Job Sequencing Problem with Deadlines

Given a set of tasks with deadlines and total profit earned on completing a task, find maximum profit earned by executing the tasks within the specified deadlines. Assume that a task takes one unit of time to execute, and it can't execute beyond its deadline. Also, only a single task will be executed at a time.

For example, consider the following set of tasks with a deadline and the profit associated with it. If we choose tasks 1, 3, 4, 5, 6, 7, 8, and 9, we can achieve a maximum profit of 109. Note that task 2 and task 10 are left out.

Tasks	Deadlines	Profit
1	9	15
2	2	2
3	5	18
4	7	1
5	4	25
6	2	20
7	5	8

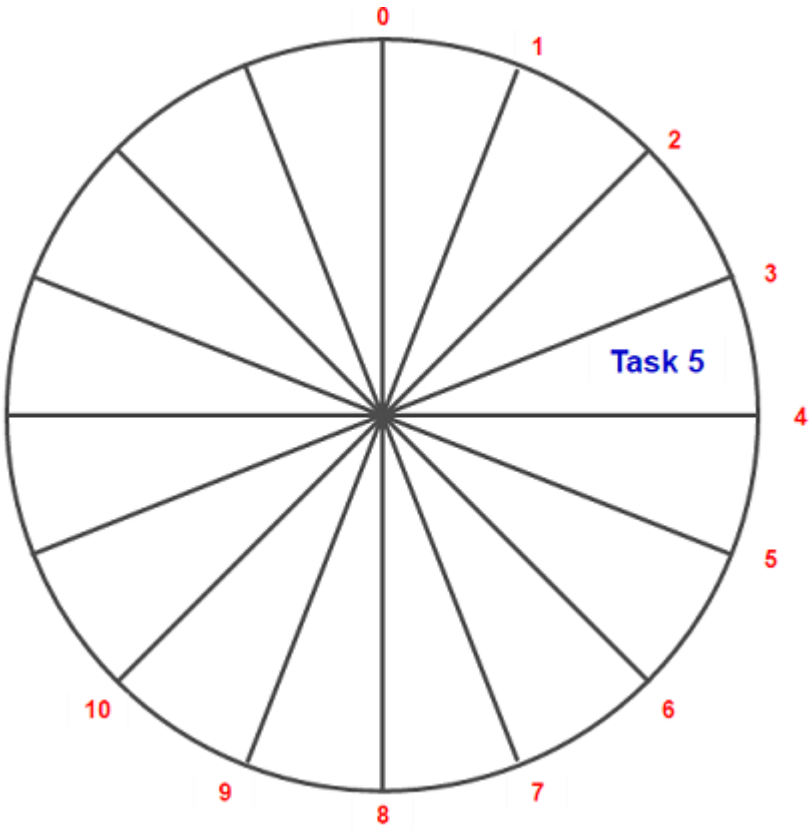
8	7	10
9	4	12
10	3	5

We can easily solve this problem by following a Greedy approach. The idea is simple – consider each task decreasing order of their profits and schedule it in the latest possible free slot that meets its deadline. If no such slot is there, don't schedule the task.

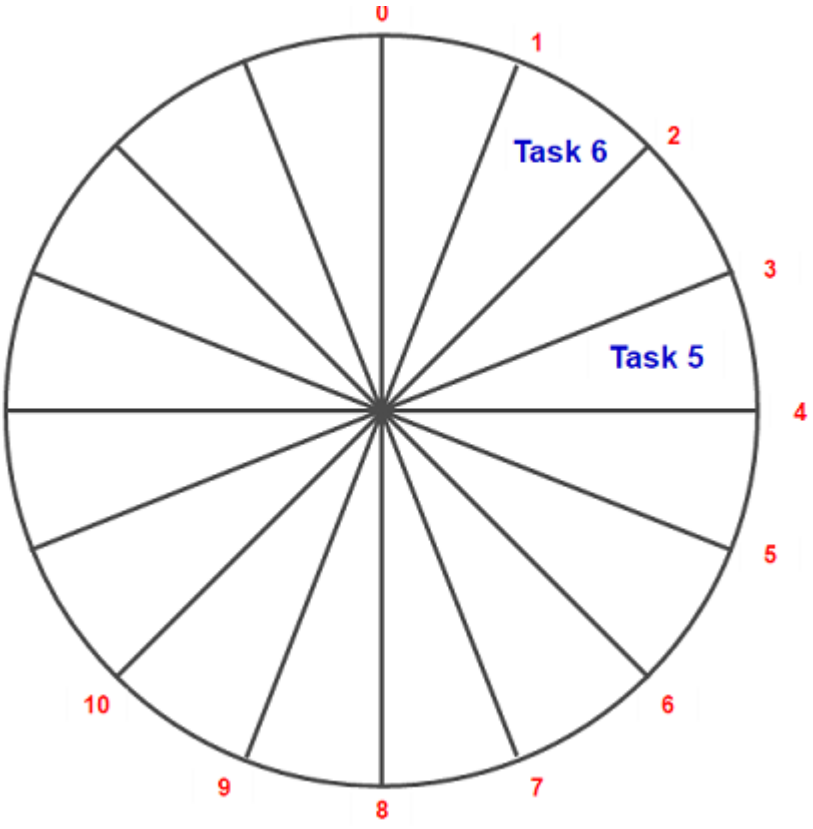
The following table shows the tasks arranged based on their associated profits. Here, task 5 has a maximum priority associated with it as it has a maximum gain of 30. Similarly, task 4 has the least priority. The greedy approach will consider the tasks in decreasing order of their priority.

Tasks	Deadlines	Profit (Maximum first)
5	4	25
6	2	20
3	5	18
1	9	15
9	4	12
8	7	10
7	5	8
10	3	5
2	2	2
4	7	1

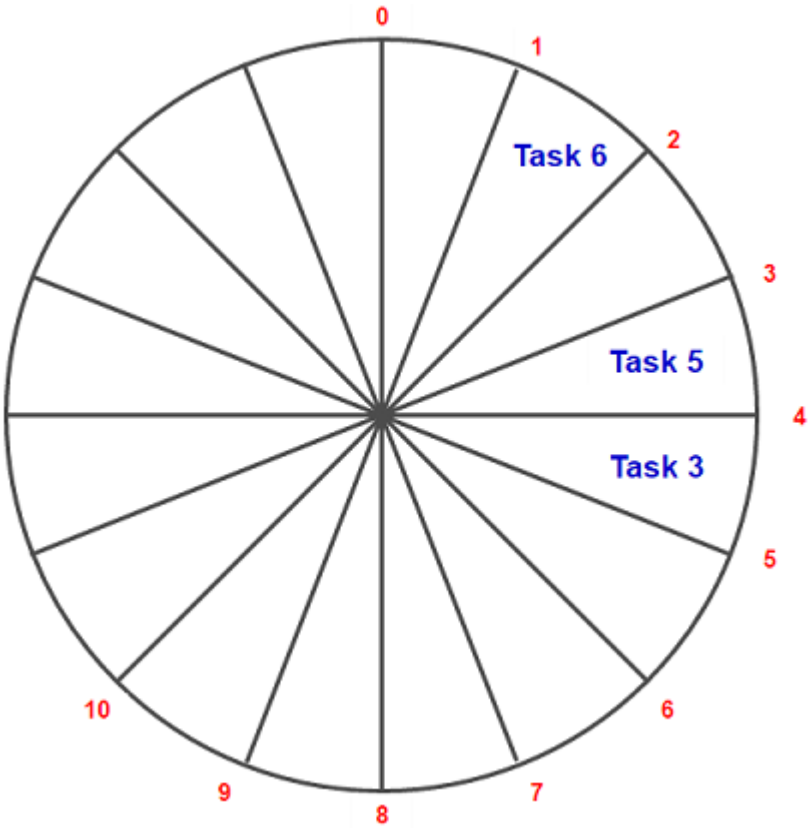
To demonstrate the greedy approach, let's consider the deadlines in the form of a circular structure, as shown below. A given task can fill each slot. Now, let's start allocated the tasks based on the deadlines. We will start with task 5, having a deadline of 4, and fill it empty slot `3–4`.



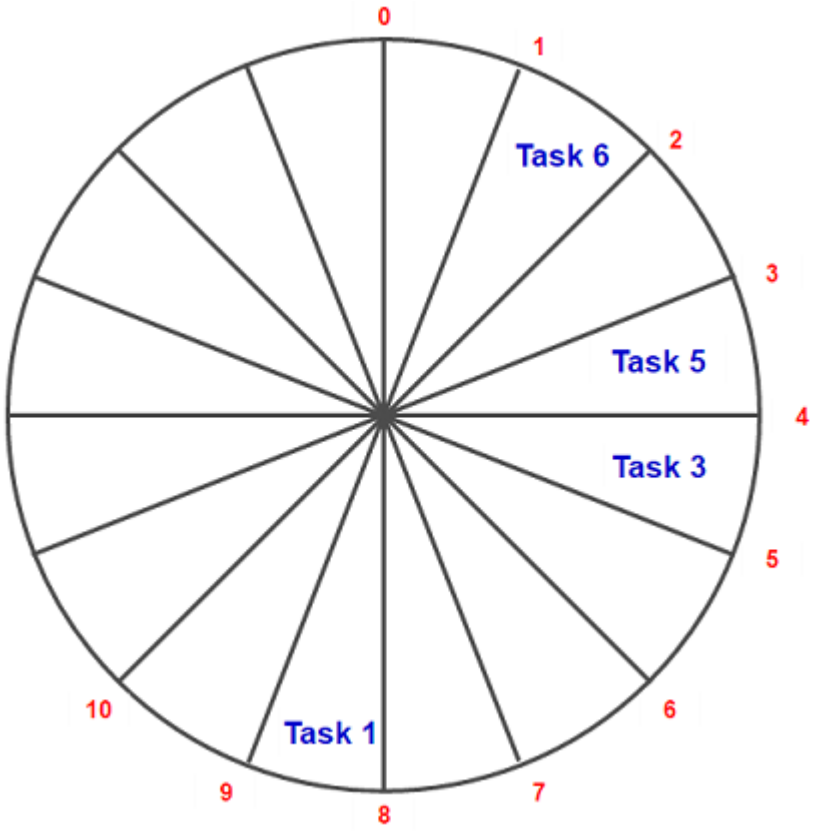
Next, consider task 6 having deadline 2 and fill it empty slot `1–2`.



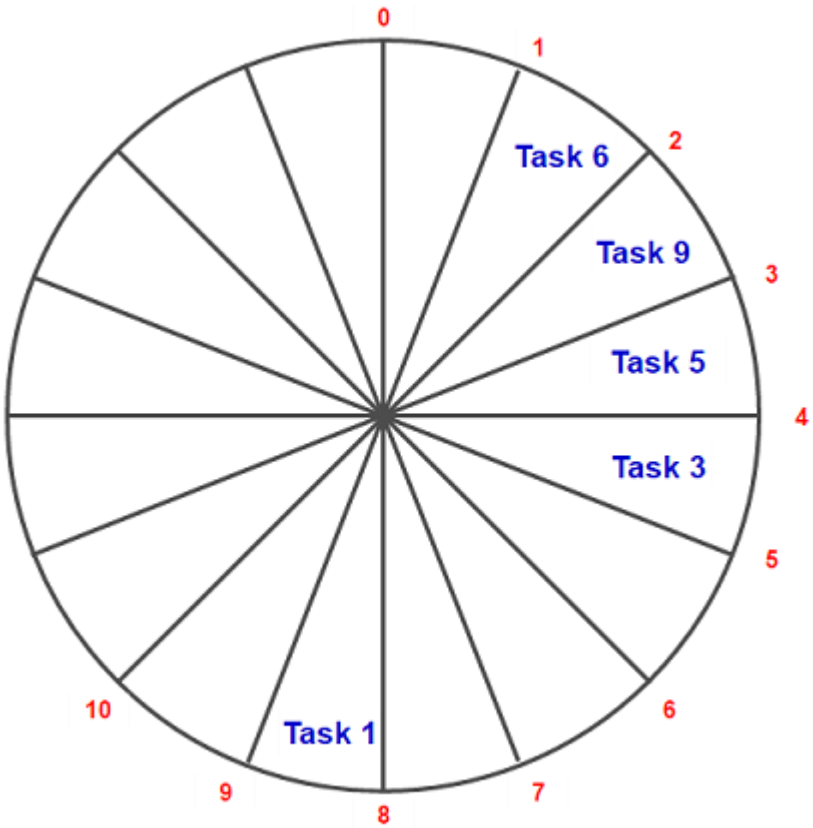
Next, consider task 3 having deadline 5 and fill it empty slot `4–5` .



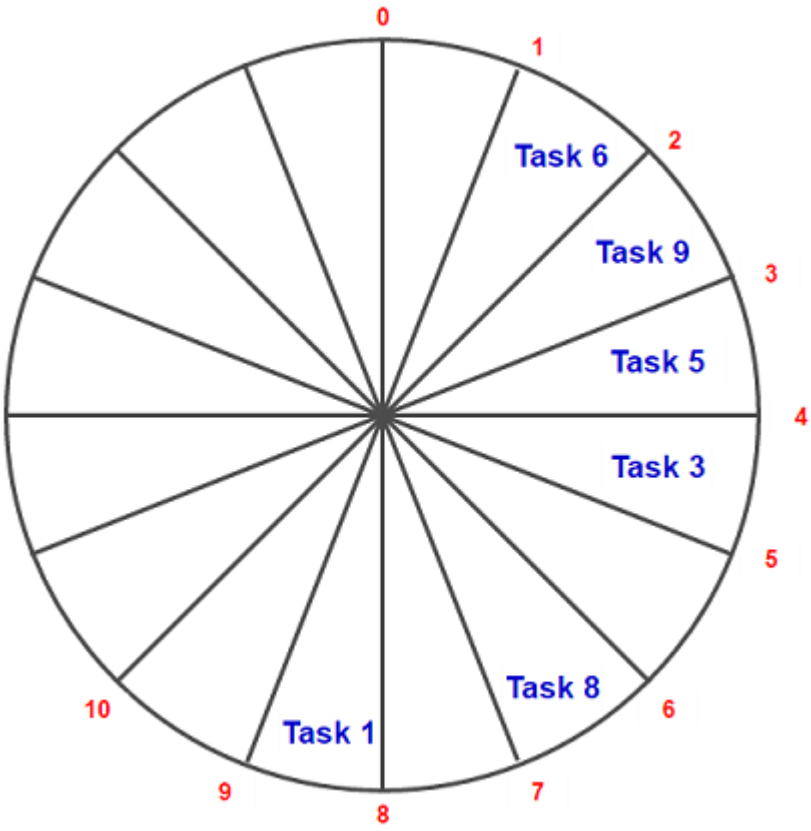
Next, consider task 1 having deadline 9 and fill it empty slot `8–9` .



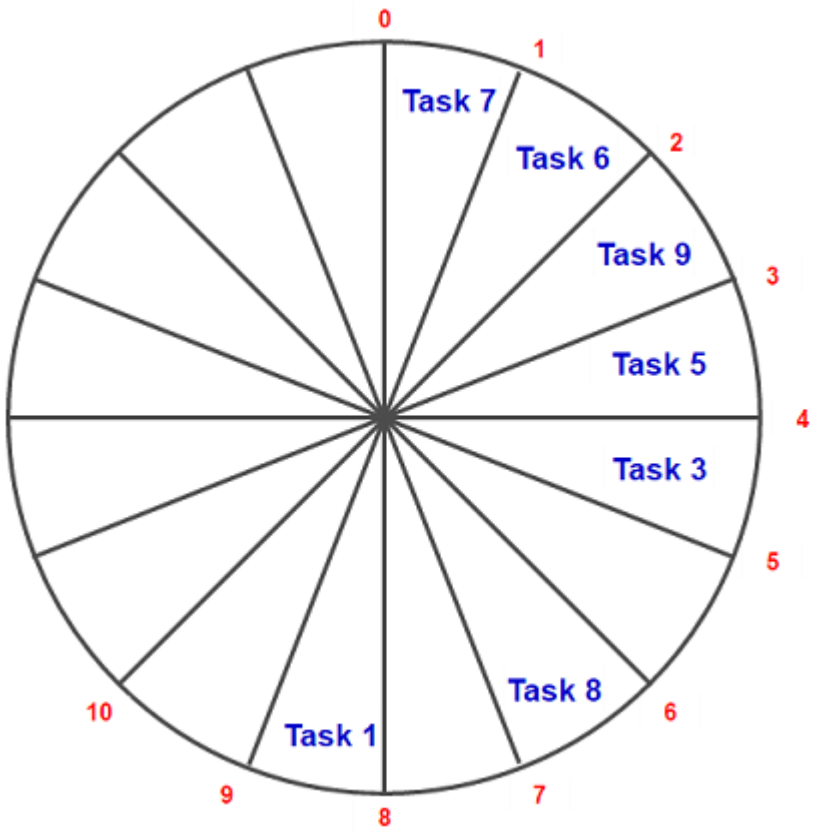
Next, consider task 9 having deadline 4. As slot `3–4` is already filled with task 5, we will consider the next free slot `2–3` and assign task 9.



Next, consider task 8 having deadline 7 and fill it empty slot `6–7` .



Next, consider task 7 having a deadline of 5. As slot `4–5` is already filled with task 3, we will consider the next free slot `0–1` and assign task 7.



Next, consider task 10 having a deadline of 3. Since all slots before deadline 3 are filled, ignore the task. Similarly, ignore the next task 2 having deadline 2.

The last task is task 4, having deadline 7, gets the next empty slot `6–5`.