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**800**  
ANNI

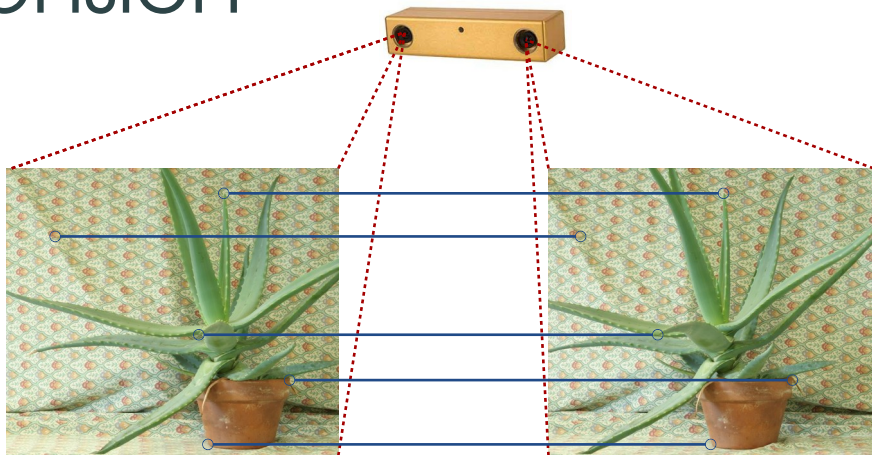


UNIVERSITÀ  
DEGLI STUDI  
DI PADOVA

# 3D Data Processing Lab 1

# Problem Formulation

- Search in a pair of stereo images for corresponding pixels that are projections of the same 3D point
- Epipolar constraints reduces the search to one dimension



# Dense Matching

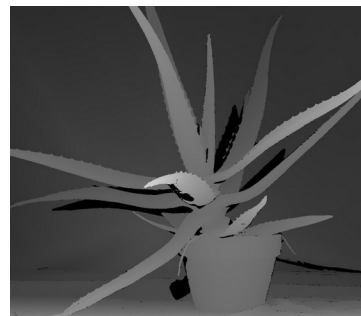
Find correspondences for all points that are projected in both images



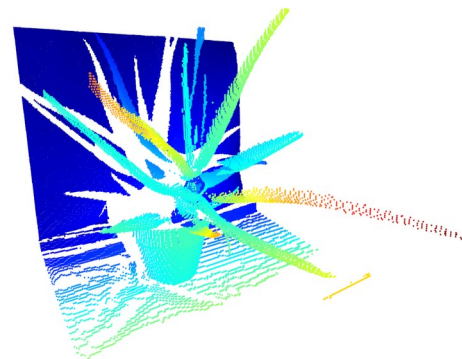
Left image



Right image



Disparity map

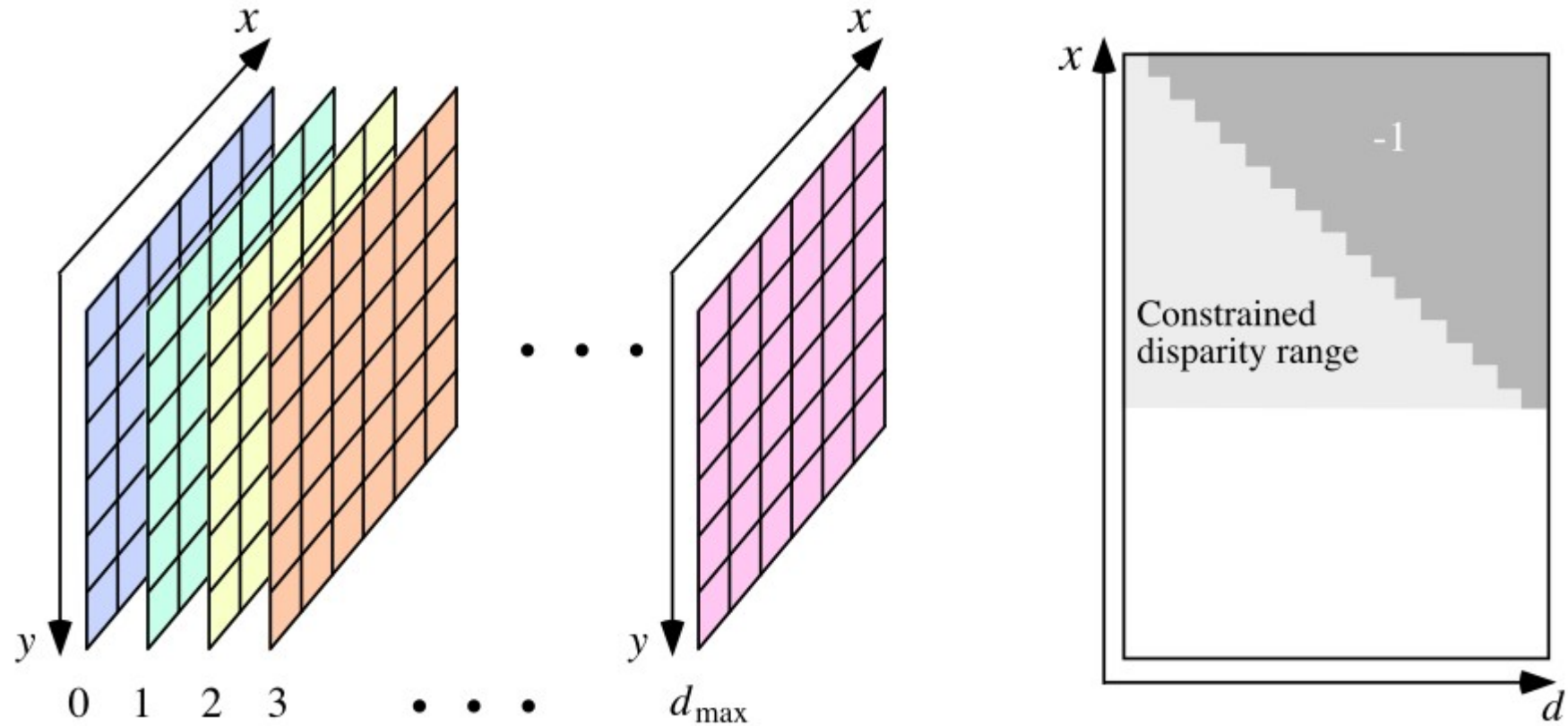


Point cloud

# Assumptions

- Most scene points are visible from both viewpoints
- Corresponding image regions are similar
- We consider rectified rectilinear stereo rigs: the epipolar lines are the rows of the images (**scanlines**)

# Cost Volume



# Cost Volume

```
for(int r = window_height_/2 + 1; r < height_ - window_height_/2 - 1; r++)
{
    for(int c = window_width_/2 + 1; c < width_ - window_width_/2 - 1; c++)
    {
        for(int d=0; d<disparity_range_; d++)
        {
            long cost = 0;
            // do stuff
            cost_[r][c][d]=cost;
        }
    }
}
```

# Global Methods

- Enforce smoothness constraints, i.e. disparity is **piecewise smooth**
- Enforce ordering constraints
- Greater computational complexity

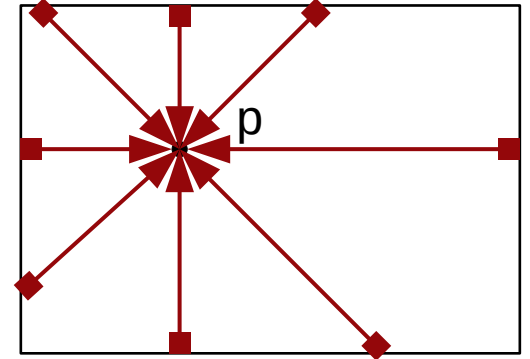
# Semi-Global Matching

- Approximate global methods by aggregating costs for a number of directions (from 2D to multiple 1D areas of interest)
- Minimization along individual image rows can be performed efficiently in polynomial time using Dynamic Programming



# Semi-Global Matching

- For each direction, start from one end point and go toward **p**
- For each pixel along the direction, update the following dynamic programming equation (the result at step i depends on the result at step i-1):



$$E(p_i, d) = E_{data}(p_i, d) + E_{smooth}(p_i, p_{i-1}) - \min_{0 \leq \Delta \leq d_{\max}} E(p_{i-1}, \Delta)$$

where:

$$E_{smooth}(p, q) = \min \begin{cases} E(q, f_q) & \text{if } f_p = f_q \\ E(q, f_q) + c_1 & \text{if } |f_p - f_q| = 1 \\ \min_{0 \leq \Delta \leq d_{\max}} E(q, \Delta) + c_2(p, q) & \text{if } |f_p - f_q| > 1 \end{cases}$$

▲ Restrict the range of resulting values, without affecting the minimization procedure

# Exercise: Semi-Global Matching

Consider the stereo matching problem for the following 7 x 1 left and right images:

2	3	1	2	3	3	1
---	---	---	---	---	---	---

1	2	3	1	4	0	2
---	---	---	---	---	---	---

- 1) Compute the right to left **cost volume**, considering positive disparities,  $d \geq 0$  with  $d_{\max} = 3$  and data cost defined by the sum of absolute differences (SAD) computed in  $1 \times 1$  windows. Use value -1 to set "no disparity assigned"
- 2) Given the matching cost computed in 1), consider the Semi-Global Matching method with the following simplified dynamic programming equations:

$$E(p_i, d) = E_{\text{data}}(p_i, d) + E_{\text{smooth}}(p_i, p_{i-1})$$

$$E_{\text{smooth}}(p, q) = \min \begin{cases} E(q, f_q) & \text{if } f_p = f_q \\ E(q, f_q) + c_1 & \text{if } |f_p - f_q| = 1 \\ \min_{0 \leq \Delta \leq d_{\max}} E(q, \Delta) + c_2 & \text{if } |f_p - f_q| > 1 \end{cases}$$

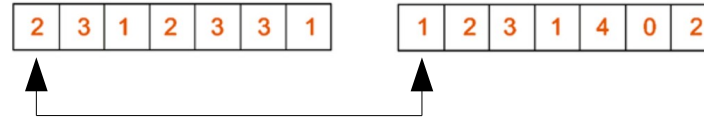
where  $c_1 = 1$  and  $c_2 = 2$ . Compute the integration matrix for the highlighted pixel and scanline:

2	3	1	2	3	3	1
---	---	---	---	---	---	---

1	2	3	1	4	0	2
---	---	---	---	---	---	---

# Exercise Solution



0	1	1	2	1	1	3	1
1	2	1	1	2	1	1	-1
2	0	0	0	2	3	-1	-1
3	1	1	0	0	-1	-1	-1

# Exercise Solution



$$E(p_i, d) = E_{data}(p_i, d) + E_{smooth}(p_i, p_{i-1})$$

$$E_{smooth}(p, q) = \min \begin{cases} E(q, f_q) & \text{if } f_p = f_q \\ E(q, f_q) + c_1 & \text{if } |f_p - f_q| = 1 \\ \min_{0 \leq \Delta \leq d_{\max}} E(q, \Delta) + c_2 & \text{if } |f_p - f_q| > 1 \end{cases}$$

0				
1				
2				
3				



# Exercise Solution



$$E(p_i, d) = E_{data}(p_i, d) + E_{smooth}(p_i, p_{i-1})$$

$$E_{smooth}(p, q) = \min \begin{cases} E(q, f_q) & \text{if } f_p = f_q \\ E(q, f_q) + c_1 & \text{if } |f_p - f_q| = 1 \\ \min_{0 \leq \Delta \leq d_{\max}} E(q, \Delta) + c_2 & \text{if } |f_p - f_q| > 1 \end{cases}$$

0	1			
1	2			
2	0			
3	1			



# Exercise Solution



$$E(p_i, d) = E_{data}(p_i, d) + E_{smooth}(p_i, p_{i-1})$$

$$E_{smooth}(p, q) = \min \begin{cases} E(q, f_q) \\ E(q, f_q) + c_1 \\ \min_{0 \leq \Delta \leq d_{max}} E(q, \Delta) + c_2 \end{cases}$$

if  $f_p = f_q$   
 if  $|f_p - f_q| = 1$   
 if  $|f_p - f_q| > 1$

0	1			
1	2			
2	0			
3	1			

$$1 + \min(1, 2+1, 0+2)=2$$

$$1 + \min(2, 1+1, 0+1, 0+2)=2$$

$$0 + \min(0, 2+1, 1+1, 0+2)=0$$

$$1 + \min(1, 0+1, 0+2)=2$$



# Exercise Solution



$$E(p_i, d) = E_{data}(p_i, d) + E_{smooth}(p_i, p_{i-1})$$

$$E_{smooth}(p, q) = \min \begin{cases} E(q, f_q) \\ E(q, f_q) + c_1 \\ \min_{0 \leq \Delta \leq d_{\max}} E(q, \Delta) + c_2 \end{cases}$$

if  $f_p = f_q$   
 if  $|f_p - f_q| = 1$   
 if  $|f_p - f_q| > 1$

0	1			
1	2			
2	0			
3	1			

$$1 + \min(1, 2+1, 0+2)=2$$

$$1 + \min(2, 1+1, 0+1, 0+2)=2$$

$$0 + \min(0, 2+1, 1+1, 0+2)=0$$

$$1 + \min(1, 0+1, 0+2)=2$$



# Exercise Solution



$$E(p_i, d) = E_{data}(p_i, d) + E_{smooth}(p_i, p_{i-1})$$

$$E_{smooth}(p, q) = \min \begin{cases} E(q, f_q) \\ E(q, f_q) + c_1 \\ \min_{0 \leq \Delta \leq d_{\max}} E(q, \Delta) + c_2 \end{cases}$$

if  $f_p = f_q$   
 if  $|f_p - f_q| = 1$   
 if  $|f_p - f_q| > 1$

0	1	2		
1	2	2		
2	0	0		
3	1	2		

$$2 + \min(2, 2+1, 0+2)=4$$

$$1 + \min(2, 2+1, 0+1, 0+2)=2$$

$$0 + \min(0, 2+1, 2+1, 0+2)=0$$

$$0 + \min(2, 0+1, 0+2)=1$$





# Exercise Solution



$$E(p_i, d) = E_{data}(p_i, d) + E_{smooth}(p_i, p_{i-1})$$

$$E_{smooth}(p, q) = \min \begin{cases} E(q, f_q) \\ E(q, f_q) + c_1 \\ \min_{0 \leq \Delta \leq d_{\max}} E(q, \Delta) + c_2 \end{cases}$$

if  $f_p = f_q$   
 if  $|f_p - f_q| = 1$   
 if  $|f_p - f_q| > 1$

0	1	2	4	
1	2	2	2	
2	0	0	0	
3	1	2	1	

$$1 + \min(4, 2+1, 0+2)=3$$

$$2 + \min(2, 4+1, 0+1, 0+2)=3$$

$$2 + \min(0, 2+1, 1+1, 0+2)=2$$

$$0 + \min(1, 0+1, 0+2)=1$$



# Exercise Solution



$$E(p_i, d) = E_{data}(p_i, d) + E_{smooth}(p_i, p_{i-1})$$

$$E_{smooth}(p, q) = \min \begin{cases} E(q, f_q) & \text{if } f_p = f_q \\ E(q, f_q) + c_1 & \text{if } |f_p - f_q| = 1 \\ \min_{0 \leq \Delta \leq d_{\max}} E(q, \Delta) + c_2 & \text{if } |f_p - f_q| > 1 \end{cases}$$

0	1	2	4	3
1	2	2	2	3
2	0	0	0	2
3	1	2	1	1