

Algorithms Mid-term Exam (In-class 60%)

PM 5:30 前繳交

1. [5 pts] State whether each of the following is **true or false**, Why?

$$5^{\frac{3n}{2}} = O(5^n)$$

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✓

✓

2. [5 pts] Solve the recurrence and represented with *asymptotic notation*, $T(1) = c$ and
 $T(n) = 10T(n/5) + \Theta(n)$

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3. [10 pts] Compute *time complexity* of the following algorithm:

(Need to show the recurrence of time complexity and solve it)

sum (A, p, r) {

if p = r

return A[p]

else {

k = $\lfloor (p + r) / 2 \rfloor$

return sum(A, p, k) + sum(A, k+1, r)

}

}

✓

4. [10 pts] A teacher has sorted the exam sheets of all the students at a class on their student ID number. Now he would like to take that list and sort by exam score. However, he wishes to be certain that if there are several students same score, at the end of the sort by score, these students are still sorted by student ID number. **What sorting property is required for the algorithm and what is the time complexity of the algorithm?**

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5. [10 pts] Using MAX-HEAPIFY algorithm,

(a) Show the operation on the array:

I A = [17, 86, 41, 13, 65, 32, 29, 9, 10, 44, 23, 21]

(b) Show the worst-case running time.

6. [10 pts] Design and analysis a variant QUICKSORT algorithm, which runs $O(n \lg n)$ in **worst-case**.

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7. [10 pts] For the following input array $A = [10, 80, 30, 90, 40, 50, 70]$, show the operation of PARTITION on the array A .

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PARTITION( $A, p, r$ )
1   $x \leftarrow A[r]$ 
2   $i \leftarrow p - 1$ 
3  for  $j \leftarrow p$  to  $r - 1$ 
4      do if  $A[j] \leq x$ 
5          then  $i \leftarrow i + 1$ 
6              exchange  $A[i] \leftrightarrow A[j]$ 
7  exchange  $A[i + 1] \leftrightarrow A[r]$ 
8  return  $i + 1$ 
```

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