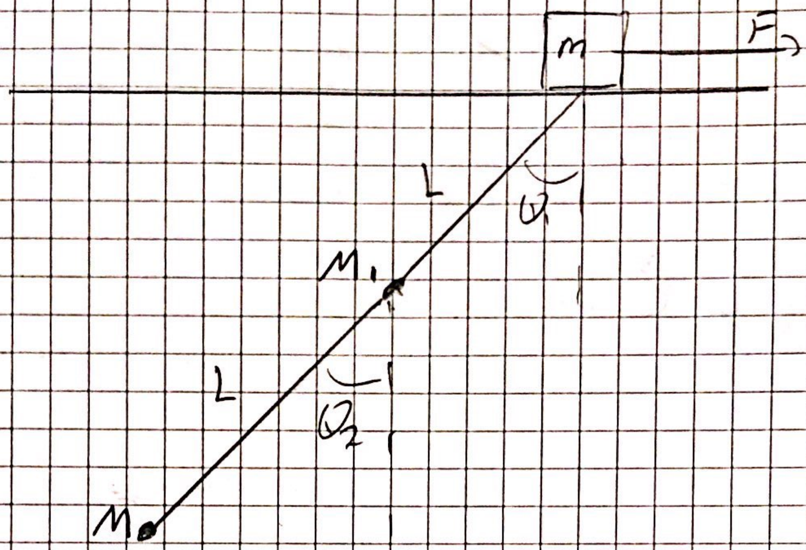
Modsim øving 4

- Problem 1



$m = 1kg$

$M_1 = M_2 = 1\ kg$

$L = 1\ m$

- x position of M_1
  $= x + L \sin\alpha_1$

  y position of M_1
  $= -L\cos\alpha_1$

x position of M_2
$=$ x pos of $M_1 + L\sin(\alpha_2)$

y position of M_2
$=$ y pos $M_1 - L\cos\alpha_2$

Kinetic energy of cart
$T = \frac{1}{2} m\dot{x}^2$

Kinetic energy of mass k

$$T_k = \frac{1}{2} m_k (\dot{x}^2 + \dot{y}^2)$$

Potential energy of mass k

$$V_1 = -Mg\cos\alpha_1$$

$$V_2 = +Mg(-L\cos\alpha_1 - L\cos\alpha_2)$$

$$= -MLgL(\cos\alpha_1 + \cos\alpha_2)$$

b) $F = -10x - \dot{x}$

The functions returned

state = $\begin{bmatrix} y \\ \dot{y} \end{bmatrix}$, param = $\begin{bmatrix} m \\ M \\ L \\ g \end{bmatrix}$

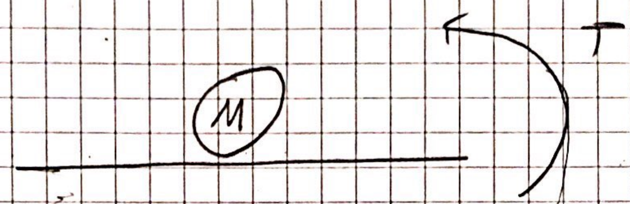
c) The results are reasonable. The pendulums and the cart oscillate at a reasonable velocity and displacement. And it comes to rest after some time problem 2

- pure rotation
- Torque T acts on the beam

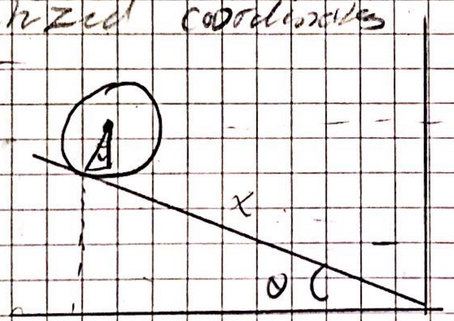
$J = 1\,kg\,m^2$
$M = 10\,kg$
$R = 0.25\,m$

position of ball respect to the joint of the rail is x

$q = \begin{bmatrix} x \\ \theta \end{bmatrix}$



a) position of ball's center as a function of the generalized coordinates



x position of ball : $x\cos\alpha - r\sin\alpha$

y position of ball : $x\sin\alpha + r\cos\alpha$

$P = \begin{bmatrix} x\cos\alpha - R\sin\alpha \\ x\sin\alpha + R\cos\alpha \end{bmatrix}$

b) Angular velocity of ball : $\dot{\theta}$

$\omega = \dot{\theta} + \frac{\dot{x}}{R}$ from 7.117

c) Translation : $\frac{1}{2} m v^2$

Rotation : $\frac{1}{2} I \omega^2$

Kinetic = Translation + Rotation

$$= \frac{1}{2} m \dot{p}^T p + \frac{1}{2} I \left(\dot{\alpha} + \frac{\dot{x}}{R}\right)^2$$

$$\dot{p} = \begin{bmatrix} -x\sin\alpha\,\dot{\alpha} + \dot{x}\cos\alpha & - R\cos\alpha\,\dot{\alpha} \\ x\dot{\alpha}\cos\alpha + \dot{x}\sin\alpha & - R\sin\alpha \end{bmatrix}$$

d) Only rotational

$$K = \frac{1}{2} J \dot{\alpha}^2$$

g) The result is reasonable.