NOVA SCHOOL OF SCIENCE & TECHNOLOGY

Master's in Electrical and Computer Engineering

Circuitos e Sistemas para Rádio-Frequência

Design of an RF-CMOS front-end for a Low Data Rate SDR.

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1 Introduction

This labwork investigates the use of linear precoding and beamforming techniques in multiple-input multiple-output (MIMO) communication systems, with particular focus on their impact on downlink performance. In multi-user and multi-stream MIMO scenarios, the simultaneous transmission of several data streams leads to inter-stream and inter-user interference, which must be properly managed in order to fully exploit the available spatial degrees of freedom while maintaining reliable communication.

In the first phase of the work, a MIMO-OFDM downlink system is analysed under the assumption of perfect channel state information (CSI) at both the transmitter and the receiver, as well as ideal time and frequency synchronisation. Two linear precoding techniques are considered and compared: Zero-Forcing (ZF) precoding and Singular Value Decomposition (SVD) based precoding. A Rayleigh fading channel model is assumed, and full spatial multiplexing is adopted, requiring precoding to ensure independence between the transmitted data streams. System performance is evaluated in terms of bit-error rate (BER), achievable capacity, and computational complexity.

In the second phase, the analysis is extended to a beamforming-based transmission scenario in a multi-user MIMO system, where users are equipped with multiple antennas. Both perfect and imperfect channel estimation are considered in order to assess the impact of channel estimation errors on inter-user interference and overall system performance. The influence of beamforming on BER and interference mitigation is analysed and compared with scenarios where beamforming is not employed.

Throughout the work, transmission is based on an OFDM waveform, and Monte Carlo simulations are used to obtain statistically meaningful performance results. This laboratory work aims to provide insight into the trade-offs between performance, robustness, and computational complexity associated with different linear precoding and beamforming strategies in MIMO systems.

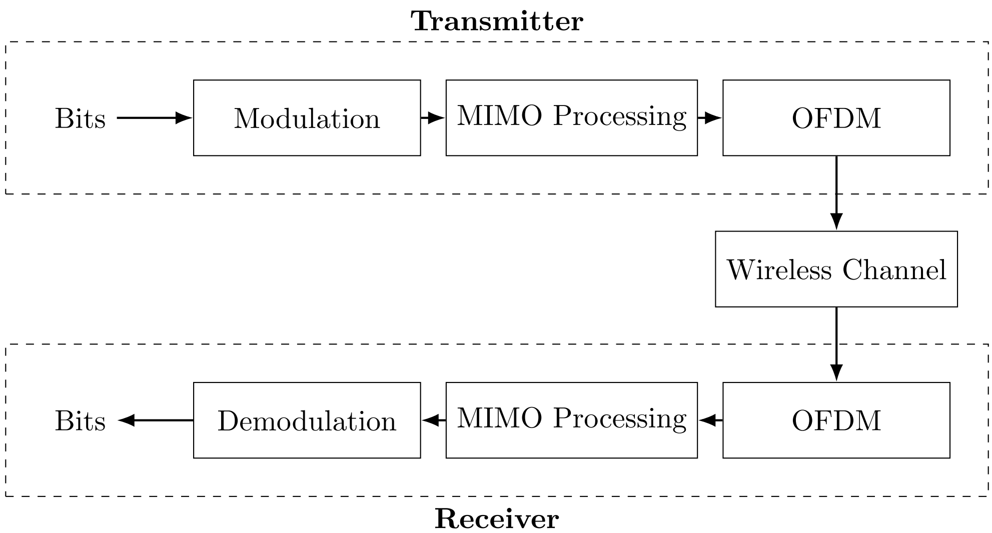

Figure 1: Generic block diagram of the MIMO-OFDM transmission system considered in this work.

Figure 1 shows a generic block diagram of the MIMO-OFDM communication system considered in this work. The transmitter processes the information bits through modulation, MIMO spatial processing and OFDM prior to transmission over the wireless channel. At the receiver, the inverse operations are applied in order to recover the transmitted data. More detailed processing blocks are introduced and analysed in the subsequent phases.

2 Phase I – Downlink Precoding with Perfect CSI

2.1 System Implemented and Parameters

In this first phase, a downlink MIMO-OFDM transmission system is considered under the assumption of perfect channel state information (CSI) at both the transmitter and the receiver. Ideal time and frequency synchronisation are also assumed. The objective of this phase is to evaluate and compare different linear precoding techniques in terms of bit-error rate, achievable capacity, and computational complexity.

The simulated system consists of a base station equipped with $N_{\text{tx}} = 16$ transmit antennas and a receiver with $N_{\text{rx}} = 16$ receive antennas. Full spatial multiplexing is employed, with $N_s = 16$ independent data streams transmitted simultaneously. An OFDM waveform with $N = 512$ subcarriers is used, and Monte Carlo simulations are performed over $N_{\text{slot}} = 100$ independent channel realisations for each signal-to-noise ratio point.

The wireless channel is modelled as a Rayleigh fading MIMO channel, where the channel matrix is assumed to remain constant over all subcarriers within a given OFDM symbol (flat fading per slot). Additive white Gaussian noise (AWGN) is added at the receiver. The signal-to-noise ratio is defined in terms of the energy per bit to noise power spectral density ratio E_b/N_0, which is swept from -5 dB to 22 dB.

At the transmitter, information bits are mapped onto 16-QAM symbols with unit average power. The symbols are then spatially multiplexed and processed by a linear precoder prior to OFDM modulation and transmission over the MIMO channel. Two linear precoding techniques are considered in this phase: Zero-Forcing (ZF) precoding and Singular Value Decomposition (SVD) based precoding.

At the receiver, OFDM demodulation is performed followed by linear combining and equalisation according to the adopted precoding scheme. The estimated symbols are finally demapped to recover the transmitted bit stream.

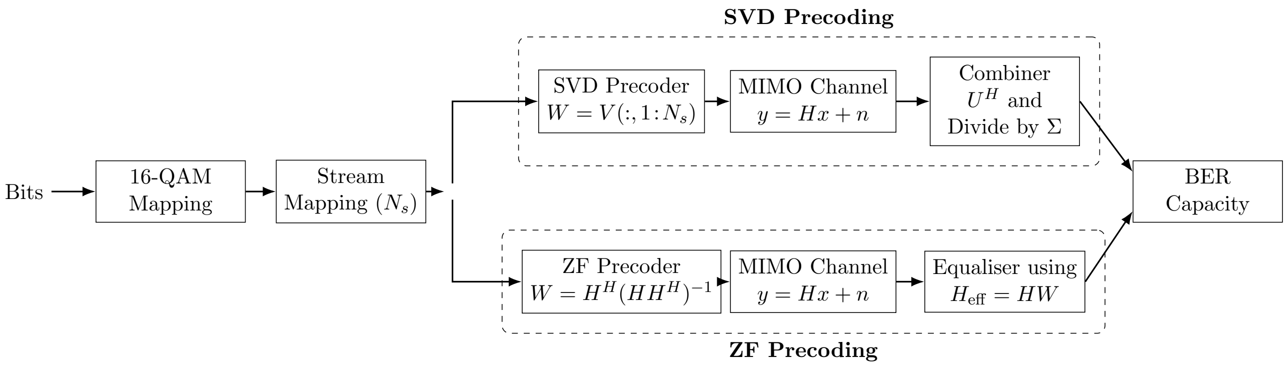

Figure 2: Phase I processing chain highlighting the differences between SVD and ZF precoding in the downlink.

2.2 Signals

Let $\mathbf{s}_k \in \mathbb{C}^{N_s \times 1}$ denote the vector of modulated symbols transmitted on the k-th OFDM subcarrier, where N_s is the number of spatial streams. After stream mapping, the transmit signal is obtained by applying a linear precoder $\mathbf{W} \in \mathbb{C}^{N_{\text{tx}} \times N_s}$, resulting in

$$\mathbf{x}_k = \mathbf{W}\mathbf{s}_k. \tag{1}$$

The transmitted signal propagates through a flat Rayleigh fading MIMO channel $\mathbf{H} \in \mathbb{C}^{N_{\text{rx}} \times N_{\text{tx}}}$ and is corrupted by additive white Gaussian noise. The received signal on the k-th subcarrier is therefore given by

$$\mathbf{y}_k = \mathbf{H}\mathbf{x}_k + \mathbf{n}_k, \tag{2}$$

where $\mathbf{n}_k$ is a complex Gaussian noise vector with independent entries.

2.2.1 SVD Precoding

For SVD-based precoding, the channel matrix is decomposed as

$$\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H, \tag{3}$$

where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices and $\mathbf{\Sigma}$ contains the singular values of the channel. The precoder is chosen as $\mathbf{W}_{\text{SVD}} = \mathbf{V}(:, 1 : N_s)$, while the receiver applies the corresponding linear combiner $\mathbf{U}^H$. This processing diagonalises the MIMO channel, resulting in N_s parallel and independent subchannels. Each stream is then equalised by dividing by the corresponding singular value.

2.2.2 ZF Precoding

In the case of Zero-Forcing precoding, the precoder is designed as

$$\mathbf{W}_{\text{ZF}} = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1}, \tag{4}$$

followed by power normalisation. The resulting effective channel is given by $\mathbf{H}_{\text{eff}} = \mathbf{H}\mathbf{W}_{\text{ZF}}$. At the receiver, a linear equaliser matched to $\mathbf{H}_{\text{eff}}$ is applied in order to mitigate inter-stream interference.

2.3 Results

In this subsection, the performance of the considered precoding techniques is evaluated and compared in terms of bit-error rate (BER), achievable capacity, and computational complexity. All results are obtained through Monte Carlo simulations under the assumptions described in the previous sections.

2.3.1 Bit-Error Rate Performance

The bit-error rate (BER) performance of SVD and Zero-Forcing (ZF) precoding is evaluated as a function of the energy-per-bit to noise power spectral density ratio E_b/N_0. For each SNR point, more than one million bits are transmitted, and the BER is computed by direct comparison between the transmitted and detected bit streams.

Figure 3 shows the BER performance for a 16×16 MIMO-OFDM system employing full spatial multiplexing with $N_s = 16$ streams. It can be observed that SVD precoding consistently outperforms ZF precoding across the entire SNR range. This behaviour is expected, since SVD precoding diagonalises the MIMO channel, completely eliminating inter-stream interference and resulting in independent parallel subchannels.

In contrast, ZF precoding relies on channel inversion, which leads to noise enhancement, particularly in ill-conditioned channel realisations. As a consequence, ZF exhibits a higher BER, especially at low and moderate E_b/N_0 values.

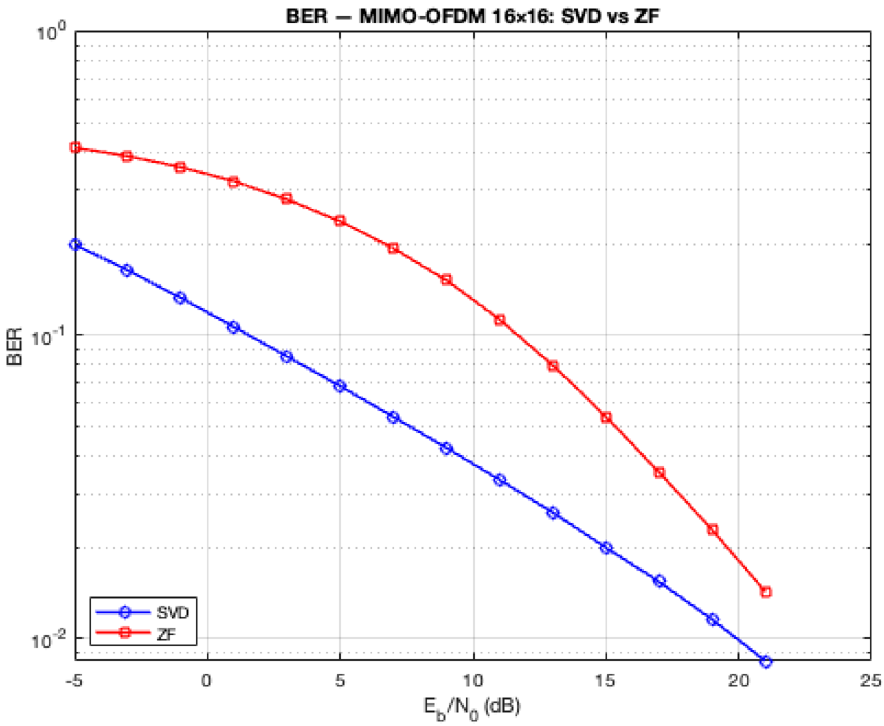

Figure 3: BER performance of SVD and ZF precoding for a 16×16 MIMO-OFDM system.

Overall, the results confirm that SVD precoding provides a clear BER advantage over ZF for the considered full spatial multiplexing configuration. The performance gap is mainly associated with the noise enhancement inherent to channel inversion in ZF precoding, while SVD processing leads to decoupled spatial subchannels.

2.3.2 Achievable Capacity

In addition to BER, the achievable spectral efficiency of the system is evaluated through the Shannon capacity. For a generic MIMO channel at the k-th OFDM subcarrier, the capacity is computed as

$$C_k = \log_2 \det \left(\mathbf{I} + \frac{\rho}{N_s} \mathbf{H}_k \mathbf{H}_k^H \right), \tag{5}$$

where ρ denotes the signal-to-noise ratio (SNR) and N_s is the number of spatial streams. Since an OFDM waveform with N subcarriers is considered, the overall capacity is obtained as the average across subcarriers,

$$C = \frac{1}{N} \sum_{k=1}^{N} C_k. \tag{6}$$

When SVD precoding is applied, the channel can be decomposed as $\mathbf{H}_k = \mathbf{U}_k \mathbf{\Sigma}_k \mathbf{V}_k^H$, which diagonalises the MIMO channel into independent spatial modes. In this case, the capacity per subcarrier reduces to

$$C_{\text{SVD}}(k) = \sum_{i=1}^{N_s} \log_2 \left(1 + \frac{\rho}{N_s}\sigma_i^2\right), \tag{7}$$

where σ_i denotes the i-th singular value of the channel.

Figure 4 shows the average achievable capacity as a function of E_b/N_0 for SVD and ZF precoding. The results indicate that SVD achieves a significantly higher capacity across the entire SNR range. This behaviour is expected because SVD fully diagonalises the channel and preserves the spatial gains associated with the strongest singular modes. In contrast, ZF precoding requires channel inversion, which reduces the effective channel gain and leads to a noticeable capacity loss.

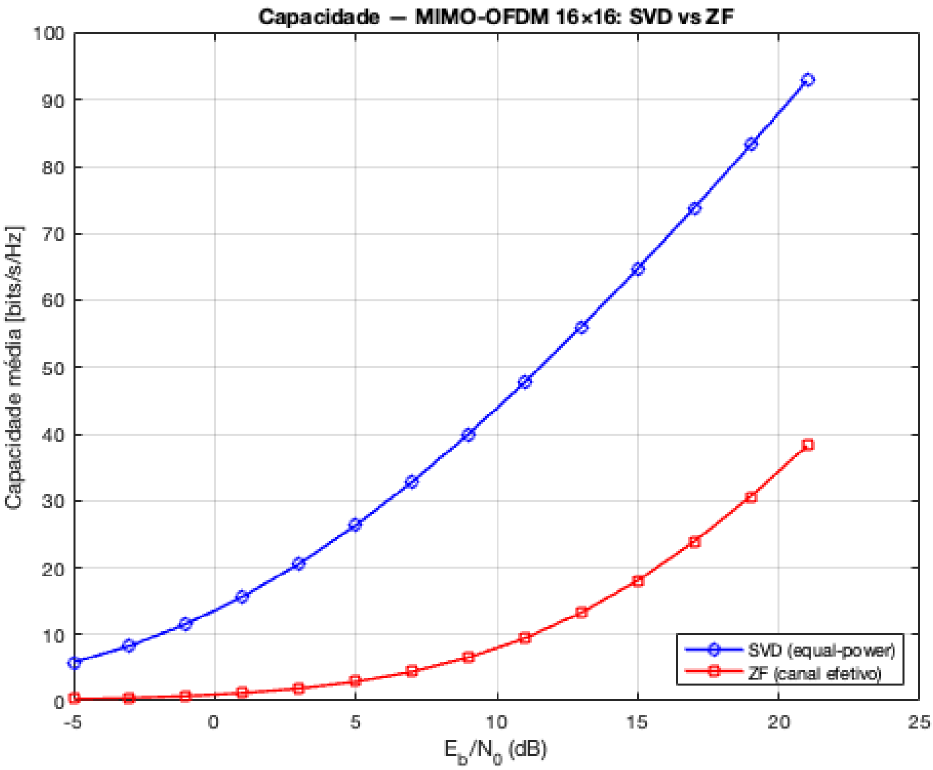

Figure 4: Average achievable capacity of SVD and ZF precoding for a 16×16 MIMO-OFDM system.

In the implemented simulator, the channel matrix is constant across all subcarriers within each slot; therefore, the averaging over k yields the same value per slot, while Monte Carlo averaging over multiple slots captures the channel variability.

2.3.3 Computational Complexity

The computational complexity of the considered precoding techniques was evaluated empirically by measuring execution time in MATLAB using the `tic` and `toc` functions. The objective was to estimate the average time required to compute each precoder matrix, including the Frobenius-norm power normalisation used in the simulator, for a 16×16 Rayleigh fading MIMO channel.

For each technique, the measurement was repeated over $N_{\text{reps}} = 200$ independent random channel realisations. Since the channel matrix is randomly generated at each repetition, small variations in runtime are expected due to differences in channel conditioning and due to the fact that MATLAB relies on highly optimised numerical libraries (e.g., multi-threaded BLAS/LAPACK routines). To mitigate these fluctuations and obtain a representative estimate, the reported execution times correspond to the average across all repetitions.

Table 1 summarises the measured average execution times for SVD and ZF precoding. For the considered setup, the SVD-based computation exhibited a lower average runtime than the ZF computation, with a ratio of $T_{\text{SVD}}/T_{\text{ZF}} \approx 0.33$. It should be noted that these measurements reflect practical runtime behaviour for the specific matrix dimensions and software environment used, rather than asymptotic algorithmic complexity.

Table 1: Average computational time of SVD and ZF precoding measured using MATLAB's `tic/toc` over $N_{\text{reps}} = 200$ random channel realisations.

Precoding Scheme	Average Time [s]	Relative Cost
SVD Precoding	1.609×10^{-4}	0.33
ZF Precoding	4.804×10^{-4}	1.00

2.4 Phase I Conclusions

In this first phase, a downlink MIMO-OFDM system with perfect channel state information was analysed, focusing on the comparison between SVD-based and Zero-Forcing (ZF) linear precoding techniques. The evaluation was carried out in terms of bit-error rate (BER), achievable capacity, and computational complexity, using Monte Carlo simulations for a 16×16 MIMO configuration with full spatial multiplexing.

From the BER results, it was observed that SVD precoding consistently outperforms ZF precoding across the entire range of E_b/N_0 values considered. This behaviour is a direct consequence of the channel diagonalisation provided by SVD, which transforms the MIMO channel into a set of independent parallel subchannels and effectively eliminates inter-stream interference. In contrast, ZF precoding relies on channel inversion, which leads to noise enhancement, particularly for ill-conditioned channel realisations, resulting in a higher BER, especially at low and moderate SNR values.

A similar trend is observed in terms of achievable capacity. SVD precoding achieves a significantly higher spectral efficiency when compared to ZF precoding. By aligning transmission with the dominant singular modes of the channel, SVD enables a more efficient exploitation of the available spatial degrees of freedom. On the other hand, the effective channel gains obtained with ZF precoding are reduced due to the inversion process, leading to a noticeable loss in capacity, particularly at low SNR.

Regarding computational complexity, an empirical analysis based on execution time measurements using MATLAB's `tic/toc` functions

3 Phase II – Multi-User Beamforming with Imperfect CSI

3.1 System Implemented and Parameters

In this second phase, a multi-user downlink MIMO-OFDM system is considered in order to analyse the impact of beamforming and imperfect channel state information (CSI) on system performance. The transmission scenario corresponds to configuration (b) defined in the problem statement, where a base station equipped with $N_{\text{tx}} = 16$ antennas serves two users, each equipped with 8 receive antennas.

Each user is allocated a single spatial data stream, resulting in a total of $N_s = 2$ simultaneously transmitted streams. Linear precoding techniques are employed at the transmitter to mitigate inter-user and inter-stream interference. In particular, Zero-Forcing (ZF) and Singular Value Decomposition (SVD) based precoding are considered. The transmission waveform is OFDM, and a frequency-flat Rayleigh fading channel is assumed per OFDM symbol, with additive white Gaussian noise at the receiver.

Unlike Phase I, this phase explicitly considers the effect of imperfect channel knowledge at the transmitter. Channel estimation errors are modelled by introducing a mismatch between the true channel matrix $\mathbf{H}$ and the channel estimate $\widehat{\mathbf{H}}$ used for precoder computation. The precoder is designed based on $\widehat{\mathbf{H}}$, while signal propagation occurs through the true channel $\mathbf{H}$, leading to residual inter-user interference.

In order to model an additional beamforming or array gain, a constant gain factor is applied to the transmitted symbol streams. System performance is evaluated primarily in terms of bit-error rate (BER) as a function of the signal-to-noise ratio, with particular emphasis on the impact of CSI imperfections on inter-user interference.

3.2 Signals

3.2.1 Multi-User Signal Model

Let $\mathbf{s}_k \in \mathbb{C}^{N_s \times 1}$ denote the vector of transmitted symbols on subcarrier k, where each element corresponds to one user data stream. In the considered configuration, $N_s = 2$, with one spatial stream allocated per user. The information symbols are drawn from a 16-QAM constellation with unit average power.

At the transmitter, linear precoding is applied to spatially multiplex the user streams across the $N_{\text{tx}} = 16$ transmit antennas. The transmitted signal vector on subcarrier k is given by

$$\mathbf{x}_k = \mathbf{W}\mathbf{s}_k, \tag{8}$$

where $\mathbf{W} \in \mathbb{C}^{N_{\text{tx}} \times N_s}$ denotes the precoding matrix.

To model an additional beamforming or array gain, the transmitted symbol vector is scaled by a constant factor, such that $\mathbf{s}_k \leftarrow \sqrt{G_{\text{BF}}}\,\mathbf{s}_k$, where G_{BF} represents the beamforming gain applied in the simulation.

The received signal at the users is expressed as

$$\mathbf{y}_k = \mathbf{H}\mathbf{x}_k + \mathbf{n}_k, \tag{9}$$

where $\mathbf{H} \in \mathbb{C}^{N_{\text{rx}} \times N_{\text{tx}}}$ is the Rayleigh fading MIMO channel matrix and $\mathbf{n}_k$ is additive white Gaussian noise. Due to the multi-user transmission, the received signal generally contains contributions from both intended and interfering user streams.

In addition, a simplified beamforming gain model is adopted in this phase in order to capture the array gain provided by transmit beamforming. This effect is modelled by scaling the transmitted symbol vector by a constant factor $\sqrt{G_{\text{BF}}}$, where $G_{\text{BF}} = 10$ in the performed simulations. This approach allows isolating the impact of beamforming gain on system performance while maintaining a simple and transparent simulation model.

3.2.2 Impact of Imperfect CSI

In this phase, imperfect channel state information at the transmitter is explicitly considered. The transmitter is assumed to have access only to an estimate of the channel matrix,

$$\widehat{\mathbf{H}} = \mathbf{H} + \mathbf{E}, \tag{10}$$

where $\mathbf{E}$ models the channel estimation error. The estimation error is assumed to be independent of the true channel and is controlled by a predefined variance parameter.

The precoding matrix $\mathbf{W}$ is computed using the estimated channel $\widehat{\mathbf{H}}$, while signal propagation occurs through the true channel $\mathbf{H}$. This mismatch prevents perfect interference suppression and leads to residual inter-user interference at the receiver.

For Zero-Forcing (ZF) precoding, the precoder is obtained by inverting the estimated channel matrix, followed by power normalisation. Due to the channel inversion operation, ZF is particularly sensitive to estimation errors, which result in noise enhancement and significant performance degradation.

For SVD-based precoding, the estimated channel is decomposed as $\widehat{\mathbf{H}} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H$, and the precoder is chosen as $\mathbf{W}_{\text{SVD}} = \mathbf{V}(:, 1 : N_s)$. Although SVD-based precoding aims at diagonalising the channel, imperfect CSI prevents perfect diagonalisation of the true channel, and residual inter-user interference remains.

As a consequence, the effective channel experienced by the transmitted streams is no longer interference-free, directly impacting both the bit-error rate and the achievable capacity, as analysed in the following section.

3.3 Results

In this subsection, the performance of the considered linear precoding techniques in a multi-user downlink MIMO-OFDM system is evaluated under imperfect channel state information. The results are obtained through Monte Carlo simulations and are presented in terms of bit-error rate (BER) and achievable capacity as functions of the energy-per-bit to noise power spectral density ratio E_b/N_0.

3.3.1 Bit-Error Rate Performance

Figure 5 shows the BER performance of SVD and Zero-Forcing (ZF) precoding for the considered two-user scenario with imperfect CSI. Each user is served by one spatial stream, and the precoder is designed based on an estimated channel matrix, while signal transmission occurs through the true channel.

The results indicate that SVD-based precoding significantly outperforms ZF precoding across the entire E_b/N_0 range. SVD exhibits a steep BER decay as the signal-to-noise ratio increases, whereas ZF presents substantially higher BER values, even at moderate and high E_b/N_0.

This behaviour is mainly attributed to the sensitivity of ZF precoding to channel estimation errors. Since ZF relies on channel inversion, inaccuracies in the estimated channel result in residual inter-user interference and noise enhancement. In contrast, SVD-based precoding is more robust to CSI imperfections, maintaining better interference mitigation and therefore superior BER performance.

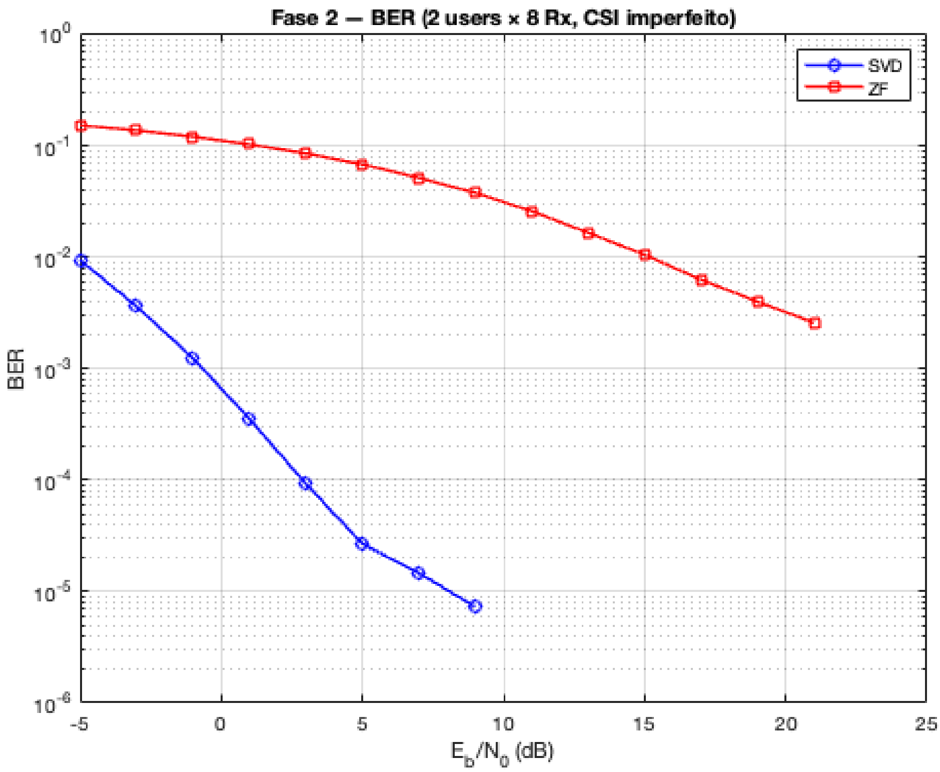

Figure 5: Phase II BER performance for a two-user (2×8 Rx) MIMO-OFDM system with imperfect CSI.

3.3.2 Achievable Capacity

Figure 6 presents the average achievable capacity as a function of E_b/N_0 for SVD and ZF precoding under the same imperfect CSI conditions. The capacity is computed using the effective channel formed by the combination of the true channel and the precoder derived from the estimated channel, ensuring that residual inter-user interference is properly accounted for.

The results show that SVD-based precoding achieves a significantly higher spectral efficiency compared to ZF across the evaluated SNR range. Despite the presence of channel estimation errors, SVD preserves stronger effective channel gains, resulting in a nearly linear capacity growth with increasing E_b/N_0.

In contrast, ZF precoding suffers from a pronounced capacity degradation. Residual inter-user interference and noise enhancement caused by imperfect channel inversion limit the achievable spectral efficiency and prevent ZF from fully exploiting the available spatial degrees of freedom.

These results confirm that, in multi-user downlink scenarios with imperfect CSI, SVD-based precoding provides superior robustness and spectral efficiency when compared to ZF precoding.

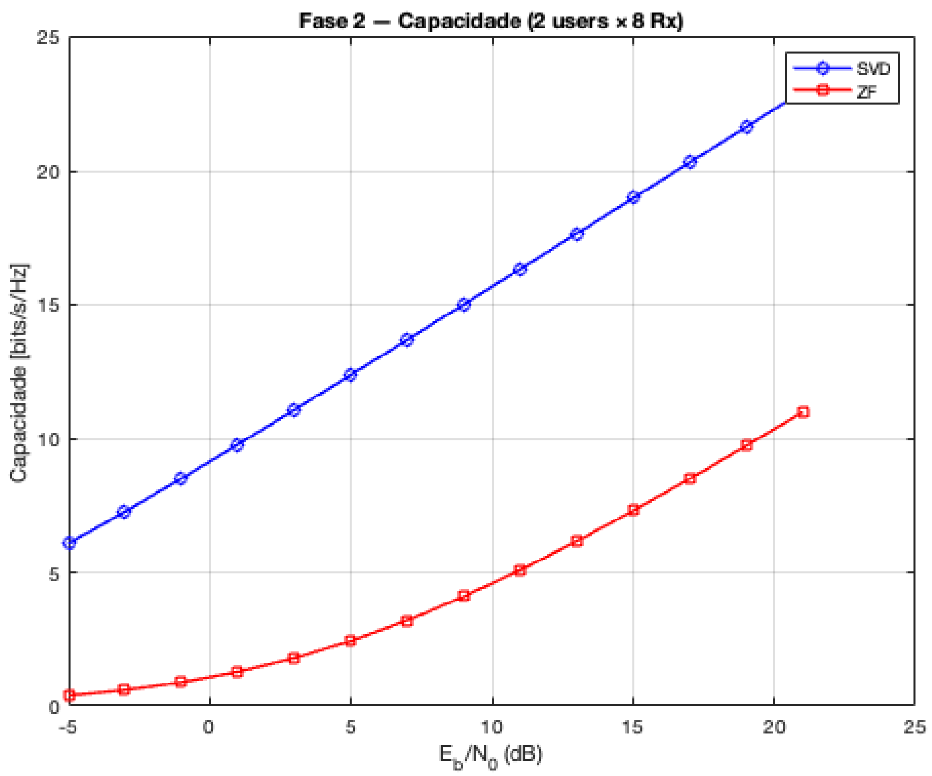

Figure 6: Phase II achievable capacity for a two-user (2 × 8 Rx) MIMO-OFDM system with imperfect CSI.

3.4 Phase II Conclusions

In this phase, a multi-user downlink MIMO-OFDM system was analysed in order to assess the impact of beamforming and imperfect channel state information on system performance. A two-user scenario was considered, with each user equipped with eight receive antennas, and linear precoding was applied at the transmitter.

The obtained results show that imperfect CSI has a significant impact on system performance due to residual inter-user interference. When the precoder is computed using an inaccurate channel estimate, perfect interference cancellation can no longer be achieved, leading to performance degradation.

In terms of bit-error rate, SVD-based precoding consistently outperforms Zero-Forcing precoding across the entire E_b/N_0 range. While SVD maintains a steep BER decay even in the presence of channel estimation errors, ZF suffers from a pronounced degradation, particularly due to its sensitivity to channel inversion errors and noise enhancement.

The achievable capacity results further confirm these observations. SVD-based precoding achieves higher spectral efficiency and exhibits a more robust behaviour under imperfect CSI, whereas ZF experiences a limited capacity growth as a consequence of persistent inter-user interference.

Overall, the results demonstrate that, in multi-user downlink scenarios with imperfect channel knowledge, SVD-based precoding provides superior robustness and performance compared to ZF precoding, at the expense of increased computational complexity.

4 Overall Conclusions

This work investigated linear precoding and beamforming techniques for downlink MIMO-OFDM systems under different channel state information (CSI) assumptions. Two complementary scenarios were analysed. In Phase I, a single-user MIMO-OFDM system with perfect CSI was considered, while Phase II extended the analysis to a multi-user downlink scenario with beamforming and imperfect CSI.

In Phase I, both Zero-Forcing (ZF) and Singular Value Decomposition (SVD) based precoding were evaluated in terms of bit-error rate (BER), achievable capacity, and computational complexity. The results showed that SVD-based precoding consistently outperforms ZF precoding in terms of BER and achievable capacity across the entire E_b/N_0 range. This behaviour is explained by the ability of SVD to diagonalise the MIMO channel, creating independent parallel subchannels and effectively eliminating inter-stream interference. In contrast, ZF precoding relies on channel inversion, which leads to noise enhancement, particularly in ill-conditioned channel realisations. The computational complexity analysis confirmed that SVD precoding requires a higher execution time than ZF precoding, as expected due to the higher computational cost of singular value decomposition.

Phase II addressed a more realistic multi-user downlink scenario, where a base station equipped with multiple antennas simultaneously serves multiple users in the presence of imperfect CSI. The impact of channel estimation errors and beamforming gain was explicitly considered. The obtained results demonstrate that imperfect CSI has a significant impact on system performance, particularly due to residual inter-user interference that cannot be fully cancelled when the precoder is designed based on an inaccurate channel estimate.

Under imperfect CSI, SVD-based precoding remains more robust than ZF precoding, exhibiting substantially lower BER and higher achievable capacity. While ZF precoding suffers from severe performance degradation due to its sensitivity to channel estimation errors and noise enhancement, SVD maintains better effective channel gains and interference mitigation capabilities. The achievable capacity results further confirm that SVD-based precoding provides superior spectral efficiency in multi-user scenarios with imperfect CSI.

Overall, the results of this work highlight the fundamental trade-offs between performance, robustness, and computational complexity in linear precoding techniques. While SVD-based precoding entails higher computational complexity, it offers significant gains in reliability and spectral efficiency, particularly in challenging scenarios with imperfect channel knowledge. These conclusions underscore the importance of robust precoding strategies in practical multi-user MIMO-OFDM systems, such as those employed in modern wireless communication standards. Table 2 provides a consolidated qualitative comparison of the main performance trends observed for SVD and ZF precoding across both analysed phases.

Table 2: Qualitative comparison of SVD and ZF precoding across the two analysed phases.

Criterion	**SVD Precoding**	**ZF Precoding**
BER (Phase I, CSI perfect)	Lower BER across all E_b/N_0	Higher BER due to noise enhancement
Achievable Capacity (Phase I)	High, near-linear growth	Significantly reduced
Computational Complexity (Phase I)	Higher (SVD decomposition)	Lower (matrix inversion)
BER Robustness to CSI Errors (Phase II)	High robustness	Strong degradation
Achievable Capacity (Phase II)	Preserved spatial efficiency	Severely limited
Sensitivity to Channel Estimation Errors	Moderate	High

References