# Objective: Learn to do clustering and noise reduction in data using PCA

In [334]:
```
import matplotlib.pyplot as plt
import numpy as np
from numpy.linalg import svd
from sklearn.datasets import load_digits
digits = load_digits()
digits.data.shape
```

Out[334]:
```
(1797, 64)
```

## PCA using SVD

In [335]:
```
def pca(X):
    U, S, PTrans = svd(X, full_matrices=False)
    Sigma = np.diag(S)
    T=np.dot(U,Sigma)
    P=PTrans.T
    return T, Sigma,  P #Score, Variace_matrix, Loadings
```

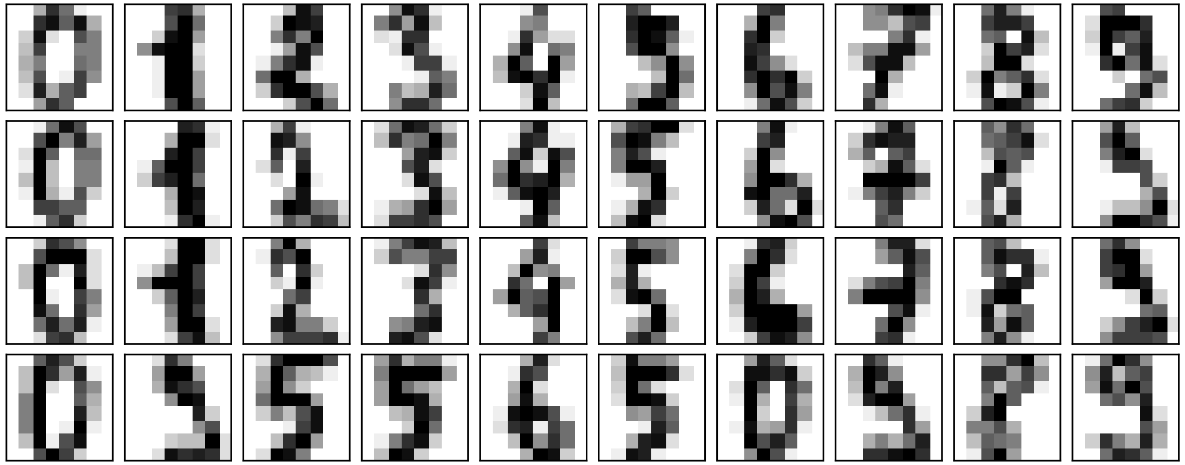
In [336]:
```
def plot_digits(data):
    fig, axes = plt.subplots(4, 10, figsize=(10, 4),
                             subplot_kw={'xticks':[], 'yticks':[]},
                             gridspec_kw=dict(hspace=0.1, wspace=0.1))
    for i, ax in enumerate(axes.flat):
        ax.imshow(data[i].reshape(8, 8),
                  cmap='binary', interpolation='nearest',
                  clim=(0, 16))
```

In [337]:
```
# Find out the original dimension of the data
```

```
X=digits.data
y=digits.target
print("Shape of X",X.shape)
print("Shape of y", y.shape)
```

```
Shape of X (1797, 64)
Shape of y (1797,)
```

In [338]:
```
#Visualize the original data
plot_digits(X)
```



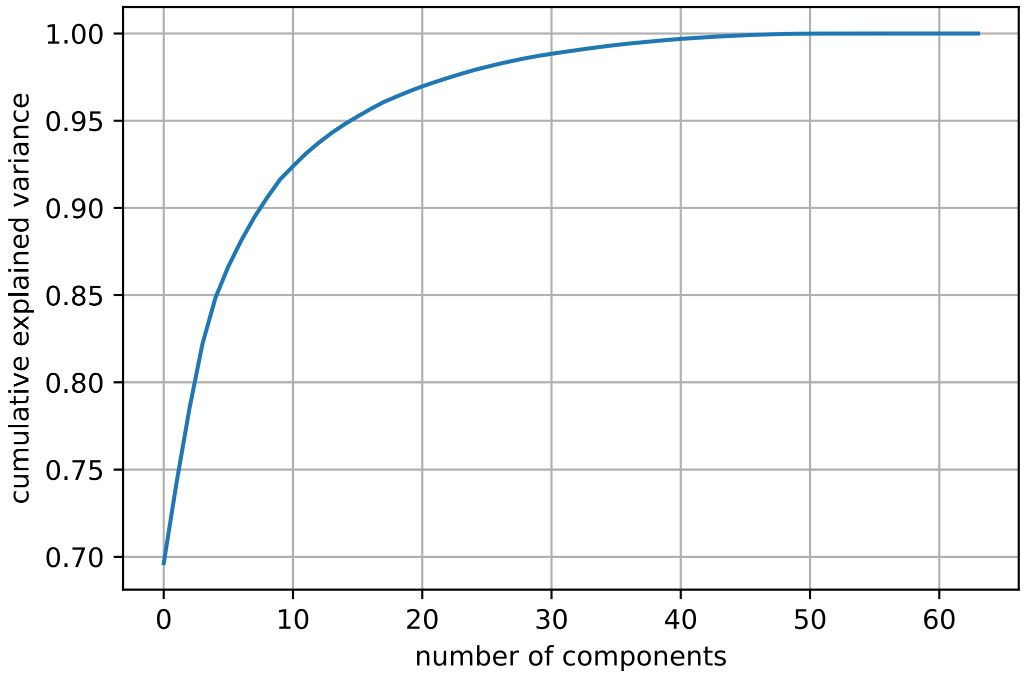
**Task 1: Dimensionality reduction: Conduct PCA on the the matrix $X$ to find out the dimension required to capture 80% of the variance.**

In [339]:
```
#Seems like we only need like 4 components to capture 80% of variance
T, Sigma, P = pca(X)
SS = np.diag(Sigma)

explained_variance = (SS ** 2) / 4
explained_variance_ratio = (explained_variance / explained_variance.sum
())
```

```
plt.plot(np.cumsum(explained_variance_ratio))
plt.grid()
plt.xlabel('number of components')
plt.ylabel('cumulative explained variance')
```

Out[339]:
```
Text(0, 0.5, 'cumulative explained variance')
```

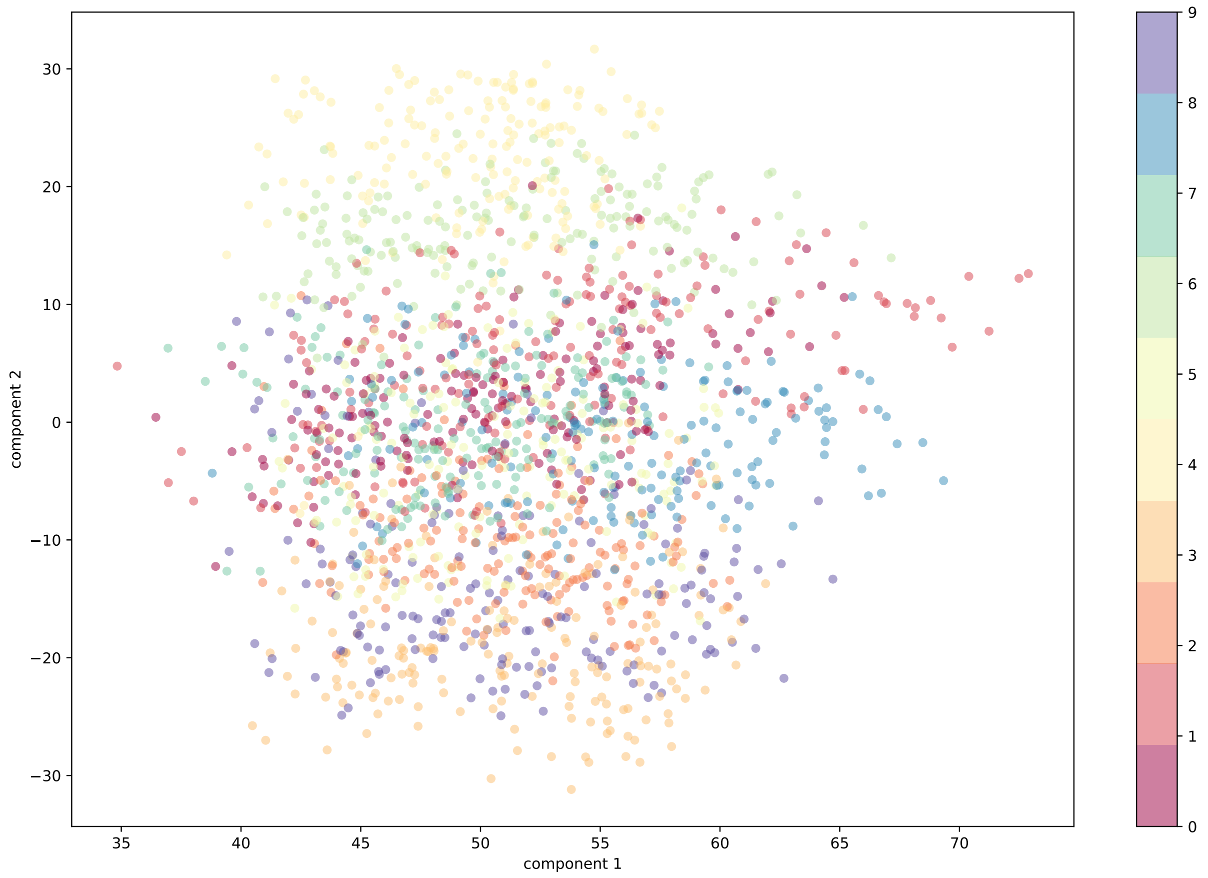


### Task 2: Clustering: Project the original data matrix X on the first two PCs and draw the scalar plot

In [340]:
```
t1= np.dot(X,P[:,0])
t2= np.dot(X,P[:,1])
plt.figure(figsize=(15,10))
plt.scatter(t1, t2,
            c=digits.target, edgecolor='none', alpha=0.5,
            cmap=plt.cm.get_cmap('Spectral', 10))
```

```
plt.xlabel('component 1')
plt.ylabel('component 2')
plt.colorbar()
```

```
Out[340]: <matplotlib.colorbar.Colorbar at 0x18b9703deb0>
```

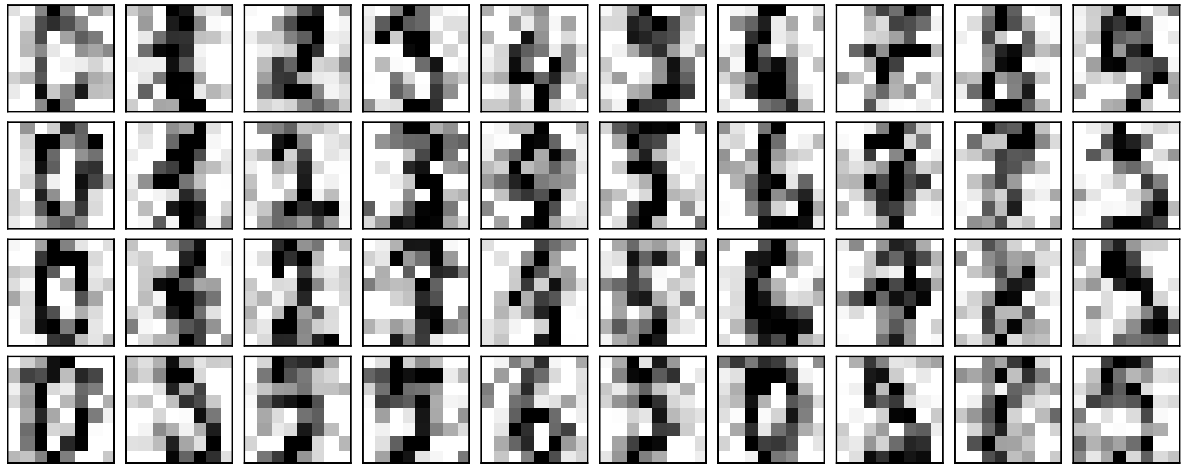


## Task 3: Denoising: Remove noise from the noisy data

```
In [341]: # Adding noise to the original data
X=digits.data
y=digits.target
```

```
np.random.seed(42)
noisy = np.random.normal(X, 4)
print(noisy.shape)
plot_digits(noisy)
```

```
(1797, 64)
```



Tips:

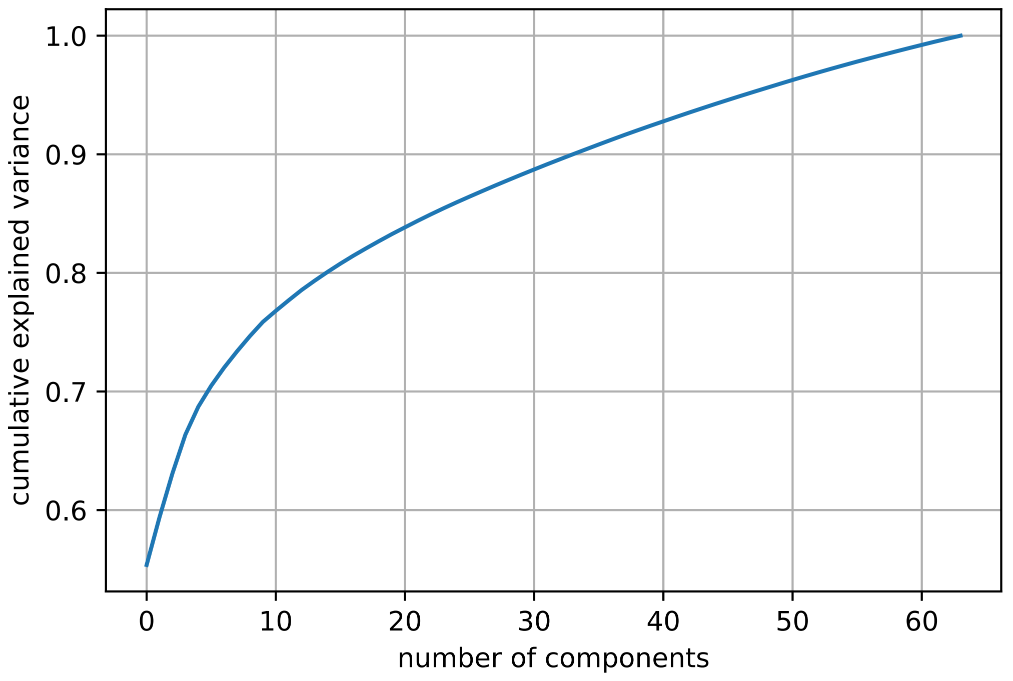
- Decompose the noisy data using PCA
- Reconstruct the data using just a few dominant components.For eg. check the variance plot

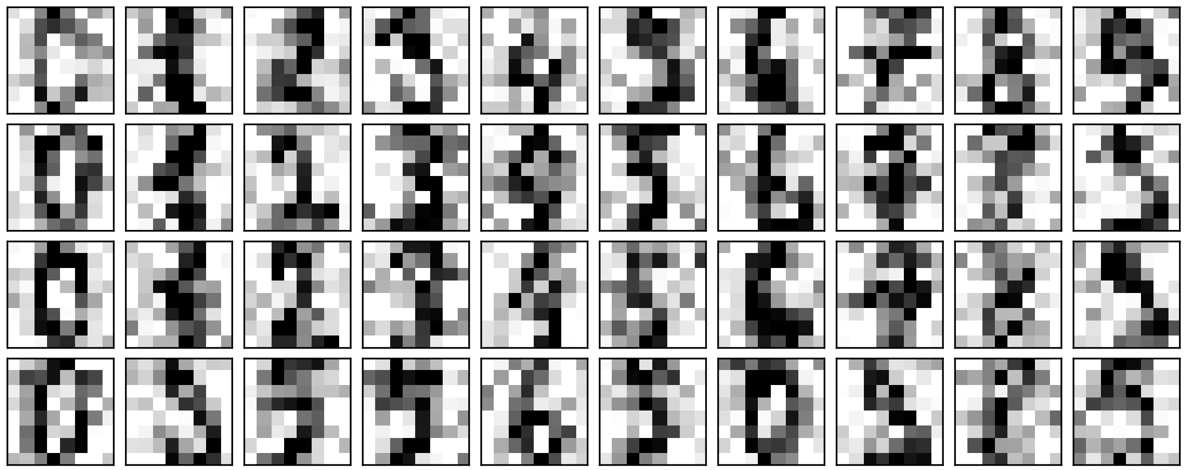
Since the nature of the noise is more or less similar across all the digits, they are not the fearues with enough variance to discriminate between the digits.

In [342]:
```
T, Sigma, P = pca(noisy)
SS = np.diag(Sigma)
explained_variance = (SS ** 2) / 4
explained_variance_ratio = (explained_variance / explained_variance.sum
())
plt.plot(np.cumsum(explained_variance_ratio))
plt.grid()
plt.xlabel('number of components')
plt.ylabel('cumulative explained variance')

P_inv = np.linalg.inv(P)
X_denoised = np.dot(T,P_inv)
print(X_denoised.shape)
plot_digits(X_denoised)
```

```
(1797, 64)
```





**Task 4: Study the impact of normalization of the dataset before conducting PCA. Discuss if it is critical to normalize this particular data compared to the dataset in other notebooks**

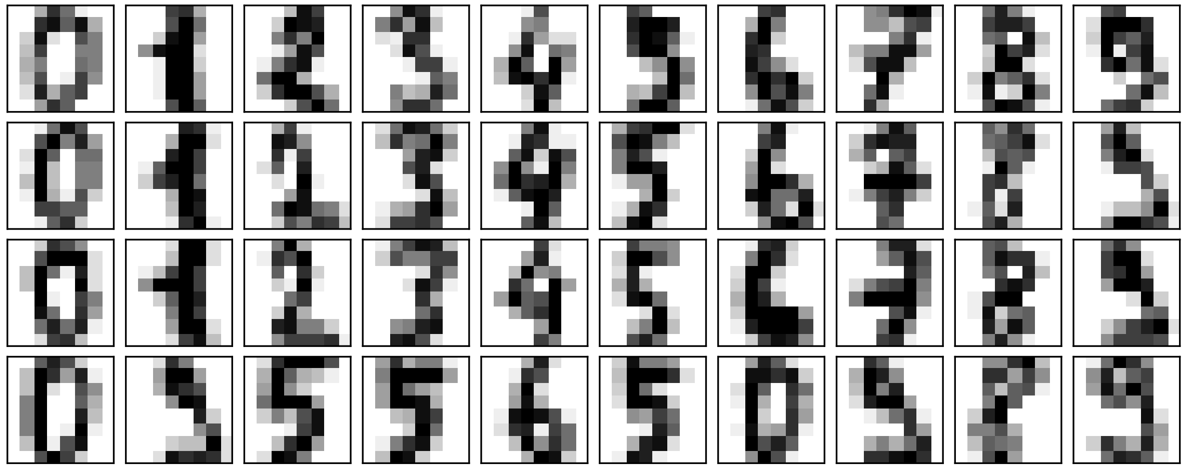
## All the above excercise can be done using the SKLEAR library as follows

In [343]:
```
from sklearn.decomposition import PCA
X=digits.data
y=digits.target
```

In [344]:
```
pca = PCA(2)  # project from 64 to 2 dimensions
projected = pca.fit_transform(digits.data)
print(digits.data.shape)
print(projected.shape)
plot_digits(digits.data)
```

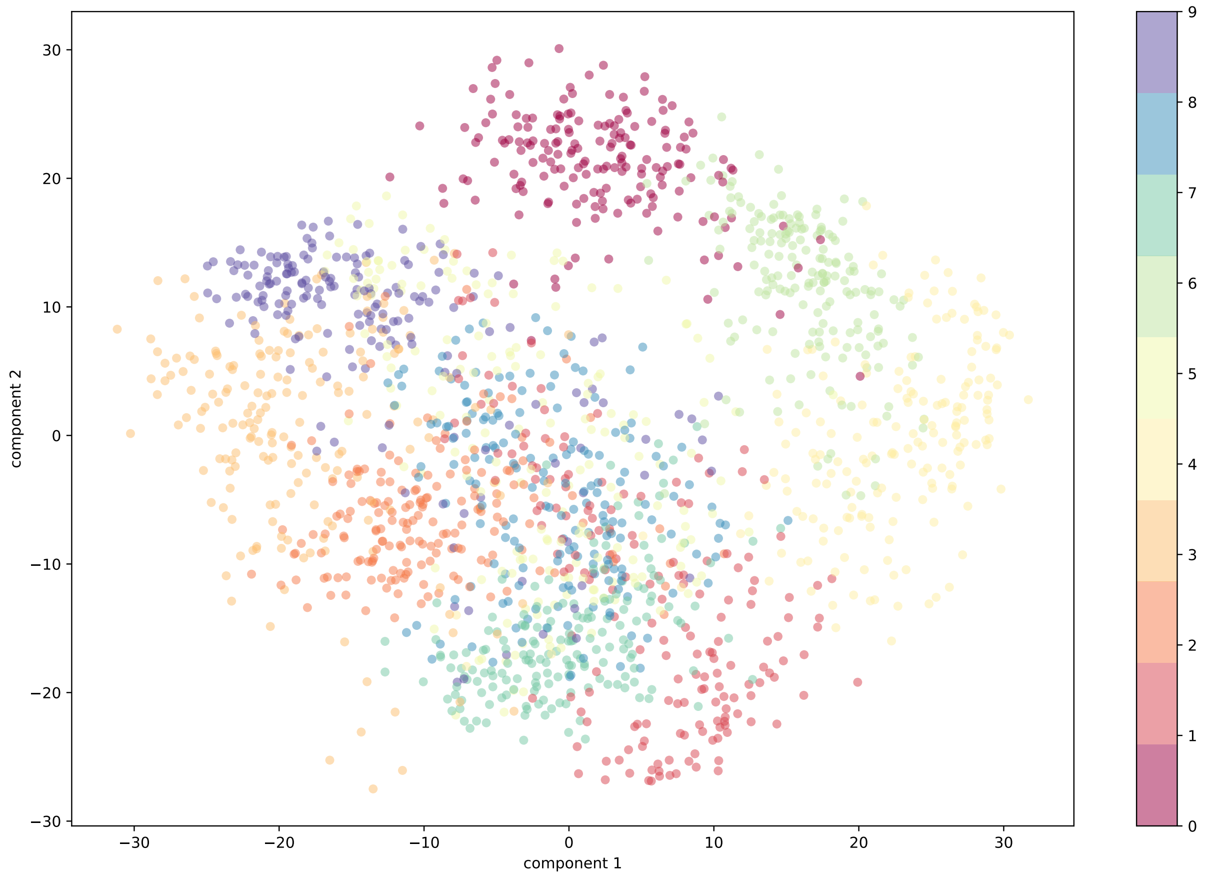
```
(1797, 64)
```

```
(1797, 2)
```



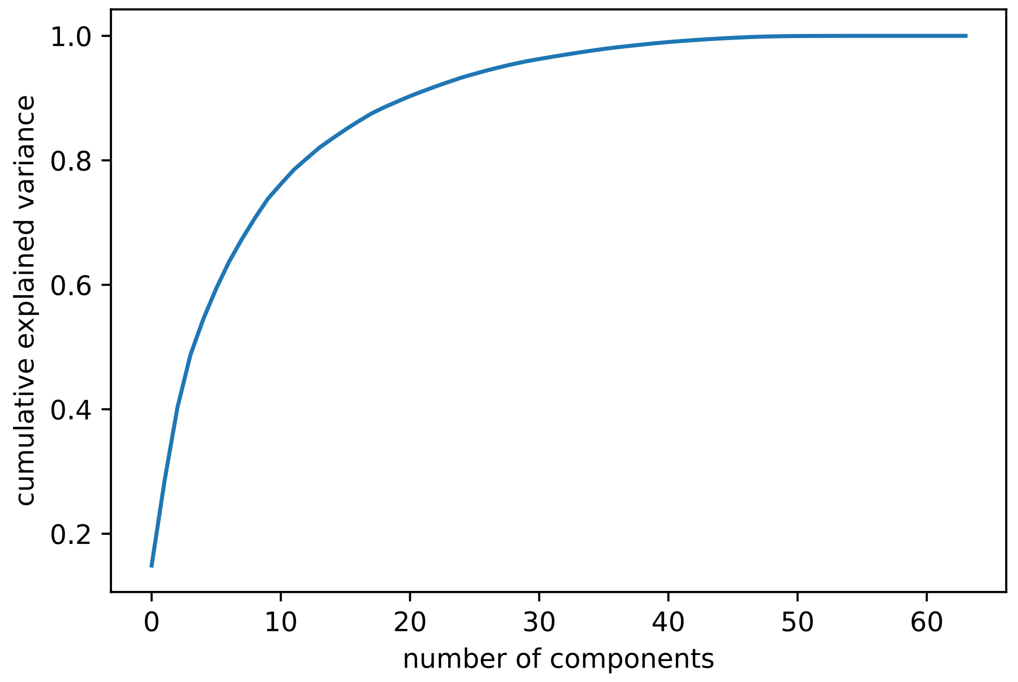
In [345]:
```
plt.figure(figsize=(15,10))
plt.scatter(projected[:, 0], projected[:, 1],
            c=digits.target, edgecolor='none', alpha=0.5,
            cmap=plt.cm.get_cmap('Spectral', 10))
plt.xlabel('component 1')
plt.ylabel('component 2')
plt.colorbar()
```

Out[345]:
```
<matplotlib.colorbar.Colorbar at 0x18b99bc6100>
```

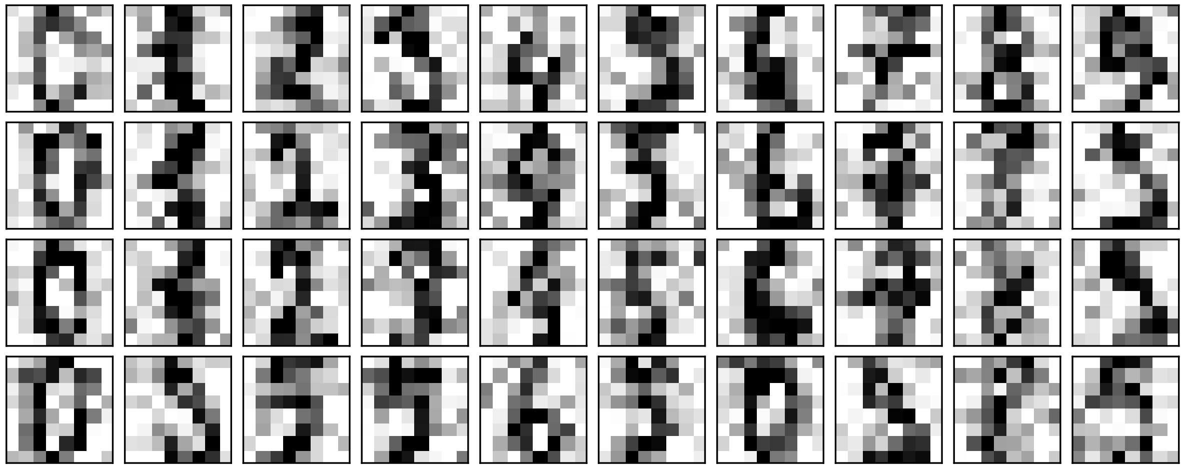


In [346]:
```
pca = PCA().fit(digits.data)
plt.plot(np.cumsum(pca.explained_variance_ratio_))
plt.xlabel('number of components')
plt.ylabel('cumulative explained variance')
```

Out[346]:
```
Text(0, 0.5, 'cumulative explained variance')
```

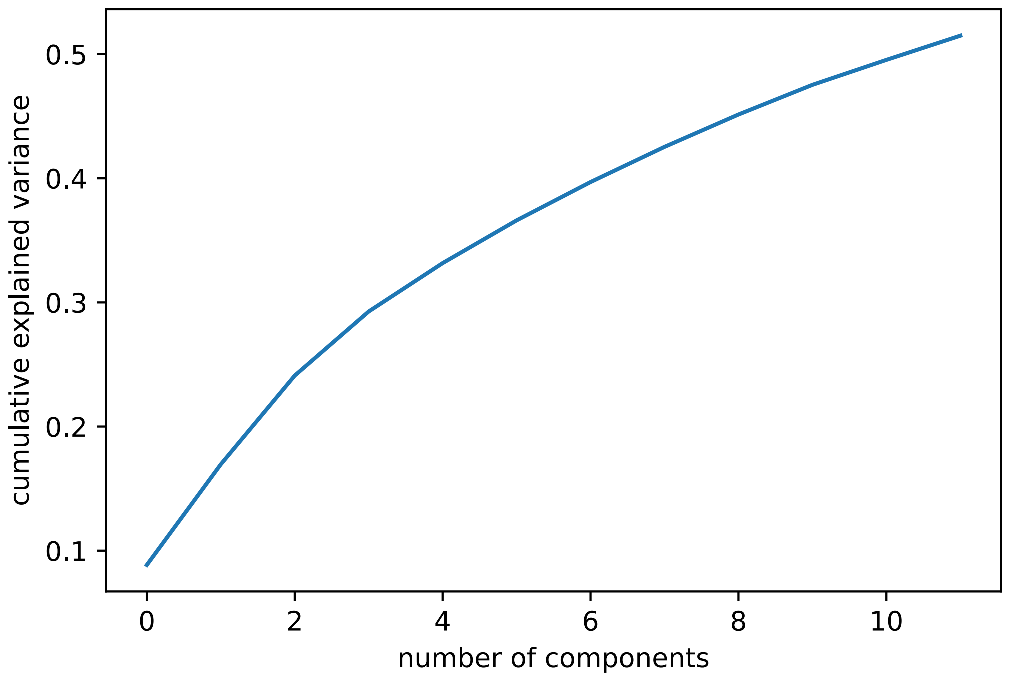


In [347]:
```
np.random.seed(42)
noisy = np.random.normal(digits.data, 4)
plot_digits(noisy)
```

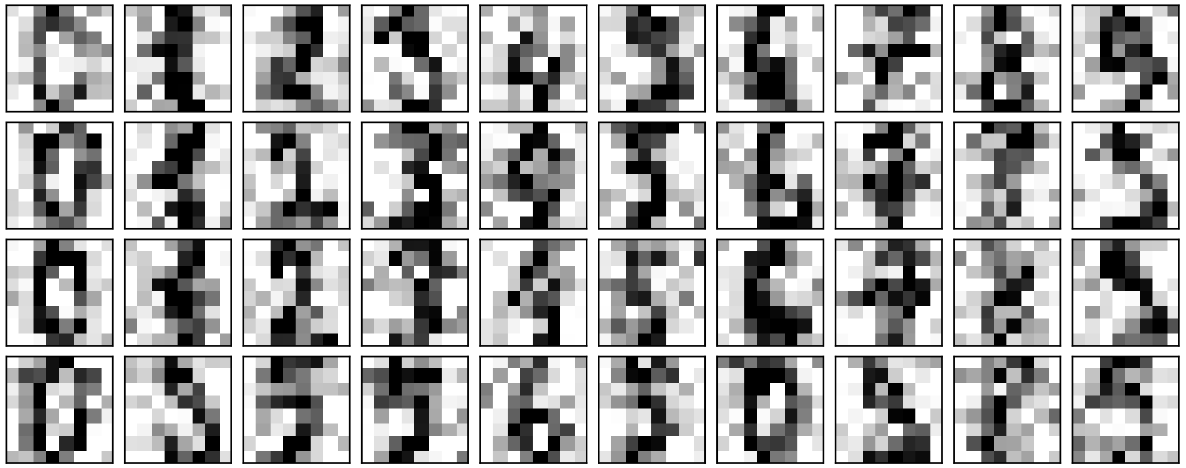


In [348]:
```
pca = PCA(0.50).fit(noisy) # 50% of the variance amounts to 12 principa
l components.
pca.n_components_
plt.plot(np.cumsum(pca.explained_variance_ratio_))
plt.xlabel('number of components')
plt.ylabel('cumulative explained variance')
```

Out[348]:
```
Text(0, 0.5, 'cumulative explained variance')
```



In [332]:
```
components = pca.transform(noisy)
filtered = pca.inverse_transform(components)
plot_digits(filtered)
```



In [ ]: