Algorithms Mid-term Exam (Take home 40%) Due: 2024/04/26

1. [10 pts] Consider a modification to **MERGE-SORT** in which n/k sublists of length k are sorted using **INSERTION-SORT** and then merged using the standard merging mechanism, where k is a value to be determined.

   (a) Show that the n/k sublists, each of length k, can be sorted by **INSERTION-SORT** in $\Theta(nk)$ worst-case time.

   (b) Show that the sublists can be merged in $\Theta(n \lg (n/k))$ worst-case time.

   (c) Given that the modified algorithm runs in $\Theta(nk + n \lg (n/k))$ worst-case time, what is the largest asymptotic value of k as a function of n for which the modified algorithm has the same asymptotic running time as standard **MERGE-SORT.**

   (d) How should k be chosen in practice?

2. [10 pts]

   (a) Write pseudocode for **Binary Search.**

   (b) Write a recurrence for the time complexity T(n) and represent the time complexity using asymptotic notation.

3. [10 pts] Use the **substitution method** to prove that the recurrence $T(n) = T(n-1) + \Theta(n)$ has the solution $T(n) = \Theta(n^2)$.

4. [10 pts] Using **MAX-HEAPIFY** algorithm,

   (a) Show the operation on the array:
   A= [17, 86, 41, 13, 65, 32, 29, 9, 10, 44, 23, 21]

   (b) Show the worst-case running time.