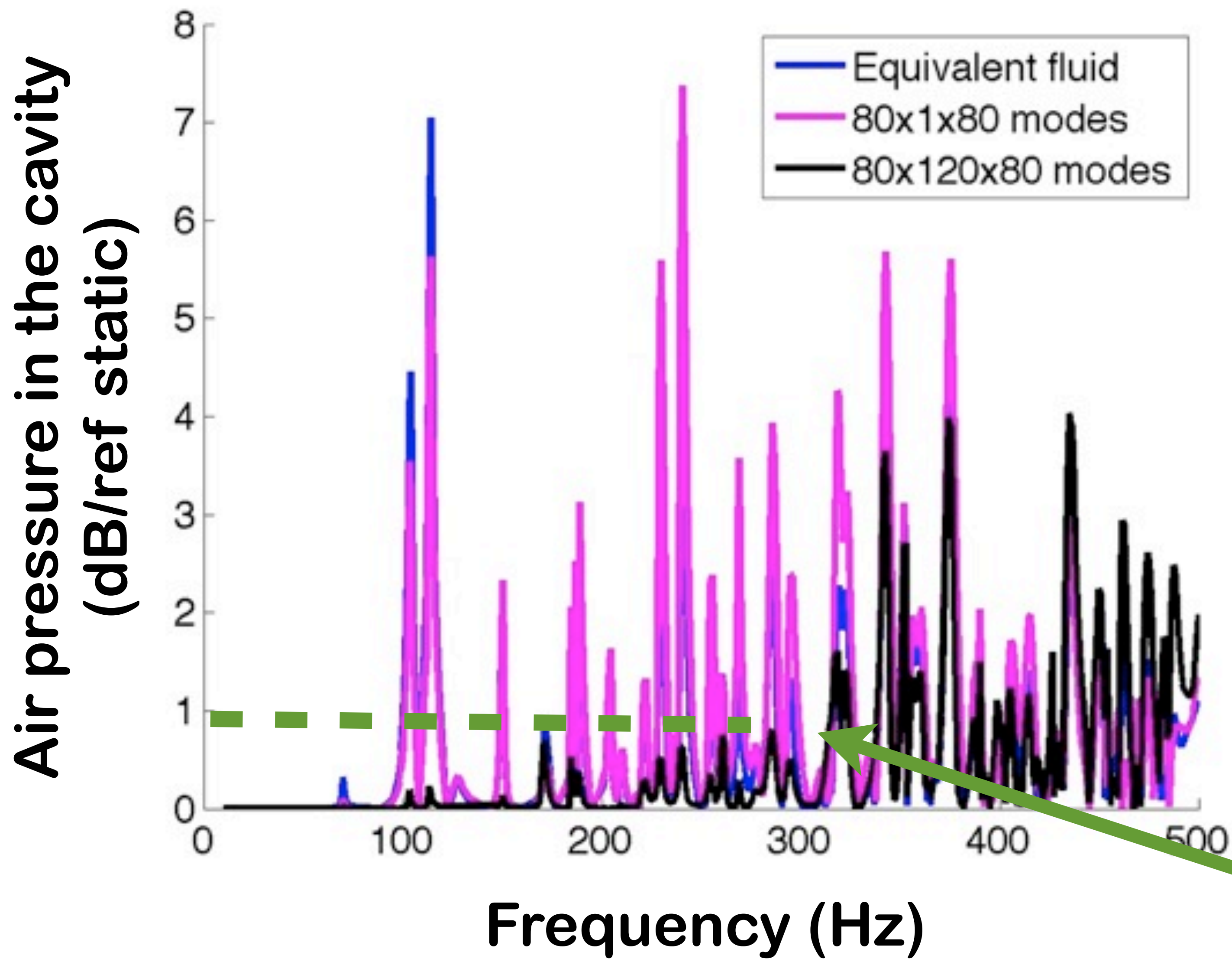
Modes in the cavity



Biot uP HSL ME57	18 802 s
Limp HSL ME57	3 012 s
Modes comp. time	255 s
Modal frequency loop	112 s
40x1x1 total time	367 s

Chosen cutoff frequency at 400 Hz

Modes of the PEM



Biot uP HSL ME57	18 802 s
Limp HSL ME57	3 012 s
80 x 1x80	
Modes comp. time	826 s
Modal frequency loop	260 s
80x120x80 total time	1086 s
80 x 120x80	
Modes comp. time	2196 s
Modal frequency loop	505 s
80x120x80 total time	2701 s

Use of solid modes to control error

Summary

Remembering Bradford

Can porous materials be treated with a “modal” approach

- Use of decoupled normal modes
  - Easy to calculate (Real and DP matrices)
  - Static correction through analytical calculations
    - Neglect inertia
    - Solid-fluid higher modes coupling
    - All of them are real
- Main results
  - Reduction of the size of systems
  - Prediction of resonances
  - Modes are geometric (test of an adequat material)
- Future works
  - Coupling with plates (I have it on a paper !)
  - Industrial cases
- Limits
  - Transition for the mid-frequency range of the solid phase