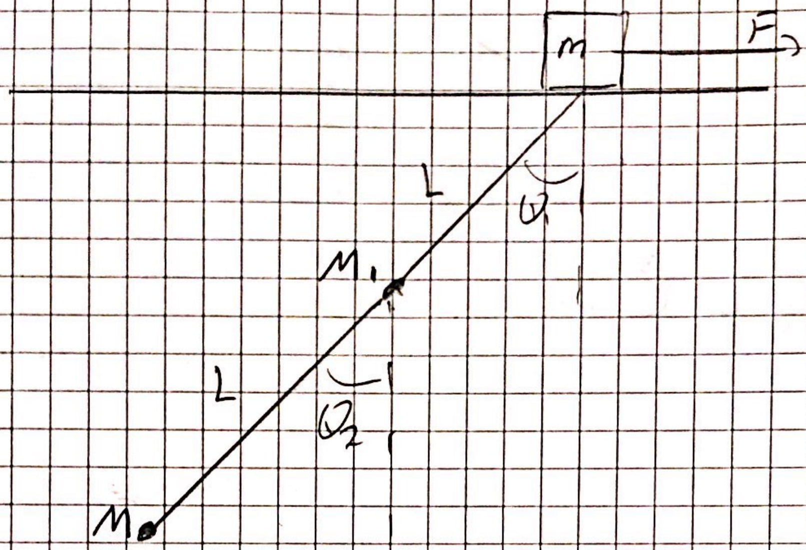
# Modsim øving 4

## Problem 1




$m = 1 kg$

$M_1 = M_2 = 1 kg$

$L = 1 m$

x position of $M_1$
$= x + L \sin \alpha_1$

y position of $M_1$
$= -L \cos \alpha_1$

x position of $M_2$
$=$ x pos of $M_1 + L \sin(\theta_2)$

y position of $M_2$
$=$ y pos $M_1 - L \cos \theta_2$

Kinetic energy of cart

$$T = \frac{1}{2} m \dot{x}^2$$

Kinetic energy of mass k

$$\cancel{T}_k = \frac{1}{2} m_k (\dot{x}^2 + \dot{y}^2)$$

Potential energy of mass k

$$V_1 = +Mg\cos\alpha_1$$

$$V_2 = +Mg(-L\cos\alpha_1 - L\cos\alpha_2)$$

$$= -MLgL(\cos\alpha_1 + \cos\alpha_2)$$

b) $F = -10x - \dot{x}$

The functions returned

$$\text{state} = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}, \quad \text{param} = \begin{bmatrix} m \\ M \\ L \\ g \end{bmatrix}$$

c) The results are reasonable. The pendulum and the cart oscillate at a reasonable velocity and displacement. And it comes to rest after some time problem 2

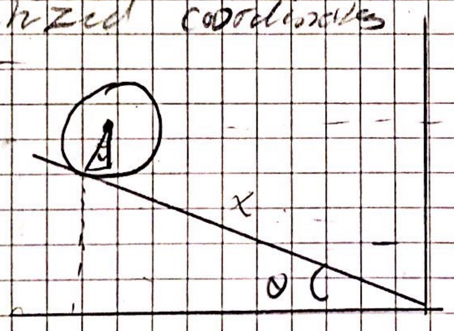
- pure rotation
- Torque $T$ acts on the beam

$J = 1\,kg\,m^2$
$M = 10\,kg$
$R = 0.75\,m$

position of ball respect to the joint of the rail is $x$

$$q = \begin{bmatrix} x \\ \theta \end{bmatrix}$$

a) position of ball's center as a function of the generalized coordinates



$x$ position of ball : $x\cos\alpha - r\sin\alpha$

$y$ position of ball : $x\sin\alpha + r\cos\alpha$

$$P = \begin{bmatrix} x\cos\alpha - R\sin\alpha \\ x\sin\alpha + R\cos\alpha \end{bmatrix}$$

b) Angular velocity of ball : $\dot{Q}$

$$\omega = \dot{Q} + \frac{\dot{x}}{R} \quad \text{from 7.117}$$

c) Translation : $\frac{1}{2} m v^2$


Rotation : $\frac{1}{2} I \omega^2$

Kinetic = Translation + Rotation

$$= \frac{1}{2} m \dot{p}^T p + \frac{1}{2} I \left(\dot{\alpha} + \frac{\dot{x}}{R}\right)^2$$

$$\dot{p} = \begin{bmatrix} -x\sin\alpha\,\dot{\alpha} + \dot{x}\cos\alpha - R\cos\alpha\,\dot{\alpha} \\ x\dot{\alpha}\cos\alpha + \dot{x}\sin\alpha - R\sin\alpha \end{bmatrix}$$

d) Only rotational

$$k = \frac{1}{2} J \dot{\alpha}^2$$

g) The result is reasonable.

```
clear all
clc

% Parameters
syms m M g L F real
% Variables
syms x  theta1  theta2 real
syms dx dtheta1 dtheta2 real

% Define symbolic variable q for the generalized coordinates
% x, theta1 and theta2
q = [x, theta1, theta2]';
% Define symbolic variable dq for the derivatives
% of the generalized coordinates
dq = [dx, dtheta1, dtheta2]';
% Write the expressions for the positions of the masses
p{1} = [x-L*sin(theta1); -L*cos(theta1)];
p{2} = p{1} + [-L*sin(theta2); -L*cos(theta2)];

% Kinetic energy of the cart
T = (m*(dx^2))/2;
% For loop that adds the kinetic energies of the masses
for k = 1:length(p)
    dp{k} = jacobian(p{k},q); % velocity of mass k
```

```
for k = 1:length(p)
    dp{k} = jacobian(p{k},q); % velocity of mass k
    %T = T + 0.5*M*norm(dp{k})^2; % add kinetic energy of mass k
    T = T + 0.5*M*dq'*dp{k}'*dp{k}*dq;
end
T = simplify(T);

% Potential energy of the cart
V = 0;
% For loop that adds the potential energies of the masses
for k = 1:length(p)
    V = V + M*g*k*L + M*g*p{k}(2); % add potential energy of mass k
end
V = simplify(V);

% Generalized forces
Q = [F; 0; 0];

% Lagrangian
Lag = T - V;

Lag_q = simplify(jacobian(Lag,q)).';
Lag_qdq = simplify(jacobian(Lag_q.',dq));
Lag_dq = simplify(jacobian(Lag,dq)).';
```

```matlab
% The equations have the form W*q_dotdot = RHS, with
W = Lag_dqdq;
RHS = Q + simplify(Lag_q - Lag_qdq*dq);

state = [q;dq];
param = [m;M;L;g];

matlabFunction(p{1},p{2}, 'file','PendulumPosition','vars',{state, L});
matlabFunction(W,RHS, 'file','PendulumODEMatrices','vars',{state,F,param});
```

```matlab
function [ state_dot ] = PendulumDynamics(t, state, param )
%state = [q; dq] = [x theta1 theta2 dx dtheta1 dtheta2]'
%param = [m M L g]'

F = -10*state(1) - state(4);

[W, RHS] = PendulumODEMatrices(state, F, param);

state_dot = [state(4:6);
             W\RHS];
end
```

```
clear all
clc

% Parameters
syms J M R g To real
% Variables
syms x theta  real
syms dx dtheta real

% Define symbolic variable q for the generalized coordinates
% x and theta
q  = [x theta]';
% Define symbolic variable dq for the derivatives
% of the generalized coordinates
dq = [dx dtheta]';
% Write the expressions for the position of
% the center of the ball:
p = [x*cos(theta)-R*sin(theta), x*sin(theta)+ R*cos(theta)]';

% Kinetic energy
T = (J*(dtheta)^2)/2; % kinetic energy of beam

dp = jacobian(p,q)*dq; % linear velocity of ball
T  = T + 0.5*M*((dx+dtheta*R)^2 + (dtheta*x)^2); % add linear kinetic energy of ball
```

```
I     = 0.4*M*R^2; % inertia in rotation of ball
omega = dtheta + dx/R; % angular velocity of ball

T  = T +0.5*I*omega^2; % add rotational kinetic energy of ball

T = simplify(T);

% Potential energy
V = M*g*(R*cos(theta)-x*sin(theta));

% Generalized forces
Q = [0; To];

% Lagrangian
Lag = T - V;

Lag_q = simplify(jacobian(Lag,q)).';
Lag_qdq = simplify(jacobian(Lag_q.',dq));
Lag_dq = simplify(jacobian(Lag,dq)).';
Lag_dqdq = simplify(jacobian(Lag_dq.',dq));

% The equations have the form W*q_dotdot = RHS, with
W = Lag_dqdq;
```

```matlab
% The equations have the form W*q_dotdot = RHS, with
W = Lag_dqdq;
RHS = Q + simplify(Lag_q - Lag_qdq*dq);

state = [q;dq];
param = [J; M; R; g];

matlabFunction(p, 'file','BallPosition','vars',{state,param});
matlabFunction(W,RHS, 'file','BallAndBeamODEMatrices','vars',{state,To,param});
```