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ITEC - Institute of Information Technology

ALPEN-ADRIA UNIVERSITÄT KLAGENFURT

Dimensionality Reduction and Data Fusion

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Key questions

- How to transform a high-dimensional data set into a small set?
- Which methods can be used to combine and reduce the high-dimensional data?
- Are these methods among the learning-based schemes?
- Do I need some pre-processing techniques before data fusion exploitation?

Data fusion

- Humans, who rely on their senses as the vision, smell, taste, voice and physical movement, are a principal example of data-fusion system.
- A major tool is to remove the dependencies among the collected data.
- In computer science, it is also required to combine various data sets into a unified (fused) data set which includes all data points.

Data fusion (cont.)

- To store, analyze and summarize the vast amounts of generated data, one may reduce the dimension of data by dimensionality-reduction data fusion.

- This transformation finds a subspace whose vectors are a combination of the old subspace and projects a *t*-dimensional space onto an *k*-dimensional subspace of the original features, where k<<t.

- The raw data sets collected from our implementations are not available before running the algorithm.

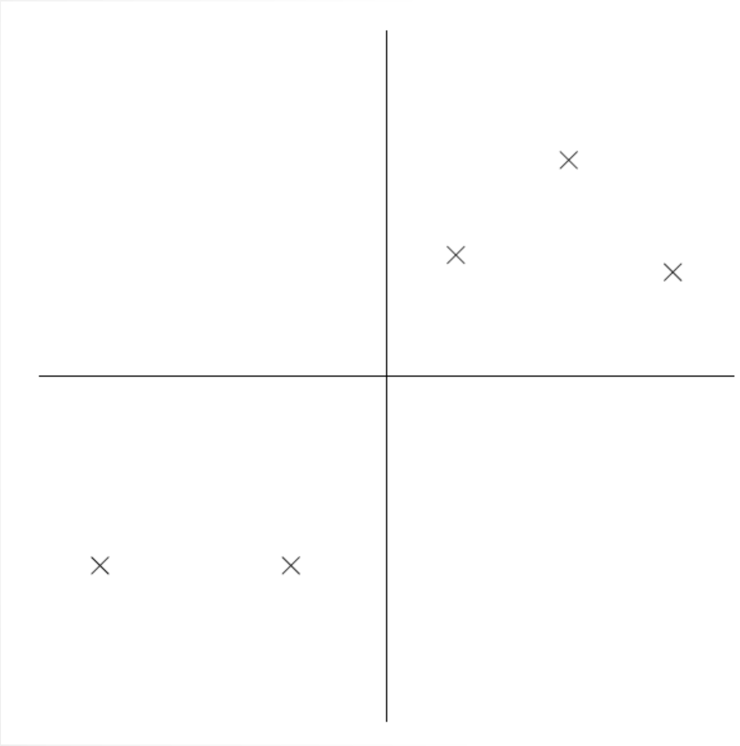
Principal component analysis

- PCA is mathematically defined as an orthogonal linear transformation that transforms the data to a new coordinate system,

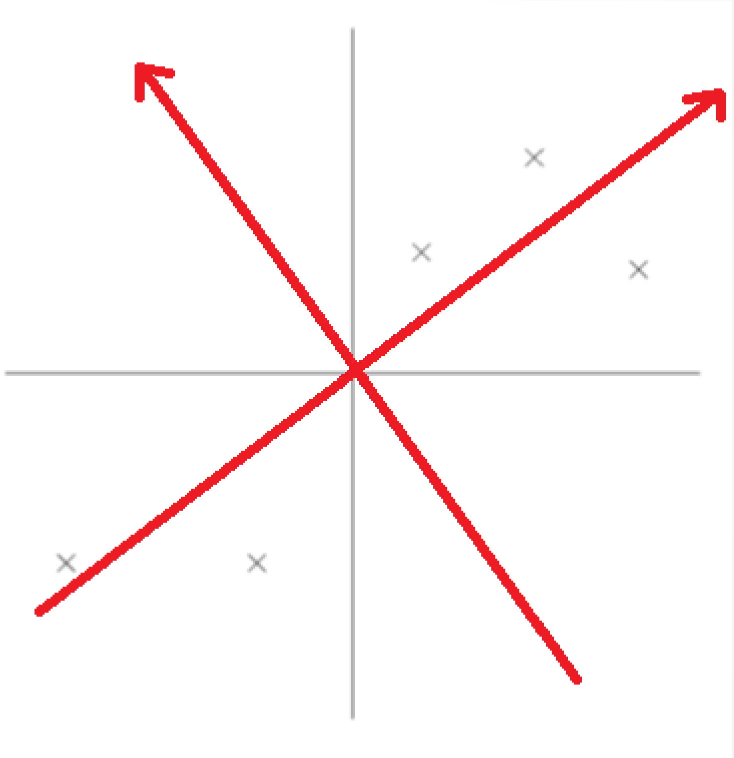
- PCA
  - helps to find relevant structure in data,
  - helps to throw away things that won't matter

- The projection of the data comes to lie on the new coordinate system,
  - the greatest variance on the first coordinate (called the first principal component),
  - the second greatest variance on the second coordinate,
  - and so on

Transforming the data set to the new space

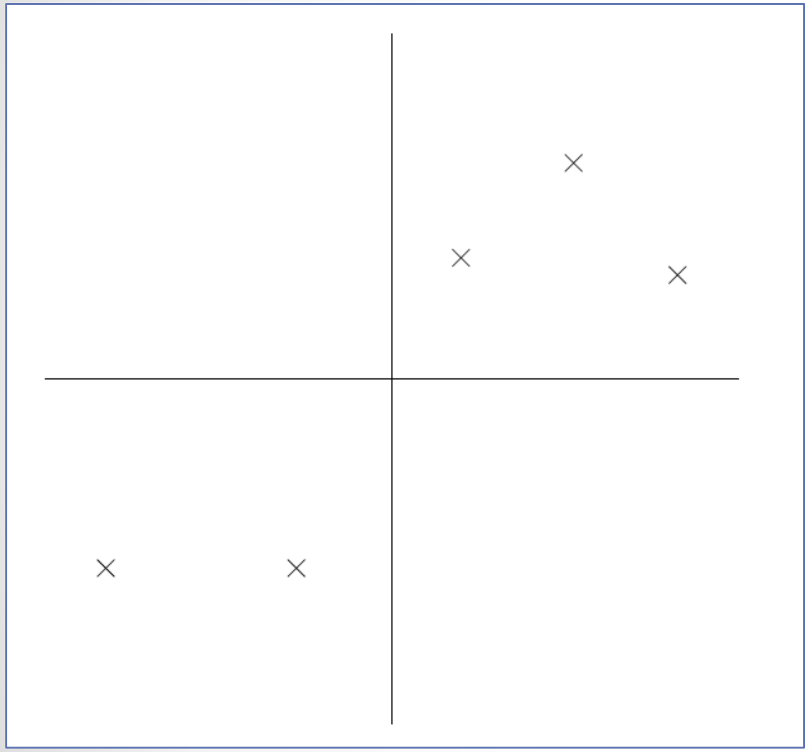


Data in the old space

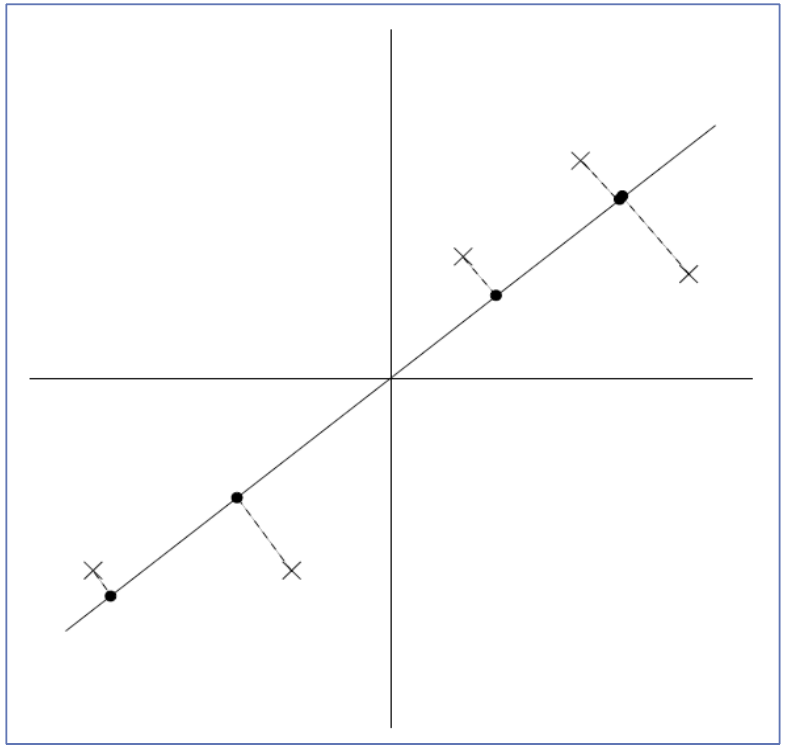


Data in new space after PCA Transformation

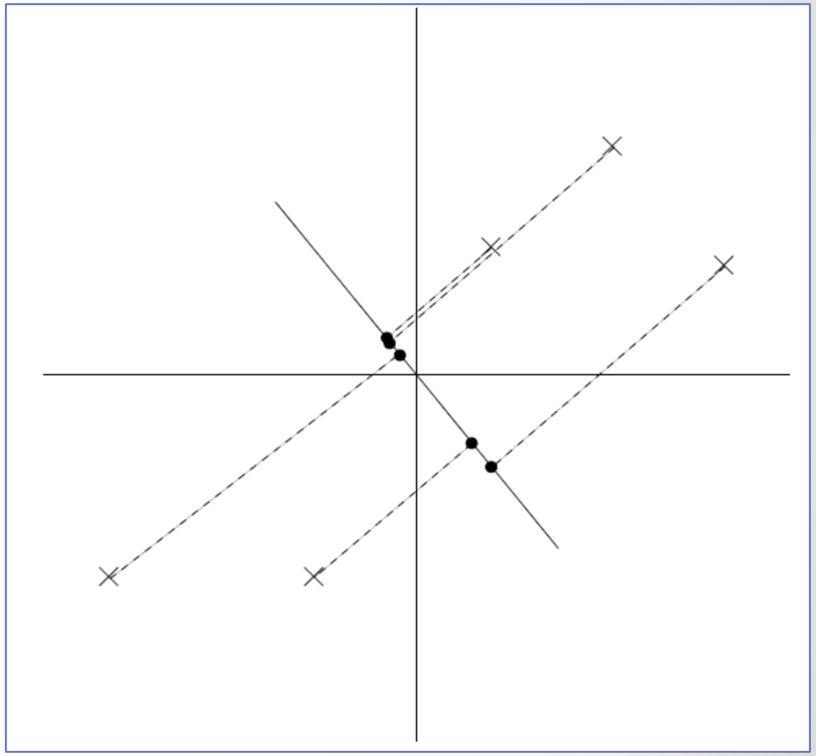
Projection of the data



Data in the old space



Projection on the line with higher variance



Projection on the line with lower variance

PCA Calculation

- Vectors of data *X*: $X_1, X_2, X_3, \ldots, X_J$
  - Dimension of every vector of data *X*: $I \times 1$
- Matrix dimension: *I*×*J* → *I* is the number of samples,
  J shows the attribute for every sample.
- First step is to calculate the average of samples and *normalizing them*:

$$\mu_j = \frac{1}{I}\left(\sum_{i=1}^{I} X_{ij}\right) \quad \rightarrow \quad X = [X_1 - \mu_1, X_2 - \mu_2, \ldots, X_J - \mu_J,]$$

PCA Calculation (cont.)

- Second step is to calculate the principal components of the new subspace:
  1) Calculating the co-variance matrix:
     - $C = \frac{1}{l}(X^T X)$
     - $V_i C = \lambda_i C$
  2) Calculating by the singular value decomposition: SVD (X) = [U,Σ,V] = U Σ V^T
     - Σ is the diagonal matrix
     - *U* and *V* are unitary matrices
- Third step is choosing a few number of eigenvectors of *V* and projecting *X* on this new subspace.
  - $X_K = X.V_k$

Randomized-SVD algorithm

- For reducing the size of information and combining features with different qualities, Truncated-SVD is exploited; Truncated-SVD has the ability to extract only the most important information from the data matrix by using just the first several components estimated from the original matrix of data set.

- Randomized-SVD implements a type of Truncated Singular Value Decomposition (Truncated-SVD) that only computes the k-largest singular values with a randomized algorithm, where k is a user-specified parameter.

- Randomized-SVD is similar to PCA, but differs in that it works on sample matrix X directly instead of their covariance matrices. When the column-wise (per-feature) means of X is subtracted from the feature values, Randomized-SVD on the resulting matrix is equivalent to PCA.

Randomized-SVD algorithm (cont.)

- Given an m×n matrix *X*, a target number *k* of singular vectors, this procedure computes an approximate factorization *UV*, where *U* and *V* are orthonormal matrices whose columns are eigenvectors of X.X* and X*.X respectively, and is nonnegative and diagonal matrix containing the eigenvalues of X. X* in the diagonal being sorted in descending order.

- By considering the problem of finding the k principal components of the SVD of an m×n input matrix, randomized algorithms involve O(mnlog(k)) floating-point operations (flops) in distinction to O(mnk) for classical algorithms.

- Randomized-SVD can generate a structure from an unstructured input data matrix by using a subsampled random Fourier Transform (SRFT) and QR decomposition:

Randomized-SVD algorithm (cont.)

Algorithm 1 Randomized-SVD's Pseudo Code

GOAL: GIVEN AN $m \times n$ INPUT MATRIX X, COMPUTE AN APPROXIMATE RANK-K SVD: $X \approx U_k.\Sigma_k.V_k{}^T$

1: Draw an $n \times k$ Gaussian random matrix Ω,
2: Form an $m \times k$ orthonormal matrix Q by using (subsampled) FFT and QR factorization,
3: Form the $k \times n$ matrix $B = Q^T.X$
4: Compute the SVD of the small matrix B: $B = \tilde{U}.\Sigma_k.V_k{}^T$,
5: Form the matrix $U_k = Q.\tilde{U}$,
6: Calculate $X_k = U_k.\Sigma_k.V_k{}^T$.

✓ N. Mehran and N. Movahhedinia "Randomized SVD Based Probabilistic Caching Strategy in Named Data Networks," (2018).

Reaching the fused data in a nutshell

- Using the Randomized SVD algorithm as a data fusion model as follows:

At first, normalizing the data

$$x = \frac{x - mean}{std\_dev}$$

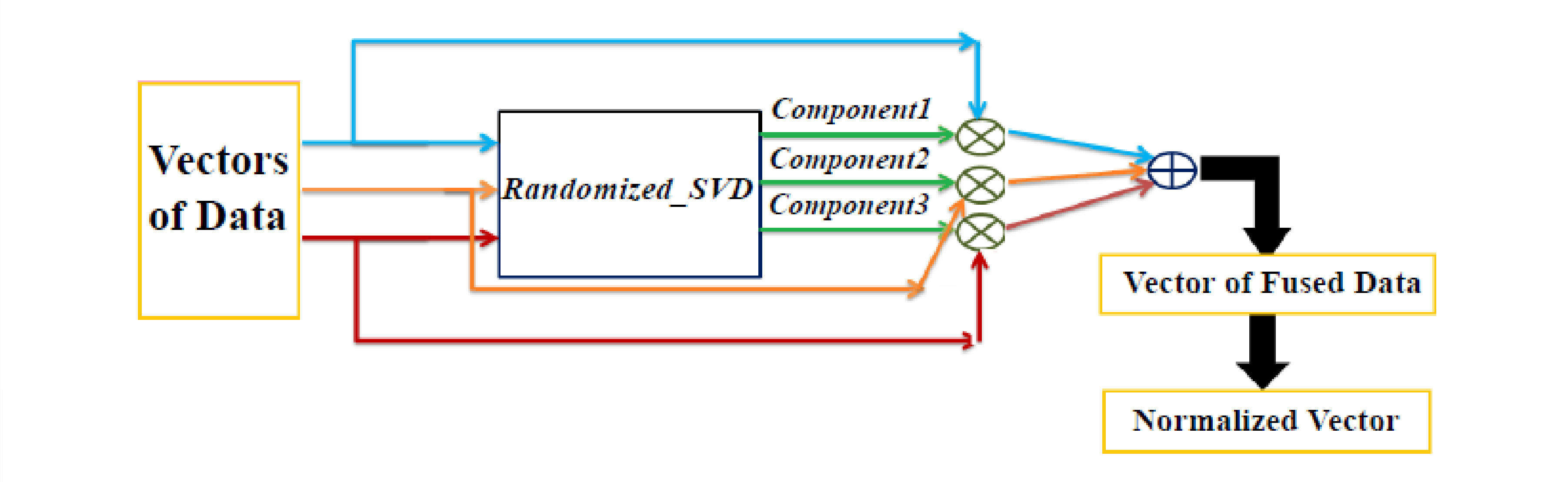
Second, Randomized-SVD, applied to the training samples X, produces a low-rank approximation X_k

$$X \approx X_k = U_k.\, \Sigma_k.\, {V_k}^T$$

$U_k \Sigma_k$ is a transformed training set with k features. To also transform the original set X, we multiply it with V_k (the normalized eigenvectors of a new sub-space)

$$X_{fused} = X \,.\, V_k$$

A Data Fusion Diagram



- ✓ Raol, Jitendra R. "Multi-Sensor Data Fusion with MATLAB," (2009).
- ✓ N. Mehran and N. Movahhedinia "Randomized SVD Based Probabilistic Caching Strategy in Named Data Networks," (2018).

- https://scikit-learn.org/stable/
- ***Scikit-learn***, an open source *library* in Python, can be exploited for:

https://scikit-learn.org/stable/

Classification

Identifying to which category an object belongs to.

Applications: Spam detection, Image recognition.
Algorithms: SVM, nearest neighbors, random forest, …

— Examples

Regression

Predicting a continuous-valued attribute associated with an object.

Applications: Drug response, Stock prices.
Algorithms: SVR, ridge regression, Lasso, …

— Examples

Clustering

Automatic grouping of similar objects into sets.

Applications: Customer segmentation, Grouping experiment outcomes
Algorithms: k-Means, spectral clustering, mean-shift, …

— Examples

Dimensionality reduction

Reducing the number of random variables to consider.

Applications: Visualization, Increased efficiency
Algorithms: PCA, feature selection, non-negative matrix factorization.

— Examples

Model selection

Comparing, validating and choosing parameters and models.

Goal: Improved accuracy via parameter tuning
Modules: grid search, cross validation, metrics.

— Examples

Preprocessing

Feature extraction and normalization.

Application: Transforming input data such as text for use with machine learning algorithms.
Modules: preprocessing, feature extraction.

— Examples

Randomized-SVD.py pca.py

```
        X = array2d(X)
        n_samples, n_features = X.shape
        X = as_float_array(X, copy=self.copy)
        # Center data
        self.mean_ = np.mean(X, axis=0)
        X -= self.mean_
        U, S, V = linalg.svd(X, full_matrices=False)
        self.explained_variance_ = (S ** 2) / n_samples
        self.explained_variance_ratio_ = (self.explained_variance_ /
                                          self.explained_variance_.sum())

        if self.whiten:
            self.components_ = V / S[:, np.newaxis] * np.sqrt(n_samples)
        else:
            self.components_ = V


    Cov_ = np.cov(X.transpose())
    print "\nCovariance matirx is:"
    print Cov_
        if self.n_components == 'mle':
            if n_samples < n_features:
                raise ValueError("n_components='mle' is only supported "
                                 "if n_samples >= n_features")
            self.n_components = _infer_dimension_(self.explained_variance_,
                                                  n_samples, n_features)

        elif (self.n_components is not None
              and 0 < self.n_components
              and self.n_components < 1.0):
            # number of components for which the cumulated explained variance
            # percentage is superior to the desired threshold
            ratio_cumsum = self.explained_variance_ratio_.cumsum()
            self.n_components = np.sum(ratio_cumsum < self.n_components) + 1

        if self.n_components is not None:
            self.components_ = self.components_[:self.n_components, :]
            self.explained_variance_ = \
                self.explained_variance_[:self.n_components]
            self.explained_variance_ratio_ = \
                self.explained_variance_ratio_[:self.n_components]

        return (U, S, V)
```

PCA

Randomized-SVD.py pca.py

Randomized SVD

```python
    * An implementation of a randomized algorithm for principal component
      analysis
      A. Szlam et al. 2014
    """
    random_state = check_random_state(random_state)
    n_random = n_components + n_oversamples
    n_samples, n_features = M.shape

    if n_iter == 'auto':
        # Checks if the number of iterations is explicitely specified
        # Adjust n_iter. 7 was found a good compromise for PCA. See #5299
        n_iter = 7 if n_components < .1 * min(M.shape) else 4

    if transpose == 'auto':
        transpose = n_samples < n_features
    if transpose:
        # this implementation is a bit faster with smaller shape[1]
        M = M.T

    Q = randomized_range_finder(M, n_random, n_iter,
                                power_iteration_normalizer, random_state)

    # project M to the (k + p) dimensional space using the basis vectors
    B = safe_sparse_dot(Q.T, M)

    # compute the SVD on the thin matrix: (k + p) wide
    Uhat, s, V = linalg.svd(B, full_matrices=False)
    del B
    U = np.dot(Q, Uhat)

    if flip_sign:
        if not transpose:
            U, V = svd_flip(U, V)
        else:
            # In case of transpose u_based_decision=false
            # to actually flip based on u and not v.
            U, V = svd_flip(U, V, u_based_decision=False)

    if transpose:
        # transpose back the results according to the input convention
        return V[:n_components, :].T, s[:n_components], U[:, :n_components].T
    else:
        return U[:, :n_components], s[:n_components], V[:n_components, :]
```

A sample code of Python

```
from sklearn.decomposition import PCA
from sklearn.decomposition import TruncatedSVD

X_std = StandardScaler().fit_transform(X)

sklearn_X = TruncatedSVD(n_components=1)
sklearn_transf = sklearn_X.fit(X_std)
```

Ref.

- http://cs229.stanford.edu/notes/cs229-notes10.pdf
- J. Novakovic and . S. Rankov, "Classification performance using principal component analysis and different value of the ratio R," *International Journal of Computers Communications & Control,* vol. 6, no. 2, pp. 317-327, 2011.
- A. Janecek, W. Gansterer, M. Demel and G. Ecker, "On the relationship between feature selection and classification accuracy," in *New Challenges for Feature Selection in Data Mining and Knowledge Discovery*, 2008.
- H. Abdi and L. J. Williams, "Principal component analysis," *Wiley interdisciplinary reviews: computational statistics,* vol. 2, no. 4, pp. 433-459, 2010.
- Huamin Li, George C. Linderman, Arthur Szlam, Kelly P. Stanton, Yuval Kluger, and Mark Tygert, "Algorithm 971: An implementation of a randomized algorithm for principal component analysis," *Math. Softw.,* 43 (3): 28: 1-28: 14, January 2017.
- Christopher D. Manning, Prabhakar Raghavan, and Hinrich Schuetze. Matrix decompositions & latent semantic indexing in Introduction to information Retrieval, pages 220{235. Cambridge University Press, New York, NY, USA, 2008.

Thank you