In [85]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm, laplace, bernoulli
import scipy.optimize as optimize
import math
```

Defining constant and writing the function for generating measured data wihtout the noise

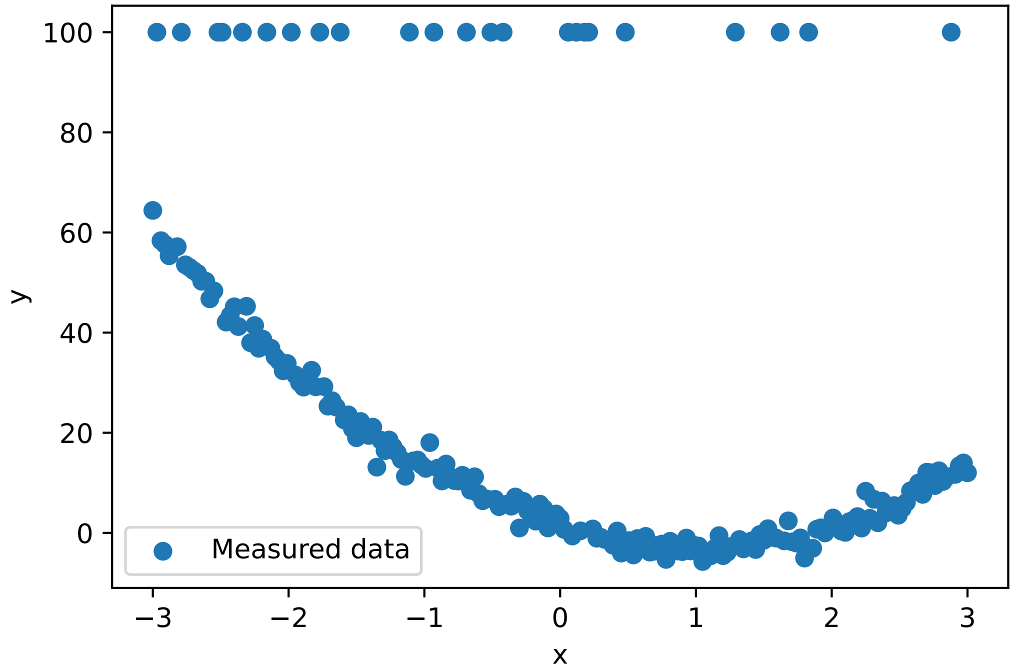
In [87]:
```
N = 201
magnitude = 1.2
mu = 0  # loc
sigma = 1  # beta
alpha = 0
gamma = 0.1
x = np.linspace(-3, 3, N)
params = [1, -8, 4]

def create_y_model(params):
    def y(x): return sum([p*(x**i) for i, p in enumerate(params)])
    return y
```

The Laplacian noise function

In [89]:
```
def add_noise_to_true_model(x,N, params, magnitude, alpha, mu, sigma):
    y = create_y_model(params)
    y_true = y(x)
    y_added_noise = y_true + magnitude * \
        (alpha*np.random.normal(mu, sigma, N) +
         (1-alpha)*np.random.laplace(mu, sigma, N))
    return y_added_noise
```

Adding the outlier noise function

In [91]:
```
def add_outlier(params, N, magnitude, alpha, mu, sigma, gamma):
    y_added_noise = add_noise_to_true_model(x,
        N, params, magnitude, alpha, mu, sigma)
    data = bernoulli.rvs(size=N, p=gamma)
    for i in range(N):
        if data[i] == 1:
            y_added_noise[i] = 100
    return y_added_noise


y_measured_data = add_outlier(params, N, magnitude, alpha, mu, sigma, g
amma)
plt.figure()
plt.scatter(x, y_measured_data, label="Measured data")
plt.legend()
plt.xlabel("x")
plt.ylabel("y")
plt.show()
```



Splitting the measured data into training, testing and validation

In [93]:
```
y_traning_set = y_measured_data[0:math.floor(len(y_measured_data)/3)]
x_traning_set = x[0:math.floor(len(x)/3)]

y_testing_set = y_measured_data[math.floor(len(y_measured_data)/3):math
.floor(len(y_measured_data)*2/3)]
x_testing_set = x[math.floor(len(x)/3):math.floor(len(x)*2/3)]

y_validation_set = y_measured_data[math.floor(len(y_measured_data)*2/3
):]
x_validation_set = x[math.floor(len(x)*2/3):]
```

Estimting the LS estimator for each mode

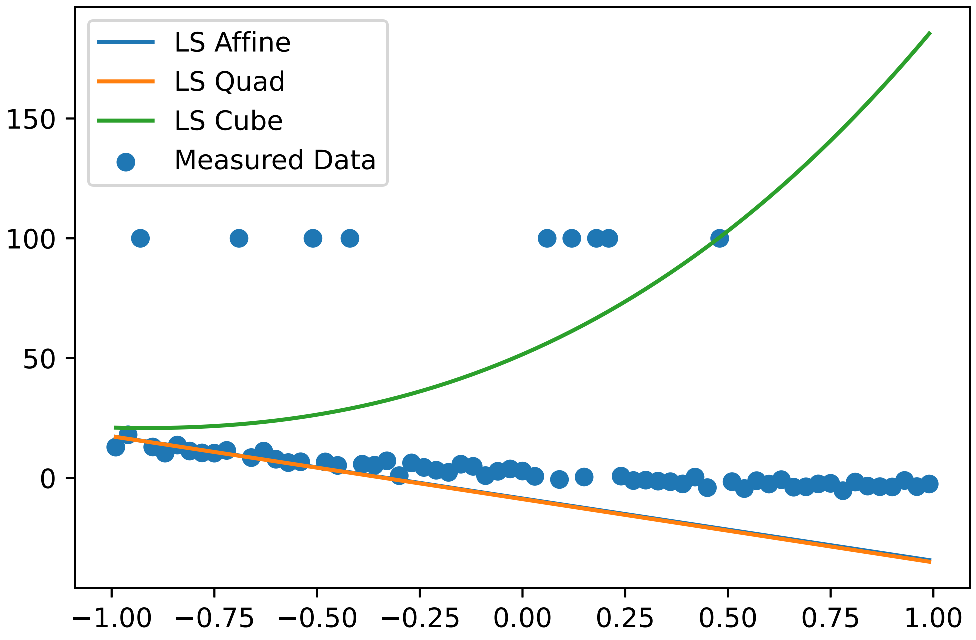
In [95]:
```
def LS_estimator_given_mode(mode, x, y):
    u_tensor_0 = np.reshape(x, (len(x), 1))

    ones_vec = np.ones((len(x), 1))
    u_tensor = np.append(ones_vec, u_tensor_0, axis=1)

    for i in range(2, mode):
        u_tensor = np.append(u_tensor, np.power(u_tensor_0, i), axis=1)

    u_transpose_dot_u = np.dot(u_tensor.T, u_tensor)  # calculating dot
product
    u_transpose_dot_u_inv = np.linalg.inv(
        u_transpose_dot_u)  # calculating inverse
    u_transpose_dot_y = np.dot(u_tensor.T, y)  # calculating dot produc
t

    LS_params = np.dot(u_transpose_dot_u_inv, u_transpose_dot_y)
    # Recreate model based on LS estimate:
    LS_params = LS_params.tolist()
    return LS_params

LS_params_affine = LS_estimator_given_mode(2,x_traning_set, y_traning_s
```

```
et)
LS_params_quad = LS_estimator_given_mode(3,x_traning_set,y_traning_set)
LS_params_cube = LS_estimator_given_mode(4,x_traning_set,y_traning_set)

y_hat_LS_affine = create_y_model(LS_params_affine)
y_hat_LS_quad = create_y_model(LS_params_quad)
y_hat_LS_cube = create_y_model(LS_params_cube)
```

When plotting the different LS estimator modes for the training set, we can clearly see that the affine and quadratic estimator is performing the best, while the cubic estimator has terrible performance

In [97]:

```
plt.figure()
plt.scatter(x_testing_set,y_testing_set, label= "Measured Data")
plt.plot(x_testing_set,y_hat_LS_affine(x_testing_set), label= "LS Affin
e")
plt.plot(x_testing_set,y_hat_LS_quad(x_testing_set), label="LS Quad")
plt.plot(x_testing_set,y_hat_LS_cube(x_testing_set),label= "LS Cube")
plt.legend()
plt.show()
```



The code for estimating the ML estimator for each mode

In [99]:
```
def log_lik(par_vec,x,y):
    pdf = laplace.pdf
    # If the standard deviation parameter is negative, return a large v
alue:
    if par_vec[-1] < 0:
        return(1e8)
    # The likelihood function values:
    lik = pdf(y,
              loc=sum([p*(x**i) for i, p in enumerate(par_vec[:-1])]),
              scale=par_vec[-1])

    if all(v == 0 for v in lik):
        return(1e8)
    # Logarithm of zero = -Inf
    return(-sum(np.log(lik[np.nonzero(lik)])))
```

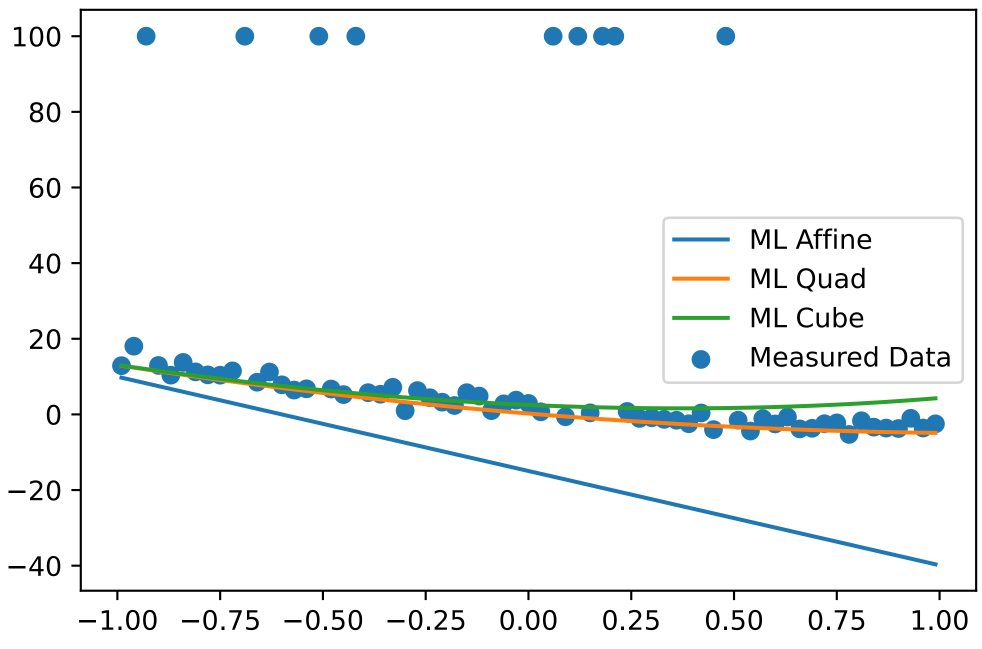
```python
def ML_estimator_given_mode(mode, N, x,y):
    init_guess = np.zeros(mode+1)
    init_guess[-1] = len(x)

    opt_res = optimize.minimize(fun=log_lik,
                                x0=init_guess,
                                options={'disp': True},
                                args=(x,y))
    MLE_params = opt_res.x[:-1]
    MLE_params = MLE_params.tolist()
    return MLE_params

ML_params_affine = ML_estimator_given_mode(2,N,x_traning_set,y_traning_set)
ML_params_quad = ML_estimator_given_mode(3,N,x_traning_set,y_traning_set)
ML_params_cube = ML_estimator_given_mode(4,N,x_traning_set,y_traning_set)

y_hat_ML_affine = create_y_model(ML_params_affine)
y_hat_ML_quad = create_y_model(ML_params_quad)
y_hat_ML_cube = create_y_model(ML_params_cube)
```

```
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 270.900525
         Iterations: 17
         Function evaluations: 336
         Gradient evaluations: 81
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 269.065481
         Iterations: 35
         Function evaluations: 623
         Gradient evaluations: 123
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 269.015023
         Iterations: 23
         Function evaluations: 596
         Gradient evaluations: 98
```

Plotting the different ML estimators we can see that the quadratic and cubic estimators performed really well. Although the affine estimator was not yielding good result it was still performing reasonaly well compared to the ML estimators

In [100]:
```
plt.figure()
plt.scatter(x_testing_set,y_testing_set, label= "Measured Data")
plt.plot(x_testing_set,y_hat_ML_affine(x_testing_set), label= "ML Affin
e")
plt.plot(x_testing_set,y_hat_ML_quad(x_testing_set), label="ML Quad")
plt.plot(x_testing_set,y_hat_ML_cube(x_testing_set),label= "ML Cube")
plt.legend()
```

Out[100]: <matplotlib.legend.Legend at 0x2b3d403bd00>



Renning the performance index on the testing data for alle the different estimators, we can clearly observe that the best estimator is the quadratic ML estimator as was evident in the plots we did earlier. The second best estimator is the cubic ML estimator. The worst performances came from the LS estimators.

In [101]:
```
def performance_index(x_data,y_data,params):
    y_hat_model = create_y_model(params)
    y_hat = y_hat_model(x_data)

    performance = 0
    for i in range(len(y_data)):
        performance += abs(y_data[i] - y_hat[i])
    return performance

performance_LS_affine = performance_index(x_testing_set,y_testing_set,
LS_params_affine)
performance_LS_quad = performance_index(x_testing_set,y_testing_set, LS
_params_quad)
performance_LS_cube = performance_index(x_testing_set,y_testing_set, LS
_params_cube)
performance_ML_affine = performance_index(x_testing_set,y_testing_set,
ML_params_affine)
performance_ML_quad = performance_index(x_testing_set,y_testing_set, ML
_params_quad)
performance_ML_cube = performance_index(x_testing_set,y_testing_set, ML
_params_cube)

print("LS Affine: ", performance_LS_affine)
print("LS Quad: ", performance_LS_quad)
print("LS Cube: ", performance_LS_cube)
print("ML Affine: ", performance_ML_affine)
print("ML Quad: ", performance_ML_quad)
print("ML Cube: ", performance_ML_cube)
```

```
LS Affine:  1662.5771103211305
LS Quad:  1681.5744939718413
LS Cube:  4475.585107256561
ML Affine:  2066.210291534851
ML Quad:  973.7625424503735
ML Cube:  1021.0821063426866
```

In the final part of our assignment i have plotted all the estimators as a function of gamma. If one scrolls past all the warnings one can see the plot at the end. I have choose to exclude the LS cubic estimator from the plots because it was too large and so streched out the graph too much and made it too difficult to compare the perforamance index of the other estimators. The most robust estimators seems to be the LS affine estimator which has the least amount and least rapid spikes in it's performance. However it is clear that the LS affine estimator is not the best estimator if gamma is less than 0.2. The ML quadratic estimator have really good performance in the range gamma = 0 to 0.15, after that it spikes and,but then for gamma > 0.25, the spike ends and it becomes a relativelly good estimator. To me it becomes clear that the ML estimators have the best performances on average when gamma is either < 0.2 or > 0.4. In between there the LS affine estimator is sometimes good. For gamma > 0.5 all the estimators becomes equally good. At least from what i can see in my plots. To answer the quesiton a) The LS affine estimator degrades least and b) the ML quadratic estimator performes the best

In [102]:

```
def plot_LS_ML_func_of_gamma(x,params, N, magnitude, alpha, mu, sigma):
    gamma = np.linspace(0,1,30)
    LS_affine_performance_vector = [0]*len(gamma)
    LS_quad_performance_vector = [0]*len(gamma)
    LS_cube_performance_vector = [0]*len(gamma)
    ML_affine_performance_vector = [0]*len(gamma)
    ML_quad_performance_vector = [0]*len(gamma)
    ML_cube_performance_vector = [0]*len(gamma)


    y = create_y_model(params)
    y_true = y(x)
    y_added_noise = y_true + magnitude * \
        (alpha*np.random.normal(mu, sigma, N) +
        (1-alpha)*np.random.laplace(mu, sigma, N))

    x_traning_set = x[0:math.floor(len(x)/3)]
    x_testing_set = x[math.floor(len(x)/3):math.floor(len(x)*2/3)]
    x_validation_set = x[math.floor(len(x)*2/3):]

    for j in range(len(gamma)):
        data = bernoulli.rvs(size=N, p=gamma[j])
        for i in range(N):
```

```
            if data[i] == 1:
                y_added_noise[i] = 100

        y_measured_data = y_added_noise
        y_traning_set = y_measured_data[0:math.floor(len(y_measured_dat
a)/3)]
        y_testing_set = y_measured_data[math.floor(len(y_measured_data)
/3):math.floor(len(y_measured_data)*2/3)]
        y_validation_set = y_measured_data[math.floor(len(y_measured_da
ta)*2/3):]

        LS_params_affine = LS_estimator_given_mode(2,x_traning_set, y_t
raning_set)
        LS_params_quad = LS_estimator_given_mode(3,x_traning_set,y_tran
ing_set)
        LS_params_cube = LS_estimator_given_mode(4,x_traning_set,y_tran
ing_set)

        ML_params_affine = ML_estimator_given_mode(2,N,x_traning_set,y_
traning_set)
        ML_params_quad = ML_estimator_given_mode(3,N,x_traning_set,y_tr
aning_set)
        ML_params_cube = ML_estimator_given_mode(4,N,x_traning_set,y_tr
aning_set)


        LS_affine_performance_vector[j] = performance_index(x_validatio
n_set,y_validation_set, LS_params_affine)
        LS_quad_performance_vector[j] = performance_index(x_validation_
set,y_validation_set, LS_params_quad)
        LS_cube_performance_vector[j] = performance_index(x_validation_
set,y_validation_set, LS_params_cube)
        ML_affine_performance_vector[j] = performance_index(x_validatio
n_set,y_validation_set, ML_params_affine)
        ML_quad_performance_vector[j] = performance_index(x_validation_
set,y_validation_set, ML_params_quad)
        ML_cube_performance_vector[j] = performance_index(x_validation_
set,y_validation_set, ML_params_cube)
```

```python
    plt.figure()
    plt.plot(gamma, LS_affine_performance_vector, label="LS Affine")
    plt.plot(gamma, LS_quad_performance_vector, label="LS Quad")
    #plt.plot(gamma, LS_cube_performance_vector, label="LS Cube")
    plt.plot(gamma, ML_affine_performance_vector, label="ML Affine")
    plt.plot(gamma, ML_quad_performance_vector, label="ML Quad")
    plt.plot(gamma, ML_cube_performance_vector, label="ML Cube")
    plt.legend()
    plt.xlabel('gamma')
    plt.ylabel('Performance index')
    plt.show()

plot_LS_ML_func_of_gamma(x,params, N, magnitude, alpha, mu, sigma)
```

```
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 142.102576
         Iterations: 23
         Function evaluations: 420
         Gradient evaluations: 102
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 115.435949
         Iterations: 28
         Function evaluations: 626
         Gradient evaluations: 123
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 115.008290
         Iterations: 53
         Function evaluations: 1037
         Gradient evaluations: 172
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 201.454826
         Iterations: 25
         Function evaluations: 415
         Gradient evaluations: 101
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 191.707810
         Iterations: 35
         Function evaluations: 672
         Gradient evaluations: 133
Warning: Desired error not necessarily achieved due to precision loss.
```

```
         Current function value: 191.776975
         Iterations: 20
         Function evaluations: 601
         Gradient evaluations: 99
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 221.214440
         Iterations: 22
         Function evaluations: 480
         Gradient evaluations: 117
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 214.093968
         Iterations: 52
         Function evaluations: 1025
         Gradient evaluations: 203
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 214.144882
         Iterations: 22
         Function evaluations: 672
         Gradient evaluations: 110
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 257.719842
         Iterations: 17
         Function evaluations: 288
         Gradient evaluations: 71
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 253.871864
         Iterations: 16
         Function evaluations: 400
         Gradient evaluations: 79
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 253.835530
         Iterations: 25
         Function evaluations: 726
         Gradient evaluations: 119
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 303.302256
         Iterations: 22
         Function evaluations: 484
         Gradient evaluations: 117
Warning: Desired error not necessarily achieved due to precision loss.
```

```
         Current function value: 301.725019
         Iterations: 32
         Function evaluations: 676
         Gradient evaluations: 134
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 301.718902
         Iterations: 27
         Function evaluations: 1074
         Gradient evaluations: 177
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 323.996536
         Iterations: 20
         Function evaluations: 327
         Gradient evaluations: 80
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 323.210440
         Iterations: 17
         Function evaluations: 325
         Gradient evaluations: 64
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 323.195353
         Iterations: 28
         Function evaluations: 585
         Gradient evaluations: 97
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 340.657697
         Iterations: 29
         Function evaluations: 580
         Gradient evaluations: 142
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 340.404941
         Iterations: 11
         Function evaluations: 377
         Gradient evaluations: 73
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 340.300297
         Iterations: 57
         Function evaluations: 2059
         Gradient evaluations: 342
Warning: Desired error not necessarily achieved due to precision loss.
```

```
         Current function value: 338.894398
         Iterations: 20
         Function evaluations: 462
         Gradient evaluations: 113
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 335.435681
         Iterations: 24
         Function evaluations: 397
         Gradient evaluations: 78
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 334.645775
         Iterations: 16
         Function evaluations: 438
         Gradient evaluations: 72
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 314.566871
         Iterations: 45
         Function evaluations: 608
         Gradient evaluations: 149
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 314.566875
         Iterations: 53
         Function evaluations: 674
         Gradient evaluations: 133
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 314.613445
         Iterations: 33
         Function evaluations: 611
         Gradient evaluations: 101
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 298.584614
         Iterations: 46
         Function evaluations: 536
         Gradient evaluations: 131
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 298.584615
         Iterations: 52
         Function evaluations: 752
         Gradient evaluations: 148
Warning: Desired error not necessarily achieved due to precision loss.
```

```
         Current function value: 298.584622
         Iterations: 77
         Function evaluations: 1068
         Gradient evaluations: 176
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 265.509467
         Iterations: 48
         Function evaluations: 608
         Gradient evaluations: 149
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 265.509471
         Iterations: 50
         Function evaluations: 566
         Gradient evaluations: 111
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 265.509501
         Iterations: 67
         Function evaluations: 960
         Gradient evaluations: 158
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 248.038481
         Iterations: 39
         Function evaluations: 411
         Gradient evaluations: 100
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 248.038481
         Iterations: 55
         Function evaluations: 913
         Gradient evaluations: 180
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 248.038503
         Iterations: 55
         Function evaluations: 780
         Gradient evaluations: 128
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 208.632371
         Iterations: 51
         Function evaluations: 531
         Gradient evaluations: 130
Warning: Desired error not necessarily achieved due to precision loss.
```

```
         Current function value: 208.632384
         Iterations: 53
         Function evaluations: 664
         Gradient evaluations: 131
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 208.632381
         Iterations: 69
         Function evaluations: 1092
         Gradient evaluations: 180
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 153.704912
         Iterations: 50
         Function evaluations: 600
         Gradient evaluations: 147
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 153.704915
         Iterations: 46
         Function evaluations: 766
         Gradient evaluations: 151
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 153.704964
         Iterations: 71
         Function evaluations: 990
         Gradient evaluations: 163
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 153.704912
         Iterations: 50
         Function evaluations: 600
         Gradient evaluations: 147
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 153.704915
         Iterations: 46
         Function evaluations: 766
         Gradient evaluations: 151
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: 153.704964
         Iterations: 71
         Function evaluations: 990
         Gradient evaluations: 163
Warning: Desired error not necessarily achieved due to precision loss.
```

```
         Current function value: -843.811314
         Iterations: 53
         Function evaluations: 910
         Gradient evaluations: 225
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: -747.379070
         Iterations: 74
         Function evaluations: 1026
         Gradient evaluations: 203
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: -250.957523
         Iterations: 59
         Function evaluations: 1086
         Gradient evaluations: 179
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: -843.811314
         Iterations: 53
         Function evaluations: 910
         Gradient evaluations: 225
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: -747.379070
         Iterations: 74
         Function evaluations: 1026
         Gradient evaluations: 203
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: -250.957523
         Iterations: 59
         Function evaluations: 1086
         Gradient evaluations: 179
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: -843.811314
         Iterations: 53
         Function evaluations: 910
         Gradient evaluations: 225
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: -747.379070
         Iterations: 74
         Function evaluations: 1026
         Gradient evaluations: 203
Warning: Desired error not necessarily achieved due to precision loss.
```

```
         Current function value: -250.957523
         Iterations: 59
         Function evaluations: 1086
         Gradient evaluations: 179
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: -843.811314
         Iterations: 53
         Function evaluations: 910
         Gradient evaluations: 225
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: -747.379070
         Iterations: 74
         Function evaluations: 1026
         Gradient evaluations: 203
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: -250.957523
         Iterations: 59
         Function evaluations: 1086
         Gradient evaluations: 179
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: -843.811314
         Iterations: 53
         Function evaluations: 910
         Gradient evaluations: 225
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: -747.379070
         Iterations: 74
         Function evaluations: 1026
         Gradient evaluations: 203
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: -250.957523
         Iterations: 59
         Function evaluations: 1086
         Gradient evaluations: 179
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: -843.811314
         Iterations: 53
         Function evaluations: 910
         Gradient evaluations: 225
Warning: Desired error not necessarily achieved due to precision loss.
```

```
         Current function value: -747.379070
         Iterations: 74
         Function evaluations: 1026
         Gradient evaluations: 203
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: -250.957523
         Iterations: 59
         Function evaluations: 1086
         Gradient evaluations: 179
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: -843.811314
         Iterations: 53
         Function evaluations: 910
         Gradient evaluations: 225
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: -747.379070
         Iterations: 74
         Function evaluations: 1026
         Gradient evaluations: 203
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: -250.957523
         Iterations: 59
         Function evaluations: 1086
         Gradient evaluations: 179
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: -843.811314
         Iterations: 53
         Function evaluations: 910
         Gradient evaluations: 225
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: -747.379070
         Iterations: 74
         Function evaluations: 1026
         Gradient evaluations: 203
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: -250.957523
         Iterations: 59
         Function evaluations: 1086
         Gradient evaluations: 179
Warning: Desired error not necessarily achieved due to precision loss.
```

```
         Current function value: -843.811314
         Iterations: 53
         Function evaluations: 910
         Gradient evaluations: 225
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: -747.379070
         Iterations: 74
         Function evaluations: 1026
         Gradient evaluations: 203
Warning: Desired error not necessarily achieved due to precision loss.
         Current function value: -250.957523
         Iterations: 59
         Function evaluations: 1086
         Gradient evaluations: 179
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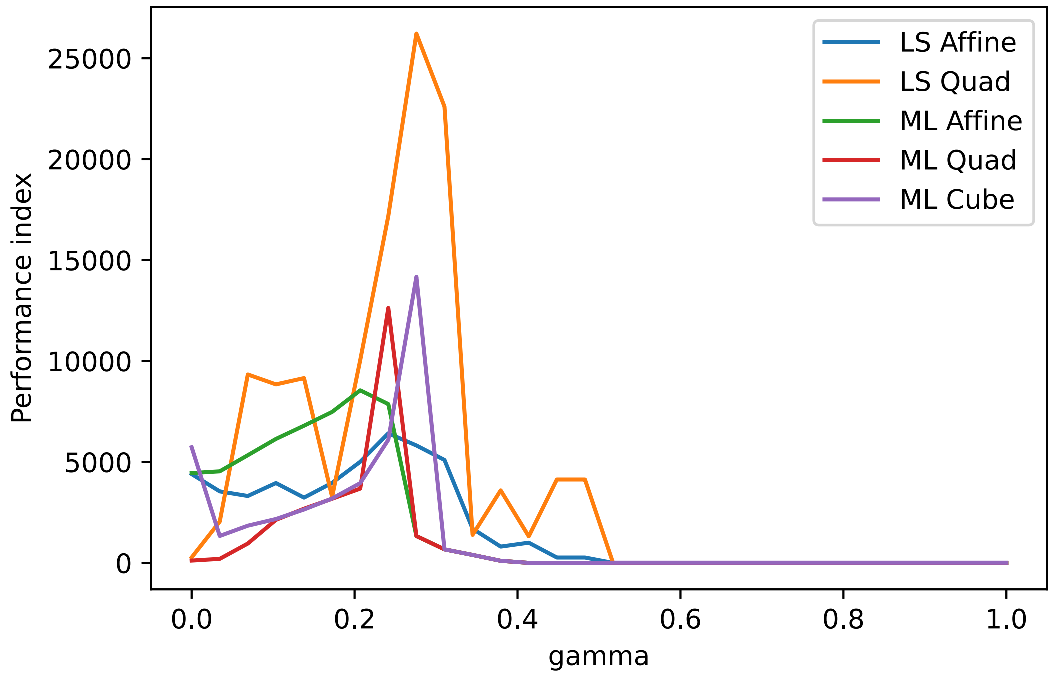
LS Affine
LS Quad
ML Affine
ML Quad
ML Cube
Performance index
0
5000
10000
15000
20000
25000
0.0
0.2
0.4
0.6
0.8
1.0
gamma