Python code for Artificial Intelligence

Foundations of Computational Agents

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https://aipython.org https://artint.info

Contents

Chapter 1

Python for Artificial Intelligence

AIPython contains runnable code for the book *Artificial Intelligence, foundations of computational agents, 3rd Edition* [Poole and Mackworth, 2023]. It has the following design goals:

- Readability is more important than efficiency, although the asymptotic complexity is not compromised. AIPython is not a replacement for well-designed libraries, or optimized tools. Think of it like a model of an engine made of glass, so you can see the inner workings; don't expect it to power a big truck, but it lets you see how a metal engine can power a truck.

- It uses as few libraries as possible. A reader only needs to understand Python. Libraries hide details that we make explicit. The only library used is matplotlib for plotting and drawing.

1.1 Why Python?

We use Python because Python programs can be close to pseudo-code. It is designed for humans to read.

Python is reasonably efficient. Efficiency is usually not a problem for small examples. If your Python code is not efficient enough, a general procedure to improve it is to find out what is taking most of the time, and implement just that part more efficiently in some lower-level language. Most of these lower-level languages interoperate with Python nicely. This will result in much less programming and more efficient code (because you will have more time to optimize) than writing everything in a low-level language. You will not have to do that for the code here if you are using it for larger projects.

1.2 Getting Python

You need Python 3.9 or later (`https://python.org/`) and a compatible version of matplotlib (`https://matplotlib.org/`). This code is *not* compatible with Python 2 (e.g., with Python 2.7).

Download and install the latest Python 3 release from `https://python.org/` or `https://www.anaconda.com/download`. This should also install `pip3`. You can install matplotlib using

```
pip3 install matplotlib
```

in a terminal shell (not in Python). That should "just work". If not, try using `pip` instead of `pip3`.

The command `python` or `python3` should then start the interactive Python shell. You can quit Python with a control-D or with `quit()`.

To upgrade matplotlib to the latest version (which you should do if you install a new version of Python) do:

```
pip3 install --upgrade matplotlib
```

We recommend using the enhanced interactive python **ipython** (`https://ipython.org/`) [Pérez and Granger, 2007]. To install ipython after you have installed python do:

```
pip3 install ipython
```

1.3 Running Python

We assume that everything is done with an interactive Python shell. You can either do this with an IDE, such as IDLE that comes with standard Python distributions, or just running ipython3 or python3 (or perhaps just ipython or python) from a shell.

Here we describe the most simple version that uses no IDE. If you download the zip file, and cd to the "`aipython`" folder where the .py files are, you should be able to do the following, with user input in bold. The first `python` command is in the operating system shell; the `-i` is important to enter interactive mode.

```
python -i searchGeneric.py
Testing problem 1:
7 paths have been expanded and 4 paths remain in the frontier
Path found: A --> C --> B --> D --> G
Passed unit test
>>> searcher2 = AStarSearcher(searchProblem.acyclic_delivery_problem) #A*
>>> searcher2.search()  # find first path
16 paths have been expanded and 5 paths remain in the frontier
o103 --> o109 --> o119 --> o123 --> r123
>>> searcher2.search()  # find next path
```

```
21 paths have been expanded and 6 paths remain in the frontier
o103 --> b3 --> b4 --> o109 --> o119 --> o123 --> r123
>>> searcher2.search()  # find next path
28 paths have been expanded and 5 paths remain in the frontier
o103 --> b3 --> b1 --> b2 --> b4 --> o109 --> o119 --> o123 --> r123
>>> searcher2.search()  # find next path
No (more) solutions. Total of 33 paths expanded.
>>>
```

You can then interact at the last prompt.

There are many textbooks for Python. The best source of information about python is `https://www.python.org/`. The documentation is at `https://docs.python.org/3/`.

The rest of this chapter is about what is special about the code for AI tools. We will only use the standard Python library and matplotlib. All of the exercises can be done (and should be done) without using other libraries; the aim is for you to spend your time thinking about how to solve the problem rather than searching for pre-existing solutions.

1.4 Pitfalls

It is important to know when side effects occur. Often AI programs consider what would/might happen given certain conditions. In many such cases, we don't want side effects. When an agent acts in the world, side effects are appropriate.

In Python, you need to be careful to understand side effects. For example, the inexpensive function to add an element to a list, namely `append`, changes the list. In a functional language like Haskell or Lisp, adding a new element to a list, without changing the original list, is a cheap operation. For example if x is a list containing n elements, adding an extra element to the list in Python (using `append`) is fast, but it has the side effect of changing the list x. To construct a new list that contains the elements of x plus a new element, without changing the value of x, entails copying the list, or using a different representation for lists. In the searching code, we will use a different representation for lists for this reason.

1.5 Features of Python

1.5.1 f-strings

Python can use matching `'`, `"`, `'''` or `"""`, the latter two respecting line breaks in the string. We use the convention that when the string denotes a unique symbol, we use single quotes, and when it is designed to be for printing, we use double quotes.

We make extensive use of f-strings https://docs.python.org/3/tutorial/inputoutput.html. In its simplest form

```
f"str1{e1}str2{e2}str3"
```

where e1 and e2 are expressions, is an abbreviation for

```
"str1"+str(e2)+"str2"+str(e2)+"str3"
```

where + is string concatenation, and str is the function that returns a string representation of its expression argument.

1.5.2 Lists, Tuples, Sets, Dictionaries and Comprehensions

We make extensive uses of lists, tuples, sets and dictionaries (dicts). See https://docs.python.org/3/library/stdtypes.html

One of the nice features of Python is the use of **comprehensions**[1] (and also list, tuple, set and dictionary comprehensions). A generator expression is of the form

(*fe* for *e* in *iter* if *cond*)

enumerates the values *fe* for each *e* in *iter* for which *cond* is true. The "if *cond*" part is optional, but the "for" and "in" are not optional. Here *e* is a variable (or a pattern that can be on the left side of =), *iter* is an iterator, which can generate a stream of data, such as a list, a set, a range object (to enumerate integers between ranges) or a file. *cond* is an expression that evaluates to either True or False for each *e*, and *fe* is an expression that will be evaluated for each value of *e* for which *cond* returns True.

The result can go in a list or used in another iteration, or can be called directly using next. The procedure next takes an iterator and returns the next element (advancing the iterator); it raises a StopIteration exception if there is no next element. The following shows a simple example, where user input is prepended with >>>

```
>>> [e*e for e in range(20) if e%2==0]
[0, 4, 16, 36, 64, 100, 144, 196, 256, 324]
>>> a = (e*e for e in range(20) if e%2==0)
>>> next(a)
0
>>> next(a)
4
>>> next(a)
16
>>> list(a)
[36, 64, 100, 144, 196, 256, 324]
```

[1] https://docs.python.org/3/reference/expressions.html#displays-for-lists-sets-and-dictionaries

```
>>> next(a)
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
StopIteration
```

Notice how `list(a)` continued on the enumeration, and got to the end of it.

Comprehensions can also be used for dictionaries. The following code creates an index for list a:

```
>>> a = ["a","f","bar","b","a","aaaaa"]
>>> ind = {a[i]:i for i in range(len(a))}
>>> ind
{'a': 4, 'f': 1, 'bar': 2, 'b': 3, 'aaaaa': 5}
>>> ind['b']
3
```

which means that `'b'` is the 3rd element of the list.

The assignment of `ind` could have also be written as:

```
>>> ind = {val:i for (i,val) in enumerate(a)}
```

where `enumerate` is a built-in function that, given a dictionary, returns an iterator of $(index, value)$ pairs.

1.5.3 Functions as first-class objects

Python can create lists and other data structures that contain functions. There is an issue that tricks many newcomers to Python. For a local variable in a function, the function uses the last value of the variable when the function is *called*, not the value of the variable when the function was defined (this is called "late binding"). This means if you want to use the value a variable has when the function is created, you need to save the current value of that variable. Whereas Python uses "late binding" by default, the alternative that newcomers often expect is "early binding", where a function uses the value a variable had when the function was defined. The following examples show how early binding can be implemented.

Consider the following programs designed to create a list of 5 functions, where the ith function in the list is meant to add i to its argument:[2]

pythonDemo.py — Some tricky examples

```
11 fun_list1 = []
12 for i in range(5):
13     def fun1(e):
14         return e+i
15     fun_list1.append(fun1)
```

[2]Numbered lines are Python code available in the code-directory, `aipython`. The name of the file is given in the gray text above the listing. The numbers correspond to the line numbers in that file.

```
16
17 fun_list2 = []
18 for i in range(5):
19     def fun2(e,iv=i):
20         return e+iv
21     fun_list2.append(fun2)
22
23 fun_list3 = [lambda e: e+i for i in range(5)]
24
25 fun_list4 = [lambda e,iv=i: e+iv for i in range(5)]
26
27 i=56
```

Try to predict, and then test to see the output, of the output of the following calls, remembering that the function uses the latest value of any variable that is not bound in the function call:

pythonDemo.py — (continued)

```
29 # in Shell do
30 ## ipython -i pythonDemo.py
31 # Try these (copy text after the comment symbol and paste in the Python
       prompt):
32 # print([f(10) for f in fun_list1])
33 # print([f(10) for f in fun_list2])
34 # print([f(10) for f in fun_list3])
35 # print([f(10) for f in fun_list4])
```

In the first for-loop, the function `fun1` uses `i`, whose value is the last value it was assigned. In the second loop, the function `fun2` uses `iv`. There is a separate `iv` variable for each function, and its value is the value of `i` when the function was defined. Thus `fun1` uses late binding, and `fun2` uses early binding. `fun_list3` and `fun_list4` are equivalent to the first two (except `fun_list4` uses a different `i` variable).

One of the advantages of using the embedded definitions (as in `fun1` and `fun2` above) over the lambda is that is it possible to add a `__doc__` string, which is the standard for documenting functions in Python, to the embedded definitions.

1.5.4 Generators

Python has generators which can be used for a form of lazy evaluation – only computing values when needed.

The `yield` command returns a value that is obtained with `next`. It is typically used to enumerate the values for a `for` loop or in generators. (The `yield` command can also be used for coroutines, but AIPython only uses it for generators.)

A version of the built-in `range`, with 2 or 3 arguments (and positive steps) can be implemented as:

pythonDemo.py — (continued)

```
def myrange(start, stop, step=1):
    """enumerates the values from start in steps of size step that are
    less than stop.
    """
    assert step>0, f"only positive steps implemented in myrange: {step}"
    i = start
    while i<stop:
        yield i
        i += step

print("list(myrange(2,30,3)):",list(myrange(2,30,3)))
```

Note that the built-in `range` is unconventional in how it handles a single argument, as the single argument acts as the second argument of the function. Note also that the built-in range also allows for indexing (e.g., `range(2,30,3)[2]` returns 8), but the above implementation does not. However `myrange` also works for floats, whereas the built-in range does not.

Exercise 1.1 Implement a version of `myrange` that acts like the built-in version when there is a single argument. (Hint: make the second argument have a default value that can be recognized in the function.) There is no need to make it with indexing.

Yield can be used to generate the same sequence of values as in the example of Section 1.5.2:

pythonDemo.py — (continued)

```
def ga(n):
    """generates square of even nonnegative integers less than n"""
    for e in range(n):
        if e%2==0:
            yield e*e
a = ga(20)
```

The sequence of next(a), and list(a) gives exactly the same results as the comprehension in Section 1.5.2.

It is straightforward to write a version of the built-in `enumerate` called `myenumerate`:

pythonDemo.py — (continued)

```
def myenumerate(enum):
    for i in range(len(enum)):
        yield i,enum[i]
```

Exercise 1.2 Write a version of `enumerate` where the only iteration is "for val in enum". Hint: keep track of the index.

1.6 Useful Libraries

1.6.1 Timing Code

In order to compare algorithms, we often want to compute how long a program takes; this is called the **run time** of the program. The most straightforward way to compute run time is to use `time.perf_counter()`, as in:

```
import time
start_time = time.perf_counter()
compute_for_a_while()
end_time = time.perf_counter()
print("Time:", end_time - start_time, "seconds")
```

Note that `time.perf_counter()` measures clock time; so this should be done without user interaction between the calls. On the interactive python shell, you should do:

```
start_time = time.perf_counter(); compute_for_a_while(); end_time = time.perf_counter()
```

If this time is very small (say less than 0.2 second), it is probably very inaccurate, and it may be better to run your code many times to get a more accurate count. For this you can use `timeit` (`https://docs.python.org/3/library/timeit.html`). To use timeit to time the call to `foo.bar(aaa)` use:

```
import timeit
time = timeit.timeit("foo.bar(aaa)",
                     setup="from __main__ import foo,aaa", number=100)
```

The setup is needed so that Python can find the meaning of the names in the string that is called. This returns the number of seconds to execute `foo.bar(aaa)` 100 times. The variable `number` should be set so that the run time is at least 0.2 seconds.

You should not trust a single measurement as that can be confounded by interference from other processes. `timeit.repeat` can be used for running `timit` a few (say 3) times. When reporting the time of any computation, you should be explicit and explain what you are reporting. Usually the minimum time is the one to report.

1.6.2 Plotting: Matplotlib

The standard plotting for Python is matplotlib (`https://matplotlib.org/`). We will use the most basic plotting using the pyplot interface.

Here is a simple example that uses everything we will use. The output is shown in Figure 1.1.

pythonDemo.py — (continued)

```
60 import matplotlib.pyplot as plt
61
```

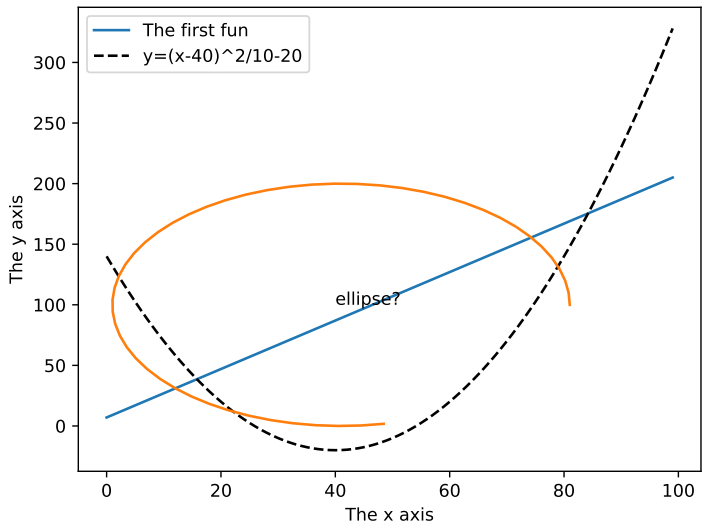


Figure 1.1: Result of pythonDemo code

```
def myplot(minv,maxv,step,fun1,fun2):
    plt.ion() # make it interactive
    plt.xlabel("The x axis")
    plt.ylabel("The y axis")
    plt.xscale('linear') # Makes a 'log' or 'linear' scale
    xvalues = range(minv,maxv,step)
    plt.plot(xvalues,[fun1(x) for x in xvalues],
             label="The first fun")
    plt.plot(xvalues,[fun2(x) for x in xvalues], linestyle='--',color='k',
             label=fun2.__doc__) # use the doc string of the function
    plt.legend(loc="upper right") # display the legend

def slin(x):
    """y=2x+7"""
    return 2*x+7
def sqfun(x):
    """y=(x-40)^2/10-20"""
    return (x-40)**2/10-20

# Try the following:
# from pythonDemo import myplot, slin, sqfun
# import matplotlib.pyplot as plt
# myplot(0,100,1,slin,sqfun)
# plt.legend(loc="best")
# import math
# plt.plot([41+40*math.cos(th/10) for th in range(50)],
#          [100+100*math.sin(th/10) for th in range(50)])
```

```
# plt.text(40,100,"ellipse?")
# plt.xscale('log')
```

At the end of the code are some commented-out commands you should try in interactive mode. Cut from the file and paste into Python (and remember to remove the comments symbol and leading space).

1.7 Utilities

1.7.1 Display

In this distribution, to keep things simple, using only standard Python, we use a text-oriented tracing of the code. A graphical depiction of the code can override the definition of `display` (e.g., see `SearcherGUI` in Section 3.2.2 and `ConsistencyGUI` in Section 4.4.2).

The method `self.display` is used to trace the program. Any call

`self.display`(*level*, *to_print* ...)

where the *level* is less than or equal to the value for `max_display_level` will be printed. The *to_print* ... can be anything that is accepted by the built-in `print` (including any keyword arguments).

The definition of `display` is:

display.py — A simple way to trace the intermediate steps of algorithms.

```
class Displayable(object):
    """Class that uses 'display'.
    The amount of detail is controlled by max_display_level
    """
    max_display_level = 1 # can be overridden in subclasses or instances

    def display(self,level,*args,**nargs):
        """print the arguments if level is less than or equal to the
        current max_display_level.
        level is an integer.
        the other arguments are whatever arguments print can take.
        """
        if level <= self.max_display_level:
            print(*args, **nargs) ##if error you are using Python2 not
                Python3
```

(Note that `args` gets a tuple of the positional arguments, and `nargs` gets a dictionary of the keyword arguments). This will not work in Python 2, and will give an error.

Any class that wants to use `display` can be made a subclass of `Displayable`.

To change the maximum display level to 3 for a class do:

Classname.`max_display_level` = 3

which will make calls to `display` in that class print when the value of `level` is less-than-or-equal to 3. The default display level is 1. It can also be changed for individual objects (the object value overrides the class value).

The value of `max_display_level` by convention is:

0 display nothing

1 display solutions (nothing that happens repeatedly)

2 also display the values as they change (little detail through a loop)

3 also display more details

4 and above even more detail

1.7.2 Argmax

Python has a built-in `max` function that takes a generator (or a list or set) and returns the maximum value. The `argmax` method returns the index of an element that has the maximum value. If there are multiple elements with the maximum value, one of the indexes to that value is returned at random. `argmaxe` assumes an enumeration; a generator of $(element, value)$ pairs, as for example is generated by the built-in `enumerate`$(list)$ for lists or $dict$`.items()` for dictionaries.

utilities.py — AIPython useful utilities

```
import random
import math

def argmaxall(gen):
    """gen is a generator of (element,value) pairs, where value is a real.
    argmaxall returns a list of all of the elements with maximal value.
    """
    maxv = -math.inf     # negative infinity
    maxvals = []    # list of maximal elements
    for (e,v) in gen:
        if v>maxv:
            maxvals,maxv = [e], v
        elif v==maxv:
            maxvals.append(e)
    return maxvals

def argmaxe(gen):
    """gen is a generator of (element,value) pairs, where value is a real.
    argmaxe returns an element with maximal value.
    If there are multiple elements with the max value, one is returned at
        random.
    """
    return random.choice(argmaxall(gen))

def argmax(lst):
```

```
35      """returns maximum index in a list"""
36      return argmaxe(enumerate(lst))
37  # Try:
38  # argmax([1,6,3,77,3,55,23])
39
40  def argmaxd(dct):
41     """returns the arg max of a dictionary dct"""
42     return argmaxe(dct.items())
43  # Try:
44  # arxmaxd({2:5,5:9,7:7})
```

Exercise 1.3 Change argmaxall to have an optional argument that specifies whether you want the "first", "last" or a "random" index of the maximum value returned. If you want the first or the last, you don't need to keep a list of the maximum elements. Enable the other methods to have this optional argument.

1.7.3 Probability

For many of the simulations, we want to make a variable True with some probability. `flip(p)` returns True with probability `p`, and otherwise returns False.

utilities.py — (continued)

```
45  def flip(prob):
46      """return true with probability prob"""
47      return random.random() < prob
```

The `select_from_dist` method takes in a *item* : *probability* dictionary, and returns one of the items in proportion to its probability. The probabilities should sum to 1 or more. If they sum to more than one, the excess is ignored.

utilities.py — (continued)

```
49  def select_from_dist(item_prob_dist):
50      """ returns a value from a distribution.
51      item_prob_dist is an item:probability dictionary, where the
52          probabilities sum to 1.
53      returns an item chosen in proportion to its probability
54      """
55      ranreal = random.random()
56      for (it,prob) in item_prob_dist.items():
57          if ranreal < prob:
58              return it
59          else:
60              ranreal -= prob
61      raise RuntimeError(f"{item_prob_dist} is not a probability
            distribution")
```

1.8 Testing Code

It is important to test code early and test it often. We include a simple form of **unit test**. The value of the current module is in `__name__` and if the module is run at the top-level, its value is `"__main__"`. See `https://docs.python.org/3/library/__main__.html`.

The following code tests `argmax` and `dict_union`, but only when if `utilities` is loaded in the top-level. If it is loaded in a module the test code is not run.

In your code, you should do more substantial testing than done here. In particular, you should also test boundary cases.

utilities.py — (continued)

```
63 def test():
64     """Test part of utilities"""
65     assert argmax([1,6,55,3,55,23]) in [2,4]
66     print("Passed unit test in utilities")
67     print("run test_aipython() to test (almost) everything")
68 
69 if __name__ == "__main__":
70     test()
```

The following does a simple check of all of AIPython that has automatic checks. If you develop new algorithms or tests, add them here!

utilities.py — (continued)

```
72 def test_aipython():
73     # Agents: currently no tests
74     # Search:
75     print("***** testing Search *****")
76     import searchGeneric, searchBranchAndBound, searchExample, searchTest
77     searchGeneric.test(searchGeneric.AStarSearcher)
78     searchBranchAndBound.test(searchBranchAndBound.DF_branch_and_bound)
79     searchTest.run(searchExample.problem1,"Problem 1")
80     # CSP
81     print("\n***** testing CSP *****")
82     import cspExamples, cspDFS, cspSearch, cspConsistency, cspSLS
83     cspExamples.test_csp(cspDFS.dfs_solve1)
84     cspExamples.test_csp(cspSearch.solver_from_searcher)
85     cspExamples.test_csp(cspConsistency.ac_solver)
86     cspExamples.test_csp(cspConsistency.ac_search_solver)
87     cspExamples.test_csp(cspSLS.sls_solver)
88     cspExamples.test_csp(cspSLS.any_conflict_solver)
89     # Propositions
90     print("\n***** testing Propositional Logic *****")
91     import logicBottomUp, logicTopDown, logicExplain, logicNegation
92     logicBottomUp.test()
93     logicTopDown.test()
94     logicExplain.test()
95     logicNegation.test()
96     # Planning
```

```
    print("\n***** testing Planning *****")
    import stripsHeuristic
    stripsHeuristic.test_forward_heuristic()
    stripsHeuristic.test_regression_heuristic()
    # Learning
    print("\n***** testing Learning *****")
    import learnProblem, learnNoInputs, learnDT, learnLinear
    learnNoInputs.test_no_inputs(training_sizes=[4])
    data = learnProblem.Data_from_file('data/carbool.csv', target_index=-1,
        seed=123)
    learnDT.testDT(data, print_tree=False)
    learnLinear.test()
    # Deep Learning: currently no tests
    # Uncertainty
    print("\n***** testing Uncertainty *****")
    import probGraphicalModels, probRC, probVE, probStochSim
    probGraphicalModels.InferenceMethod.testIM(probRC.ProbSearch)
    probGraphicalModels.InferenceMethod.testIM(probRC.ProbRC)
    probGraphicalModels.InferenceMethod.testIM(probVE.VE)
    probGraphicalModels.InferenceMethod.testIM(probStochSim.RejectionSampling,
        threshold=0.1)
    probGraphicalModels.InferenceMethod.testIM(probStochSim.LikelihoodWeighting,
        threshold=0.1)
    probGraphicalModels.InferenceMethod.testIM(probStochSim.ParticleFiltering,
        threshold=0.1)
    probGraphicalModels.InferenceMethod.testIM(probStochSim.GibbsSampling,
        threshold=0.1)
    # Learning under uncertainty: currently no tests
    # Causality: currently no tests
    # Planning under uncertainty
    print("\n***** testing Planning under Uncertainty *****")
    import decnNetworks
    decnNetworks.test(decnNetworks.fire_dn)
    import mdpExamples
    mdpExamples.test_MDP(mdpExamples.partyMDP)
    # Reinforement Learning:
    print("\n***** testing Reinforcement Learning *****")
    import rlQLearner
    rlQLearner.test_RL(rlQLearner.Q_learner, alpha_fun=lambda k:10/(9+k))
    import rlQExperienceReplay
    rlQLearner.test_RL(rlQExperienceReplay.Q_ER_learner, alpha_fun=lambda
        k:10/(9+k))
    import rlStochasticPolicy
    rlQLearner.test_RL(rlStochasticPolicy.StochasticPIAgent,
        alpha_fun=lambda k:10/(9+k))
    import rlModelLearner
    rlQLearner.test_RL(rlModelLearner.Model_based_reinforcement_learner)
    import rlFeatures
    rlQLearner.test_RL(rlFeatures.SARSA_LFA_learner,
        es_kwargs={'epsilon':1}, eps=4)
```

```
    # Multiagent systems: currently no tests
    # Individuals and Relations
    print("\n***** testing Datalog and Logic Programming *****")
    import relnExamples
    relnExamples.test_ask_all()
    # Knowledge Graphs and Onologies
    print("\n***** testing Knowledge Graphs and Onologies *****")
    import knowledgeGraph
    knowledgeGraph.test_kg()
    # Relational Learning: currently no tests
```

Chapter 2

Agent Architectures and Hierarchical Control

This implements the controllers described in Chapter 2 of Poole and Mackworth [2023].

These provide sequential implementations of the control. More sophisticated version may have them run concurrently (either as coroutines or in parallel).

In this version the higher-levels call the lower-levels. The higher-levels calling the lower-level works in simulated environments when there is a single agent, and where the lower-level are written to make sure they return (and don't go on forever), and the higher level doesn't take too long (as the lower-levels will wait until called again).

2.1 Representing Agents and Environments

In the initial implementation, both agents and the environment are treated as objects in the send of object-oriented programs: they can have an internal state they maintain, and can evaluate methods that can provide answers. This is the same representation used for the reinforcement learning algorithms (Chapter 13).

An **environment** takes in actions of the agents, updates its internal state and returns the next percept, using the method `do`.

An **agent** takes the precept, updates its internal state, and output it next action. An agent implements the method `select_action` that takes percept and returns its next action.

The methods do and select_action are chained together to build a simulator. In order to start this, we need either an action or a percept. There are two variants used:

- An agent implements the initial_action() method which is used initially. This is the method used in the reinforcement learning chapter (page 307).
- The environment implements the initial_percept() method which gives the initial percept. This is the method used in this chapter.

In this implementation, the state of the agent and the state of the environment are represented using standard Python variables, which are updated as the state changes. The percept and the actions are represented as variable-value dictionaries. When agent has only a limited number of actions, the action can be a single value.

In the following code raise NotImplementedError() is a way to specify an abstract method that needs to be overridden in any implemented agent or environment.

agents.py — Agent and Controllers

```
from display import Displayable

class Agent(Displayable):

    def initial_action(self, percept):
        """return the initial action."""
        return self.select_action(percept) # same as select_action

    def select_action(self, percept):
        """return the next action (and update internal state) given percept
        percept is variable:value dictionary
        """
        raise NotImplementedError("go") # abstract method
```

The environment implements a do(action) method where action is a variable-value dictionary. This returns a percept, which is also a variable-value dictionary. The use of dictionaries allows for structured actions and percepts.

Note that Environment is a subclass of Displayable so that it can use the display method described in Section 1.7.1.

agents.py — (continued)

```
class Environment(Displayable):
    def initial_percept(self):
        """returns the initial percept for the agent"""
        raise NotImplementedError("initial_percept") # abstract method

    def do(self, action):
        """does the action in the environment
```

```
32        returns the next percept """
33        raise NotImplementedError("Environment.do") # abstract method
```

The simulator lets the agent and the environment take turns in updating their states and returning the action and the percept.

The first implementation is a simple procedure to carry out `n` steps of the simulation and return the agent state and the environment state at the end.

agents.py — (continued)

```
35 class Simulate(Displayable):
36     """simulate the interaction between the agent and the environment
37     for n time steps.
38     Returns a pair of the agent state and the environment state.
39     """
40     def __init__(self,agent, environment):
41         self.agent = agent
42         self.env = environment
43         self.percept = self.env.initial_percept()
44         self.percept_history = [self.percept]
45         self.action_history = []
46 
47     def go(self, n):
48         for i in range(n):
49             action = self.agent.select_action(self.percept)
50             self.display(2,f"i={i} action={action}")
51             self.percept = self.env.do(action)
52             self.display(2,f"    percept={self.percept}")
```

2.2 Paper buying agent and environment

> To run the demo, in folder "aipython", load "agents.py", using e.g., `ipython -i agentBuying.py`, and copy and paste the commented-out commands at the bottom of that file.

This is an implementation of Example 2.1 of Poole and Mackworth [2023]. You might get different plots to Figures 2.2 and 2.3 as there is randomness in the environment.

2.2.1 The Environment

The environment state is given in terms of the `time` and the amount of paper in `stock`. It also remembers the in-stock history and the price history. The percept consists of the price and the amount of paper in stock. The action of the agent is the number to buy.

Here we assume that the prices are obtained from the `prices` list (which cycles) plus a random integer in range $[0, \texttt{max\_price\_addon})$ plus a linear "in-

flation". The agent cannot access the price model; it just observes the prices and the amount in stock.

agentBuying.py — Paper-buying agent

```
import random
from agents import Agent, Environment, Simulate
from utilities import select_from_dist

class TP_env(Environment):
    price_delta = [0, 0, 0, 21, 0, 20, 0, -64, 0, 0, 23, 0, 0, 0, -35,
        0, 76, 0, -41, 0, 0, 0, 21, 0, 5, 0, 5, 0, 0, 0, 5, 0, -15, 0, 5,
       0, 5, 0, -115, 0, 115, 0, 5, 0, -15, 0, 5, 0, 5, 0, 0, 0, 5, 0,
       -59, 0, 44, 0, 5, 0, 5, 0, 0, 0, 5, 0, -65, 50, 0, 5, 0, 5, 0, 0,
       0, 5, 0]
    sd = 5 # noise standard deviation

    def __init__(self):
        """paper buying agent"""
        self.time=0
        self.stock=20
        self.stock_history = [] # memory of the stock history
        self.price_history = [] # memory of the price history

    def initial_percept(self):
        """return initial percept"""
        self.stock_history.append(self.stock)
        self.price = round(234+self.sd*random.gauss(0,1))
        self.price_history.append(self.price)
        return {'price': self.price,
                'instock': self.stock}

    def do(self, action):
        """does action (buy) and returns percept consisting of price and
            instock"""
        used = select_from_dist({6:0.1, 5:0.1, 4:0.1, 3:0.3, 2:0.2, 1:0.2})
        # used = select_from_dist({7:0.1, 6:0.2, 5:0.2, 4:0.3, 3:0.1,
            2:0.1}) # uses more paper
        bought = action['buy']
        self.stock = self.stock+bought-used
        self.stock_history.append(self.stock)
        self.time += 1
        self.price = round(self.price
                       + self.price_delta[self.time%len(self.price_delta)] #
                           repeating pattern
                       + self.sd*random.gauss(0,1)) # plus randomness
        self.price_history.append(self.price)
        return {'price': self.price,
                'instock': self.stock}
```

2.2.2 The Agent

The agent does not have access to the price model but can only observe the current price and the amount in stock. It has to decide how much to buy.

The belief state of the agent is an estimate of the average price of the paper, and the total amount of money the agent has spent.

agentBuying.py — (continued)

```
53 class TP_agent(Agent):
54     def __init__(self):
55         self.spent = 0
56         percept = env.initial_percept()
57         self.ave = self.last_price = percept['price']
58         self.instock = percept['instock']
59         self.buy_history = []
60 
61     def select_action(self, percept):
62         """return next action to carry out
63         """
64         self.last_price = percept['price']
65         self.ave = self.ave+(self.last_price-self.ave)*0.05
66         self.instock = percept['instock']
67         if self.last_price < 0.9*self.ave and self.instock < 60:
68             tobuy = 48
69         elif self.instock < 12:
70             tobuy = 12
71         else:
72             tobuy = 0
73         self.spent += tobuy*self.last_price
74         self.buy_history.append(tobuy)
75         return {'buy': tobuy}
```

Set up an environment and an agent. Uncomment the last lines to run the agent for 90 steps, and determine the average amount spent.

agentBuying.py — (continued)

```
77 env = TP_env()
78 ag = TP_agent()
79 sim = Simulate(ag,env)
80 #sim.go(90)
81 #ag.spent/env.time ## average spent per time period
```

2.2.3 Plotting

The following plots the price and number in stock history:

agentBuying.py — (continued)

```
83 import matplotlib.pyplot as plt
84 
85 class Plot_history(object):
```

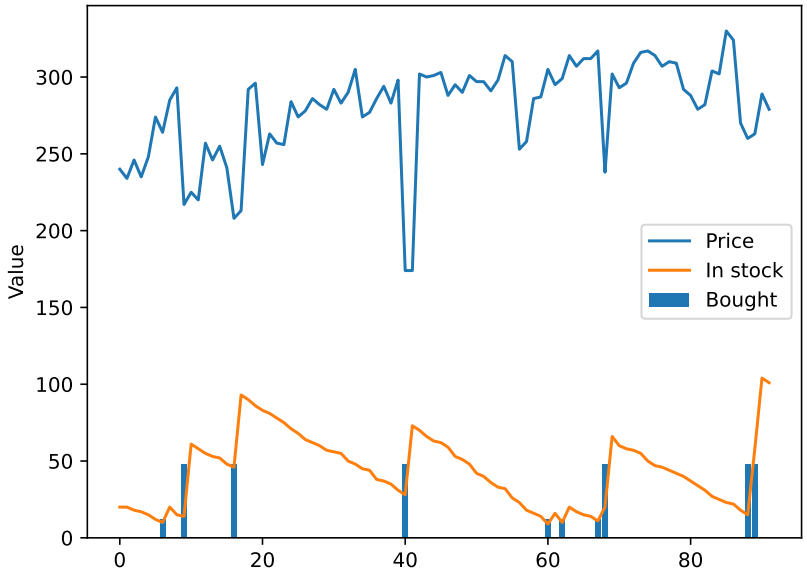


Figure 2.1: Percept and command traces for the paper-buying agent

```
86      """Set up the plot for history of price and number in stock"""
87      def __init__(self, ag, env):
88          self.ag = ag
89          self.env = env
90          plt.ion()
91          plt.xlabel("Time")
92          plt.ylabel("Value")
93
94
95      def plot_env_hist(self):
96          """plot history of price and instock"""
97          num = len(env.stock_history)
98          plt.plot(range(num),env.price_history,label="Price")
99          plt.plot(range(num),env.stock_history,label="In stock")
100         plt.legend()
101         #plt.draw()
102
103     def plot_agent_hist(self):
104         """plot history of buying"""
105         num = len(ag.buy_history)
106         plt.bar(range(1,num+1), ag.buy_history, label="Bought")
107         plt.legend()
108         #plt.draw()
109
110 # sim.go(100); print(f"agent spent ${ag.spent/100}")
111 # pl = Plot_history(ag,env); pl.plot_env_hist(); pl.plot_agent_hist()
```

Figure 2.1 shows the result of the plotting in the previous code.

Exercise 2.1 Design a better controller for a paper-buying agent.

- Justify a performance measure that is a fair comparison. Note that minimizing the total amount of money spent may be unfair to agents who have built up a stockpile, and favors agents that end up with no paper.
- Give a controller that can work for many different price histories. An agent can use other local state variables, but does not have access to the environment model.
- Is it worthwhile trying to infer the amount of paper that the home uses? (Try your controller with the different paper consumption commented out in `TP_env.do`.)

2.3 Hierarchical Controller

To run the hierarchical controller, in folder "aipython", load "agentTop.py", using e.g., `ipython -i agentTop.py`, and copy and paste the commands near the bottom of that file.

In this implementation, each layer, including the top layer, implements the environment class, because each layer is seen as an environment from the layer above.

We arbitrarily divide the environment and the body, so that the environment just defines the walls, and the body includes everything to do with the agent. Note that the named locations are part of the (top-level of the) agent, not part of the environment, although they could have been.

2.3.1 Environment

The environment defines the walls.

agentEnv.py — Agent environment

```
11 import math
12 from display import Displayable
13 from agents import Environment
14
15 class Rob_env(Environment):
16     def __init__(self,walls = {}):
17         """walls is a set of line segments
18               where each line segment is of the form ((x0,y0),(x1,y1))
19         """
20         self.walls = walls
```

2.3.2 Body

The body defines everything about the agent body.

agentEnv.py — (continued)

```
import math
from agents import Environment
import matplotlib.pyplot as plt
import time

class Rob_body(Environment):
    def __init__(self, env, init_pos=(0,0,90)):
        """ env is the current environment
        init_pos is a triple of (x-position, y-position, direction)
            direction is in degrees; 0 is to right, 90 is straight-up, etc
        """
        self.env = env
        self.rob_x, self.rob_y, self.rob_dir = init_pos
        self.turning_angle = 18 # degrees that a left makes
        self.whisker_length = 6 # length of the whisker
        self.whisker_angle = 30 # angle of whisker relative to robot
        self.crashed = False
        # The following control how it is plotted
        self.plotting = True    # whether the trace is being plotted
        self.sleep_time = 0.05  # time between actions (for real-time
            plotting)
        # The following are data structures maintained:
        self.history = [(self.rob_x, self.rob_y)] # history of (x,y)
            positions
        self.wall_history = []  # history of hitting the wall

    def percept(self):
        return {'rob_x_pos':self.rob_x, 'rob_y_pos':self.rob_y,
                'rob_dir':self.rob_dir, 'whisker':self.whisker(),
                    'crashed':self.crashed}
    initial_percept = percept # use percept function for initial percept too

    def do(self,action):
        """ action is {'steer':direction}
        direction is 'left', 'right' or 'straight'
        """
        if self.crashed:
            return self.percept()
        direction = action['steer']
        compass_deriv =
            {'left':1,'straight':0,'right':-1}[direction]*self.turning_angle
        self.rob_dir = (self.rob_dir + compass_deriv +360)%360 # make in
            range [0,360)
        rob_x_new = self.rob_x + math.cos(self.rob_dir*math.pi/180)
        rob_y_new = self.rob_y + math.sin(self.rob_dir*math.pi/180)
        path = ((self.rob_x,self.rob_y),(rob_x_new,rob_y_new))
```

```
63          if any(line_segments_intersect(path,wall) for wall in
                self.env.walls):
64              self.crashed = True
65              if self.plotting:
66                  plt.plot([self.rob_x],[self.rob_y],"r*",markersize=20.0)
67                  plt.draw()
68          self.rob_x, self.rob_y = rob_x_new, rob_y_new
69          self.history.append((self.rob_x, self.rob_y))
70          if self.plotting and not self.crashed:
71              plt.plot([self.rob_x],[self.rob_y],"go")
72              plt.draw()
73              plt.pause(self.sleep_time)
74          return self.percept()
```

The Boolean `whisker` method returns True when the whisker and the wall intersect.

agentEnv.py — (continued)

```
76      def whisker(self):
77          """returns true whenever the whisker sensor intersects with a wall
78          """
79          whisk_ang_world = (self.rob_dir-self.whisker_angle)*math.pi/180
80              # angle in radians in world coordinates
81          wx = self.rob_x + self.whisker_length * math.cos(whisk_ang_world)
82          wy = self.rob_y + self.whisker_length * math.sin(whisk_ang_world)
83          whisker_line = ((self.rob_x,self.rob_y),(wx,wy))
84          hit = any(line_segments_intersect(whisker_line,wall)
85                    for wall in self.env.walls)
86          if hit:
87              self.wall_history.append((self.rob_x, self.rob_y))
88              if self.plotting:
89                  plt.plot([self.rob_x],[self.rob_y],"ro")
90                  plt.draw()
91          return hit
92
93  def line_segments_intersect(linea,lineb):
94      """returns true if the line segments, linea and lineb intersect.
95      A line segment is represented as a pair of points.
96      A point is represented as a (x,y) pair.
97      """
98      ((x0a,y0a),(x1a,y1a)) = linea
99      ((x0b,y0b),(x1b,y1b)) = lineb
100     da, db = x1a-x0a, x1b-x0b
101     ea, eb = y1a-y0a, y1b-y0b
102     denom = db*ea-eb*da
103     if denom==0:  # line segments are parallel
104         return False
105     cb = (da*(y0b-y0a)-ea*(x0b-x0a))/denom # position along line b
106     if cb<0 or cb>1:
107         return False
108     ca = (db*(y0b-y0a)-eb*(x0b-x0a))/denom # position along line a
```

```
    return 0<=ca<=1

# Test cases:
# assert line_segments_intersect(((0,0),(1,1)),((1,0),(0,1)))
# assert not line_segments_intersect(((0,0),(1,1)),((1,0),(0.6,0.4)))
# assert line_segments_intersect(((0,0),(1,1)),((1,0),(0.4,0.6)))
```

2.3.3 Middle Layer

The middle layer acts like both a controller (for the environment layer) and an environment for the upper layer. It has to tell the environment how to steer. Thus it calls $env.do(\cdot)$. It also is told the position to go to and the timeout. Thus it also has to implement $do(\cdot)$.

agentMiddle.py — Middle Layer

```
from agents import Environment
import math

class Rob_middle_layer(Environment):
    def __init__(self,env):
        self.env=env
        self.percept = env.initial_percept()
        self.straight_angle = 11 # angle that is close enough to straight
            ahead
        self.close_threshold = 2 # distance that is close enough to arrived
        self.close_threshold_squared = self.close_threshold**2 # just
            compute it once

    def initial_percept(self):
        return {}

    def do(self, action):
        """action is {'go_to':target_pos,'timeout':timeout}
        target_pos is (x,y) pair
        timeout is the number of steps to try
        returns {'arrived':True} when arrived is true
             or {'arrived':False} if it reached the timeout
        """
        if 'timeout' in action:
            remaining = action['timeout']
        else:
            remaining = -1  # will never reach 0
        target_pos = action['go_to']
        arrived = self.close_enough(target_pos)
        while not arrived and remaining != 0:
            self.percept = self.env.do({"steer":self.steer(target_pos)})
            remaining -= 1
            arrived = self.close_enough(target_pos)
        return {'arrived':arrived}
```

The following method determines how to steer depending on whether the goal is to the right or the left of where the robot is facing.

agentMiddle.py — (continued)

```
44      def steer(self,target_pos):
45          if self.percept['whisker']:
46              self.display(3,'whisker on', self.percept)
47              return "left"
48          else:
49              return self.head_towards(target_pos)
50
51      def head_towards(self,target_pos):
52              """ given a target position, return the action that heads
                    towards that position
53              """
54              gx,gy = target_pos
55              rx,ry = self.percept['rob_x_pos'],self.percept['rob_y_pos']
56              goal_dir = math.acos((gx-rx)/math.sqrt((gx-rx)*(gx-rx)
57                                                     +(gy-ry)*(gy-ry)))*180/math.pi
58              if ry>gy:
59                  goal_dir = -goal_dir
60              goal_from_rob = (goal_dir - self.percept['rob_dir']+540)%360-180
61              assert -180 < goal_from_rob <= 180
62              if goal_from_rob > self.straight_angle:
63                  return "left"
64              elif goal_from_rob < -self.straight_angle:
65                  return "right"
66              else:
67                  return "straight"
68
69      def close_enough(self,target_pos):
70          gx,gy = target_pos
71          rx,ry = self.percept['rob_x_pos'],self.percept['rob_y_pos']
72          return (gx-rx)**2 + (gy-ry)**2 <= self.close_threshold_squared
```

2.3.4 Top Layer

The top layer treats the middle layer as its environment. Note that the top layer is an environment for us to tell it what to visit.

agentTop.py — Top Layer

```
11  from display import Displayable
12  from agentMiddle import Rob_middle_layer
13  from agents import Environment
14
15  class Rob_top_layer(Environment):
16      def __init__(self, middle, timeout=200, locations = {'mail':(-5,10),
17                           'o103':(50,10), 'o109':(100,10),'storage':(101,51)}
                                 ):
18          """middle is the middle layer
```

```
19          timeout is the number of steps the middle layer goes before giving
                up
20          locations is a loc:pos dictionary
21              where loc is a named location, and pos is an (x,y) position.
22          """
23          self.middle = middle
24          self.timeout = timeout # number of steps before the middle layer
                should give up
25          self.locations = locations
26
27      def do(self,plan):
28          """carry out actions.
29          actions is of the form {'visit':list_of_locations}
30          It visits the locations in turn.
31          """
32          to_do = plan['visit']
33          for loc in to_do:
34              position = self.locations[loc]
35              arrived = self.middle.do({'go_to':position,
                    'timeout':self.timeout})
36              self.display(1,"Arrived at",loc,arrived)
```

2.3.5 Plotting

The following is used to plot the locations, the walls and (eventually) the movement of the robot. It can either plot the movement if the robot as it is going (with the default *env.plotting* = *True*), or not plot it as it is going (setting *env.plotting* = *False*; in this case the trace can be plotted using *pl.plot_run*()).

agentTop.py — (continued)

```
38  import matplotlib.pyplot as plt
39
40  class Plot_env(Displayable):
41      def __init__(self, body,top):
42          """sets up the plot
43          """
44          self.body = body
45          self.top = top
46          plt.ion()
47          plt.axes().set_aspect('equal')
48          self.redraw()
49
50      def redraw(self):
51          plt.clf()
52          for wall in body.env.walls:
53              ((x0,y0),(x1,y1)) = wall
54              plt.plot([x0,x1],[y0,y1],"-k",linewidth=3)
55          for loc in top.locations:
56              (x,y) = top.locations[loc]
```

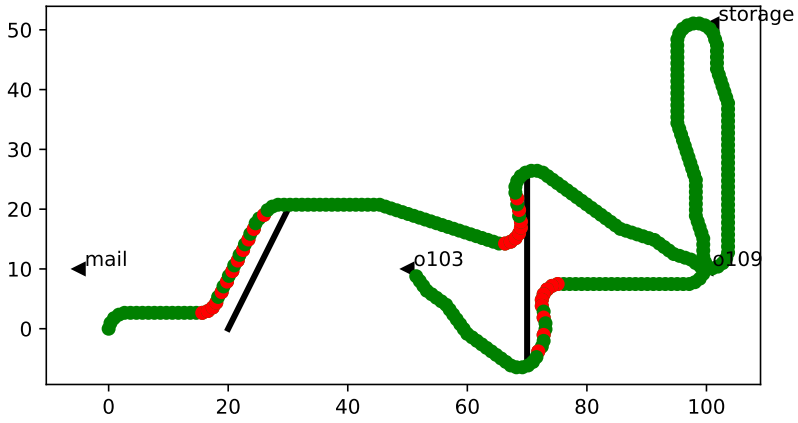


Figure 2.2: A trace of the trajectory of the agent. Red dots correspond to the whisker sensor being on; the green dot to the whisker sensor being off. The agent starts at position $(0,0)$ facing up.

```
            plt.plot([x],[y],"k<")
            plt.text(x+1.0,y+0.5,loc) # print the label above and to the
                right
        plt.plot([body.rob_x],[body.rob_y],"go")
        plt.gca().figure.canvas.draw()
        if self.body.history or self.body.wall_history:
            self.plot_run()

    def plot_run(self):
        """plots the history after the agent has finished.
        This is typically only used if body.plotting==False
        """
        if self.body.history:
            xs,ys = zip(*self.body.history)
            plt.plot(xs,ys,"go")
        if self.body.wall_history:
            wxs,wys = zip(*self.body.wall_history)
            plt.plot(wxs,wys,"ro")
```

The following code plots the agent as it acts in the world. Figure 2.2 shows the result of the `top.do`

agentTop.py — (continued)

```
from agentEnv import Rob_body, Rob_env

env = Rob_env({((20,0),(30,20)), ((70,-5),(70,25))})
body = Rob_body(env)
middle = Rob_middle_layer(body)
top = Rob_top_layer(middle)

# try:
```

Figure 2.3: Robot trap

```
# pl=Plot_env(body,top)
# top.do({'visit':['o109','storage','o109','o103']})
# You can directly control the middle layer:
# middle.do({'go_to':(30,-10), 'timeout':200})
# Can you make it crash?
```

Exercise 2.2 The following code implements a robot trap (Figure 2.3). Write a controller that can escape the "trap" and get to the goal. See Exercise 2.4 in the textbook for hints.

agentTop.py — (continued)

```
# Robot Trap for which the current controller cannot escape:
trap_env = Rob_env({((10,-21),(10,0)), ((10,10),(10,31)),
    ((30,-10),(30,0)),
                   ((30,10),(30,20)), ((50,-21),(50,31)),
                       ((10,-21),(50,-21)),
                   ((10,0),(30,0)), ((10,10),(30,10)), ((10,31),(50,31))})
trap_body = Rob_body(trap_env,init_pos=(-1,0,90))
trap_middle = Rob_middle_layer(trap_body)
trap_top = Rob_top_layer(trap_middle,locations={'goal':(71,0)})

# Robot trap exercise:
# pl=Plot_env(trap_body,trap_top)
# trap_top.do({'visit':['goal']})
```

Plotting for Moving Targets

Exercise 2.5 refers to targets that can move. The following implements targets than can be moved by the user (using the mouse).

agentFollowTarget.py — Plotting for moving targets

```
import matplotlib.pyplot as plt
from agentTop import Plot_env, body, top
```

```
13
14 class Plot_follow(Plot_env):
15     def __init__(self, body, top, epsilon=2.5):
16         """plot the agent in the environment.
17         epsilon is the threshold how how close someone needs to click to
               select a location.
18         """
19         Plot_env.__init__(self, body, top)
20         self.epsilon = epsilon
21         self.canvas = plt.gca().figure.canvas
22         self.canvas.mpl_connect('button_press_event', self.on_press)
23         self.canvas.mpl_connect('button_release_event', self.on_release)
24         self.canvas.mpl_connect('motion_notify_event', self.on_move)
25         self.pressloc = None
26         self.pressevent = None
27         for loc in self.top.locations:
28             self.display(2,f"  loc {loc} at {self.top.locations[loc]}")
29
30     def on_press(self, event):
31         self.display(2,'v',end="")
32         self.display(2,f"Press at ({event.xdata},{event.ydata}")
33         for loc in self.top.locations:
34             lx,ly = self.top.locations[loc]
35             if abs(event.xdata- lx) <= self.epsilon and abs(event.ydata-
                   ly) <= self.epsilon :
36                 self.pressloc = loc
37                 self.pressevent = event
38                 self.display(2,"moving",loc)
39
40     def on_release(self, event):
41         self.display(2,'^',end="")
42         if self.pressloc is not None: #and event.inaxes ==
                self.pressevent.inaxes:
43             self.top.locations[self.pressloc] = (event.xdata, event.ydata)
44             self.display(1,f"Placing {self.pressloc} at {(event.xdata,
                   event.ydata)}")
45         self.pressloc = None
46         self.pressevent = None
47
48     def on_move(self, event):
49         if self.pressloc is not None: # and event.inaxes ==
                self.pressevent.inaxes:
50             self.display(2,'-',end="")
51             self.top.locations[self.pressloc] = (event.xdata, event.ydata)
52             self.redraw()
53         else:
54             self.display(2,'.',end="")
55
56 # try:
57 # pl=Plot_follow(body,top)
```

```
58 # top.do({'visit':['o109','storage','o109','o103']})
```

Exercise 2.3 Change the code to also allow walls to move.

Chapter 3

Searching for Solutions

3.1 Representing Search Problems

A search problem consists of:

- a start node
- a *neighbors* function that given a node, returns an enumeration of the arcs from the node
- a specification of a goal in terms of a Boolean function that takes a node and returns true if the node is a goal
- a (optional) heuristic function that, given a node, returns a non-negative real number. The heuristic function defaults to zero.

As far as the searcher is concerned a node can be anything. If multiple-path pruning is used, a node must be `hashable`. In the simple examples, it is a string, but in more complicated examples (in later chapters) it can be a tuple, a frozen set, or a Python object.

In the following code, "`raise NotImplementedError()`" is a way to specify that this is an abstract method that needs to be overridden to define an actual search problem.

searchProblem.py — representations of search problems

```
11 from display import Displayable
12 import matplotlib.pyplot as plt
13 import random
14
15 class Search_problem(Displayable):
16     """A search problem consists of:
```

```
    * a start node
    * a neighbors function that gives the neighbors of a node
    * a specification of a goal
    * a (optional) heuristic function.
    The methods must be overridden to define a search problem."""

    def start_node(self):
        """returns start node"""
        raise NotImplementedError("start_node") # abstract method

    def is_goal(self,node):
        """is True if node is a goal"""
        raise NotImplementedError("is_goal") # abstract method

    def neighbors(self,node):
        """returns a list (or enumeration) of the arcs for the neighbors of
            node"""
        raise NotImplementedError("neighbors") # abstract method

    def heuristic(self,n):
        """Gives the heuristic value of node n.
        Returns 0 if not overridden."""
        return 0
```

The neighbors is a list of arcs. A (directed) arc consists of a `from_node` node and a `to\_node` node. The arc is the pair `(from_node,to_node)`, but can also contain a non-negative `cost` (which defaults to 1) and can be labeled with an `action`.

searchProblem.py — (continued)

```
class Arc(object):
    """An arc has a from_node and a to_node node and a (non-negative)
        cost"""
    def __init__(self, from_node, to_node, cost=1, action=None):
        self.from_node = from_node
        self.to_node = to_node
        self.action = action
        self.cost = cost
        assert cost >= 0, (f"Cost cannot be negative: {self}, cost={cost}")

    def __repr__(self):
        """string representation of an arc"""
        if self.action:
            return f"{self.from_node} --{self.action}--> {self.to_node}"
        else:
            return f"{self.from_node} --> {self.to_node}"
```

3.1.1 Explicit Representation of Search Graph

The first representation of a search problem is from an explicit graph (as opposed to one that is generated as needed).

An **explicit graph** consists of

- a list or set of nodes
- a list or set of arcs
- a start node
- a list or set of goal nodes
- (optionally) a dictionary that maps a node to a heuristic value for that node

To define a search problem, we need to define the start node, the goal predicate, the neighbors function and the heuristic function.

searchProblem.py — (continued)

```
56 class Search_problem_from_explicit_graph(Search_problem):
57     """A search problem from an explicit graph.
58     """
59
60     def __init__(self, title, nodes, arcs, start=None, goals=set(), hmap={},
61                  positions=None, show_costs = True):
62         """ A search problem consists of:
63         * list or set of nodes
64         * list or set of arcs
65         * start node
66         * list or set of goal nodes
67         * hmap: dictionary that maps each node into its heuristic value.
68         * positions: dictionary that maps each node into its (x,y) position
69         * show_costs is used for show()
70         """
71         self.title = title
72         self.neighs = {}
73         self.nodes = nodes
74         for node in nodes:
75             self.neighs[node]=[]
76         self.arcs = arcs
77         for arc in arcs:
78             self.neighs[arc.from_node].append(arc)
79         self.start = start
80         self.goals = goals
81         self.hmap = hmap
82         if positions is None:
83             self.positions = {node:(random.random(),random.random()) for
                   node in nodes}
84         else:
```

```
85              self.positions = positions
86          self.show_costs = show_costs
87
88
89      def start_node(self):
90          """returns start node"""
91          return self.start
92
93      def is_goal(self,node):
94          """is True if node is a goal"""
95          return node in self.goals
96
97      def neighbors(self,node):
98          """returns the neighbors of node (a list of arcs)"""
99          return self.neighs[node]
100
101     def heuristic(self,node):
102         """Gives the heuristic value of node n.
103         Returns 0 if not overridden in the hmap."""
104         if node in self.hmap:
105             return self.hmap[node]
106         else:
107             return 0
108
109     def __repr__(self):
110         """returns a string representation of the search problem"""
111         res=""
112         for arc in self.arcs:
113             res += f"{arc}. "
114         return res
```

Graphical Display of a Search Graph

searchProblem.py — (continued)

```
116     def show(self, fontsize=10, node_color='orange', show_costs = None):
117         """Show the graph as a figure
118         """
119         self.fontsize = fontsize
120         if show_costs is not None: # override default definition
121             self.show_costs = show_costs
122         plt.ion()  # interactive
123         ax = plt.figure().gca()
124         ax.set_axis_off()
125         plt.title(self.title, fontsize=fontsize)
126         self.show_graph(ax, node_color)
127
128     def show_graph(self, ax, node_color='orange'):
129         bbox =
                dict(boxstyle="round4,pad=1.0,rounding_size=0.5",facecolor=node_color)
```

```
        for arc in self.arcs:
            self.show_arc(ax, arc)
        for node in self.nodes:
            self.show_node(ax, node, node_color = node_color)

    def show_node(self, ax, node, node_color):
            x,y = self.positions[node]
            ax.text(x,y,node,bbox=dict(boxstyle="round4,pad=1.0,rounding_size=0.5",
                                       facecolor=node_color),
                        ha='center',va='center',
                    fontsize=self.fontsize)

    def show_arc(self, ax, arc, arc_color='black', node_color='white'):
            from_pos = self.positions[arc.from_node]
            to_pos = self.positions[arc.to_node]
            ax.annotate(arc.to_node, from_pos, xytext=to_pos,
                                # arrowprops=dict(facecolor='black',
                                    shrink=0.1, width=2),
                                arrowprops={'arrowstyle':'<|-', 'linewidth':
                                    2, 'color':arc_color},
                                bbox=dict(boxstyle="round4,pad=1.0,rounding_size=0.5",
                                          facecolor=node_color),
                                ha='center',va='center',
                                fontsize=self.fontsize)
            # Add costs to middle of arcs:
            if self.show_costs:
                ax.text((from_pos[0]+to_pos[0])/2, (from_pos[1]+to_pos[1])/2,
                        arc.cost, bbox=dict(pad=1,fc='w',ec='w'),
                        ha='center',va='center',fontsize=self.fontsize)
```

3.1.2 Paths

A searcher will return a path from the start node to a goal node. A Python list is not a suitable representation for a path, as many search algorithms consider multiple paths at once, and these paths should share initial parts of the path. If we wanted to do this with Python lists, we would need to keep copying the list, which can be expensive if the list is long. An alternative representation is used here in terms of a recursive data structure that can share subparts.

A path is either:

- a node (representing a path of length 0) or
- a path, `initial` and an arc, where the `from_node` of the arc is the node at the end of `initial`.

These cases are distinguished in the following code by having `arc=None` if the path has length 0, in which case `initial` is the node of the path. Note that we only use the most basic form of Python's `yield` for enumerations (Section 1.5.4).

searchProblem.py — (continued)

```
157 class Path(object):
158     """A path is either a node or a path followed by an arc"""
159 
160     def __init__(self,initial,arc=None):
161         """initial is either a node (in which case arc is None) or
162         a path (in which case arc is an object of type Arc)"""
163         self.initial = initial
164         self.arc=arc
165         if arc is None:
166             self.cost=0
167         else:
168             self.cost = initial.cost+arc.cost
169 
170     def end(self):
171         """returns the node at the end of the path"""
172         if self.arc is None:
173             return self.initial
174         else:
175             return self.arc.to_node
176 
177     def nodes(self):
178         """enumerates the nodes for the path.
179         This enumerates the nodes in the path from the last elements
                backwards.
180         """
181         current = self
182         while current.arc is not None:
183             yield current.arc.to_node
184             current = current.initial
185         yield current.initial
186 
187     def initial_nodes(self):
188         """enumerates the nodes for the path before the end node.
189         This calls nodes() for the initial part of the path.
190         """
191         if self.arc is not None:
192             yield from self.initial.nodes()
193 
194     def __repr__(self):
195         """returns a string representation of a path"""
196         if self.arc is None:
197             return str(self.initial)
198         elif self.arc.action:
199             return f"{self.initial}\n --{self.arc.action}-->
                    {self.arc.to_node}"
200         else:
201             return f"{self.initial} --> {self.arc.to_node}"
```

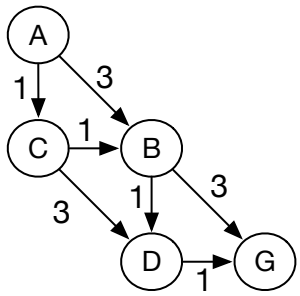


Figure 3.1: problem1

3.1.3 Example Search Problems

The first search problem is one with 5 nodes where the least-cost path is one with many arcs. See Figure 3.1. Note that this example is used for the unit tests, so the test (in `searchGeneric`) will need to be changed if this is changed.

searchExample.py — Search Examples

```
from searchProblem import Arc, Search_problem_from_explicit_graph,
    Search_problem

problem1 = Search_problem_from_explicit_graph('Problem 1',
    {'A','B','C','D','G'},
    [Arc('A','B',3), Arc('A','C',1), Arc('B','D',1), Arc('B','G',3),
        Arc('C','B',1), Arc('C','D',3), Arc('D','G',1)],
    start = 'A',
    goals = {'G'},
    positions={'A': (0, 1), 'B': (0.5, 0.5), 'C': (0,0.5), 'D': (0.5,0),
        'G': (1,0)})
```

The second search problem is one with 8 nodes where many paths do not lead to the goal. See Figure 3.2.

searchExample.py — (continued)

```
problem2 = Search_problem_from_explicit_graph('Problem 2',
    {'A','B','C','D','E','G','H','J'},
    [Arc('A','B',1), Arc('B','C',3), Arc('B','D',1), Arc('D','E',3),
       Arc('D','G',1), Arc('A','H',3), Arc('H','J',1)],
    start = 'A',
    goals = {'G'},
    positions={'A': (0, 1), 'B': (0, 3/4), 'C': (0,0), 'D': (1/4,3/4), 'E':
        (1/4,0),
                'G': (2/4,3/4), 'H': (3/4,1), 'J': (3/4,3/4)})
```

The third search problem is a disconnected graph (contains no arcs), where the start node is a goal node. This is a boundary case to make sure that weird cases work.

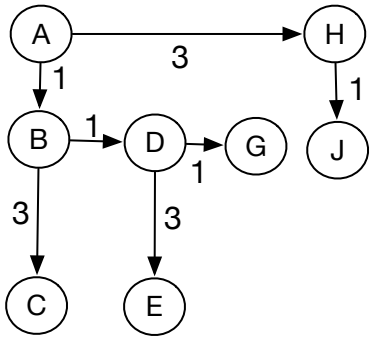


Figure 3.2: problem2

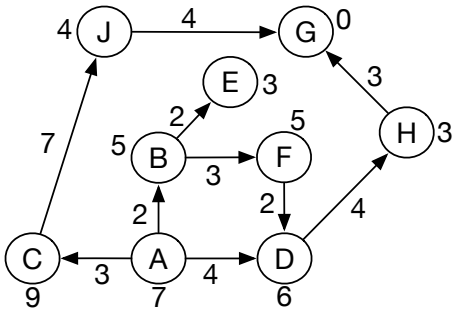


Figure 3.3: simp_delivery_graph with arc costs and h values of nodes

searchExample.py — (continued)

```
30 problem3 = Search_problem_from_explicit_graph('Problem 3',
31     {'a','b','c','d','e','g','h','j'},
32     [],
33     start = 'g',
34     goals = {'k','g'})
```

The simp_delivery_graph is the graph shown Figure 3.3. This is Figure 3.3 with the heuristics of Figure 3.1 as shown in Figure 3.13 of Poole and Mackworth [2023],

searchExample.py — (continued)

```
36 simp_delivery_graph = Search_problem_from_explicit_graph("Acyclic Delivery
       Graph",
37     {'A', 'B', 'C', 'D', 'E', 'F', 'G', 'H', 'J'},
38     [   Arc('A', 'B', 2),
39         Arc('A', 'C', 3),
40         Arc('A', 'D', 4),
41         Arc('B', 'E', 2),
```

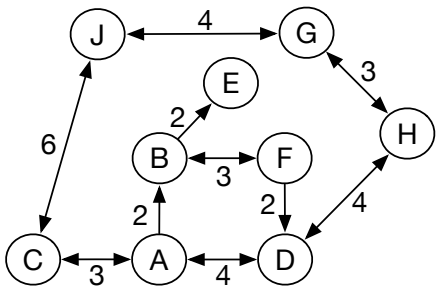


Figure 3.4: cyclic_simp_delivery_graph with arc costs

```
        Arc('B', 'F', 3),
        Arc('C', 'J', 7),
        Arc('D', 'H', 4),
        Arc('F', 'D', 2),
        Arc('H', 'G', 3),
        Arc('J', 'G', 4)],
    start = 'A',
    goals = {'G'},
    hmap = {
        'A': 7,
        'B': 5,
        'C': 9,
        'D': 6,
        'E': 3,
        'F': 5,
        'G': 0,
        'H': 3,
        'J': 4,
     },
     positions = {
        'A': (0.4,0.1),
        'B': (0.4,0.4),
        'C': (0.1,0.1),
        'D': (0.7,0.1),
        'E': (0.6,0.7),
        'F': (0.7,0.4),
        'G': (0.7,0.9),
        'H': (0.9,0.6),
        'J': (0.3,0.9)
        }
    )
```

cyclic_simp_delivery_graph is the graph shown Figure 3.4. This is the graph of Figure 3.10 of [Poole and Mackworth, 2023]. The heuristic values are the same as in simp_delivery_graph.

searchExample.py — (continued)

```
cyclic_simp_delivery_graph = Search_problem_from_explicit_graph("Cyclic
    Delivery Graph",
    {'A', 'B', 'C', 'D', 'E', 'F', 'G', 'H', 'J'},
    [   Arc('A', 'B', 2),
        Arc('A', 'C', 3),
        Arc('A', 'D', 4),
        Arc('B', 'E', 2),
        Arc('B', 'F', 3),
        Arc('C', 'A', 3),
        Arc('C', 'J', 6),
        Arc('D', 'A', 4),
        Arc('D', 'H', 4),
        Arc('F', 'B', 3),
        Arc('F', 'D', 2),
        Arc('G', 'H', 3),
        Arc('G', 'J', 4),
        Arc('H', 'D', 4),
        Arc('H', 'G', 3),
        Arc('J', 'C', 6),
        Arc('J', 'G', 4)],
   start = 'A',
   goals = {'G'},
   hmap = {
        'A': 7,
        'B': 5,
        'C': 9,
        'D': 6,
        'E': 3,
        'F': 5,
        'G': 0,
        'H': 3,
        'J': 4,
    },
    positions = {
        'A': (0.4,0.1),
        'B': (0.4,0.4),
        'C': (0.1,0.1),
        'D': (0.7,0.1),
        'E': (0.6,0.7),
        'F': (0.7,0.4),
        'G': (0.7,0.9),
        'H': (0.9,0.6),
        'J': (0.3,0.9)
        })
```

The next problem is the tree graph shown in Figure 3.6, and is Figure 3.15 in Poole and Mackworth [2023].

searchExample.py — (continued)

```
tree_graph = Search_problem_from_explicit_graph("Tree Graph",
```

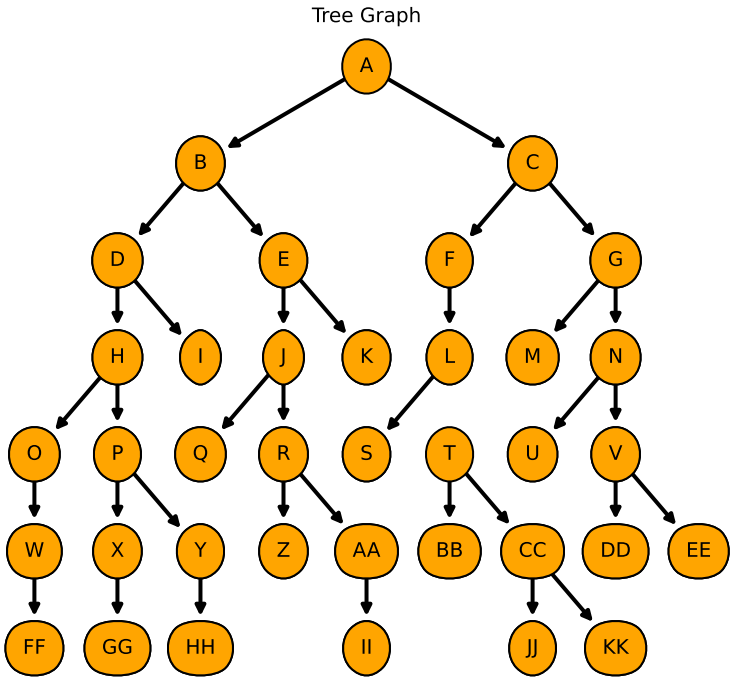


Figure 3.5: `tree_graph.show(show_costs = False)`

```
118     {'A', 'B', 'C', 'D', 'E', 'F', 'G', 'H', 'I', 'J', 'K', 'L', 'M', 'N',
            'O',
119          'P', 'Q', 'R', 'S', 'T', 'U', 'V', 'W', 'X', 'Y', 'Z', 'AA', 'BB',
                 'CC',
120          'DD', 'EE', 'FF', 'GG', 'HH', 'II', 'JJ', 'KK'},
121     [    Arc('A', 'B', 1),
122          Arc('A', 'C', 1),
123          Arc('B', 'D', 1),
124          Arc('B', 'E', 1),
125          Arc('C', 'F', 1),
126          Arc('C', 'G', 1),
127          Arc('D', 'H', 1),
128          Arc('D', 'I', 1),
129          Arc('E', 'J', 1),
130          Arc('E', 'K', 1),
131          Arc('F', 'L', 1),
132          Arc('G', 'M', 1),
133          Arc('G', 'N', 1),
```

```
        Arc('H', 'O', 1),
        Arc('H', 'P', 1),
        Arc('J', 'Q', 1),
        Arc('J', 'R', 1),
        Arc('L', 'S', 1),
        Arc('L', 'T', 1),
        Arc('N', 'U', 1),
        Arc('N', 'V', 1),
        Arc('O', 'W', 1),
        Arc('P', 'X', 1),
        Arc('P', 'Y', 1),
        Arc('R', 'Z', 1),
        Arc('R', 'AA', 1),
        Arc('T', 'BB', 1),
        Arc('T', 'CC', 1),
        Arc('V', 'DD', 1),
        Arc('V', 'EE', 1),
        Arc('W', 'FF', 1),
        Arc('X', 'GG', 1),
        Arc('Y', 'HH', 1),
        Arc('AA', 'II', 1),
        Arc('CC', 'JJ', 1),
        Arc('CC', 'KK', 1)
    ],
    start = 'A',
    goals = {'K', 'M', 'T', 'X', 'Z', 'HH'},
     positions = {
        'A': (0.5,0.95),
        'B': (0.3,0.8),
        'C': (0.7,0.8),
        'D': (0.2,0.65),
        'E': (0.4,0.65),
        'F': (0.6,0.65),
        'G': (0.8,0.65),
        'H': (0.2,0.5),
        'I': (0.3,0.5),
        'J': (0.4,0.5),
        'K': (0.5,0.5),
        'L': (0.6,0.5),
        'M': (0.7,0.5),
        'N': (0.8,0.5),
        'O': (0.1,0.35),
        'P': (0.2,0.35),
        'Q': (0.3,0.35),
        'R': (0.4,0.35),
        'S': (0.5,0.35),
        'T': (0.6,0.35),
        'U': (0.7,0.35),
        'V': (0.8,0.35),
        'W': (0.1,0.2),
```

```
        'X': (0.2,0.2),
        'Y': (0.3,0.2),
        'Z': (0.4,0.2),
        'AA': (0.5,0.2),
        'BB': (0.6,0.2),
        'CC': (0.7,0.2),
        'DD': (0.8,0.2),
        'EE': (0.9,0.2),
        'FF': (0.1,0.05),
        'GG': (0.2,0.05),
        'HH': (0.3,0.05),
        'II': (0.5,0.05),
        'JJ': (0.7,0.05),
        'KK': (0.8,0.05)
        },
        show_costs = False
    )

# tree_graph.show(show_costs = False)
```

3.2 Generic Searcher and Variants

> To run the search demos, in folder "aipython", load "searchGeneric.py" , using e.g., ipython -i searchGeneric.py, and copy and paste the example queries at the bottom of that file.

3.2.1 Searcher

A *Searcher* for a problem can be asked repeatedly for the next path. To solve a problem, you can construct a *Searcher* object for the problem and then repeatedly ask for the next path using *search*. If there are no more paths, *None* is returned.

searchGeneric.py — Generic Searcher, including depth-first and A*

```
from display import Displayable

class Searcher(Displayable):
    """returns a searcher for a problem.
    Paths can be found by repeatedly calling search().
    This does depth-first search unless overridden
    """
    def __init__(self, problem):
        """creates a searcher from a problem
        """
        self.problem = problem
        self.initialize_frontier()
        self.num_expanded = 0
```

```
24          self.add_to_frontier(Path(problem.start_node()))
25          super().__init__()
26
27      def initialize_frontier(self):
28          self.frontier = []
29
30      def empty_frontier(self):
31          return self.frontier == []
32
33      def add_to_frontier(self,path):
34          self.frontier.append(path)
35
36      def search(self):
37          """returns (next) path from the problem's start node
38          to a goal node.
39          Returns None if no path exists.
40          """
41          while not self.empty_frontier():
42              self.path = self.frontier.pop()
43              self.num_expanded += 1
44              if self.problem.is_goal(self.path.end()): # solution found
45                  self.solution = self.path # store the solution found
46                  self.display(1, f"Solution: {self.path} (cost:
                        {self.path.cost})\n",
47                      self.num_expanded, "paths have been expanded and",
48                             len(self.frontier), "paths remain in the
                                frontier")
49                  return self.path
50              else:
51                  self.display(4,f"Expanding: {self.path} (cost:
                        {self.path.cost})")
52                  neighs = self.problem.neighbors(self.path.end())
53                  self.display(2,f"Expanding: {self.path} with neighbors
                        {neighs}")
54                  for arc in reversed(list(neighs)):
55                      self.add_to_frontier(Path(self.path,arc))
56                  self.display(3, f"New frontier: {[p.end() for p in
                        self.frontier]}")
57
58          self.display(0,"No (more) solutions. Total of",
59                       self.num_expanded,"paths expanded.")
```

Note that this reverses the neighbors so that it implements depth-first search in an intuitive manner (expanding the first neighbor first). The call to *list* is for the case when the neighbors are generated (and not already in a list). Reversing the neighbors might not be required for other methods. The calls to *reversed* and *list* can be removed, and the algorithm still implements depth-first search.

To use depth-first search to find multiple paths for `problem1` and `simp_delivery_graph`, copy and paste the following into Python's read-evaluate-print loop; keep finding next solutions until there are no more:

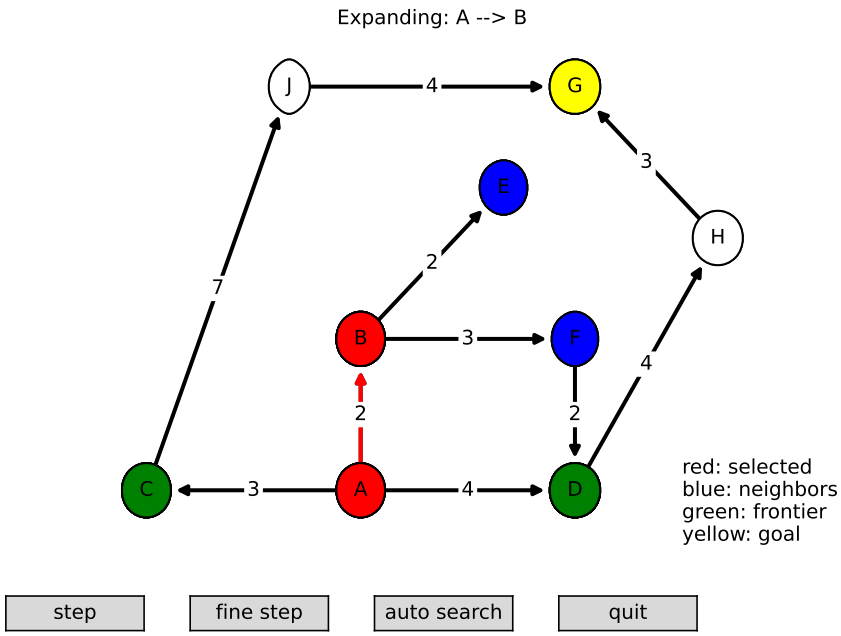


Figure 3.6: `SearcherGUI(Searcher, simp_delivery_graph).go()`

searchGeneric.py — (continued)

```
61 # Depth-first search for problem1; do the following:
62 # searcher1 = Searcher(searchExample.problem1)
63 # searcher1.search() # find first solution
64 # searcher1.search() # find next solution (repeat until no solutions)
65 # searcher_sdg = Searcher(searchExample.simp_delivery_graph)
66 # searcher_sdg.search() # find first or next solution
```

Exercise 3.1 Implement breadth-first search. Only *add_to_frontier* and/or *pop* need to be modified to implement a first-in first-out queue.

3.2.2 GUI for Tracing Search

This GUI implements most of the functionality of the AISpace.org search app.

Figure 3.6 shows the GUI to step through various algorithms. Here the path $A \rightarrow B$ is being expanded, and the neighbors are E and F. The other nodes at the end of paths of the frontier are C and D. Thus the frontier contains paths to C and D, used to also contain $A \rightarrow B$, and now will contain $A \rightarrow B \rightarrow E$ and $A \rightarrow B \rightarrow F$.

`SearcherGUI` takes a search class and a problem, and lets one explore the search space after calling `go()`. A GUI can only be used for one search; at the end of the search the loop ends and the buttons no longer work.

This is implemented by redefining `display`. The search algorithms don't need to be modified. If you modify them (or create your own), you just have to be careful to use the appropriate number for the display. The first argument to display has the following meanings:

1. a solution has been found
2. what is shown for a "step" on a GUI; here it is assumed to be the path, the neighbors of the end of the path, and the other nodes at the end of paths on the frontier
3. (shown with "fine step" but not with "step") the frontier and the path selected
4. (shown with "fine step" but not with "step") the frontier.

It is also useful to look at the Python console, as the display information is printed there.

searchGUI.py — GUI for search

```
import matplotlib.pyplot as plt
from matplotlib.widgets import Button
import time

class SearcherGUI(object):
    def __init__(self, SearchClass, problem, fontsize=10,
                 colors = {'selected':'red', 'neighbors':'blue',
                     'frontier':'green', 'goal':'yellow'}):
        self.problem = problem
        self.searcher = SearchClass(problem)
        self.problem.fontsize = fontsize
        self.colors = colors
        #self.go()

    def go(self):
        fig, self.ax = plt.subplots()
        plt.ion()  # interactive
        self.ax.set_axis_off()
        plt.subplots_adjust(bottom=0.15)
        step_butt = Button(plt.axes([0.05,0.02,0.15,0.05]), "step")
        step_butt.on_clicked(self.step)
        fine_butt = Button(plt.axes([0.25,0.02,0.15,0.05]), "fine step")
        fine_butt.on_clicked(self.finestep)
        auto_butt = Button(plt.axes([0.45,0.02,0.15,0.05]), "auto search")
        auto_butt.on_clicked(self.auto)
        quit_butt = Button(plt.axes([0.65,0.02,0.15,0.05]), "quit")
        quit_butt.on_clicked(self.quit)
        self.ax.text(0.85,0, '\n'.join(self.colors[a]+": "+a for a in
            self.colors))
        self.problem.show_graph(self.ax, node_color='white')
```

```
39          self.problem.show_node(self.ax, self.problem.start,
                self.colors['frontier'])
40          for node in self.problem.nodes:
41              if self.problem.is_goal(node):
42                  self.problem.show_node(self.ax, node,self.colors['goal'])
43          plt.show()
44          self.click = 7 # bigger than any display!
45          #while self.click == 0:
46          #    plt.pause(0.1)
47          self.searcher.display = self.display
48          try:
49              while self.searcher.frontier:
50                  path = self.searcher.search()
51          except ExitToPython:
52              print("Exited")
53          else:
54              print("No more solutions")
55
56      def display(self, level,*args,**nargs):
57          if level <= self.click: #step
58              print(*args, **nargs)
59              self.ax.set_title(f"Expanding:
                    {self.searcher.path}",fontsize=self.problem.fontsize)
60              if level == 1:
61                  self.show_frontier(self.colors['frontier'])
62                  self.show_path(self.colors['selected'])
63                  self.ax.set_title(f"Solution Found:
                        {self.searcher.path}",fontsize=self.problem.fontsize)
64              elif level == 2: # what should be shown if a node is in all
                    three?
65                  self.show_frontier(self.colors['frontier'])
66                  self.show_path(self.colors['selected'])
67                  self.show_neighbors(self.colors['neighbors'])
68              elif level == 3:
69                  self.show_frontier(self.colors['frontier'])
70                  self.show_path(self.colors['selected'])
71              elif level == 4:
72                  self.show_frontier(self.colors['frontier'])
73
74
75              # wait for a button click
76              self.click = 0
77              plt.draw()
78              while self.click == 0:
79                  plt.pause(0.1)
80              # undo coloring:
81              self.ax.set_title("")
82              self.show_frontier('white')
83              self.show_neighbors('white')
84              path_show = self.searcher.path
```

```
85              while path_show.arc:
86                  self.problem.show_arc(self.ax, path_show.arc, 'black')
87                  self.problem.show_node(self.ax, path_show.end(), 'white')
88                  path_show = path_show.initial
89              self.problem.show_node(self.ax, path_show.end(), 'white')
90              if self.problem.is_goal(self.searcher.path.end()):
91                  self.problem.show_node(self.ax, self.searcher.path.end(),
                        self.colors['goal'])
92              plt.draw()
93
94      def show_frontier(self, color):
95          for path in self.searcher.frontier:
96              self.problem.show_node(self.ax, path.end(), color)
97
98      def show_path(self, color):
99          """color selected path"""
100         path_show = self.searcher.path
101         while path_show.arc:
102                 self.problem.show_arc(self.ax, path_show.arc, color)
103                 self.problem.show_node(self.ax, path_show.end(), color)
104                 path_show = path_show.initial
105         self.problem.show_node(self.ax, path_show.end(), color)
106
107     def show_neighbors(self, color):
108         for neigh in self.problem.neighbors(self.searcher.path.end()):
109             self.problem.show_node(self.ax, neigh.to_node, color)
110
111     def auto(self,event):
112         self.click = 1
113     def step(self,event):
114         self.click = 2
115     def finestep(self,event):
116         self.click = 3
117     def quit(self,event):
118         quit()
119
120 class ExitToPython(Exception):
121     pass
```

searchGUI.py — (continued)

```
123 from searchGeneric import Searcher, AStarSearcher
124 from searchMPP import SearcherMPP
125 import searchExample
126 from searchBranchAndBound import DF_branch_and_bound
127
128 # to demonstrate depth-first search:
129 # sdfs = SearcherGUI(Searcher, searchExample.tree_graph); sdfs.go()
130
131 # delivery graph examples:
132 # sh = SearcherGUI(Searcher, searchExample.simp_delivery_graph); sh.go()
```

```
# sha = SearcherGUI(AStarSearcher, searchExample.simp_delivery_graph); sha.go()
# shac = SearcherGUI(AStarSearcher, searchExample.cyclic_simp_delivery_graph); shac.go()
# shm = SearcherGUI(SearcherMPP, searchExample.cyclic_simp_delivery_graph); shm.go()
# shb = SearcherGUI(DF_branch_and_bound, searchExample.simp_delivery_graph); shb.go()

# The following is AI:FCA figure 3.15, and is useful to show branch&bound:
# shbt = SearcherGUI(DF_branch_and_bound, searchExample.tree_graph); shbt.go()
```

3.2.3 Frontier as a Priority Queue

In many of the search algorithms, such as A^* and other best-first searchers, the frontier is implemented as a priority queue. The following code uses the Python's built-in priority queue implementations, `heapq`.

Following the lead of the Python documentation, `https://docs.python.org/3/library/heapq.html`, a frontier is a list of triples. The first element of each triple is the value to be minimized. The second element is a unique index which specifies the order that the elements were added to the queue, and the third element is the path that is on the queue. The use of the unique index ensures that the priority queue implementation does not compare paths; whether one path is less than another is not defined. It also lets us control what sort of search (e.g., depth-first or breadth-first) occurs when the value to be minimized does not give a unique next path.

The variable *frontier_index* is the total number of elements of the frontier that have been created. As well as being used as the unique index, it is useful for statistics, particularly in conjunction with the current size of the frontier.

searchGeneric.py — (continued)

```
import heapq      # part of the Python standard library
from searchProblem import Path

class FrontierPQ(object):
    """A frontier consists of a priority queue (heap), frontierpq, of
        (value, index, path) triples, where
    * value is the value we want to minimize (e.g., path cost + h).
    * index is a unique index for each element
    * path is the path on the queue
    Note that the priority queue always returns the smallest element.
    """

    def __init__(self):
        """constructs the frontier, initially an empty priority queue
        """
        self.frontier_index = 0 # the number of items added to the frontier
```

```
84          self.frontierpq = [] # the frontier priority queue
85
86      def empty(self):
87          """is True if the priority queue is empty"""
88          return self.frontierpq == []
89
90      def add(self, path, value):
91          """add a path to the priority queue
92          value is the value to be minimized"""
93          self.frontier_index += 1 # get a new unique index
94          heapq.heappush(self.frontierpq,(value, -self.frontier_index, path))
95
96      def pop(self):
97          """returns and removes the path of the frontier with minimum value.
98          """
99          (_,_,path) = heapq.heappop(self.frontierpq)
100         return path
```

The following methods are used for finding and printing information about the frontier.

searchGeneric.py — (continued)

```
102     def count(self,val):
103         """returns the number of elements of the frontier with value=val"""
104         return sum(1 for e in self.frontierpq if e[0]==val)
105
106     def __repr__(self):
107         """string representation of the frontier"""
108         return str([(n,c,str(p)) for (n,c,p) in self.frontierpq])
109
110     def __len__(self):
111         """length of the frontier"""
112         return len(self.frontierpq)
113
114     def __iter__(self):
115         """iterate through the paths in the frontier"""
116         for (_,_,path) in self.frontierpq:
117             yield path
```

3.2.4 A^* Search

For an A^* **Search** the frontier is implemented using the FrontierPQ class.

searchGeneric.py — (continued)

```
119 class AStarSearcher(Searcher):
120     """returns a searcher for a problem.
121     Paths can be found by repeatedly calling search().
122     """
123
124     def __init__(self, problem):
```

```
125         super().__init__(problem)
126 
127     def initialize_frontier(self):
128         self.frontier = FrontierPQ()
129 
130     def empty_frontier(self):
131         return self.frontier.empty()
132 
133     def add_to_frontier(self,path):
134         """add path to the frontier with the appropriate cost"""
135         value = path.cost+self.problem.heuristic(path.end())
136         self.frontier.add(path, value)
```

Code should always be tested. The following provides a simple **unit test**, using `problem1` as the default problem.

searchGeneric.py — (continued)

```
138 import searchExample
139 
140 def test(SearchClass, problem=searchExample.problem1,
        solutions=[['G','D','B','C','A']] ):
141     """Unit test for aipython searching algorithms.
142     SearchClass is a class that takes a problem and implements search()
143     problem is a search problem
144     solutions is a list of optimal solutions
145     """
146     print("Testing problem 1:")
147     schr1 = SearchClass(problem)
148     path1 = schr1.search()
149     print("Path found:",path1)
150     assert path1 is not None, "No path is found in problem1"
151     assert list(path1.nodes()) in solutions, "Shortest path not found in
            problem1"
152     print("Passed unit test")
153 
154 if __name__ == "__main__":
155     #test(Searcher)    # what needs to be changed to make this succeed?
156     test(AStarSearcher)
157 
158 # example queries:
159 # searcher1 = Searcher(searchExample.simp_delivery_graph) # DFS
160 # searcher1.search() # find first path
161 # searcher1.search() # find next path
162 # searcher2 = AStarSearcher(searchExample.simp_delivery_graph) # A*
163 # searcher2.search() # find first path
164 # searcher2.search() # find next path
165 # searcher3 = Searcher(searchExample.cyclic_simp_delivery_graph) # DFS
166 # searcher3.search() # find first path with DFS. What do you expect to
        happen?
167 # searcher4 = AStarSearcher(searchExample.cyclic_simp_delivery_graph) # A*
168 # searcher4.search() # find first path
```

Exercise 3.2 Change the code so that it implements (i) best-first search and (ii) lowest-cost-first search. For each of these methods compare it to A^* in terms of the number of paths expanded, and the path found.

Exercise 3.3 The searcher acts like a Python iterator, in that it returns one value (here a path) and then returns other values (paths) on demand, but does not implement the iterator interface. Change the code so it implements the iterator interface. What does this enable us to do?

3.2.5 Multiple Path Pruning

To run the multiple-path pruning demo, in folder "aipython", load "searchMPP.py" , using e.g., `ipython -i searchMPP.py`, and copy and paste the example queries at the bottom of that file.

The following implements A^* with multiple-path pruning. It overrides *search*() in *Searcher*.

searchMPP.py — Searcher with multiple-path pruning

```
from searchGeneric import AStarSearcher
from searchProblem import Path

class SearcherMPP(AStarSearcher):
    """returns a searcher for a problem.
    Paths can be found by repeatedly calling search().
    """
    def __init__(self, problem):
        super().__init__(problem)
        self.explored = set()

    def search(self):
        """returns next path from an element of problem's start nodes
        to a goal node.
        Returns None if no path exists.
        """
        while not self.empty_frontier():
            self.path = self.frontier.pop()
            if self.path.end() not in self.explored:
                self.explored.add(self.path.end())
                self.num_expanded += 1
                if self.problem.is_goal(self.path.end()):
                    self.solution = self.path # store the solution found
                    self.display(1, f"Solution: {self.path} (cost:
                        {self.path.cost})\n",
                    self.num_expanded, "paths have been expanded and",
                           len(self.frontier), "paths remain in the
                               frontier")
                    return self.path
                else:
```

```
                    self.display(4,f"Expanding: {self.path} (cost:
                        {self.path.cost})")
                    neighs = self.problem.neighbors(self.path.end())
                    self.display(2,f"Expanding: {self.path} with neighbors
                        {neighs}")
                    for arc in neighs:
                        self.add_to_frontier(Path(self.path,arc))
                    self.display(3, f"New frontier: {[p.end() for p in
                        self.frontier]}")
        self.display(0,"No (more) solutions. Total of",
                     self.num_expanded,"paths expanded.")

from searchGeneric import test
if __name__ == "__main__":
    test(SearcherMPP)

import searchExample
# searcherMPPcdp = SearcherMPP(searchExample.cyclic_simp_delivery_graph)
# searcherMPPcdp.search() # find first path
```

Exercise 3.4 Chris was very puzzled as to why there was a minus ("$-$") in the second element of the tuple added to the heap in the add method in `FrontierPQ` in `searchGeneric.py`.

Sam suggested the following example would demonstrate the importance of the minus. Consider an infinite integer grid, where the states are pairs of integers, the start is (0,0), and the goal is (10,10). The neighbors of (i, j) are $(i+1, j)$ and $(i, j+1)$. Consider the heuristic function $h((i, j)) = |10-i| + |10-j|$. Sam suggested you compare how many paths are expanded with the minus and without the minus. `searchGrid` is a representation of Sam's graph. If something takes too long, you might consider changing the size.

searchGrid.py — A grid problem to demonstrate A*

```
from searchProblem import Search_problem, Arc

class GridProblem(Search_problem):
    """a node is a pair (x,y)"""
    def __init__(self, size=10):
        self.size = size

    def start_node(self):
        """returns the start node"""
        return (0,0)

    def is_goal(self,node):
        """returns True when node is a goal node"""
        return node == (self.size,self.size)

    def neighbors(self,node):
        """returns a list of the neighbors of node"""
        (x,y) = node
```

```
29          return [Arc(node,(x+1,y)), Arc(node,(x,y+1))]
30
31      def heuristic(self,node):
32          (x,y) = node
33          return abs(x-self.size)+abs(y-self.size)
34
35  class GridProblemNH(GridProblem):
36      """Grid problem with a heuristic of 0"""
37      def heuristic(self,node):
38          return 0
39
40  from searchGeneric import Searcher, AStarSearcher
41  from searchMPP import SearcherMPP
42  from searchBranchAndBound import DF_branch_and_bound
43
44  def testGrid(size = 10):
45      print("\nWith MPP")
46      gridsearchermpp = SearcherMPP(GridProblem(size))
47      print(gridsearchermpp.search())
48      print("\nWithout MPP")
49      gridsearchera = AStarSearcher(GridProblem(size))
50      print(gridsearchera.search())
51      print("\nWith MPP and a heuristic = 0 (Dijkstra's algorithm)")
52      gridsearchermppnh = SearcherMPP(GridProblemNH(size))
53      print(gridsearchermppnh.search())
```

Explain to Chris what the minus does and why it is there. Give evidence for your claims. It might be useful to refer to other search strategies in your explanation. As part of your explanation, explain what is special about Sam's example.

Exercise 3.5 Implement a searcher that implements cycle pruning instead of multiple-path pruning. You need to decide whether to check for cycles when paths are added to the frontier or when they are removed. (Hint: either method can be implemented by only changing one or two lines in SearcherMPP. Hint: there is a cycle if `path.end() in path.initial_nodes()`) Compare no pruning, multiple path pruning and cycle pruning for the cyclic delivery problem. Which works better in terms of number of paths expanded, computational time or space?

3.3 Branch-and-bound Search

To run the demo, in folder "`aipython`", load "`searchBranchAndBound.py`", and copy and paste the example queries at the bottom of that file.

Depth-first search methods do not need a priority queue, but can use a list as a stack. In this implementation of branch-and-bound search, we call *search* to find an optimal solution with cost less than bound. This uses depth-first search to find a path to a goal that extends *path* with cost less than the bound.

Once a path to a goal has been found, that path is remembered as the *best_path*, the bound is reduced, and the search continues.

searchBranchAndBound.py — Branch and Bound Search

```
from searchProblem import Path
from searchGeneric import Searcher
from display import Displayable

class DF_branch_and_bound(Searcher):
    """returns a branch and bound searcher for a problem.
    An optimal path with cost less than bound can be found by calling search()
    """
    def __init__(self, problem, bound=float("inf")):
        """creates a searcher than can be used with search() to find an optimal path.
        bound gives the initial bound. By default this is infinite - meaning there
        is no initial pruning due to depth bound
        """
        super().__init__(problem)
        self.best_path = None
        self.bound = bound

    def search(self):
        """returns an optimal solution to a problem with cost less than bound.
        returns None if there is no solution with cost less than bound."""
        self.frontier = [Path(self.problem.start_node())]
        self.num_expanded = 0
        while self.frontier:
            self.path = self.frontier.pop()
            if self.path.cost+self.problem.heuristic(self.path.end()) < self.bound:
                # if self.path.end() not in self.path.initial_nodes(): # for cycle pruning
                self.display(2,"Expanding:",self.path,"cost:",self.path.cost)
                self.num_expanded += 1
                if self.problem.is_goal(self.path.end()):
                    self.best_path = self.path
                    self.bound = self.path.cost
                    self.display(1,"New best path:",self.path," cost:",self.path.cost)
                else:
                    neighs = self.problem.neighbors(self.path.end())
                    self.display(4,"Neighbors are", neighs)
                    for arc in reversed(list(neighs)):
                        self.add_to_frontier(Path(self.path, arc))
                    self.display(3, f"New frontier: {[p.end() for p in self.frontier]}")
        self.path = self.best_path
```

```
50        self.solution = self.best_path
51        self.display(1,f"Optimal solution is {self.best_path}." if
              self.best_path
52                         else "No solution found.",
53                     f"Number of paths expanded: {self.num_expanded}.")
54        return self.best_path
```

Note that this code used *reversed* in order to expand the neighbors of a node in the left-to-right order one might expect. It does this because *pop*() removes the rightmost element of the list. The call to *list* is there because *reversed* only works on lists and tuples, but the neighbors can be generated.

Here is a unit test and some queries:

searchBranchAndBound.py — (continued)

```
56 from searchGeneric import test
57 if __name__ == "__main__":
58     test(DF_branch_and_bound)
59
60 # Example queries:
61 import searchExample
62 # searcherb1 = DF_branch_and_bound(searchExample.simp_delivery_graph)
63 # searcherb1.search()     # find optimal path
64 # searcherb2 =
       DF_branch_and_bound(searchExample.cyclic_simp_delivery_graph,
       bound=100)
65 # searcherb2.search()     # find optimal path
```

Exercise 3.6 In searcherb2, in the code above, what happens if the bound is smaller, say 10? What if it is larger, say 1000?

Exercise 3.7 Implement a branch-and-bound search using recursion. Hint: you don't need an explicit frontier, but can do a recursive call for the children.

Exercise 3.8 After the branch-and-bound search found a solution, Sam ran search again, and noticed a different count. Sam hypothesized that this count was related to the number of nodes that an $A*$ search would use (either expand or be added to the frontier). Or maybe, Sam thought, the count for a number of nodes when the bound is slightly above the optimal path case is related to how A^* would work. Is there a relationship between these counts? Are there different things that it could count so they are related? Try to find the most specific statement that is true, and explain why it is true.

To test the hypothesis, Sam wrote the following code, but isn't sure it is helpful:

searchTest.py — code that may be useful to compare A* and branch-and-bound

```
11 from searchGeneric import Searcher, AStarSearcher
12 from searchBranchAndBound import DF_branch_and_bound
13 from searchMPP import SearcherMPP
14
15 DF_branch_and_bound.max_display_level = 1
16 Searcher.max_display_level = 1
```

```
17
18 def run(problem,name):
19     print("\n\n*******",name)
20
21     print("\nA*:")
22     asearcher = AStarSearcher(problem)
23     print("Path found:",asearcher.search()," cost=",asearcher.solution.cost)
24     print("there are",asearcher.frontier.count(asearcher.solution.cost),
25           "elements remaining on the queue with
                f-value=",asearcher.solution.cost)
26
27     print("\nA* with MPP:"),
28     msearcher = SearcherMPP(problem)
29     print("Path found:",msearcher.search()," cost=",msearcher.solution.cost)
30     print("there are",msearcher.frontier.count(msearcher.solution.cost),
31           "elements remaining on the queue with
                f-value=",msearcher.solution.cost)
32
33     bound = asearcher.solution.cost+0.01
34     print("\nBranch and bound (with too-good initial bound of", bound,")")
35     tbb = DF_branch_and_bound(problem,bound) # cheating!!!!
36     print("Path found:",tbb.search()," cost=",tbb.solution.cost)
37     print("Rerunning B&B")
38     print("Path found:",tbb.search())
39
40     bbound = asearcher.solution.cost*2+10
41     print("\nBranch and bound (with not-very-good initial bound of",
           bbound, ")")
42     tbb2 = DF_branch_and_bound(problem,bbound)
43     print("Path found:",tbb2.search()," cost=",tbb2.solution.cost)
44     print("Rerunning B&B")
45     print("Path found:",tbb2.search())
46
47     print("\nDepth-first search: (Use ^C if it goes on forever)")
48     tsearcher = Searcher(problem)
49     print("Path found:",tsearcher.search()," cost=",tsearcher.solution.cost)
50
51
52 import searchExample
53 from searchTest import run
54 if __name__ == "__main__":
55     run(searchExample.problem1,"Problem 1")
56 #   run(searchExample.simp_delivery_graph,"Acyclic Delivery")
57 #   run(searchExample.cyclic_simp_delivery_graph,"Cyclic Delivery")
58 # also test some graphs with cycles, and some with multiple least-cost
      paths
```

Chapter 4

Reasoning with Constraints

4.1 Constraint Satisfaction Problems

4.1.1 Variables

A **variable** consists of a name, a domain and an optional (x,y) position (for displaying). The domain of a variable is a list or a tuple, as the ordering will matter in the representation of constraints.

variable.py — Representations of a variable in CSPs and probabilistic models

```
11 import random
12
13 class Variable(object):
14     """A random variable.
15     name (string) - name of the variable
16     domain (list) - a list of the values for the variable.
17     Variables are ordered according to their name.
18     """
19
20     def __init__(self, name, domain, position=None):
21         """Variable
22         name a string
23         domain a list of printable values
24         position of form (x,y)
25         """
26         self.name = name # string
27         self.domain = domain # list of values
28         self.position = position if position else (random.random(),
               random.random())
29         self.size = len(domain)
30
31     def __str__(self):
```

```
32          return self.name
33
34      def __repr__(self):
35          return self.name # f"Variable({self.name})"
```

4.1.2 Constraints

A **constraint** consists of:

- A tuple (or list) of variables called the **scope**.
- A **condition**, a Boolean function that takes the same number of arguments as there are variables in the scope. The condition must have a `__name__` property that gives a printable name of the function; built-in functions and functions that are defined using *def* have such a property; for other functions you may need to define this property.
- An optional name
- An optional (x, y) position

cspProblem.py — Representations of a Constraint Satisfaction Problem

```
11  from variable import Variable
12
13  # for showing csps:
14  import matplotlib.pyplot as plt
15  import matplotlib.lines as lines
16
17  class Constraint(object):
18      """A Constraint consists of
19      * scope: a tuple of variables
20      * condition: a Boolean function that can applied to a tuple of values
            for variables in scope
21      * string: a string for printing the constraints. All of the strings
            must be unique.
22      for the variables
23      """
24      def __init__(self, scope, condition, string=None, position=None):
25          self.scope = scope
26          self.condition = condition
27          if string is None:
28              self.string = f"{self.condition.__name__}({self.scope})"
29          else:
30              self.string = string
31          self.position = position
32
33      def __repr__(self):
34          return self.string
```

An **assignment** is a *variable:value* dictionary.

If *con* is a constraint, *con.holds*(*assignment*) returns True or False depending on whether the condition is true or false for that assignment. The assignment *assignment* must assign a value to every variable in the scope of the constraint *con* (and could also assign values to other variables); *con.holds* gives an error if not all variables in the scope of *con* are assigned in the assignment. It ignores variables in *assignment* that are not in the scope of the constraint.

In Python, the $*$ notation is used for unpacking a tuple. For example, $F(*(1,2,3))$ is the same as $F(1,2,3)$. So if t has value $(1,2,3)$, then $F(*t)$ is the same as $F(1,2,3)$.

cspProblem.py — (continued)

```
36      def can_evaluate(self, assignment):
37          """
38          assignment is a variable:value dictionary
39          returns True if the constraint can be evaluated given assignment
40          """
41          return all(v in assignment for v in self.scope)
42
43      def holds(self,assignment):
44          """returns the value of Constraint con evaluated in assignment.
45
46          precondition: all variables are assigned in assignment, ie
                self.can_evaluate(assignment) is true
47          """
48          return self.condition(*tuple(assignment[v] for v in self.scope))
```

4.1.3 CSPs

A constraint satisfaction problem (CSP) requires:

- *variables*: a list or set of variables
- *constraints*: a set or list of constraints.

Other properties are inferred from these:

- *var_to_const* is a mapping from variables to set of constraints, such that *var_to_const*[*var*] is the set of constraints with *var* in the scope.

cspProblem.py — (continued)

```
50  class CSP(object):
51      """A CSP consists of
52      * a title (a string)
53      * variables, a set of variables
54      * constraints, a list of constraints
55      * var_to_const, a variable to set of constraints dictionary
56      """
```

```
57      def __init__(self, title, variables, constraints):
58          """title is a string
59          variables is set of variables
60          constraints is a list of constraints
61          """
62          self.title = title
63          self.variables = variables
64          self.constraints = constraints
65          self.var_to_const = {var:set() for var in self.variables}
66          for con in constraints:
67              for var in con.scope:
68                  self.var_to_const[var].add(con)
69
70      def __str__(self):
71          """string representation of CSP"""
72          return str(self.title)
73
74      def __repr__(self):
75          """more detailed string representation of CSP"""
76          return f"CSP({self.title}, {self.variables}, {([str(c) for c in
                self.constraints])})"
```

csp.consistent(*assignment*) returns true if the assignment is consistent with each of the constraints in *csp* (i.e., all of the constraints that can be evaluated evaluate to true). Note that this is a local consistency with each constraint; it does *not* imply the CSP is consistent or has a solution.

cspProblem.py — (continued)

```
78      def consistent(self,assignment):
79          """assignment is a variable:value dictionary
80          returns True if all of the constraints that can be evaluated
81                          evaluate to True given assignment.
82          """
83          return all(con.holds(assignment)
84                     for con in self.constraints
85                     if con.can_evaluate(assignment))
```

The **show** method uses matplotlib to show the graphical structure of a constraint network. If the node positions are not specified, this gives different positions each time it is run; if you don't like the graph, try again.

cspProblem.py — (continued)

```
87      def show(self, linewidth=3, showDomains=False, showAutoAC = False):
88          self.linewidth = linewidth
89          self.picked = None
90          plt.ion()  # interactive
91          self.arcs = {} # arc: (con,var) dictionary
92          self.thelines = {} # (con,var):arc dictionary
93          self.nodes = {} # node: variable dictionary
94          self.fig, self.ax= plt.subplots(1, 1)
95          self.ax.set_axis_off()
```

```
        for var in self.variables:
            if var.position is None:
                var.position = (random.random(), random.random())
        self.showAutoAC = showAutoAC # used for consistency GUI
        self.autoAC = False
        domains = {var:var.domain for var in self.variables} if showDomains
            else {}
        self.draw_graph(domains=domains)

    def draw_graph(self, domains={}, to_do = {}, title=None, fontsize=10):
        self.ax.clear()
        self.ax.set_axis_off()
        if title:
            plt.title(title, fontsize=fontsize)
        else:
            plt.title(self.title, fontsize=fontsize)
        var_bbox = dict(boxstyle="round4,pad=1.0,rounding_size=0.5")
        con_bbox = dict(boxstyle="square,pad=1.0",color="green")
        self.autoACtext = plt.text(0,0,"Auto AC" if self.showAutoAC else "",
                                   bbox={'boxstyle':'square','color':'yellow'},
                                   picker=True, fontsize=fontsize)
        for con in self.constraints:
            if con.position is None:
                con.position = tuple(sum(var.position[i] for var in
                    con.scope)/len(con.scope)
                                     for i in range(2))
            cx,cy = con.position
            bbox = dict(boxstyle="square,pad=1.0",color="green")
            for var in con.scope:
                vx,vy = var.position
                if (var,con) in to_do:
                    color = 'blue'
                else:
                    color = 'limegreen'
                line = lines.Line2D([cx,vx], [cy,vy], axes=self.ax,
                    color=color,
                                    picker=True, pickradius=10,
                                        linewidth=self.linewidth)
                self.arcs[line]= (var,con)
                self.thelines[(var,con)] = line
                self.ax.add_line(line)
            plt.text(cx,cy,con.string,
                                 bbox=con_bbox,
                                 ha='center',va='center', fontsize=fontsize)
        for var in self.variables:
            x,y = var.position
            if domains:
                node_label = f"{var.name}\n{domains[var]}"
            else:
                node_label = var.name
```

```
142             node = plt.text(x, y, node_label, bbox=var_bbox, ha='center',
                    va='center',
143                             picker=True, fontsize=fontsize)
144             self.nodes[node] = var
145         self.fig.canvas.mpl_connect('pick_event', self.pick_handler)
146 
147     def pick_handler(self,event):
148         mouseevent = event.mouseevent
149         self.last_artist = artist = event.artist
150         #print('***picker handler:',artist, 'mouseevent:', mouseevent)
151         if artist in self.arcs:
152             #print('### selected arc',self.arcs[artist])
153             self.picked = self.arcs[artist]
154         elif artist in self.nodes:
155             #print('### selected node',self.nodes[artist])
156             self.picked = self.nodes[artist]
157         elif artist==self.autoACtext:
158             self.autoAC = True
159             #print("*** autoAC")
160         else:
161             print("### unknown click")
```

4.1.4 Examples

In the following code *ne_*, when given a number, returns a function that is true when its argument is not that number. For example, if $f = ne\_(3)$, then $f(2)$ is True and $f(3)$ is False. That is, $ne\_(x)(y)$ is true when $x \neq y$. Allowing a function of multiple arguments to use its arguments one at a time is called **currying**, after the logician Haskell Curry. Functions used as conditions in constraints require names (so they can be printed).

cspExamples.py — Example CSPs

```
11 from cspProblem import Variable, CSP, Constraint
12 from operator import lt,ne,eq,gt
13 
14 def ne_(val):
15     """not equal value"""
16     # nev = lambda x: x != val # alternative definition
17     # nev = partial(neq,val)  # another alternative definition
18     def nev(x):
19         return val != x
20     nev.__name__ = f"{val} != "   # name of the function
21     return nev
```

Similarly $is\_(x)(y)$ is true when $x = y$.

cspExamples.py — (continued)

```
23 def is_(val):
24     """is a value"""
25     # isv = lambda x: x == val # alternative definition
```

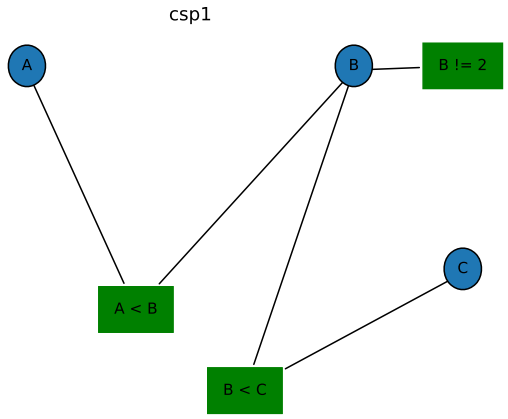


Figure 4.1: csp1.show()

```
26      # isv = partial(eq,val)   # another alternative definition
27      def isv(x):
28          return val == x
29      isv.__name__ = f"{val} == "
30      return isv
```

The CSP, *csp*0 has variables X, Y and Z, each with domain $\{1,2,3\}$. The constraints are $X < Y$ and $Y < Z$.

cspExamples.py — (continued)

```
32  X = Variable('X', {1,2,3})
33  Y = Variable('Y', {1,2,3})
34  Z = Variable('Z', {1,2,3})
35  csp0 = CSP("csp0", {X,Y,Z},
36            [ Constraint([X,Y],lt),
37              Constraint([Y,Z],lt)])
```

The CSP, *csp*1 has variables A, B and C, each with domain $\{1,2,3,4\}$. The constraints are $A < B$, $B \neq 2$, and $B < C$. This is slightly more interesting than *csp*0 as it has more solutions. This example is used in the unit tests, and so if it is changed, the unit tests need to be changed. The CSP *csp*1*s* is the same, but with only the constraints $A < B$ and $B < C$

cspExamples.py — (continued)

```
39  A = Variable('A', {1,2,3,4}, position=(0.2,0.9))
40  B = Variable('B', {1,2,3,4}, position=(0.8,0.9))
41  C = Variable('C', {1,2,3,4}, position=(1,0.4))
42  C0 = Constraint([A,B], lt, "A < B", position=(0.4,0.3))
43  C1 = Constraint([B], ne_(2), "B != 2", position=(1,0.9))
44  C2 = Constraint([B,C], lt, "B < C", position=(0.6,0.1))
```

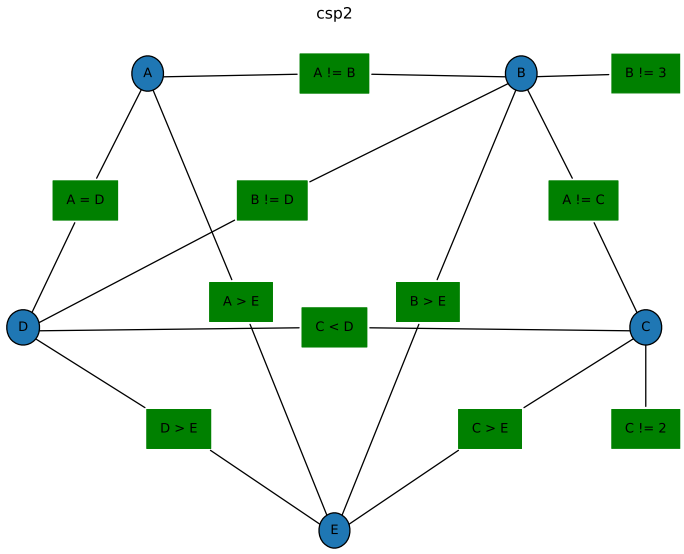


Figure 4.2: csp2.show()

```
45 csp1 = CSP("csp1", {A, B, C},
46           [C0, C1, C2])
47
48 csp1s = CSP("csp1s", {A, B, C},
49           [C0, C2]) # A<B, B<C
```

The next CSP, *csp*2 is Example 4.9 of Poole and Mackworth [2023]; the domain consistent network (after applying the unary constraints) is shown in Figure 4.2. Note that we use the same variables as the previous example and add two more.

cspExamples.py — (continued)

```
51 D = Variable('D', {1,2,3,4}, position=(0,0.4))
52 E = Variable('E', {1,2,3,4}, position=(0.5,0))
53 csp2 = CSP("csp2", {A,B,C,D,E},
54           [ Constraint([B], ne_(3), "B != 3", position=(1,0.9)),
55            Constraint([C], ne_(2), "C != 2", position=(1,0.2)),
56            Constraint([A,B], ne, "A != B"),
57            Constraint([B,C], ne, "A != C"),
58            Constraint([C,D], lt, "C < D"),
59            Constraint([A,D], eq, "A = D"),
60            Constraint([E,A], lt, "E < A"),
61            Constraint([E,B], lt, "E < B"),
62            Constraint([E,C], lt, "E < C"),
63            Constraint([E,D], lt, "E < D"),
```

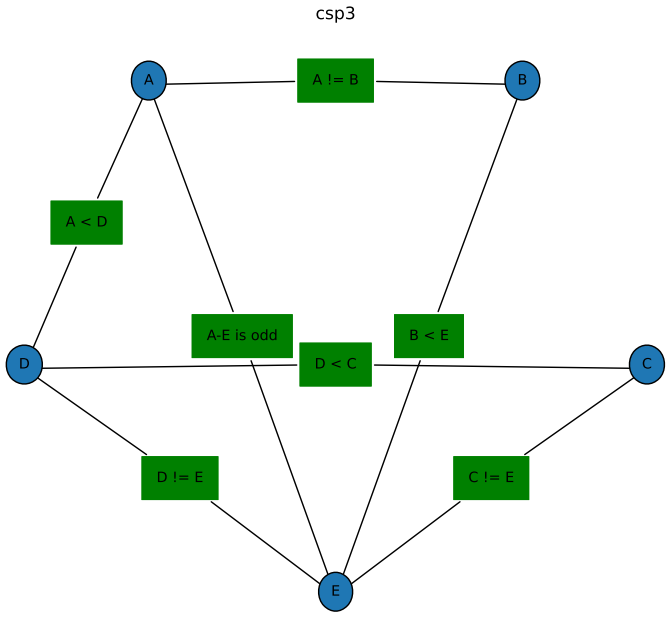


Figure 4.3: csp3.show()

```
64            Constraint([B,D], ne, "B != D")])
```

The following example is another scheduling problem (but with multiple answers). This is the same as "scheduling 2" in the original AIspace.org consistency app.

cspExamples.py — (continued)

```
66 csp3 = CSP("csp3", {A,B,C,D,E},
67           [Constraint([A,B], ne, "A != B"),
68            Constraint([A,D], lt, "A < D"),
69            Constraint([A,E], lambda a,e: (a-e)%2 == 1, "A-E is odd"),
70            Constraint([B,E], lt, "B < E"),
71            Constraint([D,C], lt, "D < C"),
72            Constraint([C,E], ne, "C != E"),
73            Constraint([D,E], ne, "D != E")])
```

The following example is another abstract scheduling problem. What are the solutions?

cspExamples.py — (continued)

```
75 def adjacent(x,y):
76     """True when x and y are adjacent numbers"""
77     return abs(x-y) == 1
```

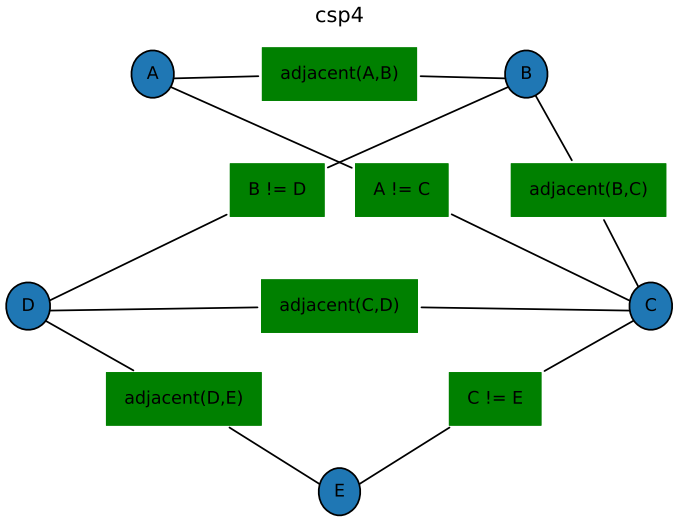


Figure 4.4: csp4.show()

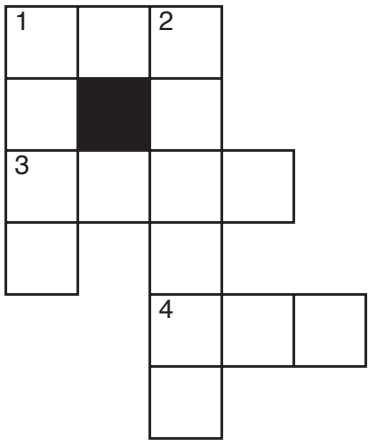
```
78
79 csp4 = CSP("csp4", {A,B,C,D},
80           [Constraint([A,B], adjacent, "adjacent(A,B)"),
81            Constraint([B,C], adjacent, "adjacent(B,C)"),
82            Constraint([C,D], adjacent, "adjacent(C,D)"),
83            Constraint([A,C], ne, "A != C"),
84            Constraint([B,D], ne, "B != D") ])
```

The following examples represent the crossword shown in Figure 4.5.

In the first representation, the variables represent words. The constraint imposed by the crossword is that where two words intersect, the letter at the intersection must be the same. The method `meet_at` is used to test whether two words intersect with the same letter. For example, the constraint `meet_at(2,0)` means that the third letter (at position 2) of the first argument is the same as the first letter of the second argument. This is shown in Figure 4.6.

cspExamples.py — (continued)

```
86 def meet_at(p1,p2):
87     """returns a function of two words that is true
88                 when the words intersect at positions p1, p2.
89     The positions are relative to the words; starting at position 0.
90     meet_at(p1,p2)(w1,w2) is true if the same letter is at position p1 of
           word w1
91          and at position p2 of word w2.
92     """
93     def meets(w1,w2):
94         return w1[p1] == w2[p2]
```



Words:
ant, big, bus, car, has, book, buys, hold, lane, year, ginger, search, symbol, syntax.

Figure 4.5: crossword1: a crossword puzzle to be solved

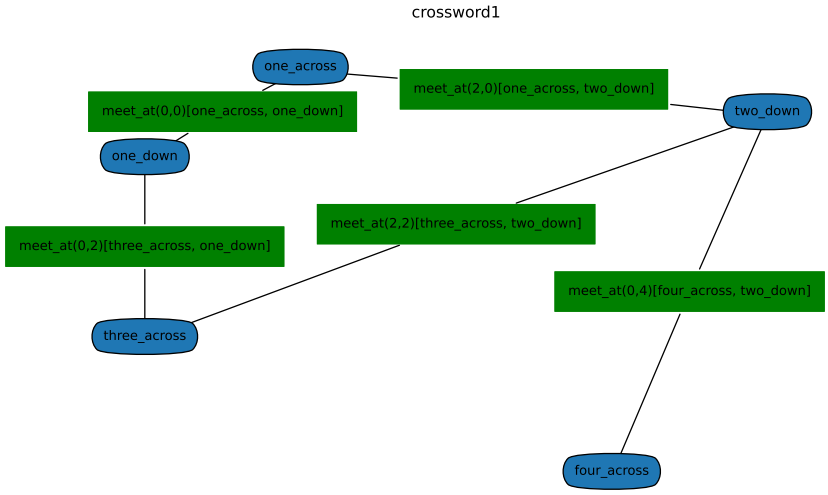


Figure 4.6: crossword1.show()

```
95      meets.__name__ = f"meet_at({p1},{p2})"
96      return meets
97
98  one_across = Variable('one_across', {'ant', 'big', 'bus', 'car', 'has'},
        position=(0.3,0.9))
99  one_down = Variable('one_down', {'book', 'buys', 'hold', 'lane', 'year'},
        position=(0.1,0.7))
100 two_down = Variable('two_down', {'ginger', 'search', 'symbol', 'syntax'},
        position=(0.9,0.8))
101 three_across = Variable('three_across', {'book', 'buys', 'hold', 'land',
        'year'}, position=(0.1,0.3))
102 four_across = Variable('four_across',{'ant', 'big', 'bus', 'car', 'has'},
        position=(0.7,0.0))
103 crossword1 = CSP("crossword1",
104                 {one_across, one_down, two_down, three_across,
                        four_across},
105                 [Constraint([one_across,one_down], meet_at(0,0)),
106                  Constraint([one_across,two_down], meet_at(2,0)),
107                  Constraint([three_across,two_down], meet_at(2,2)),
108                  Constraint([three_across,one_down], meet_at(0,2)),
109                  Constraint([four_across,two_down], meet_at(0,4))])
```

In an alternative representation of a crossword (the "dual" representation), the variables represent letters, and the constraints are that adjacent sequences of letters form words. This is shown in Figure 4.7.

cspExamples.py — (continued)

```
111 words = {'ant', 'big', 'bus', 'car', 'has','book', 'buys', 'hold',
112         'lane', 'year', 'ginger', 'search', 'symbol', 'syntax'}
113
114 def is_word(*letters, words=words):
115     """is true if the letters concatenated form a word in words"""
116     return "".join(letters) in words
117
118 letters = {"a", "b", "c", "d", "e", "f", "g", "h", "i", "j", "k", "l",
119   "m", "n", "o", "p", "q", "r", "s", "t", "u", "v", "w", "x", "y",
120   "z"}
121
122 # pij is the variable representing the letter i from the left and j down
        (starting from 0)
123 p00 = Variable('p00', letters, position=(0.1,0.85))
124 p10 = Variable('p10', letters, position=(0.3,0.85))
125 p20 = Variable('p20', letters, position=(0.5,0.85))
126 p01 = Variable('p01', letters, position=(0.1,0.7))
127 p21 = Variable('p21', letters, position=(0.5,0.7))
128 p02 = Variable('p02', letters, position=(0.1,0.55))
129 p12 = Variable('p12', letters, position=(0.3,0.55))
130 p22 = Variable('p22', letters, position=(0.5,0.55))
131 p32 = Variable('p32', letters, position=(0.7,0.55))
132 p03 = Variable('p03', letters, position=(0.1,0.4))
133 p23 = Variable('p23', letters, position=(0.5,0.4))
```

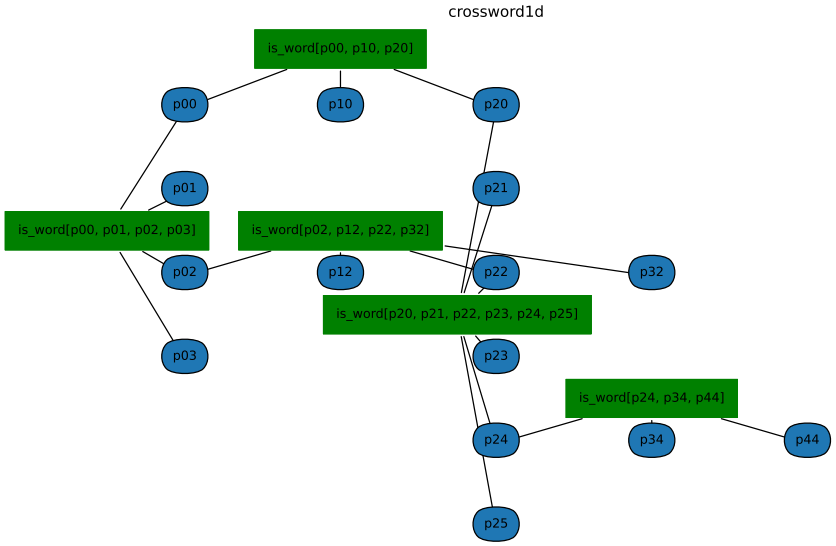


Figure 4.7: crossword1d.show()

```
p24 = Variable('p24', letters, position=(0.5,0.25))
p34 = Variable('p34', letters, position=(0.7,0.25))
p44 = Variable('p44', letters, position=(0.9,0.25))
p25 = Variable('p25', letters, position=(0.5,0.1))

crossword1d = CSP("crossword1d",
                {p00, p10, p20, # first row
                 p01, p21, # second row
                 p02, p12, p22, p32, # third row
                 p03, p23, #fourth row
                 p24, p34, p44, # fifth row
                 p25 # sixth row
                 },
                [Constraint([p00, p10, p20], is_word,
                    position=(0.3,0.95)), #1-across
                 Constraint([p00, p01, p02, p03], is_word,
                     position=(0,0.625)), # 1-down
                 Constraint([p02, p12, p22, p32], is_word,
                     position=(0.3,0.625)), # 3-across
                 Constraint([p20, p21, p22, p23, p24, p25], is_word,
                     position=(0.45,0.475)), # 2-down
                 Constraint([p24, p34, p44], is_word,
                     position=(0.7,0.325)) # 4-across
```

```
152                 ])
```

Exercise 4.1 How many assignments of a value to each variable are there for each of the representations of the above crossword? Do you think an exhaustive enumeration will work for either one?

The queens problem is a puzzle on a chess board, where the idea is to place a queen on each column so the queens cannot take each other: there are no two queens on the same row, column or diagonal. The **n-queens problem** is a generalization where the size of the board is an $n \times n$, and n queens have to be placed.

Here is a representation of the n-queens problem, where the variables are the columns and the values are the rows in which the queen is placed. The original queens problem on a standard (8×8) chess board is `n_queens(8)`

cspExamples.py — (continued)

```
154 def queens(ri,rj):
155     """ri and rj are different rows, return the condition that the queens
            cannot take each other"""
156     def no_take(ci,cj):
157         """is true if queen at (ri,ci) cannot take a queen at (rj,cj)"""
158         return ci != cj and abs(ri-ci) != abs(rj-cj)
159     return no_take
160 
161 def n_queens(n):
162     """returns a CSP for n-queens"""
163     columns = list(range(n))
164     variables = [Variable(f"R{i}",columns) for i in range(n)]
165     return CSP("n-queens",
166                variables,
167                 [Constraint([variables[i], variables[j]], queens(i,j))
168                     for i in range(n) for j in range(n) if i != j])
169 
170 # try the CSP n_queens(8) in one of the solvers.
171 # What is the smallest n for which there is a solution?
```

Exercise 4.2 How many constraints does this representation of the n-queens problem produce? Can it be done with fewer constraints? Either explain why it can't be done with fewer constraints, or give a solution using fewer constraints.

Unit tests

The following defines a **unit test** for csp solvers, by default using example csp1.

cspExamples.py — (continued)

```
173 def test_csp(CSP_solver, csp=csp1,
174               solutions=[{A: 1, B: 3, C: 4}, {A: 2, B: 3, C: 4}]):
175     """CSP_solver is a solver that takes a csp and returns a solution
176     csp is a constraint satisfaction problem
177     solutions is the list of all solutions to csp
```

```
178     This tests whether the solution returned by CSP_solver is a solution.
179     """
180     print("Testing csp with",CSP_solver.__doc__)
181     sol0 = CSP_solver(csp)
182     print("Solution found:",sol0)
183     assert sol0 in solutions, f"Solution not correct for {csp}"
184     print("Passed unit test")
```

Exercise 4.3 Modify *test* so that instead of taking in a list of solutions, it checks whether the returned solution actually is a solution.

Exercise 4.4 Propose a test that is appropriate for CSPs with no solutions. Assume that the test designer knows there are no solutions. Consider what a CSP solver should return if there are no solutions to the CSP.

Exercise 4.5 Write a unit test that checks whether all solutions (e.g., for the search algorithms that can return multiple solutions) are correct, and whether all solutions can be found.

4.2 A Simple Depth-first Solver

The first solver carries out a depth-first search through the space of partial assignments. This takes in a CSP problem and an optional variable ordering (a list of the variables in the CSP). It returns a generator of the solutions (see Section 1.5.4 on `yield` for enumerations).

cspDFS.py — Solving a CSP using depth-first search.

```
11  import cspExamples
12
13  def dfs_solver(constraints, context, var_order):
14      """generator for all solutions to csp.
15      context is an assignment of values to some of the variables.
16      var_order is  a list of the variables in csp that are not in context.
17      """
18      to_eval = {c for c in constraints if c.can_evaluate(context)}
19      if all(c.holds(context) for c in to_eval):
20          if var_order == []:
21              yield context
22          else:
23              rem_cons = [c for c in constraints if c not in to_eval]
24              var = var_order[0]
25              for val in var.domain:
26                  yield from dfs_solver(rem_cons, context|{var:val},
                        var_order[1:])
27
28  def dfs_solve_all(csp, var_order=None):
29      """depth-first CSP solver to return a list of all solutions to csp.
30      """
31      if var_order == None:  # use an arbitrary variable order
32          var_order = list(csp.variables)
```

```
33      return list( dfs_solver(csp.constraints, {}, var_order))
34
35  def dfs_solve1(csp, var_order=None):
36      """depth-first CSP solver"""
37      if var_order == None:  # use an arbitrary variable order
38          var_order = list(csp.variables)
39      for sol in dfs_solver(csp.constraints, {}, var_order):
40          return sol #return first one
41
42  if __name__ == "__main__":
43      cspExamples.test_csp(dfs_solve1)
44
45  #Try:
46  # dfs_solve_all(cspExamples.csp1)
47  # dfs_solve_all(cspExamples.csp2)
48  # dfs_solve_all(cspExamples.crossword1)
49  # dfs_solve_all(cspExamples.crossword1d) # warning: may take a *very* long
        time!
```

Exercise 4.6 Instead of testing all constraints at every node, change it so each constraint is only tested when all of its variables are assigned. Given an elimination ordering, it is possible to determine when each constraint needs to be tested. Implement this. Hint: create a parallel list of sets of constraints, where at each position i in the list, the constraints at position i can be evaluated when the variable at position i has been assigned.

Exercise 4.7 Estimate how long `dfs_solve_all(crossword1d)` will take on your computer. To do this, reduce the number of variables that need to be assigned, so that the simplified problem can be solved in a reasonable time (between 0.1 second and 10 seconds). This can be done by reducing the number of variables in `var_order`, as the program only splits on these. How much more time will it take if the number of variables is increased by 1? (Try it!) Then extrapolate to all of the variables. See Section 1.6.1 for how to time your code. Would making the code 100 times faster or using a computer 100 times faster help?

4.3 Converting CSPs to Search Problems

To run the demo, in folder "`aipython`", load "`cspSearch.py`", and copy and paste the example queries at the bottom of that file.

The next solver constructs a search space that can be solved using the search methods of the previous chapter. This takes in a CSP problem and an optional variable ordering, which is a list of the variables in the CSP. In this search space:

- A node is a *variable* : *value* dictionary which does not violate any constraints (so that dictionaries that violate any conmtratints are not added).

- An arc corresponds to an assignment of a value to the next variable. This assumes a static ordering; the next variable chosen to split does not depend on the context. If no variable ordering is given, this makes no attempt to choose a good ordering.

cspSearch.py — Representations of a Search Problem from a CSP.

```
from cspProblem import CSP, Constraint
from searchProblem import Arc, Search_problem

class Search_from_CSP(Search_problem):
    """A search problem directly from the CSP.

    A node is a variable:value dictionary"""
    def __init__(self, csp, variable_order=None):
        self.csp=csp
        if variable_order:
            assert set(variable_order) == set(csp.variables)
            assert len(variable_order) == len(csp.variables)
            self.variables = variable_order
        else:
            self.variables = list(csp.variables)

    def is_goal(self, node):
        """returns whether the current node is a goal for the search
        """
        return len(node)==len(self.csp.variables)

    def start_node(self):
        """returns the start node for the search
        """
        return {}
```

The *neighbors*(*node*) method uses the fact that the length of the node, which is the number of variables already assigned, is the index of the next variable to split on. Note that we do not need to check whether there are no more variables to split on, as the nodes are all consistent, by construction, and so when there are no more variables we have a solution, and so don't need the neighbors.

cspSearch.py — (continued)

```
    def neighbors(self, node):
        """returns a list of the neighboring nodes of node.
        """
        var = self.variables[len(node)] # the next variable
        res = []
        for val in var.domain:
            new_env = node|{var:val} #dictionary union
            if self.csp.consistent(new_env):
                res.append(Arc(node,new_env))
        return res
```

The unit tests relies on a solver. The following procedure creates a solver using search that can be tested.

cspSearch.py — (continued)

```
48 import cspExamples
49 from searchGeneric import Searcher
50
51 def solver_from_searcher(csp):
52     """depth-first search solver"""
53     path = Searcher(Search_from_CSP(csp)).search()
54     if path is not None:
55         return path.end()
56     else:
57         return None
58
59 if __name__ == "__main__":
60     test_csp(solver_from_searcher)
61
62 ## Test Solving CSPs with Search:
63 searcher1 = Searcher(Search_from_CSP(cspExamples.csp1))
64 #print(searcher1.search()) # get next solution
65 searcher2 = Searcher(Search_from_CSP(cspExamples.csp2))
66 #print(searcher2.search()) # get next solution
67 searcher3 = Searcher(Search_from_CSP(cspExamples.crossword1))
68 #print(searcher3.search()) # get next solution
69 searcher4 = Searcher(Search_from_CSP(cspExamples.crossword1d))
70 #print(searcher4.search()) # get next solution (warning: slow)
```

Exercise 4.8 What would happen if we constructed the new assignment by assigning $node[var] = val$ (with side effects) instead of using dictionary union? Give an example of where this could give a wrong answer. How could the algorithm be changed to work with side effects? (Hint: think about what information needs to be in a node).

Exercise 4.9 Change neighbors so that it returns an iterator of values rather than a list. (Hint: use *yield*.)

4.4 Consistency Algorithms

To run the demo, in folder "aipython", load "cspConsistency.py", and copy and paste the commented-out example queries at the bottom of that file.

A *Con_solver* is used to simplify a CSP using arc consistency.

cspConsistency.py — Arc Consistency and Domain splitting for solving a CSP

```
11 from display import Displayable
12
13 class Con_solver(Displayable):
```

```
    """Solves a CSP with arc consistency and domain splitting
    """
    def __init__(self, csp):
        """a CSP solver that uses arc consistency
        * csp is the CSP to be solved
        """
        self.csp = csp
        super().__init__()  # Or Displayable.__init__(self)
```

The following implementation of arc consistency maintains the set *to_do* of (variable, constraint) pairs that are to be checked. It takes in a domain dictionary and returns a new domain dictionary. It needs to be careful to avoid side effects (by copying the *domains* dictionary and the *to_do* set).

cspConsistency.py — (continued)

```
    def make_arc_consistent(self, domains=None, to_do=None):
        """Makes this CSP arc-consistent using generalized arc consistency
        domains is a variable:domain dictionary
        to_do is a set of (variable,constraint) pairs
        returns the reduced domains (an arc-consistent variable:domain
            dictionary)
        """
        if domains is None:
            self.domains = {var:var.domain for var in self.csp.variables}
        else:
            self.domains = domains.copy() # use a copy of domains
        if to_do is None:
            to_do = {(var, const) for const in self.csp.constraints
                    for var in const.scope}
        else:
            to_do = to_do.copy() # use a copy of to_do
        self.display(5,"Performing AC with domains", self.domains)
        while to_do:
            self.arc_selected = (var, const) = self.select_arc(to_do)
            self.display(5, "Processing arc (", var, ",", const, ")")
            other_vars = [ov for ov in const.scope if ov != var]
            new_domain = {val for val in self.domains[var]
                          if self.any_holds(self.domains, const, {var:
                              val}, other_vars)}
            if new_domain != self.domains[var]:
                self.add_to_do = self.new_to_do(var, const) - to_do
                self.display(3, f"Arc: ({var}, {const}) is inconsistent\n"
                             f"Domain pruned, dom({var}) ={new_domain} due to
                                 {const}")
                self.domains[var] = new_domain
                self.display(4, " adding", self.add_to_do if self.add_to_do
                             else "nothing", "to to_do.")
                to_do |= self.add_to_do   # set union
            self.display(5, f"Arc: ({var},{const}) now consistent")
        self.display(5, "AC done. Reduced domains", self.domains)
        return self.domains
```

```
56
57      def new_to_do(self, var, const):
58          """returns new elements to be added to to_do after assigning
59          variable var in constraint const.
60          """
61          return {(nvar, nconst) for nconst in self.csp.var_to_const[var]
62                  if nconst != const
63                  for nvar in nconst.scope
64                  if nvar != var}
```

The following selects an arc. Any element of *to_do* can be selected. The selected element needs to be removed from *to_do*. The default implementation just selects which ever element *pop* method for sets returns. The graphical user interface below allows the user to select an arc. Alternatively, a more sophisticated selection could be employed.

cspConsistency.py — (continued)

```
66      def select_arc(self, to_do):
67          """Selects the arc to be taken from to_do .
68          * to_do is a set of arcs, where an arc is a (variable,constraint)
              pair
69          the element selected must be removed from to_do.
70          """
71          return to_do.pop()
```

The value of `new_domain` is the subset of the domain of `var` that is consistent with the assignment to the other variables. To make it easier to understand, the following treats unary (with no other variables in the constraint) and binary (with one other variables in the constraint) constraints as special cases. These cases are not strictly necessary; the last case covers the first two cases, but is more difficult to understand without seeing the first two cases. Note that this case analysis is not in the code distribution, but can replace the assignment to `new_domain` above.

```
            if len(other_vars)==0:          # unary constraint
                new_domain = {val for val in self.domains[var]
                              if const.holds({var:val})}
            elif len(other_vars)==1:        # binary constraint
                other = other_vars[0]
                new_domain = {val for val in self.domains[var]
                       if any(const.holds({var: val,other:other_val})
                              for other_val in self.domains[other])}
            else:                           # general case
                new_domain = {val for val in self.domains[var]
                    if self.any_holds(self.domains, const, {var: val}, other_vars)}
```

any_holds is a recursive function that tries to finds an assignment of values to the other variables (*other_vars*) that satisfies constraint *const* given the assignment in *env*. The integer variable *ind* specifies which index to *other_vars* needs to be

checked next. As soon as one assignment returns *True,* the algorithm returns *True*.

cspConsistency.py — (continued)

```
    def any_holds(self, domains, const, env, other_vars, ind=0):
        """returns True if Constraint const holds for an assignment
        that extends env with the variables in other_vars[ind:]
        env is a dictionary
        """
        if ind == len(other_vars):
            return const.holds(env)
        else:
            var = other_vars[ind]
            for val in domains[var]:
                if self.any_holds(domains, const, env|{var:val}, other_vars,
                    ind + 1):
                    return True
            return False
```

4.4.1 Direct Implementation of Domain Splitting

The following is a direct implementation of domain splitting with arc consistency. It implements the generator interface of Python (see Section 1.5.4). When it has found a solution it `yield`s the result; otherwise it recursively splits a domain (using `yield from`).

cspConsistency.py — (continued)

```
    def generate_sols(self, domains=None, to_do=None, context=dict()):
        """return list of all solution to the current CSP
        to_do is the list of arcs to check
        context is a dictionary of splits made (used for display)
        """
        new_domains = self.make_arc_consistent(domains, to_do)
        if any(len(new_domains[var]) == 0 for var in new_domains):
            self.display(1,f"No solutions for context {context}")
        elif all(len(new_domains[var]) == 1 for var in new_domains):
            self.display(1, "solution:", str({var: select(
                new_domains[var]) for var in new_domains}))
            yield {var: select(new_domains[var]) for var in new_domains}
        else:
            var = self.select_var(x for x in self.csp.variables if
                len(new_domains[x]) > 1)
            dom1, dom2 = partition_domain(new_domains[var])
            self.display(5, "...splitting", var, "into", dom1, "and", dom2)
            new_doms1 = new_domains | {var:dom1}
            new_doms2 = new_domains | {var:dom2}
            to_do = self.new_to_do(var, None)
            self.display(4, " adding", to_do if to_do else "nothing", "to
                to_do.")
```

```
107             yield from self.generate_sols(new_doms1, to_do,
                    context|{var:dom1})
108             yield from self.generate_sols(new_doms2, to_do,
                    context|{var:dom1})
109
110     def solve_all(self, domains=None, to_do=None):
111         return list(self.generate_sols())
112
113     def solve_one(self, domains=None, to_do=None):
114         return select(self.generate_sols())
115
116     def select_var(self, iter_vars):
117         """return the next variable to split"""
118         return select(iter_vars)
119
120 def partition_domain(dom):
121     """partitions domain dom into two.
122     """
123     split = len(dom) // 2
124     dom1 = set(list(dom)[:split])
125     dom2 = dom - dom1
126     return dom1, dom2
```

cspConsistency.py — (continued)

```
128 def select(iterable):
129     """select an element of iterable. Returns None if there is no such
            element.
130
131     This implementation just picks the first element.
132     For many of the uses, which element is selected does not affect
            correctness,
133     but may affect efficiency.
134     """
135     for e in iterable:
136         return e # returns first element found
```

Exercise 4.10 Implement *solve_all* that returns the set of all solutions without using `yield`. Hint: it can be like `generate_sols` but returns a set of solutions; the recursive calls can be unioned; `|` is Python's union.

Exercise 4.11 Implement *solve_one* that returns one solution if one exists, or `False` otherwise, without using `yield`. Hint: Python's "or" has the behaviour `A or B` will return the value of `A` unless it is None or False, in which case the value of `B` is returned.

Unit test:

cspConsistency.py — (continued)

```
138 import cspExamples
139 def ac_solver(csp):
140     "arc consistency (ac_solver)"
```

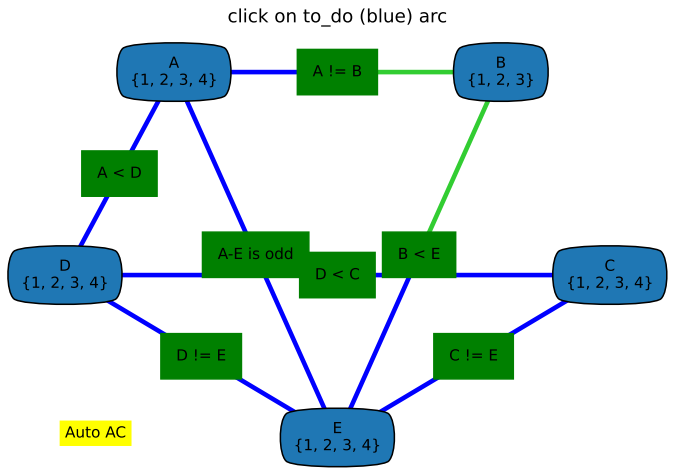


Figure 4.8: ConsistencyGUI(cspExamples.csp3).go()

```
141     for sol in Con_solver(csp).generate_sols():
142         return sol
143 
144 if __name__ == "__main__":
145     cspExamples.test_csp(ac_solver)
```

4.4.2 Consistency GUI

The consistency GUI allows students to step through the algorithm, choosing which arc to process next, and which variable to split.

Figure 4.8 shows the state of the GUI after two arcs have been made arc consistent. The arcs on the `to_do` list arc colored blue. The green arcs are those have been made arc consistent. The user can click on a blue arc to process that arc. If the arc selected is not arc consistent, it is made red, the domain is reduced, and then the arc becomes green. If the arc was already arc consistent it turns green.

This is implemented by overriding `select_arc` and `select_var` to allow the user to pick the arcs and the variables, and overriding `display` to allow for the animation. Note that the first argument of `display` (the number) in the code above is interpreted with a special meaning by the GUI and should only be changed with care.

Clicking `AutoAC` automates arc selection until the network is arc consistent.

cspConsistencyGUI.py — GUI for consistency-based CSP solving

```
11 from cspConsistency import Con_solver
12 import matplotlib.pyplot as plt
13 
```

```
class ConsistencyGUI(Con_solver):
    def __init__(self, csp, fontsize=10, speed=1, **kwargs):
        """
        csp is the csp to show
        fontsize is the size of the text
        speed is the number of animations per second (controls delay_time)
            1 (slow) and 4 (fast) seem like good values
        """
        self.fontsize = fontsize
        self.delay_time = 1/speed
        Con_solver.__init__(self, csp, **kwargs)
        csp.show(showAutoAC = True)

    def go(self):
        res = self.solve_all()
        self.csp.draw_graph(domains=self.domains,
                            title="No more solutions. GUI finished. ",
                            fontsize=self.fontsize)
        return res

    def select_arc(self, to_do):
        while True:
            self.csp.draw_graph(domains=self.domains, to_do=to_do,
                                title="click on to_do (blue) arc",
                                    fontsize=self.fontsize)
            while self.csp.picked == None and not self.csp.autoAC:
                plt.pause(0.01) # controls reaction time of GUI
            if self.csp.autoAC:
                break
            picked = self.csp.picked
            self.csp.picked = None
            if picked in to_do:
                to_do.remove(picked)
                print(f"{picked} picked")
                return picked
            else:
                print(f"{picked} not in to_do")
        if self.csp.autoAC:
            self.csp.draw_graph(domains=self.domains, to_do=to_do,
                                title="Auto AC", fontsize=self.fontsize)
            plt.pause(self.delay_time)
            return to_do.pop()

    def select_var(self, iter_vars):
        vars = list(iter_vars)
        while True:
            self.csp.draw_graph(domains=self.domains,
                                title="Arc consistent. Click node to
                                    split",
                                fontsize=self.fontsize)
```

```
            while self.csp.picked == None:
                plt.pause(0.01) # controls reaction time of GUI
            picked = self.csp.picked
            self.csp.picked = None
            self.csp.autoAC = False
            if picked in vars:
                #print("splitting",picked)
                return picked
            else:
                print(picked,"not in",vars)

    def display(self,n,*args,**nargs):
        if n <= self.max_display_level: # default display
            print(*args, **nargs)
        if n==1: # solution found or no solutions"
            self.csp.draw_graph(domains=self.domains, to_do=set(),
                                title=' '.join(args)+": click any node or
                                    arc to continue",
                                fontsize=self.fontsize)
            self.csp.autoAC = False
            while self.csp.picked == None and not self.csp.autoAC:
                plt.pause(0.01) # controls reaction time of GUI
            self.csp.picked = None
        elif n==2: # backtracking
            plt.title("backtracking: click any node or arc to continue")
            self.csp.autoAC = False
            while self.csp.picked == None and not self.csp.autoAC:
                plt.pause(0.01)
            self.csp.picked = None
        elif n==3: # inconsistent arc
            line = self.csp.thelines[self.arc_selected]
            line.set_color('red')
            line.set_linewidth(10)
            plt.pause(self.delay_time)
            line.set_color('limegreen')
            line.set_linewidth(self.csp.linewidth)
        #elif n==4 and self.add_to_do: # adding to to_do
        #    print("adding to to_do",self.add_to_do) ## highlight these arc

import cspExamples
# Try:
# ConsistencyGUI(cspExamples.csp1).go()
# ConsistencyGUI(cspExamples.csp3).go()
# ConsistencyGUI(cspExamples.csp3, speed=4, fontsize=15).go()
```

4.4.3 Domain Splitting as an interface to graph searching

An alternative implementation is to implement domain splitting in terms of the search abstraction of Chapter 3.

A node is a dictionary that maps the variables to their (pruned) domains..

cspConsistency.py — (continued)

```
from searchProblem import Arc, Search_problem

class Search_with_AC_from_CSP(Search_problem,Displayable):
    """A search problem with arc consistency and domain splitting

    A node is a CSP """
    def __init__(self, csp):
        self.cons = Con_solver(csp) #copy of the CSP
        self.domains = self.cons.make_arc_consistent()

    def is_goal(self, node):
        """node is a goal if all domains have 1 element"""
        return all(len(node[var])==1 for var in node)

    def start_node(self):
        return self.domains

    def neighbors(self,node):
        """returns the neighboring nodes of node.
        """
        neighs = []
        var = select(x for x in node if len(node[x])>1)
        if var:
            dom1, dom2 = partition_domain(node[var])
            self.display(2,"Splitting", var, "into", dom1, "and", dom2)
            to_do = self.cons.new_to_do(var,None)
            for dom in [dom1,dom2]:
                newdoms = node | {var:dom}
                cons_doms = self.cons.make_arc_consistent(newdoms,to_do)
                if all(len(cons_doms[v])>0 for v in cons_doms):
                    # all domains are non-empty
                    neighs.append(Arc(node,cons_doms))
                else:
                    self.display(2,"...",var,"in",dom,"has no solution")
        return neighs
```

Exercise 4.12 When splitting a domain, this code splits the domain into half, approximately in half (without any effort to make a sensible choice). Does it work better to split one element from a domain?

Unit test:

cspConsistency.py — (continued)

```
import cspExamples
from searchGeneric import Searcher

def ac_search_solver(csp):
    """arc consistency (search interface)"""
```

```
188     sol = Searcher(Search_with_AC_from_CSP(csp)).search()
189     if sol:
190         return {v:select(d) for (v,d) in sol.end().items()}
191 
192 if __name__ == "__main__":
193     cspExamples.test_csp(ac_search_solver)
```

Testing:

cspConsistency.py — (continued)

```
195 ## Test Solving CSPs with Arc consistency and domain splitting:
196 #Con_solver.max_display_level = 4 # display details of AC (0 turns off)
197 #Con_solver(cspExamples.csp1).solve_all()
198 #searcher1d = Searcher(Search_with_AC_from_CSP(cspExamples.csp1))
199 #print(searcher1d.search())
200 #Searcher.max_display_level = 2 # display search trace (0 turns off)
201 #searcher2c = Searcher(Search_with_AC_from_CSP(cspExamples.csp2))
202 #print(searcher2c.search())
203 #searcher3c = Searcher(Search_with_AC_from_CSP(cspExamples.crossword1))
204 #print(searcher3c.search())
205 #searcher4c = Searcher(Search_with_AC_from_CSP(cspExamples.crossword1d))
206 #print(searcher4c.search())
```

4.5 Solving CSPs using Stochastic Local Search

To run the demo, in folder "aipython", load "cspSLS.py", and copy and paste the commented-out example queries at the bottom of that file. This assumes Python 3. Some of the queries require `matplotlib`.

The following code implements the two-stage choice (select one of the variables that are involved in the most constraints that are violated, then a value), the any-conflict algorithm (select a variable that participates in a violated constraint) and a random choice of variable, as well as a probabilistic mix of the three.

Given a CSP, the stochastic local searcher (*SLSearcher*) creates the data structures:

- *variables_to_select* is the set of all of the variables with domain-size greater than one. For a variable not in this set, we cannot pick another value from that variable.
- *var_to_constraints* maps from a variable into the set of constraints it is involved in. Note that the inverse mapping from constraints into variables is part of the definition of a constraint.

cspSLS.py — Stochastic Local Search for Solving CSPs

```
11 from cspProblem import CSP, Constraint
```

```
from searchProblem import Arc, Search_problem
from display import Displayable
import random
import heapq

class SLSearcher(Displayable):
    """A search problem directly from the CSP..

    A node is a variable:value dictionary"""
    def __init__(self, csp):
        self.csp = csp
        self.variables_to_select = {var for var in self.csp.variables
                                    if len(var.domain) > 1}
        # Create assignment and conflicts set
        self.current_assignment = None # this will trigger a random restart
        self.number_of_steps = 0 #number of steps after the initialization
```

restart creates a new total assignment, and constructs the set of conflicts (the constraints that are false in this assignment).

cspSLS.py — (continued)

```
    def restart(self):
        """creates a new total assignment and the conflict set
        """
        self.current_assignment = {var:random_choice(var.domain) for
                                   var in self.csp.variables}
        self.display(2,"Initial assignment",self.current_assignment)
        self.conflicts = set()
        for con in self.csp.constraints:
            if not con.holds(self.current_assignment):
                self.conflicts.add(con)
        self.display(2,"Number of conflicts",len(self.conflicts))
        self.variable_pq = None
```

The *search* method is the top-level searching algorithm. It can either be used to start the search or to continue searching. If there is no current assignment, it must create one. Note that, when counting steps, a restart is counted as one step, which is not appropriate for CSPs with many variables, as it is a relatively expensive operation for these cases.

This method selects one of two implementations. The argument *pob_best* is the probability of selecting a best variable (one involving the most conflicts). When the value of *prob_best* is positive, the algorithm needs to maintain a priority queue of variables and the number of conflicts (using *search_with_var_pq*). If the probability of selecting a best variable is zero, it does not need to maintain this priority queue (as implemented in *search_with_any_conflict*).

The argument *prob_anycon* is the probability that the any-conflict strategy is used (which selects a variable at random that is in a conflict), assuming that it is not picking a best variable. Note that for the probability parameters, any value less that zero acts like probability zero and any value greater than 1 acts

like probability 1. This means that when *prob_anycon* $= 1.0$, a best variable is chosen with probability *prob_best*, otherwise a variable in any conflict is chosen. A variable is chosen at random with probability $1 - prob\_anycon - prob\_best$ as long as that is positive.

This returns the number of steps needed to find a solution, or *None* if no solution is found. If there is a solution, it is in *self.current_assignment*.

cspSLS.py — (continued)

```
42    def search(self,max_steps, prob_best=0, prob_anycon=1.0):
43        """
44        returns the number of steps or None if these is no solution.
45        If there is a solution, it can be found in self.current_assignment
46
47        max_steps is the maximum number of steps it will try before giving
              up
48        prob_best is the probability that a best variable (one in most
              conflict) is selected
49        prob_anycon is the probability that a variable in any conflict is
              selected
50        (otherwise a variable is chosen at random)
51        """
52        if self.current_assignment is None:
53            self.restart()
54            self.number_of_steps += 1
55            if not self.conflicts:
56                self.display(1,"Solution found:", self.current_assignment,
                      "after restart")
57                return self.number_of_steps
58        if prob_best > 0: # we need to maintain a variable priority queue
59            return self.search_with_var_pq(max_steps, prob_best,
                  prob_anycon)
60        else:
61            return self.search_with_any_conflict(max_steps, prob_anycon)
```

Exercise 4.13 This does an initial random assignment but does not do any random restarts. Implement a searcher that takes in the maximum number of walk steps (corresponding to existing *max_steps*) and the maximum number of restarts, and returns the total number of steps for the first solution found. (As in *search*, the solution found can be extracted from the variable *self.current_assignment*).

4.5.1 Any-conflict

In the any-conflict heuristic a variable that participates in a violated constraint is picked at random. The implementation need to keeps track of which variables are in conflicts. This is can avoid the need for a priority queue that is needed when the probability of picking a best variable is greter than zero.

cspSLS.py — (continued)

```
63    def search_with_any_conflict(self, max_steps, prob_anycon=1.0):
```

```
        """Searches with the any_conflict heuristic.
        This relies on just maintaining the set of conflicts;
        it does not maintain a priority queue
        """
        self.variable_pq = None # we are not maintaining the priority queue.
                                # This ensures it is regenerated if
                                #   we call search_with_var_pq.
        for i in range(max_steps):
            self.number_of_steps +=1
            if  random.random() < prob_anycon:
                con = random_choice(self.conflicts) # pick random conflict
                var = random_choice(con.scope) # pick variable in conflict
            else:
                var = random_choice(self.variables_to_select)
            if len(var.domain) > 1:
                val = random_choice([val for val in var.domain
                                     if val is not
                                         self.current_assignment[var]])
                self.display(2,self.number_of_steps,":
                    Assigning",var,"=",val)
                self.current_assignment[var]=val
                for varcon in self.csp.var_to_const[var]:
                    if varcon.holds(self.current_assignment):
                        if varcon in self.conflicts:
                            self.conflicts.remove(varcon)
                    else:
                        if varcon not in self.conflicts:
                            self.conflicts.add(varcon)
                self.display(2,"   Number of conflicts",len(self.conflicts))
            if not self.conflicts:
                self.display(1,"Solution found:", self.current_assignment,
                               "in", self.number_of_steps,"steps")
                return self.number_of_steps
        self.display(1,"No solution in",self.number_of_steps,"steps",
                   len(self.conflicts),"conflicts remain")
        return None
```

Exercise 4.14 This makes no attempt to find the best value for the variable selected. Modify the code to include an option selects a value for the selected variable that reduces the number of conflicts the most. Have a parameter that specifies the probability that the best value is chosen, and otherwise chooses a value at random.

4.5.2 Two-Stage Choice

This is the top-level searching algorithm that maintains a priority queue of variables ordered by the number of conflicts, so that the variable with the most conflicts is selected first. If there is no current priority queue of variables, one is created.

The main complexity here is to maintain the priority queue. When a variable var is assigned a value val, for each constraint that has become satisfied or unsatisfied, each variable involved in the constraint need to have its count updated. The change is recorded in the dictionary *var_differential*, which is used to update the priority queue (see Section 4.5.3).

cspSLS.py — (continued)

```
99      def search_with_var_pq(self,max_steps, prob_best=1.0, prob_anycon=1.0):
100         """search with a priority queue of variables.
101         This is used to select a variable with the most conflicts.
102         """
103         if not self.variable_pq:
104             self.create_pq()
105         pick_best_or_con = prob_best + prob_anycon
106         for i in range(max_steps):
107             self.number_of_steps +=1
108             randnum = random.random()
109             ## Pick a variable
110             if randnum < prob_best: # pick best variable
111                 var,oldval = self.variable_pq.top()
112             elif randnum < pick_best_or_con: # pick a variable in a conflict
113                 con = random_choice(self.conflicts)
114                 var = random_choice(con.scope)
115             else: #pick any variable that can be selected
116                 var = random_choice(self.variables_to_select)
117             if len(var.domain) > 1: # var has other values
118                 ## Pick a value
119                 val = random_choice([val for val in var.domain if val is not
120                                      self.current_assignment[var]])
121                 self.display(2,"Assigning",var,val)
122                 ## Update the priority queue
123                 var_differential = {}
124                 self.current_assignment[var]=val
125                 for varcon in self.csp.var_to_const[var]:
126                     self.display(3,"Checking",varcon)
127                     if varcon.holds(self.current_assignment):
128                         if varcon in self.conflicts: #was incons, now consis
129                             self.display(3,"Became consistent",varcon)
130                             self.conflicts.remove(varcon)
131                             for v in varcon.scope: # v is in one fewer
                                    conflicts
132                                 var_differential[v] =
                                        var_differential.get(v,0)-1
133                     else:
134                         if varcon not in self.conflicts: # was consis, not now
135                             self.display(3,"Became inconsistent",varcon)
136                             self.conflicts.add(varcon)
137                             for v in varcon.scope: # v is in one more
                                    conflicts
138                                 var_differential[v] =
```

```
                                    var_differential.get(v,0)+1
139             self.variable_pq.update_each_priority(var_differential)
140             self.display(2,"Number of conflicts",len(self.conflicts))
141         if not self.conflicts: # no conflicts, so solution found
142             self.display(1,"Solution found:",
                    self.current_assignment,"in",
143                         self.number_of_steps,"steps")
144             return self.number_of_steps
145     self.display(1,"No solution in",self.number_of_steps,"steps",
146                  len(self.conflicts),"conflicts remain")
147     return None
```

create_pq creates an updatable priority queue of the variables, ordered by the number of conflicts they participate in. The priority queue only includes variables in conflicts and the value of a variable is the *negative* of the number of conflicts the variable is in. This ensures that the priority queue, which picks the minimum value, picks a variable with the most conflicts.

cspSLS.py — (continued)

```
149 def create_pq(self):
150     """Create the variable to number-of-conflicts priority queue.
151     This is needed to select the variable in the most conflicts.
152
153     The value of a variable in the priority queue is the negative of the
154     number of conflicts the variable appears in.
155     """
156     self.variable_pq = Updatable_priority_queue()
157     var_to_number_conflicts = {}
158     for con in self.conflicts:
159         for var in con.scope:
160             var_to_number_conflicts[var] =
                    var_to_number_conflicts.get(var,0)+1
161     for var,num in var_to_number_conflicts.items():
162         if num>0:
163             self.variable_pq.add(var,-num)
```

cspSLS.py — (continued)

```
165 def random_choice(st):
166     """selects a random element from set st.
167     It would be more efficient to convert to a tuple or list only once
168     (left as exercise)."""
169     return random.choice(tuple(st))
```

Exercise 4.15 These implementations always select a value for the variable selected that is different from its current value (if that is possible). Change the code so that it does not have this restriction (so it can leave the value the same). Would you expect this code to be faster? Does it work worse (or better)?

4.5.3 Updatable Priority Queues

An **updatable priority queue** is a priority queue, where key-value pairs can be stored, and the pair with the smallest key can be found and removed quickly, and where the values can be updated. This implementation follows the idea of `http://docs.python.org/3.9/library/heapq.html`, where the updated elements are marked as removed. This means that the priority queue can be used unmodified. However, this might be expensive if changes are more common than popping (as might happen if the probability of choosing the best is close to zero).

In this implementation, the equal values are sorted randomly. This is achieved by having the elements of the heap being $[val, rand, elt]$ triples, where the second element is a random number. Note that Python requires this to be a list, not a tuple, as the tuple cannot be modified.

cspSLS.py — (continued)

```
class Updatable_priority_queue(object):
    """A priority queue where the values can be updated.
    Elements with the same value are ordered randomly.

    This code is based on the ideas described in
    http://docs.python.org/3.3/library/heapq.html
    It could probably be done more efficiently by
    shuffling the modified element in the heap.
    """
    def __init__(self):
        self.pq = []  # priority queue of [val,rand,elt] triples
        self.elt_map = {} # map from elt to [val,rand,elt] triple in pq
        self.REMOVED = "*removed*" # a string that won't be a legal element
        self.max_size=0

    def add(self,elt,val):
        """adds elt to the priority queue with priority=val.
        """
        assert val <= 0,val
        assert elt not in self.elt_map, elt
        new_triple = [val, random.random(),elt]
        heapq.heappush(self.pq, new_triple)
        self.elt_map[elt] = new_triple

    def remove(self,elt):
        """remove the element from the priority queue"""
        if elt in self.elt_map:
            self.elt_map[elt][2] = self.REMOVED
            del self.elt_map[elt]

    def update_each_priority(self,update_dict):
        """update values in the priority queue by subtracting the values in
        update_dict from the priority of those elements in priority queue.
```

```
204         """
205         for elt,incr in update_dict.items():
206             if incr != 0:
207                 newval = self.elt_map.get(elt,[0])[0] - incr
208                 assert newval <= 0, f"{elt}:{newval+incr}-{incr}"
209                 self.remove(elt)
210                 if newval != 0:
211                     self.add(elt,newval)
212 
213     def pop(self):
214         """Removes and returns the (elt,value) pair with minimal value.
215         If the priority queue is empty, IndexError is raised.
216         """
217         self.max_size = max(self.max_size, len(self.pq)) # keep statistics
218         triple = heapq.heappop(self.pq)
219         while triple[2] == self.REMOVED:
220             triple = heapq.heappop(self.pq)
221         del self.elt_map[triple[2]]
222         return triple[2], triple[0] # elt, value
223 
224     def top(self):
225         """Returns the (elt,value) pair with minimal value, without
                removing it.
226         If the priority queue is empty, IndexError is raised.
227         """
228         self.max_size = max(self.max_size, len(self.pq)) # keep statistics
229         triple = self.pq[0]
230         while triple[2] == self.REMOVED:
231             heapq.heappop(self.pq)
232             triple = self.pq[0]
233         return triple[2], triple[0] # elt, value
234 
235     def empty(self):
236         """returns True iff the priority queue is empty"""
237         return all(triple[2] == self.REMOVED for triple in self.pq)
```

4.5.4 Plotting Run-Time Distributions

Runtime_distribution uses matplotlib to plot run time distributions. Here the run time is a misnomer as we are only plotting the number of steps, not the time. Computing the run time is non-trivial as many of the runs have a very short run time. To compute the time accurately would require running the same code, with the same random seed, multiple times to get a good estimate of the run time. This is left as an exercise.

cspSLS.py — (continued)

```
239 import matplotlib.pyplot as plt
240 # plt.style.use('grayscale')
241 
```

```
class Runtime_distribution(object):
    def __init__(self, csp, xscale='log'):
        """Sets up plotting for csp
        xscale is either 'linear' or 'log'
        """
        self.csp = csp
        plt.ion()
        plt.xlabel("Number of Steps")
        plt.ylabel("Cumulative Number of Runs")
        plt.xscale(xscale) # Makes a 'log' or 'linear' scale

    def plot_runs(self,num_runs=100,max_steps=1000, prob_best=1.0,
        prob_anycon=1.0):
        """Plots num_runs of SLS for the given settings.
        """
        stats = []
        SLSearcher.max_display_level, temp_mdl = 0,
            SLSearcher.max_display_level # no display
        for i in range(num_runs):
            searcher = SLSearcher(self.csp)
            num_steps = searcher.search(max_steps, prob_best, prob_anycon)
            if num_steps:
                stats.append(num_steps)
        stats.sort()
        if prob_best >= 1.0:
            label = "P(best)=1.0"
        else:
            p_ac = min(prob_anycon, 1-prob_best)
            label = "P(best)=%.2f, P(ac)=%.2f" % (prob_best, p_ac)
        plt.plot(stats,range(len(stats)),label=label)
        plt.legend(loc="upper left")
        SLSearcher.max_display_level= temp_mdl #restore display
```

Figure 4.9 gives run-time distributions for 3 algorithms. It is also useful to compare the distributions of different runs of the same algorithms and settings.

4.5.5 Testing

cspSLS.py — (continued)

```
import cspExamples
def sls_solver(csp,prob_best=0.7):
    """stochastic local searcher (prob_best=0.7)"""
    se0 = SLSearcher(csp)
    se0.search(1000,prob_best)
    return se0.current_assignment
def any_conflict_solver(csp):
    """stochastic local searcher (any-conflict)"""
    return sls_solver(csp,0)

```

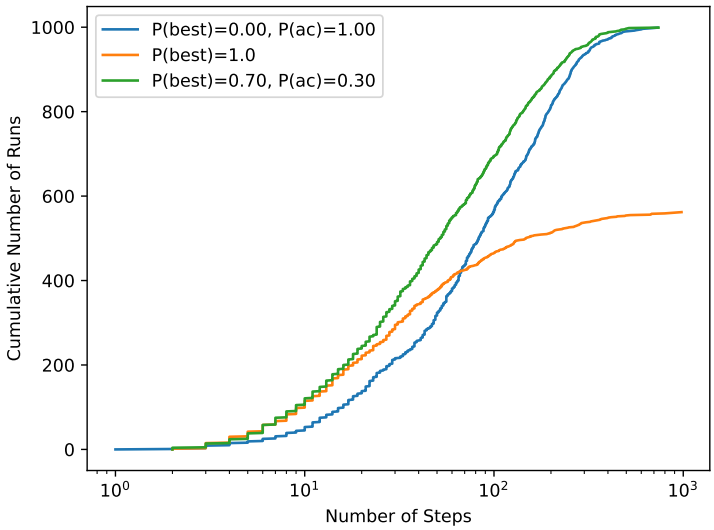


Figure 4.9: Run-time distributions for three algorithms on *csp*2.

```
283 if __name__ == "__main__":
284     cspExamples.test_csp(sls_solver)
285     cspExamples.test_csp(any_conflict_solver)
286
287 ## Test Solving CSPs with Search:
288 #se1 = SLSearcher(cspExamples.csp1); print(se1.search(100))
289 #se2 = SLSearcher(cspExamples.csp2); print(se2.search(1000,1.0)) # greedy
290 #se2 = SLSearcher(cspExamples.csp2); print(se2.search(1000,0)) #
        any_conflict
291 #se2 = SLSearcher(cspExamples.csp2); print(se2.search(1000,0.7)) # 70%
        greedy; 30% any_conflict
292 #SLSearcher.max_display_level=2 #more detailed display
293 #se3 = SLSearcher(cspExamples.crossword1); print(se3.search(100),0.7)
294 #p = Runtime_distribution(cspExamples.csp2)
295 #p.plot_runs(1000,1000,0) # any_conflict
296 #p.plot_runs(1000,1000,1.0) # greedy
297 #p.plot_runs(1000,1000,0.7) # 70% greedy; 30% any_conflict
```

Exercise 4.16 Modify this to plot the run time, instead of the number of steps. To measure run time use *timeit* (`https://docs.python.org/3.9/library/timeit.html`). Small run times are inaccurate, so timeit can run the same code multiple times. Stochastic local algorithms give different run times each time called. To make the timing meaningful, you need to make sure the random seed is the same for each repeated call (see `random.getstate` and `random.setstate` in `https://docs.python.org/3.9/library/random.html`). Because the run time for different seeds can vary a great deal, for each seed, you should start with 1 iteration and multiplying it by, say 10, until the time is greater than 0.2 seconds. Make sure you

plot the average time for each run. Before you start, try to estimate the total run time, so you will be able to tell if there is a problem with the algorithm stopping.

4.6 Discrete Optimization

A `SoftConstraint` is a constraint, but where the condition is a real-valued function. Because the definition of the constraint class did not force the condition to be Boolean, you can use the `Constraint` class for soft constraints too.

cspSoft.py — Representations of Soft Constraints

```
from cspProblem import Variable, Constraint, CSP
class SoftConstraint(Constraint):
    """A Constraint consists of
    * scope: a tuple of variables
    * function: a real-valued function that can applied to a tuple of values
    * string: a string for printing the constraints. All of the strings
        must be unique.
    for the variables
    """
    def __init__(self, scope, function, string=None, position=None):
        Constraint.__init__(self, scope, function, string, position)

    def value(self,assignment):
        return self.holds(assignment)
```

cspSoft.py — (continued)

```
A = Variable('A', {1,2}, position=(0.2,0.9))
B = Variable('B', {1,2,3}, position=(0.8,0.9))
C = Variable('C', {1,2}, position=(0.5,0.5))
D = Variable('D', {1,2}, position=(0.8,0.1))

def c1fun(a,b):
    if a==1: return (5 if b==1 else 2)
    else: return (0 if b==1 else 4 if b==2 else 3)
c1 = SoftConstraint([A,B],c1fun,"c1")
def c2fun(b,c):
    if b==1: return (5 if c==1 else 2)
    elif b==2: return (0 if c==1 else 4)
    else: return (2 if c==1 else 0)
c2 = SoftConstraint([B,C],c2fun,"c2")
def c3fun(b,d):
    if b==1: return (3 if d==1 else 0)
    elif b==2: return 2
    else: return (2 if d==1 else 4)
c3 = SoftConstraint([B,D],c3fun,"c3")

def penalty_if_same(pen):
    "returns a function that gives a penalty of pen if the arguments are
        the same"
```

```
47      return lambda x,y: (pen if (x==y) else 0)
48
49  c4 = SoftConstraint([C,A],penalty_if_same(3),"c4")
50
51  scsp1 = CSP("scsp1", {A,B,C,D}, [c1,c2,c3,c4])
52
53  ### The second soft CSP has an extra variable, and 2 constraints
54  E = Variable('E', {1,2}, position=(0.1,0.1))
55
56  c5 = SoftConstraint([C,E],penalty_if_same(3),"c5")
57  c6 = SoftConstraint([D,E],penalty_if_same(2),"c6")
58  scsp2 = CSP("scsp1", {A,B,C,D,E}, [c1,c2,c3,c4,c5,c6])
```

4.6.1 Branch-and-bound Search

Here we specialize the branch-and-bound algorithm (Section 3.3 on page 64) to solve soft CSP problems.

cspSoft.py — (continued)

```
60  from display import Displayable, visualize
61  import math
62
63  class DF_branch_and_bound_opt(Displayable):
64      """returns a branch and bound searcher for a problem.
65      An optimal assignment with cost less than bound can be found by calling
            search()
66      """
67      def __init__(self, csp, bound=math.inf):
68          """creates a searcher than can be used with search() to find an
                optimal path.
69          bound gives the initial bound. By default this is infinite -
                meaning there
70          is no initial pruning due to depth bound
71          """
72          super().__init__()
73          self.csp = csp
74          self.best_asst = None
75          self.bound = bound
76
77      def optimize(self):
78          """returns an optimal solution to a problem with cost less than
                bound.
79          returns None if there is no solution with cost less than bound."""
80          self.num_expanded=0
81          self.cbsearch({}, 0, self.csp.constraints)
82          self.display(1,"Number of paths expanded:",self.num_expanded)
83          return self.best_asst, self.bound
84
85      def cbsearch(self, asst, cost, constraints):
```

```
        """finds the optimal solution that extends path and is less the
            bound"""
        self.display(2,"cbsearch:",asst,cost,constraints)
        can_eval = [c for c in constraints if c.can_evaluate(asst)]
        rem_cons = [c for c in constraints if c not in can_eval]
        newcost = cost + sum(c.value(asst) for c in can_eval)
        self.display(2,"Evaluaing:",can_eval,"cost:",newcost)
        if newcost < self.bound:
            self.num_expanded += 1
            if rem_cons==[]:
                self.best_asst = asst
                self.bound = newcost
                self.display(1,"New best assignment:",asst," cost:",newcost)
            else:
                var = next(var for var in self.csp.variables if var not in
                    asst)
                for val in var.domain:
                    self.cbsearch({var:val}|asst, newcost, rem_cons)

# bnb = DF_branch_and_bound_opt(scsp1)
# bnb.max_display_level=3 # show more detail
# bnb.optimize()
```

Exercise 4.17 Change the stochastic-local search algorithms to work for soft constraints. Hint: The analog of a conflict is a soft constraint that is not at its lowest value. Instead of the number of constraints violated, consider how much a change in a variable affects the objective function. Instead of returning a solution, return the best assignment found.

Chapter 5

Propositions and Inference

5.1 Representing Knowledge Bases

A clause consists of a head (an atom) and a body. A body is represented as a list of atoms. Atoms are represented as strings.

logicProblem.py — Representations Logics

```
11 class Clause(object):
12     """A definite clause"""
13
14     def __init__(self,head,body=[]):
15         """clause with atom head and lost of atoms body"""
16         self.head=head
17         self.body = body
18
19     def __repr__(self):
20         """returns the string representation of a clause.
21         """
22         if self.body:
23             return f"{self.head} <- {' & '.join(str(a) for a in
                   self.body)}."
24         else:
25             return f"{self.head}."
```

An askable atom can be asked of the user. The user can respond in English or French or just with a "y".

logicProblem.py — (continued)

```
27 class Askable(object):
28     """An askable atom"""
29
30     def __init__(self,atom):
```

```
31          """clause with atom head and lost of atoms body"""
32          self.atom=atom
33
34      def __str__(self):
35          """returns the string representation of a clause."""
36          return "askable " + self.atom + "."
37
38  def yes(ans):
39      """returns true if the answer is yes in some form"""
40      return ans.lower() in ['yes', 'oui', 'y'] # bilingual
```

A knowledge base is a list of clauses and askables. In order to make top-down inference faster, this creates a dictionary that maps each atom into the set of clauses with that atom in the head.

logicProblem.py — (continued)

```
42  from display import Displayable
43
44  class KB(Displayable):
45      """A knowledge base consists of a set of clauses.
46      This also creates a dictionary to give fast access to the clauses with
            an atom in head.
47      """
48      def __init__(self, statements=[]):
49          self.statements = statements
50          self.clauses = [c for c in statements if isinstance(c, Clause)]
51          self.askables = [c.atom for c in statements if isinstance(c,
                Askable)]
52          self.atom_to_clauses = {} # dictionary giving clauses with atom as
                head
53          for c in self.clauses:
54              self.add_clause(c)
55
56      def add_clause(self, c):
57          if c.head in self.atom_to_clauses:
58              self.atom_to_clauses[c.head].append(c)
59          else:
60              self.atom_to_clauses[c.head] = [c]
61
62      def clauses_for_atom(self,a):
63          """returns list of clauses with atom a as the head"""
64          if a in self.atom_to_clauses:
65              return self.atom_to_clauses[a]
66          else:
67              return []
68
69      def __str__(self):
70          """returns a string representation of this knowledge base.
71          """
72          return '\n'.join([str(c) for c in self.statements])
```

Here is a trivial example (I think therefore I am) used in the unit tests:

logicProblem.py — (continued)

```
triv_KB = KB([
    Clause('i_am', ['i_think']),
    Clause('i_think'),
    Clause('i_smell', ['i_exist'])
    ])
```

Here is a representation of the electrical domain of the textbook:

logicProblem.py — (continued)

```
elect = KB([
    Clause('light_l1'),
    Clause('light_l2'),
    Clause('ok_l1'),
    Clause('ok_l2'),
    Clause('ok_cb1'),
    Clause('ok_cb2'),
    Clause('live_outside'),
    Clause('live_l1', ['live_w0']),
    Clause('live_w0', ['up_s2','live_w1']),
    Clause('live_w0', ['down_s2','live_w2']),
    Clause('live_w1', ['up_s1', 'live_w3']),
    Clause('live_w2', ['down_s1','live_w3' ]),
    Clause('live_l2', ['live_w4']),
    Clause('live_w4', ['up_s3','live_w3' ]),
    Clause('live_p_1', ['live_w3']),
    Clause('live_w3', ['live_w5', 'ok_cb1']),
    Clause('live_p_2', ['live_w6']),
    Clause('live_w6', ['live_w5', 'ok_cb2']),
    Clause('live_w5', ['live_outside']),
    Clause('lit_l1', ['light_l1', 'live_l1', 'ok_l1']),
    Clause('lit_l2', ['light_l2', 'live_l2', 'ok_l2']),
    Askable('up_s1'),
    Askable('down_s1'),
    Askable('up_s2'),
    Askable('down_s2'),
    Askable('up_s3'),
    Askable('down_s2')
    ])

# print(kb)
```

The following knowledge base is false in the intended interpretation. One of the clauses is wrong; can you see which one? We will show how to debug it.

logicProblem.py — (continued)

```
elect_bug = KB([
    Clause('light_l2'),
    Clause('ok_l1'),
    Clause('ok_l2'),
```

```
    Clause('ok_cb1'),
    Clause('ok_cb2'),
    Clause('live_outside'),
    Clause('live_p_2', ['live_w6']),
    Clause('live_w6', ['live_w5', 'ok_cb2']),
    Clause('light_l1'),
    Clause('live_w5', ['live_outside']),
    Clause('lit_l1', ['light_l1', 'live_l1', 'ok_l1']),
    Clause('lit_l2', ['light_l2', 'live_l2', 'ok_l2']),
    Clause('live_l1', ['live_w0']),
    Clause('live_w0', ['up_s2','live_w1']),
    Clause('live_w0', ['down_s2','live_w2']),
    Clause('live_w1', ['up_s3', 'live_w3']),
    Clause('live_w2', ['down_s1','live_w3' ]),
    Clause('live_l2', ['live_w4']),
    Clause('live_w4', ['up_s3','live_w3' ]),
    Clause('live_p_1', ['live_w3']),
    Clause('live_w3', ['live_w5', 'ok_cb1']),
    Askable('up_s1'),
    Askable('down_s1'),
    Askable('up_s2'),
    Clause('light_l2'),
    Clause('ok_l1'),
    Clause('light_l2'),
    Clause('ok_l1'),
    Clause('ok_l2'),
    Clause('ok_cb1'),
    Clause('ok_cb2'),
    Clause('live_outside'),
    Clause('live_p_2', ['live_w6']),
    Clause('live_w6', ['live_w5', 'ok_cb2']),
    Clause('ok_l2'),
    Clause('ok_cb1'),
    Clause('ok_cb2'),
    Clause('live_outside'),
    Clause('live_p_2', ['live_w6']),
    Clause('live_w6', ['live_w5', 'ok_cb2']),
    Askable('down_s2'),
    Askable('up_s3'),
    Askable('down_s2')
    ])

# print(kb)
```

5.2 Bottom-up Proofs (with askables)

fixed_point computes the fixed point of the knowledge base *kb*.

logicBottomUp.py — Bottom-up Proof Procedure for Definite Clauses

```
from logicProblem import yes

def fixed_point(kb):
    """Returns the fixed point of knowledge base kb.
    """
    fp = ask_askables(kb)
    added = True
    while added:
        added = False  # added is true when an atom was added to fp this
            iteration
        for c in kb.clauses:
            if c.head not in fp and all(b in fp for b in c.body):
                fp.add(c.head)
                added = True
                kb.display(2,c.head,"added to fp due to clause",c)
    return fp

def ask_askables(kb):
    return {at for at in kb.askables if yes(input("Is "+at+" true? "))}
```

The following provides a trivial **unit test**, by default using the knowledge base `triv_KB`:

logicBottomUp.py — (continued)

```
from logicProblem import triv_KB
def test(kb=triv_KB, fixedpt = {'i_am','i_think'}):
    fp = fixed_point(kb)
    assert fp == fixedpt, f"kb gave result {fp}"
    print("Passed unit test")
if __name__ == "__main__":
    test()

from logicProblem import elect
# elect.max_display_level=3 # give detailed trace
# fixed_point(elect)
```

Exercise 5.1 It is not very user-friendly to ask all of the askables up-front. Implement ask-the-user so that questions are only asked if useful, and are not re-asked. For example, if there is a clause $h \leftarrow a \wedge b \wedge c \wedge d \wedge e$, where c and e are askable, c and e only need to be asked if a, b, d are all in fp and they have not been asked before. Askable e only needs to be asked if the user says "yes" to c. Askable c doesn't need to be asked if the user previously replied "no" to e.

This form of ask-the-user can ask a different set of questions than the top-down interpreter that asks questions when encountered. Give an example where they ask different questions (neither set of questions asked is a subset of the other).

Exercise 5.2 This algorithm runs in time $O(n^2)$, where n is the number of clauses, for a bounded number of elements in the body; each iteration goes through each of the clauses, and in the worst case, it will do an iteration for each clause. It is possible to implement this in time $O(n)$ time by creating an index that maps an atom to the set of clauses with that atom in the body. Implement this. What is its

complexity as a function of n and b, the maximum number of atoms in the body of a clause?

Exercise 5.3 It is possible to be asymptotically more efficient (in terms of the number of elements in a body) than the method in the previous question by noticing that each element of the body of clause only needs to be checked once. For example, the clause $a \leftarrow b \wedge c \wedge d$, needs only be considered when b is added to fp. Once b is added to fp, if c is already in fp, we know that a can be added as soon as d is added. Implement this. What is its complexity as a function of n and b, the maximum number of atoms in the body of a clause?

5.3 Top-down Proofs (with askables)

The following implements the top-down proof procedure for propositional definite clauses, as described in Section 5.3.2 and Figure 5.4 of Poole and Mackworth [2023]. It implements "choose" by looping over the alternatives (using Python's any) and returning true if any choice leads to a proof.

prove(*kb*, *goal*) is used to prove *goal* from a knowledge base, *kb*, where a *goal* is a list of atoms. It returns *True* if $kb \vdash goal$. The *indent* is used when displaying the code (and doesn't need to be called initially with a non-default value).

logicTopDown.py — Top-down Proof Procedure for Definite Clauses

```
from logicProblem import yes

def prove(kb, ans_body, indent=""):
    """returns True if kb |- ans_body
    ans_body is a list of atoms to be proved
    """
    kb.display(2,indent,'yes <-',' & '.join(ans_body))
    if ans_body:
        selected = ans_body[0] # select first atom from ans_body
        if selected in kb.askables:
            return (yes(input("Is "+selected+" true? "))
                    and  prove(kb,ans_body[1:],indent+" "))
        else:
            return any(prove(kb,cl.body+ans_body[1:],indent+" ")
                       for cl in kb.clauses_for_atom(selected))
    else:
        return True  # empty body is true
```

The following provides a simple **unit test** that is hard wired for `triv_KB`:

logicTopDown.py — (continued)

```
from logicProblem import triv_KB
def test():
    a1 = prove(triv_KB,['i_am'])
    assert a1, f"triv_KB proving i_am gave {a1}"
    a2 = prove(triv_KB,['i_smell'])
    assert not a2, f"triv_KB proving i_smell gave {a2}"
```

```
35      print("Passed unit tests")
36  if __name__ == "__main__":
37      test()
38  # try
39  from logicProblem import elect
40  # elect.max_display_level=3 # give detailed trace
41  # prove(elect,['live_w6'])
42  # prove(elect,['lit_l1'])
```

Exercise 5.4 This code can re-ask a question multiple times. Implement this code so that it only asks a question once and remembers the answer. Also implement a function to forget the answers.

Exercise 5.5 What search method is this using? Implement the search interface so that it can use A^* or other searching methods. Define an admissible heuristic that is not always 0.

5.4 Debugging and Explanation

Here we modify the top-down procedure to build a proof tree than can be traversed for explanation and debugging.

`prove_atom(kb,atom)` returns a proof for *atom* from a knowledge base *kb,* where a proof is a pair of the atom and the proofs for the elements of the body of the clause used to prove the atom. `prove_body(kb,body)` returns a list of proofs for list *body* from a knowledge base, *kb*. The *indent* is used when displaying the code (and doesn't need to have a non-default value).

logicExplain.py — Explaining Proof Procedure for Definite Clauses

```
11  from logicProblem import yes # for asking the user
12
13  def prove_atom(kb, atom, indent=""):
14      """returns a pair (atom,proofs) where proofs is the list of proofs
15         of the elements of a body of a clause used to prove atom.
16      """
17      kb.display(2,indent,'proving',atom)
18      if atom in kb.askables:
19          if yes(input("Is "+atom+" true? ")):
20              return (atom,"answered")
21          else:
22              return "fail"
23      else:
24          for cl in kb.clauses_for_atom(atom):
25              kb.display(2,indent,"trying",atom,'<-',' & '.join(cl.body))
26              pr_body = prove_body(kb, cl.body, indent)
27              if pr_body != "fail":
28                  return (atom, pr_body)
29          return "fail"
30
31  def prove_body(kb, ans_body, indent=""):
```

```
32      """returns proof tree if kb |- ans_body or "fail" if there is no proof
33      ans_body is a list of atoms in a body to be proved
34      """
35      proofs = []
36      for atom in ans_body:
37          proof_at = prove_atom(kb, atom, indent+" ")
38          if proof_at == "fail":
39              return "fail" # fail if any proof fails
40          else:
41              proofs.append(proof_at)
42      return proofs
```

The following provides a simple **unit test** that is hard wired for `triv_KB`:

logicExplain.py — (continued)

```
44  from logicProblem import triv_KB
45  def test():
46      a1 = prove_atom(triv_KB,'i_am')
47      assert a1, f"triv_KB proving i_am gave {a1}"
48      a2 = prove_atom(triv_KB,'i_smell')
49      assert a2=="fail", "triv_KB proving i_smell gave {a2}"
50      print("Passed unit tests")
51
52  if __name__ == "__main__":
53      test()
54
55  # try
56  from logicProblem import elect, elect_bug
57  # elect.max_display_level=3 # give detailed trace
58  # prove_atom(elect, 'live_w6')
59  # prove_atom(elect, 'lit_l1')
```

The `interact(kb)` provides an interactive interface to explore proofs for knowledge base `kb`. The user can ask to prove atoms and can ask `how` an atom was proved.

To ask `how`, there must be a current atom for which there is a proof. This starts as the atom asked. When the user asks "how n" the current atom becomes the n-th element of the body of the clause used to prove the (previous) current atom. The command "up" makes the current atom the atom in the head of the rule containing the (previous) current atom. Thus "how n" moves down the proof tree and "up" moves up the proof tree, allowing the user to explore the full proof.

logicExplain.py — (continued)

```
61  helptext = """Commands are:
62  ask atom    ask is there is a proof for atom (atom should not be in quotes)
63  how         show the clause that was used to prove atom
64  how n       show the clause used to prove the nth element of the body
65  up          go back up proof tree to explore other parts of the proof tree
66  kb          print the knowledge base
```

```
quit        quit this interaction (and go back to Python)
help        print this text
"""

def interact(kb):
    going = True
    ups = []   # stack for going up
    proof="fail" # there is no proof to start
    while going:
        inp = input("logicExplain: ")
        inps = inp.split(" ")
        try:
            command = inps[0]
            if command == "quit":
                going = False
            elif command == "ask":
                proof = prove_atom(kb, inps[1])
                if proof == "fail":
                    print("fail")
                else:
                    print("yes")
            elif command == "how":
                if proof=="fail":
                    print("there is no proof")
                elif len(inps)==1:
                   print_rule(proof)
                else:
                    try:
                        ups.append(proof)
                        proof = proof[1][int(inps[1])] #nth argument of rule
                        print_rule(proof)
                    except:
                        print('In "how n", n must be a number between 0
                            and',len(proof[1])-1,"inclusive.")
            elif command == "up":
                if ups:
                    proof = ups.pop()
                else:
                    print("No rule to go up to.")
                print_rule(proof)
            elif command == "kb":
                 print(kb)
            elif command == "help":
                print(helptext)
            else:
                print("unknown command:", inp)
                print("use help for help")
        except:
            print("unknown command:", inp)
            print("use help for help")
```

```
116
117 def print_rule(proof):
118     (head,body) = proof
119     if body == "answered":
120         print(head,"was answered yes")
121     elif body == []:
122             print(head,"is a fact")
123     else:
124            print(head,"<-")
125            for i,a in enumerate(body):
126                print(i,":",a[0])
127
128 # try
129 # interact(elect)
130 # Which clause is wrong in elect_bug? Try:
131 # interact(elect_bug)
132 # logicExplain: ask lit_l1
```

The following shows an interaction for the knowledge base `elect`:

```
>>> interact(elect)
logicExplain: ask lit_l1
Is up_s2 true? no
Is down_s2 true? yes
Is down_s1 true? yes
yes
logicExplain: how
lit_l1 <-
0 : light_l1
1 : live_l1
2 : ok_l1
logicExplain: how 1
live_l1 <-
0 : live_w0
logicExplain: how 0
live_w0 <-
0 : down_s2
1 : live_w2
logicExplain: how 0
down_s2 was answered yes
logicExplain: up
live_w0 <-
0 : down_s2
1 : live_w2
logicExplain: how 1
live_w2 <-
0 : down_s1
1 : live_w3
```

```
logicExplain: quit
>>>
```

Exercise 5.6 The above code only ever explores one proof – the first proof found. Change the code to enumerate the proof trees (by returning a list of all proof trees, or, preferably, using `yield`). Add the command "retry" to the user interface to try another proof.

5.5 Assumables

Atom a can be made assumable by including $Assumable(a)$ in the knowledge base. A knowledge base that can include assumables is declared with *KBA*.

logicAssumables.py — Definite clauses with assumables

```
from logicProblem import Clause, Askable, KB, yes

class Assumable(object):
    """An askable atom"""

    def __init__(self,atom):
        """clause with atom head and lost of atoms body"""
        self.atom = atom

    def __str__(self):
        """returns the string representation of a clause.
        """
        return "assumable " + self.atom + "."

class KBA(KB):
    """A knowledge base that can include assumables"""
    def __init__(self,statements):
        self.assumables = [c.atom for c in statements if isinstance(c,
            Assumable)]
        KB.__init__(self,statements)
```

The top-down Horn clause interpreter, *prove_all_ass* returns a list of the sets of assumables that imply *ans_body*. This list will contain all of the minimal sets of assumables, but can also find non-minimal sets, and repeated sets, if they can be generated with separate proofs. The set *assumed* is the set of assumables already assumed.

logicAssumables.py — (continued)

```
    def prove_all_ass(self, ans_body, assumed=set()):
        """returns a list of sets of assumables that extends assumed
        to imply ans_body from self.
        ans_body is a list of atoms (it is the body of the answer clause).
        assumed is a set of assumables already assumed
        """
        if ans_body:
```

```
            selected = ans_body[0] # select first atom from ans_body
            if selected in self.askables:
                if yes(input("Is "+selected+" true? ")):
                    return self.prove_all_ass(ans_body[1:],assumed)
                else:
                    return []  # no answers
            elif selected in self.assumables:
                return self.prove_all_ass(ans_body[1:],assumed|{selected})
            else:
                return [ass
                        for cl in self.clauses_for_atom(selected)
                        for ass in
                            self.prove_all_ass(cl.body+ans_body[1:],assumed)
                       ]  # union of answers for each clause with
                            head=selected
        else:               # empty body
            return [assumed]  # one answer

    def conflicts(self):
        """returns a list of minimal conflicts"""
        return minsets(self.prove_all_ass(['false']))
```

Given a list of sets, *minsets* returns a list of the minimal sets in the list. For example, *minsets*($[\{2,3,4\},\{2,3\},\{6,2,3\},\{2,3\},\{2,4,5\}]$) returns $[\{2,3\},\{2,4,5\}]$.

logicAssumables.py — (continued)

```
def minsets(ls):
    """ls is a list of sets
    returns a list of minimal sets in ls
    """
    ans = []    # elements known to be minimal
    for c in ls:
        if not any(c1<c for c1 in ls) and not any(c1 <= c for c1 in ans):
            ans.append(c)
    return ans

# minsets([{2, 3, 4}, {2, 3}, {6, 2, 3}, {2, 3}, {2, 4, 5}])
```

Warning: *minsets* works for a list of sets or for a set of (frozen) sets, but it does not work for a generator of sets (because `ls` is referenced in the loop). For example, try to predict and then test:

```
minsets(e for e in [{2, 3, 4}, {2, 3}, {6, 2, 3}, {2, 3}, {2, 4, 5}])
```

The diagnoses can be constructed from the (minimal) conflicts as follows. This also works if there are non-minimal conflicts, but is not as efficient.

logicAssumables.py — (continued)

```
def diagnoses(cons):
    """cons is a list of (minimal) conflicts.
    returns a list of diagnoses."""
    if cons == []:
```

```
        return [set()]
    else:
        return minsets([({e}|d)            # | is set union
                        for e in cons[0]
                        for d in diagnoses(cons[1:])])
```

Test cases:

logicAssumables.py — (continued)

```
electa = KBA([
    Clause('light_l1'),
    Clause('light_l2'),
    Assumable('ok_l1'),
    Assumable('ok_l2'),
    Assumable('ok_s1'),
    Assumable('ok_s2'),
    Assumable('ok_s3'),
    Assumable('ok_cb1'),
    Assumable('ok_cb2'),
    Assumable('live_outside'),
    Clause('live_l1', ['live_w0']),
    Clause('live_w0', ['up_s2','ok_s2','live_w1']),
    Clause('live_w0', ['down_s2','ok_s2','live_w2']),
    Clause('live_w1', ['up_s1', 'ok_s1', 'live_w3']),
    Clause('live_w2', ['down_s1', 'ok_s1','live_w3' ]),
    Clause('live_l2', ['live_w4']),
    Clause('live_w4', ['up_s3','ok_s3','live_w3' ]),
    Clause('live_p_1', ['live_w3']),
    Clause('live_w3', ['live_w5', 'ok_cb1']),
    Clause('live_p_2', ['live_w6']),
    Clause('live_w6', ['live_w5', 'ok_cb2']),
    Clause('live_w5', ['live_outside']),
    Clause('lit_l1', ['light_l1', 'live_l1', 'ok_l1']),
    Clause('lit_l2', ['light_l2', 'live_l2', 'ok_l2']),
    Askable('up_s1'),
    Askable('down_s1'),
    Askable('up_s2'),
    Askable('down_s2'),
    Askable('up_s3'),
    Askable('down_s2'),
    Askable('dark_l1'),
    Askable('dark_l2'),
    Clause('false', ['dark_l1', 'lit_l1']),
    Clause('false', ['dark_l2', 'lit_l2'])
    ])
# electa.prove_all_ass(['false'])
# cs=electa.conflicts()
# print(cs)
# diagnoses(cs)      # diagnoses from conflicts
```

Exercise 5.7 To implement a version of *conflicts* that never generates non-minimal

conflicts, modify *prove_all_ass* to implement iterative deepening on the number of assumables used in a proof, and prune any set of assumables that is a superset of a conflict.

Exercise 5.8 Implement *explanations*(*self*, *body*), where *body* is a list of atoms, that returns a list of the minimal explanations of the body. This does not require modification of *prove_all_ass*.

Exercise 5.9 Implement *explanations*, as in the previous question, so that it never generates non-minimal explanations. Hint: modify *prove_all_ass* to implement iterative deepening on the number of assumptions, generating conflicts and explanations together, and pruning as early as possible.

5.6 Negation-as-failure

The negation af an atom a is written as `Not(a)` in a body.

logicNegation.py — Propositional negation-as-failure

```
from logicProblem import KB, Clause, Askable, yes

class Not(object):
    def __init__(self, atom):
        self.theatom = atom

    def atom(self):
        return self.theatom

    def __repr__(self):
        return f"Not({self.theatom})"
```

Prove with negation-as-failure (`prove_naf`) is like prove, but with the extra case to cover `Not`:

logicNegation.py — (continued)

```
def prove_naf(kb, ans_body, indent=""):
    """ prove with negation-as-failure and askables
    returns True if kb |- ans_body
    ans_body is a list of atoms to be proved
    """
    kb.display(2,indent,'yes <-',' & '.join(str(e) for e in ans_body))
    if ans_body:
        selected = ans_body[0] # select first atom from ans_body
        if isinstance(selected, Not):
            kb.display(2,indent,f"proving {selected.atom()}")
            if prove_naf(kb, [selected.atom()], indent):
                kb.display(2,indent,f"{selected.atom()} succeeded so
                    Not({selected.atom()}) fails")
                return False
            else:
                kb.display(2,indent,f"{selected.atom()} fails so
                    Not({selected.atom()}) succeeds")
```

```
            return prove_naf(kb, ans_body[1:],indent+" ")
        if selected in kb.askables:
            return (yes(input("Is "+selected+" true? "))
                    and  prove_naf(kb,ans_body[1:],indent+" "))
        else:
            return any(prove_naf(kb,cl.body+ans_body[1:],indent+" ")
                       for cl in kb.clauses_for_atom(selected))
    else:
        return True   # empty body is true
```

Test cases:

logicNegation.py — (continued)

```
triv_KB_naf = KB([
    Clause('i_am', ['i_think']),
    Clause('i_think'),
    Clause('i_smell', ['i_am', Not('dead')]),
    Clause('i_bad', ['i_am', Not('i_think')])
    ])

triv_KB_naf.max_display_level = 4
def test():
    a1 = prove_naf(triv_KB_naf,['i_smell'])
    assert a1, f"triv_KB_naf proving i_smell gave {a1}"
    a2 = prove_naf(triv_KB_naf,['i_bad'])
    assert not a2, f"triv_KB_naf proving i_bad gave {a2}"
    print("Passed unit tests")
if __name__ == "__main__":
    test()
```

Default reasoning about beaches at resorts (Example 5.28 of Poole and Mackworth [2023]):

logicNegation.py — (continued)

```
beach_KB = KB([
   Clause('away_from_beach', [Not('on_beach')]),
   Clause('beach_access', ['on_beach', Not('ab_beach_access')]),
   Clause('swim_at_beach', ['beach_access', Not('ab_swim_at_beach')]),
   Clause('ab_swim_at_beach', ['enclosed_bay', 'big_city',
       Not('ab_no_swimming_near_city')]),
   Clause('ab_no_swimming_near_city', ['in_BC', Not('ab_BC_beaches')])
    ])

# prove_naf(beach_KB, ['away_from_beach'])
# prove_naf(beach_KB, ['beach_access'])
# beach_KB.add_clause(Clause('on_beach',[]))
# prove_naf(beach_KB, ['away_from_beach'])
# prove_naf(beach_KB, ['swim_at_beach'])
# beach_KB.add_clause(Clause('enclosed_bay',[]))
# prove_naf(beach_KB, ['swim_at_beach'])
# beach_KB.add_clause(Clause('big_city',[]))
# prove_naf(beach_KB, ['swim_at_beach'])
```

```
# beach_KB.add_clause(Clause('in_BC',[]))
# prove_naf(beach_KB, ['swim_at_beach'])
```

Chapter 6

Deterministic Planning

6.1 Representing Actions and Planning Problems

The STRIPS representation of an action consists of:

- the name of the action
- preconditions: a dictionary of *feature:value* pairs that specifies that the feature must have this value for the action to be possible
- effects: a dictionary of *feature:value* pairs that are made true by this action. In particular, a feature in the dictionary has the corresponding value (and not its previous value) after the action, and a feature not in the dictionary keeps its old value.

stripsProblem.py — STRIPS Representations of Actions

```
11 class Strips(object):
12     def __init__(self, name, preconds, effects, cost=1):
13         """
14         defines the STRIPS representation for an action:
15         * name is the name of the action
16         * preconds, the preconditions, is feature:value dictionary that
               must hold
17         for the action to be carried out
18         * effects is a feature:value map that this action makes
19         true. The action changes the value of any feature specified
20         here, and leaves other features unchanged.
21         * cost is the cost of the action
22         """
```

```
23          self.name = name
24          self.preconds = preconds
25          self.effects = effects
26          self.cost = cost
27
28      def __repr__(self):
29          return self.name
```

A STRIPS domain consists of:

- A dictionary that maps each feature into a set of possible values for the feature.
- A set of actions, each representeded using the `Strips` class.

stripsProblem.py — (continued)

```
31  class STRIPS_domain(object):
32      def __init__(self, feature_domain_dict, actions):
33          """Problem domain
34          feature_domain_dict is a feature:domain dictionary,
35                 mapping each feature to its domain
36          actions
37          """
38          self.feature_domain_dict = feature_domain_dict
39          self.actions = actions
```

A planning problem consists of a planning domain, an initial state, and a goal. The goal does not need to fully specify the final state.

stripsProblem.py — (continued)

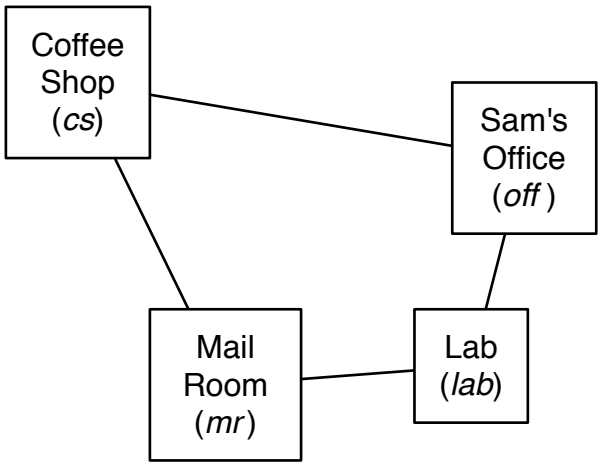
```
41  class Planning_problem(object):
42      def __init__(self, prob_domain, initial_state, goal):
43          """
44          a planning problem consists of
45          * a planning domain
46          * the initial state
47          * a goal
48          """
49          self.prob_domain = prob_domain
50          self.initial_state = initial_state
51          self.goal = goal
```

6.1.1 Robot Delivery Domain

The following specifies the robot delivery domain of Section 6.1, shown in Figure 6.1.

stripsProblem.py — (continued)

```
53  boolean = {False, True}
54  delivery_domain = STRIPS_domain(
```



Features to describe states

RLoc – Rob's location
RHC – Rob has coffee
SWC – Sam wants coffee
MW – Mail is waiting
RHM – Rob has mail

Actions

mc – move clockwise
mcc – move counterclockwise
puc – pickup coffee
dc – deliver coffee
pum – pickup mail
dm – deliver mail

Figure 6.1: Robot Delivery Domain

```
55    {'RLoc':{'cs', 'off', 'lab', 'mr'}, 'RHC':boolean, 'SWC':boolean,
56     'MW':boolean, 'RHM':boolean},       #feature:values dictionary
57    { Strips('mc_cs', {'RLoc':'cs'}, {'RLoc':'off'}),
58     Strips('mc_off', {'RLoc':'off'}, {'RLoc':'lab'}),
59     Strips('mc_lab', {'RLoc':'lab'}, {'RLoc':'mr'}),
60     Strips('mc_mr', {'RLoc':'mr'}, {'RLoc':'cs'}),
61     Strips('mcc_cs', {'RLoc':'cs'}, {'RLoc':'mr'}),
62     Strips('mcc_off', {'RLoc':'off'}, {'RLoc':'cs'}),
63     Strips('mcc_lab', {'RLoc':'lab'}, {'RLoc':'off'}),
64     Strips('mcc_mr', {'RLoc':'mr'}, {'RLoc':'lab'}),
65     Strips('puc', {'RLoc':'cs', 'RHC':False}, {'RHC':True}),
66     Strips('dc', {'RLoc':'off', 'RHC':True}, {'RHC':False, 'SWC':False}),
67     Strips('pum', {'RLoc':'mr','MW':True}, {'RHM':True,'MW':False}),
68     Strips('dm', {'RLoc':'off', 'RHM':True}, {'RHM':False})
69    } )
```

stripsProblem.py — (continued)

```
71  problem0 = Planning_problem(delivery_domain,
72                              {'RLoc':'lab', 'MW':True, 'SWC':True, 'RHC':False,
73                               'RHM':False},
```

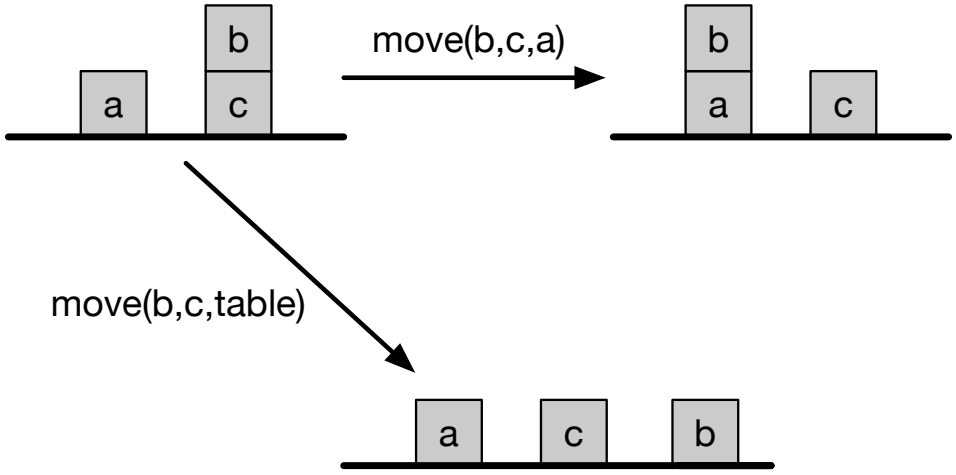


Figure 6.2: Blocks world with two actions

```
                          {'RLoc':'off'})
problem1 = Planning_problem(delivery_domain,
                          {'RLoc':'lab', 'MW':True, 'SWC':True, 'RHC':False,
                           'RHM':False},
                          {'SWC':False})
problem2 = Planning_problem(delivery_domain,
                          {'RLoc':'lab', 'MW':True, 'SWC':True, 'RHC':False,
                           'RHM':False},
                          {'SWC':False, 'MW':False, 'RHM':False})
```

6.1.2 Blocks World

The blocks world consist of blocks and a table. Each block can be on the table or on another block. A block can only have one other block on top of it. Figure 6.2 shows 3 states with some of the actions between them.

A state is defined by the two features:

- *on* where $on(x) = y$ when block x is on block or table y
- *clear* where $clear(x) = True$ when block x has nothing on it.

There is one parameterized action

- $move(x, y, z)$ move block x from y to z, where y and z could be a block or the table.

To handle parameterized actions (which depend on the blocks involved), the actions and the features are all strings, created for all the combinations of the blocks. Note that we treat moving to a block separately from moving to the

table, because the blocks needs to be clear, but the table always has room for another block.

stripsProblem.py — (continued)

```
### blocks world
def move(x,y,z):
    """string for the 'move' action"""
    return 'move_'+x+'_from_'+y+'_to_'+z
def on(x):
    """string for the 'on' feature"""
    return x+'_is_on'
def clear(x):
    """string for the 'clear' feature"""
    return 'clear_'+x
def create_blocks_world(blocks = {'a','b','c','d'}):
    blocks_and_table = blocks | {'table'}
    stmap = {Strips(move(x,y,z),{on(x):y, clear(x):True, clear(z):True},
                                {on(x):z, clear(y):True, clear(z):False})
                    for x in blocks
                    for y in blocks_and_table
                    for z in blocks
                    if x!=y and y!=z and z!=x}
    stmap.update({Strips(move(x,y,'table'), {on(x):y, clear(x):True},
                                {on(x):'table', clear(y):True})
                    for x in blocks
                    for y in blocks
                    if x!=y})
    feature_domain_dict = {on(x):blocks_and_table-{x} for x in blocks}
    feature_domain_dict.update({clear(x):boolean for x in blocks_and_table})
    return STRIPS_domain(feature_domain_dict, stmap)
```

The problem *blocks*1 is a classic example, with 3 blocks, and the goal consists of two conditions. See Figure 6.3. This example is challenging because you can't achieve one of the goals and then the other; whichever one you achieve first has to be undone to achieve the second.

stripsProblem.py — (continued)

```
blocks1dom = create_blocks_world({'a','b','c'})
blocks1 = Planning_problem(blocks1dom,
    {on('a'):'table', clear('a'):True,
     on('b'):'c', clear('b'):True,
     on('c'):'table', clear('c'):False}, # initial state
    {on('a'):'b', on('c'):'a'}) #goal
```

The problem *blocks*2 is one to invert a tower of size 4.

stripsProblem.py — (continued)

```
blocks2dom = create_blocks_world({'a','b','c','d'})
tower4 = {clear('a'):True, on('a'):'b',
         clear('b'):False, on('b'):'c',
         clear('c'):False, on('c'):'d',
```

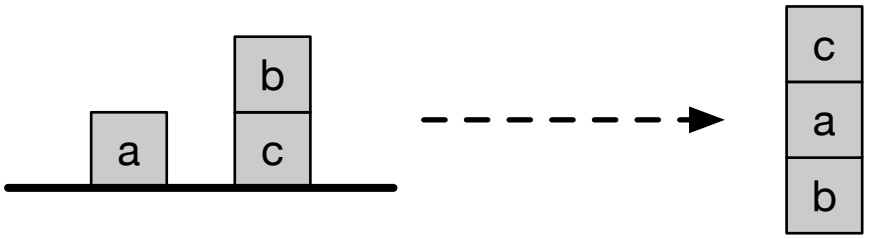


Figure 6.3: Blocks problem blocks1

```
122         clear('d'):False, on('d'):'table'}
123 blocks2 = Planning_problem(blocks2dom,
124     tower4, # initial state
125     {on('d'):'c',on('c'):'b',on('b'):'a'}) #goal
```

The problem *blocks*3 is to move the bottom block to the top of a tower of size 4.

stripsProblem.py — (continued)

```
127 blocks3 = Planning_problem(blocks2dom,
128     tower4, # initial state
129     {on('d'):'a', on('a'):'b', on('b'):'c'}) #goal
```

Exercise 6.1 Represent the problem of given a tower of 4 blocks (a on b on c on d on table), the goal is to have a tower with the previous top block on the bottom (b on c on d on a). Do not include the table in your goal (the goal does not care whether a is on the table). [Before you run the program, estimate how many steps it will take to solve this.] How many steps does an optimal planner take?

Exercise 6.2 Represent the domain so that $on(x, y)$ is a Boolean feature that is True when x is on y, Does the representation of the state need to include negative *on* facts? Why or why not? (Note that this may depend on the planner; write your answer with respect to particular planners.)

Exercise 6.3 It is possible to write the representation of the problem without using *clear*, where $clear(x)$ means nothing is on x. Change the definition of the blocks world so that it does not use *clear* but uses *on* being false instead. Does this work better for any of the planners?

6.2 Forward Planning

To run the demo, in folder "aipython", load "stripsForwardPlanner.py", and copy and paste the commented-out example queries at the bottom of that file.

In a forward planner, a node is a state. A state consists of an assignment, which is a variable:value dictionary. In order to be able to do multiple-path pruning, we need to define a hash function, and equality between states.

stripsForwardPlanner.py — Forward Planner with STRIPS actions

```
from searchProblem import Arc, Search_problem
from stripsProblem import Strips, STRIPS_domain

class State(object):
    def __init__(self,assignment):
        self.assignment = assignment
        self.hash_value = None
    def __hash__(self):
        if self.hash_value is None:
            self.hash_value = hash(frozenset(self.assignment.items()))
        return self.hash_value
    def __eq__(self,st):
        return self.assignment == st.assignment
    def __str__(self):
        return str(self.assignment)
```

In order to define a search problem (page 41), we need to define the goal condition, the start nodes, the neighbours, and (optionally) a heuristic function. Here *zero* is the default heuristic function.

stripsForwardPlanner.py — (continued)

```
def zero(*args,**nargs):
    """always returns 0"""
    return 0

class Forward_STRIPS(Search_problem):
    """A search problem from a planning problem where:
    * a node is a state object.
    * the dynamics are specified by the STRIPS representation of actions
    """
    def __init__(self, planning_problem, heur=zero):
        """creates a forward search space from a planning problem.
        heur(state,goal) is a heuristic function,
           an underestimate of the cost from state to goal, where
           both state and goals are feature:value dictionaries.
        """
        self.prob_domain = planning_problem.prob_domain
        self.initial_state = State(planning_problem.initial_state)
        self.goal = planning_problem.goal
        self.heur = heur

    def is_goal(self, state):
        """is True if node is a goal.

        Every goal feature has the same value in the state and the goal."""
        return all(state.assignment[prop]==self.goal[prop]
                   for prop in self.goal)

    def start_node(self):
        """returns start node"""
```

```
56          return self.initial_state
57
58      def neighbors(self,state):
59          """returns neighbors of state in this problem"""
60          return [ Arc(state, self.effect(act,state.assignment), act.cost,
                    act)
61                   for act in self.prob_domain.actions
62                   if self.possible(act,state.assignment)]
63
64      def possible(self,act,state_asst):
65          """True if act is possible in state.
66          act is possible if all of its preconditions have the same value in
                    the state"""
67          return all(state_asst[pre] == act.preconds[pre]
68                     for pre in act.preconds)
69
70      def effect(self,act,state_asst):
71          """returns the state that is the effect of doing act given
                    state_asst
72          Python 3.9: return state_asst | act.effects"""
73          new_state_asst = state_asst.copy()
74          new_state_asst.update(act.effects)
75          return State(new_state_asst)
76
77      def heuristic(self,state):
78          """in the forward planner a node is a state.
79          the heuristic is an (under)estimate of the cost
80          of going from the state to the top-level goal.
81          """
82          return self.heur(state.assignment, self.goal)
```

Here are some test cases to try.

stripsForwardPlanner.py — (continued)

```
84  from searchBranchAndBound import DF_branch_and_bound
85  from searchMPP import SearcherMPP
86  import stripsProblem
87
88  # SearcherMPP(Forward_STRIPS(stripsProblem.problem1)).search() #A* with MPP
89  # DF_branch_and_bound(Forward_STRIPS(stripsProblem.problem1),10).search()
        #B&B
90  # To find more than one plan:
91  # s1 = SearcherMPP(Forward_STRIPS(stripsProblem.problem1)) #A*
92  # s1.search() #find another plan
```

6.2.1 Defining Heuristics for a Planner

Each planning domain requires its own heuristics. If you change the actions, you will need to reconsider the heuristic function, as there might then be a lower-cost path, which might make the heuristic non-admissible.

Here is an example of defining heuristics for the coffee delivery planning domain.

First we define the distance between two locations, which is used for the heuristics.

stripsHeuristic.py — Planner with Heuristic Function

```
def dist(loc1, loc2):
    """returns the distance from location loc1 to loc2
    """
    if loc1==loc2:
        return 0
    if {loc1,loc2} in [{'cs','lab'},{'mr','off'}]:
        return 2
    else:
        return 1
```

Note that the current state is a complete description; there is a value for every feature. However the goal need not be complete; it does not need to define a value for every feature. Before checking the value for a feature in the goal, a heuristic needs to define whether the feature is defined in the goal.

stripsHeuristic.py — (continued)

```
def h1(state,goal):
    """ the distance to the goal location, if there is one"""
    if 'RLoc' in goal:
        return dist(state['RLoc'], goal['RLoc'])
    else:
        return 0

def h2(state,goal):
    """ the distance to the coffee shop plus getting coffee and delivering
        it
    if the robot needs to get coffee
    """
    if ('SWC' in goal and goal['SWC']==False
            and state['SWC']==True
            and state['RHC']==False):
        return dist(state['RLoc'],'cs')+3
    else:
        return 0
```

The maximum of the values of a set of admissible heuristics is also an admissible heuristic. The function `maxh` takes a number of heuristic functions as arguments, and returns a new heuristic function that takes the maximum of the values of the heuristics. For example, `h1` and `h2` are heuristic functions and so `maxh(h1,h2)` is also. `maxh` can take an arbitrary number of arguments.

stripsHeuristic.py — (continued)

```
def maxh(*heuristics):
    """Returns a new heuristic function that is the maximum of the
        functions in heuristics.
```

```
41      heuristics is the list of arguments which must be heuristic functions.
42      """
43      # return lambda state,goal: max(h(state,goal) for h in heuristics)
44      def newh(state,goal):
45          return max(h(state,goal) for h in heuristics)
46      return newh
```

The following runs the example with and without the heuristic.

stripsHeuristic.py — (continued)

```
48  ##### Forward Planner #####
49  from searchMPP import SearcherMPP
50  from stripsForwardPlanner import Forward_STRIPS
51  import stripsProblem
52
53  def test_forward_heuristic(thisproblem=stripsProblem.problem1):
54      print("\n***** FORWARD NO HEURISTIC")
55      print(SearcherMPP(Forward_STRIPS(thisproblem)).search())
56
57      print("\n***** FORWARD WITH HEURISTIC h1")
58      print(SearcherMPP(Forward_STRIPS(thisproblem,h1)).search())
59
60      print("\n***** FORWARD WITH HEURISTIC h2")
61      print(SearcherMPP(Forward_STRIPS(thisproblem,h2)).search())
62
63      print("\n***** FORWARD WITH HEURISTICs h1 and h2")
64      print(SearcherMPP(Forward_STRIPS(thisproblem,maxh(h1,h2))).search())
65
66  if __name__ == "__main__":
67      test_forward_heuristic()
```

Exercise 6.4 For more than one start-state/goal combination, test the forward planner with a heuristic function of just $h1$, with just $h2$ and with both. Explain why each one prunes or doesn't prune the search space.

Exercise 6.5 Create a better heuristic than $maxh(h1, h2)$. Try it for a number of different problems. In particular, try and include the following costs:

i) $h3$ is like $h2$ but also takes into account the case when *Rloc* is in goal.

ii) $h4$ uses the distance to the mail room plus getting mail and delivering it if the robot needs to get need to deliver mail.

iii) $h5$ is for getting mail when goal is for the robot to have mail, and then getting to the goal destination (if there is one).

Exercise 6.6 Create an admissible heuristic for the blocks world.

6.3 Regression Planning

To run the demo, in folder "aipython", load "stripsRegressionPlanner.py", and copy and paste the commented-out example queries at the bottom of that file.

In a regression planner a node is a subgoal that need to be achieved.

A *Subgoal* object consists of an assignment, which is a *variable:value* dictionary. We make it hashable so that multiple path pruning can work. The hash is only computed when necessary (and only once).

stripsRegressionPlanner.py — Regression Planner with STRIPS actions

```
from searchProblem import Arc, Search_problem

class Subgoal(object):
    def __init__(self,assignment):
        self.assignment = assignment
        self.hash_value = None
    def __hash__(self):
        if self.hash_value is None:
            self.hash_value = hash(frozenset(self.assignment.items()))
        return self.hash_value
    def __eq__(self,st):
        return self.assignment == st.assignment
    def __str__(self):
        return str(self.assignment)
```

A regression search has subgoals as nodes. The initial node is the top-level goal of the planner. The goal for the search (when the search can stop) is a subgoal that holds in the initial state.

stripsRegressionPlanner.py — (continued)

```
from stripsForwardPlanner import zero

class Regression_STRIPS(Search_problem):
    """A search problem where:
    * a node is a goal to be achieved, represented by a set of propositions.
    * the dynamics are specified by the STRIPS representation of actions
    """

    def __init__(self, planning_problem, heur=zero):
        """creates a regression search space from a planning problem.
        heur(state,goal) is a heuristic function;
           an underestimate of the cost from state to goal, where
           both state and goals are feature:value dictionaries
        """
        self.prob_domain = planning_problem.prob_domain
        self.top_goal = Subgoal(planning_problem.goal)
        self.initial_state = planning_problem.initial_state
        self.heur = heur
```

```
44
45      def is_goal(self, subgoal):
46          """if subgoal is true in the initial state, a path has been found"""
47          goal_asst = subgoal.assignment
48          return all(self.initial_state[g]==goal_asst[g]
49                     for g in goal_asst)
50
51      def start_node(self):
52          """the start node is the top-level goal"""
53          return self.top_goal
54
55      def neighbors(self,subgoal):
56          """returns a list of the arcs for the neighbors of subgoal in this
                problem"""
57          goal_asst = subgoal.assignment
58          return [ Arc(subgoal, self.weakest_precond(act,goal_asst),
                    act.cost, act)
59                   for act in self.prob_domain.actions
60                   if self.possible(act,goal_asst)]
61
62      def possible(self,act,goal_asst):
63          """True if act is possible to achieve goal_asst.
64
65          the action achieves an element of the effects and
66          the action doesn't delete something that needs to be achieved and
67          the preconditions are consistent with other subgoals that need to
                be achieved
68          """
69          return ( any(goal_asst[prop] == act.effects[prop]
70                       for prop in act.effects if prop in goal_asst)
71                   and all(goal_asst[prop] == act.effects[prop]
72                           for prop in act.effects if prop in goal_asst)
73                   and all(goal_asst[prop]== act.preconds[prop]
74                           for prop in act.preconds if prop not in act.effects
                                and prop in goal_asst)
75                   )
76
77      def weakest_precond(self,act,goal_asst):
78          """returns the subgoal that must be true so goal_asst holds after
                act
79          should be:  act.preconds | (goal_asst - act.effects)
80          """
81          new_asst = act.preconds.copy()
82          for g in goal_asst:
83              if g not in act.effects:
84                  new_asst[g] = goal_asst[g]
85          return Subgoal(new_asst)
86
87      def heuristic(self,subgoal):
88          """in the regression planner a node is a subgoal.
```

```
89          the heuristic is an (under)estimate of the cost of going from the
                initial state to subgoal.
90          """
91          return self.heur(self.initial_state, subgoal.assignment)
```

stripsRegressionPlanner.py — (continued)

```
93  from searchBranchAndBound import DF_branch_and_bound
94  from searchMPP import SearcherMPP
95  import stripsProblem
96
97  # SearcherMPP(Regression_STRIPS(stripsProblem.problem1)).search() #A* with
        MPP
98  #
        DF_branch_and_bound(Regression_STRIPS(stripsProblem.problem1),10).search()
        #B&B
```

Exercise 6.7 Multiple path pruning could be used to prune more than the current node. In particular, if the current node contains more conditions than a previously visited node, it can be pruned. For example, if $\{a : True, b : False\}$ has been visited, then any node that is a superset, e.g., $\{a : True, b : False, d : True\}$, need not be expanded. If the simpler subgoal does not lead to a solution, the more complicated one will not either. Implement this more severe pruning. (Hint: This may require modifications to the searcher.)

Exercise 6.8 It is possible that, as knowledge of the domain, that some assignment of values to variables can never be achieved. For example, the robot cannot be holding mail when there is mail waiting (assuming it isn't holding mail initially). An assignment of values to (some of the) variables is incompatible if no possible (reachable) state can include that assignment. For example, $\{'MW' : True, 'RHM' : True\}$ is an incompatible assignment. This information may be useful information for a planner; there is no point in trying to achieve these together. Define a subclass of *STRIPS_domain* that can accept a list of incompatible assignments. Modify the regression planner code to use such a list of incompatible assignments. Give an example where the search space is smaller.

Exercise 6.9 After completing the previous exercise, design incompatible assignments for the blocks world. (This should result in dramatic search improvements.)

6.3.1 Defining Heuristics for a Regression Planner

The regression planner can use the same heuristic function as the forward planner. However, just because a heuristic is useful for a forward planner does not mean it is useful for a regression planner, and vice versa. you should experiment with whether the same heuristic works well for both a regression planner and a forward planner.

The following runs the same example as the forward planner with and without the heuristic defined for the forward planner:

stripsHeuristic.py — (continued)

```
##### Regression Planner
from stripsRegressionPlanner import Regression_STRIPS

def test_regression_heuristic(thisproblem=stripsProblem.problem1):
    print("\n***** REGRESSION NO HEURISTIC")
    print(SearcherMPP(Regression_STRIPS(thisproblem)).search())

    print("\n***** REGRESSION WITH HEURISTICs h1 and h2")
    print(SearcherMPP(Regression_STRIPS(thisproblem,maxh(h1,h2))).search())

if __name__ == "__main__":
    test_regression_heuristic()
```

Exercise 6.10 Try the regression planner with a heuristic function of just $h1$ and with just $h2$ (defined in Section 6.2.1). Explain how each one prunes or doesn't prune the search space.

Exercise 6.11 Create a better heuristic than *heuristic_fun* defined in Section 6.2.1.

6.4 Planning as a CSP

> To run the demo, in folder "aipython", load "stripsCSPPlanner.py", and copy and paste the commented-out example queries at the bottom of that file. This assumes Python 3.

Here we implement the CSP planner assuming there is a single action at each step. This creates a CSP that can use any of the CSP algorithms to solve (e.g., stochastic local search or arc consistency with domain splitting).

This assumes the same action representation as before; we do not consider factored actions (action features), nor do we implement state constraints.

stripsCSPPlanner.py — CSP planner where actions are represented using STRIPS

```
from cspProblem import Variable, CSP, Constraint

class CSP_from_STRIPS(CSP):
    """A CSP where:
    * CSP variables are constructed for each feature and time, and each
        action and time
    * the dynamics are specified by the STRIPS representation of actions
    """

    def __init__(self, planning_problem, number_stages=2):
        prob_domain = planning_problem.prob_domain
        initial_state = planning_problem.initial_state
        goal = planning_problem.goal
        # self.action_vars[t] is the action variable for time t
        self.action_vars = [Variable(f"Action{t}", prob_domain.actions)
                            for t in range(number_stages)]
        # feat_time_var[f][t] is the variable for feature f at time t
```

```
        feat_time_var = {feat: [Variable(f"{feat}_{t}",dom)
                                for t in range(number_stages+1)]
                        for (feat,dom) in
                            prob_domain.feature_domain_dict.items()}

        # initial state constraints:
        constraints = [Constraint((feat_time_var[feat][0],), is_(val))
                       for (feat,val) in initial_state.items()]

        # goal constraints on the final state:
        constraints += [Constraint((feat_time_var[feat][number_stages],),
                                   is_(val))
                        for (feat,val) in goal.items()]

        # precondition constraints:
        constraints += [Constraint((feat_time_var[feat][t],
            self.action_vars[t]),
                                   if_(val,act)) # feat@t==val if action@t==act
                        for act in prob_domain.actions
                        for (feat,val) in act.preconds.items()
                        for t in range(number_stages)]

        # effect constraints:
        constraints += [Constraint((feat_time_var[feat][t+1],
            self.action_vars[t]),
                                   if_(val,act)) # feat@t+1==val if
                                       action@t==act
                        for act in prob_domain.actions
                        for feat,val in act.effects.items()
                        for t in range(number_stages)]
        # frame constraints:

        constraints += [Constraint((feat_time_var[feat][t],
            self.action_vars[t], feat_time_var[feat][t+1]),
                                   eq_if_not_in_({act for act in
                                       prob_domain.actions
                                                  if feat in act.effects}))
                        for feat in prob_domain.feature_domain_dict
                        for t in range(number_stages) ]
        variables = set(self.action_vars) | {feat_time_var[feat][t]
                                             for feat in
                                                 prob_domain.feature_domain_dict
                                             for t in range(number_stages+1)}
        CSP.__init__(self, "CSP_from_Strips", variables, constraints)

    def extract_plan(self,soln):
        return [soln[a] for a in self.action_vars]
```

The following methods return methods which can be applied to the particular environment.

For example, $is\_(3)$ returns a function that when applied to 3, returns True

and when applied to any other value returns False. So $is\_(3)(3)$ returns *True* and $is\_(3)(7)$ returns *False*.

Note that the underscore ('_') is part of the name; here we use it as the convention that it is a function that returns a function. This uses two different styles to define *is_* and *if_*; returning a function defined by *lambda* is equivalent to returning the embedded function, except that the embedded function has a name. The embedded function can also be given a docstring.

stripsCSPPlanner.py — (continued)

```
def is_(val):
    """returns a function that is true when it is it applied to val.
    """
    #return lambda x: x == val
    def is_fun(x):
        return x == val
    is_fun.__name__ = f"value_is_{val}"
    return is_fun

def if_(v1,v2):
    """if the second argument is v2, the first argument must be v1"""
    #return lambda x1,x2: x1==v1 if x2==v2 else True
    def if_fun(x1,x2):
        return x1==v1 if x2==v2 else True
    if_fun.__name__ = f"if x2 is {v2} then x1 is {v1}"
    return if_fun

def eq_if_not_in_(actset):
    """first and third arguments are equal if action is not in actset"""
    # return lambda x1, a, x2: x1==x2 if a not in actset else True
    def eq_if_not_fun(x1, a, x2):
        return x1==x2 if a not in actset else True
    eq_if_not_fun.__name__ = f"first and third arguments are equal if
        action is not in {actset}"
    return eq_if_not_fun
```

Putting it together, this returns a list of actions that solves the problem *prob* for a given horizon. If you want to do more than just return the list of actions, you might want to get it to return the solution. Or even enumerate the solutions (by using *Search_with_AC_from_CSP*).

stripsCSPPlanner.py — (continued)

```
def con_plan(prob,horizon):
    """finds a plan for problem prob given horizon.
    """
    csp = CSP_from_STRIPS(prob, horizon)
    sol = Con_solver(csp).solve_one()
    return csp.extract_plan(sol) if sol else sol
```

The following are some example queries.

stripsCSPPlanner.py — (continued)

```
from searchGeneric import Searcher
from cspConsistency import Search_with_AC_from_CSP, Con_solver
from stripsProblem import Planning_problem
import stripsProblem

# Problem 0
# con_plan(stripsProblem.problem0,1) # should it succeed?
# con_plan(stripsProblem.problem0,2) # should it succeed?
# con_plan(stripsProblem.problem0,3) # should it succeed?
# To use search to enumerate solutions
#searcher0a =
    Searcher(Search_with_AC_from_CSP(CSP_from_STRIPS(stripsProblem.problem0,
    1)))
#print(searcher0a.search()) # returns path to solution

## Problem 1
# con_plan(stripsProblem.problem1,5) # should it succeed?
# con_plan(stripsProblem.problem1,4) # should it succeed?
## To use search to enumerate solutions:
#searcher15a =
    Searcher(Search_with_AC_from_CSP(CSP_from_STRIPS(stripsProblem.problem1,
    5)))
#print(searcher15a.search()) # returns path to solution

## Problem 2
#con_plan(stripsProblem.problem2, 6) # should fail??
#con_plan(stripsProblem.problem2, 7) # should succeed???

## Example 6.13
problem3 = Planning_problem(stripsProblem.delivery_domain,
                            {'SWC':True, 'RHC':False}, {'SWC':False})
#con_plan(problem3,2) # Horizon of 2
#con_plan(problem3,3) # Horizon of 3

problem4 = Planning_problem(stripsProblem.delivery_domain,{'SWC':True},
                               {'SWC':False, 'MW':False, 'RHM':False})

# For the stochastic local search:
#from cspSLS import SLSearcher, Runtime_distribution
# cspplanning15 = CSP_from_STRIPS(stripsProblem.problem1, 5) # should
    succeed
#se0 = SLSearcher(cspplanning15); print(se0.search(100000,0.5))
#p = Runtime_distribution(cspplanning15)
#p.plot_runs(1000,1000,0.7) # warning will take a few minutes
```

6.5 Partial-Order Planning

To run the demo, in folder "aipython", load "stripsPOP.py", and copy and paste the commented-out example queries at the bottom of that file.

A partial order planner maintains a partial order of action instances. An action instance consists of a name and an index. We need action instances because the same action could be carried out at different times.

stripsPOP.py — Partial-order Planner using STRIPS representation

```
from searchProblem import Arc, Search_problem
import random

class Action_instance(object):
    next_index = 0
    def __init__(self,action,index=None):
        if index is None:
            index = Action_instance.next_index
            Action_instance.next_index += 1
        self.action = action
        self.index = index

    def __str__(self):
        return f"{self.action}#{self.index}"

    __repr__ = __str__ # __repr__ function is the same as the __str__
        function
```

A node (as in the abstraction of search space) in a partial-order planner consists of:

- *actions*: a set of action instances.
- *constraints*: a set of (a_1, a_2) pairs, where a_1 and a_2 are action instances, which represents that a_1 must come before a_2 in the partial order. There are a number of ways that this could be represented. Here we represent the set of pairs that are in transitive closure of the *before* relation. This lets us quickly determine whether some *before* relation is consistent with the current constraints.
- *agenda*: a list of (s, a) pairs, where s is a (var, val) pair and a is an action instance. This means that variable var must have value val before a can occur.
- *causal_links*: a set of $(a0, g, a1)$ triples, where a_1 and a_2 are action instances and g is a (var, val) pair. This holds when action a_0 makes g true for action a_1.

stripsPOP.py — (continued)

```
class POP_node(object):
    """a (partial) partial-order plan. This is a node in the search
        space."""
    def __init__(self, actions, constraints, agenda, causal_links):
        """
        * actions is a set of action instances
        * constraints a set of (a0,a1) pairs, representing a0<a1,
          closed under transitivity
        * agenda list of (subgoal,action) pairs to be achieved, where
          subgoal is a (variable,value) pair
        * causal_links is a set of (a0,g,a1) triples,
          where ai are action instances, and g is a (variable,value) pair
        """
        self.actions = actions # a set of action instances
        self.constraints = constraints # a set of (a0,a1) pairs
        self.agenda = agenda  # list of (subgoal,action) pairs to be
            achieved
        self.causal_links = causal_links # set of (a0,g,a1) triples

    def __str__(self):
        return ("actions: "+str({str(a) for a in self.actions})+
                "\nconstraints: "+
                str({(str(a1),str(a2)) for (a1,a2) in self.constraints})+
                "\nagenda: "+
                str([(str(s),str(a)) for (s,a) in self.agenda])+
                "\ncausal_links:"+
                str({(str(a0),str(g),str(a2)) for (a0,g,a2) in
                    self.causal_links}) )
```

extract_plan constructs a total order of action instances that is consistent with the partial order.

stripsPOP.py — (continued)

```
    def extract_plan(self):
        """returns a total ordering of the action instances consistent
        with the constraints.
        raises IndexError if there is no choice.
        """
        sorted_acts = []
        other_acts = set(self.actions)
        while other_acts:
            a = random.choice([a for a in other_acts if
                    all(((a1,a) not in self.constraints) for a1 in
                        other_acts)])
            sorted_acts.append(a)
            other_acts.remove(a)
        return sorted_acts
```

POP_search_from_STRIPS is an instance of a search problem. As such, we need to define the start nodes, the goal, and the neighbors of a node.

stripsPOP.py — (continued)

```
68 from display import Displayable
69
70 class POP_search_from_STRIPS(Search_problem, Displayable):
71     def __init__(self,planning_problem):
72         Search_problem.__init__(self)
73         self.planning_problem = planning_problem
74         self.start = Action_instance("start")
75         self.finish = Action_instance("finish")
76
77     def is_goal(self, node):
78         return node.agenda == []
79
80     def start_node(self):
81         constraints = {(self.start, self.finish)}
82         agenda = [(g, self.finish) for g in
                       self.planning_problem.goal.items()]
83         return POP_node([self.start,self.finish], constraints, agenda, [] )
```

The *neighbors* method is a coroutine that enumerates the neighbors of a given node.

stripsPOP.py — (continued)

```
85     def neighbors(self, node):
86         """enumerates the neighbors of node"""
87         self.display(3,"finding neighbors of\n",node)
88         if node.agenda:
89             subgoal,act1 = node.agenda[0]
90             self.display(2,"selecting",subgoal,"for",act1)
91             new_agenda = node.agenda[1:]
92             for act0 in node.actions:
93                 if (self.achieves(act0, subgoal) and
94                    self.possible((act0,act1),node.constraints)):
95                     self.display(2," reusing",act0)
96                     consts1 =
                           self.add_constraint((act0,act1),node.constraints)
97                     new_clink = (act0,subgoal,act1)
98                     new_cls = node.causal_links + [new_clink]
99                     for consts2 in
                           self.protect_cl_for_actions(node.actions,consts1,new_clink):
100                        yield Arc(node,
101                                  POP_node(node.actions,consts2,new_agenda,new_cls),
102                                  cost=0)
103            for a0 in self.planning_problem.prob_domain.actions: #a0 is an
                   action
104                if self.achieves(a0, subgoal):
105                    #a0 acheieves subgoal
106                    new_a = Action_instance(a0)
107                    self.display(2," using new action",new_a)
108                    new_actions = node.actions + [new_a]
```

```
109                 consts1 =
                        self.add_constraint((self.start,new_a),node.constraints)
110                 consts2 = self.add_constraint((new_a,act1),consts1)
111                 new_agenda1 = new_agenda + [(pre,new_a) for pre in
                        a0.preconds.items()]
112                 new_clink = (new_a,subgoal,act1)
113                 new_cls = node.causal_links + [new_clink]
114                 for consts3 in
                        self.protect_all_cls(node.causal_links,new_a,consts2):
115                     for consts4 in
                            self.protect_cl_for_actions(node.actions,consts3,new_clink):
116                         yield Arc(node,
117                                   POP_node(new_actions,consts4,new_agenda1,new_cls),
118                                   cost=1)
```

Given a causal link $(a0, subgoal, a1)$, the following method protects the causal link from each action in *actions*. Whenever an action deletes *subgoal*, the action needs to be before $a0$ or after $a1$. This method enumerates all constraints that result from protecting the causal link from all actions.

stripsPOP.py — (continued)

```
120     def protect_cl_for_actions(self, actions, constrs, clink):
121         """yields constraints that extend constrs and
122         protect causal link (a0, subgoal, a1)
123         for each action in actions
124         """
125         if actions:
126             a = actions[0]
127             rem_actions = actions[1:]
128             a0, subgoal, a1 = clink
129             if a != a0 and a != a1 and self.deletes(a,subgoal):
130                 if self.possible((a,a0),constrs):
131                     new_const = self.add_constraint((a,a0),constrs)
132                     for e in
                            self.protect_cl_for_actions(rem_actions,new_const,clink):
                            yield e # could be "yield from"
133                 if self.possible((a1,a),constrs):
134                     new_const = self.add_constraint((a1,a),constrs)
135                     for e in
                            self.protect_cl_for_actions(rem_actions,new_const,clink):
                            yield e
136             else:
137                 for e in
                        self.protect_cl_for_actions(rem_actions,constrs,clink):
                        yield e
138         else:
139             yield constrs
```

Given an action *act*, the following method protects all the causal links in *clinks* from *act*. Whenever *act* deletes *subgoal* from some causal link $(a0, subgoal, a1)$,

the action *act* needs to be before $a0$ or after $a1$. This method enumerates all constraints that result from protecting the causal links from *act*.

stripsPOP.py — (continued)

```
    def protect_all_cls(self, clinks, act, constrs):
        """yields constraints that protect all causal links from act"""
        if clinks:
            (a0,cond,a1) = clinks[0] # select a causal link
            rem_clinks = clinks[1:] # remaining causal links
            if act != a0 and act != a1 and self.deletes(act,cond):
                if self.possible((act,a0),constrs):
                    new_const = self.add_constraint((act,a0),constrs)
                    for e in self.protect_all_cls(rem_clinks,act,new_const):
                        yield e
                if self.possible((a1,act),constrs):
                    new_const = self.add_constraint((a1,act),constrs)
                    for e in self.protect_all_cls(rem_clinks,act,new_const):
                        yield e
            else:
                for e in self.protect_all_cls(rem_clinks,act,constrs): yield
                    e
        else:
            yield constrs
```

The following methods check whether an action (or action instance) achieves or deletes some subgoal.

stripsPOP.py — (continued)

```
    def achieves(self,action,subgoal):
        var,val = subgoal
        return var in self.effects(action) and self.effects(action)[var] ==
            val

    def deletes(self,action,subgoal):
        var,val = subgoal
        return var in self.effects(action) and self.effects(action)[var] !=
            val

    def effects(self,action):
        """returns the variable:value dictionary of the effects of action.
        works for both actions and action instances"""
        if isinstance(action, Action_instance):
            action = action.action
        if action == "start":
            return self.planning_problem.initial_state
        elif action == "finish":
            return {}
        else:
            return action.effects
```

The constraints are represented as a set of pairs closed under transitivity. Thus if (a, b) and (b, c) are the list, then (a, c) must also be in the list. This means

that adding a new constraint means adding the implied pairs, but querying whether some order is consistent is quick.

stripsPOP.py — (continued)

```
    def add_constraint(self, pair, const):
        if pair in const:
            return const
        todo = [pair]
        newconst = const.copy()
        while todo:
            x0,x1 = todo.pop()
            newconst.add((x0,x1))
            for x,y in newconst:
                if x==x1 and (x0,y) not in newconst:
                    todo.append((x0,y))
                if y==x0 and (x,x1) not in newconst:
                    todo.append((x,x1))
        return newconst

    def possible(self,pair,constraint):
        (x,y) = pair
        return (y,x) not in constraint
```

Some code for testing:

stripsPOP.py — (continued)

```
from searchBranchAndBound import DF_branch_and_bound
from searchMPP import SearcherMPP
import stripsProblem

rplanning0 = POP_search_from_STRIPS(stripsProblem.problem0)
rplanning1 = POP_search_from_STRIPS(stripsProblem.problem1)
rplanning2 = POP_search_from_STRIPS(stripsProblem.problem2)
searcher0 = DF_branch_and_bound(rplanning0,5)
searcher0a = SearcherMPP(rplanning0)
searcher1 = DF_branch_and_bound(rplanning1,10)
searcher1a = SearcherMPP(rplanning1)
searcher2 = DF_branch_and_bound(rplanning2,10)
searcher2a = SearcherMPP(rplanning2)
# Try one of the following searchers
# a = searcher0.search()
# a = searcher0a.search()
# a.end().extract_plan() # print a plan found
# a.end().constraints  # print the constraints
# SearcherMPP.max_display_level = 0 # less detailed display
# DF_branch_and_bound.max_display_level = 0 # less detailed display
# a = searcher1.search()
# a = searcher1a.search()
# a = searcher2.search()
# a = searcher2a.search()
```

Chapter 7

Supervised Machine Learning

This chapter is the first on machine learning. It covers the following topics:

- Data: how to load it, training and test sets
- Features: many of the features come directly from the data. Sometimes it is useful to construct features, e.g. $height > 1.9m$ might be a Boolean feature constructed from the real-values feature *height*. The next chapter is about neural networks and how to learn features; in this chapter we construct them explicitly in what is often known as **feature engineering**.
- Learning with no input features: this is the base case of many methods. What should we predict if we have no input features? This provides the base cases for many algorithms (e.g., decision tree algorithm) and baselines that more sophisticated algorithms need to beat. It also provides ways to test various predictors.
- Decision tree learning: one of the classic and simplest learning algorithms, which is the basis of many other algorithms.
- Cross validation and parameter tuning: methods to prevent overfitting.
- Linear regression and classification: other classic and simple techniques that often work well (particularly combined with feature learning or engineering).
- Boosting: combining simpler learning methods to make even better learners.

A good source of classic datasets is the UCI Machine Learning Repository [Lichman, 2013] [Dua and Graff, 2017]. The SPECT, IRIS, and car datasets (carbool is a Boolean version of the car dataset) are from this repository.

Dataset	# Examples	#Columns	Input Types	Target Type
SPECT	267	23	Boolean	Boolean
IRIS	150	5	numeric	categorical
carbool	1728	7	categorical/numeric	numeric
holiday	32	6	Boolean	Boolean
mail_reading	28	5	Boolean	Boolean
tv_likes	12	5	Boolean	Boolean
simp_regr	7	2	numeric	numeric

Figure 7.1: Some of the datasets used here.

7.1 Representations of Data and Predictions

The code uses the following definitions and conventions:

- A **dataset** is an enumeration of examples.
- An **example** is a list (or tuple) of values. The values can be numbers or strings.
- A **feature** is a function from examples into the range of the feature. Each feature `f` also has the following attributes:

  `f.ftype`, the type of `f`, one of: `"boolean"`, `"categorical"`, `"numeric"`

  `f.frange`, the set of values of `f` seen in the dataset, represented as a list. The `ftype` is inferred from the `frange` if not given explicitly.

  `f.__doc__`, the docstring, a string description of `f` (for printing).

  Thus for example, a **Boolean feature** is a function from the examples into $\{False, True\}$. So, if f is a Boolean feature, $f.frange == [False, True]$, and if e is an example, $f(e)$ is either *True* or *False*.

learnProblem.py — A Learning Problem

```
import math, random, statistics
import csv
from display import Displayable
from utilities import argmax

boolean = [False, True]
```

When creating a dataset, we partition the data into a training set (*train*) and a test set (*test*). The target feature is the feature that we are making a prediction of. A dataset `ds` has the following attributes

`ds.train` a list of the training examples

`ds.test` a list of the test examples

`ds.target_index` the index of the target

`ds.target` the feature corresponding to the target (a function as described above)

`ds.input_features` a list of the input features

learnProblem.py — (continued)

```
18 class Data_set(Displayable):
19     """ A dataset consists of a list of training data and a list of test
           data.
20     """
21
22     def __init__(self, train, test=None, prob_test=0.20, target_index=0,
23                  header=None, target_type= None, seed=None): #12345):
24         """A dataset for learning.
25         train is a list of tuples representing the training examples
26         test is the list of tuples representing the test examples
27         if test is None, a test set is created by selecting each
28             example with probability prob_test
29         target_index is the index of the target.
30             If negative, it counts from right.
31             If target_index is larger than the number of properties,
32             there is no target (for unsupervised learning)
33         header is a list of names for the features
34         target_type is either None for automatic detection of target type
35              or one of "numeric", "boolean", "categorical"
36         seed is for random number; None gives a different test set each time
37         """
38         if seed: # given seed makes partition consistent from run-to-run
39             random.seed(seed)
40         if test is None:
41             train,test = partition_data(train, prob_test)
42         self.train = train
43         self.test = test
44
45         self.display(1,"Training set has",len(train),"examples. Number of
               columns: ",{len(e) for e in train})
46         self.display(1,"Test set has",len(test),"examples. Number of
               columns: ",{len(e) for e in test})
47         self.prob_test = prob_test
48         self.num_properties = len(self.train[0])
49         if target_index < 0: #allows for -1, -2, etc.
50             self.target_index = self.num_properties + target_index
51         else:
52             self.target_index = target_index
53         self.header = header
54         self.domains = [set() for i in range(self.num_properties)]
55         for example in self.train:
56             for ind,val in enumerate(example):
```

```
57                 self.domains[ind].add(val)
58         self.conditions_cache = {} # cache for computed conditions
59         self.create_features()
60         if target_type:
61             self.target.ftype = target_type
62         self.display(1,"There are",len(self.input_features),"input
               features")
63
64     def __str__(self):
65         if self.train and len(self.train)>0:
66             return ("Data: "+str(len(self.train))+" training examples, "
67                    +str(len(self.test))+" test examples, "
68                    +str(len(self.train[0]))+" features.")
69         else:
70             return ("Data: "+str(len(self.train))+" training examples, "
71                    +str(len(self.test))+" test examples.")
```

A **feature** is a function that takes an example and returns a value in the range of the feature. Each feature has a **frange**, which gives the range of the feature, and an **ftype** that gives the type, one of "boolean", "numeric" or "categorical".

learnProblem.py — (continued)

```
73     def create_features(self):
74         """create the set of features
75         """
76         self.target = None
77         self.input_features = []
78         for i in range(self.num_properties):
79             def feat(e,index=i):
80                 return e[index]
81             if self.header:
82                 feat.__doc__ = self.header[i]
83             else:
84                 feat.__doc__ = "e["+str(i)+"]"
85             feat.frange = list(self.domains[i])
86             feat.ftype = self.infer_type(feat.frange)
87             if i == self.target_index:
88                 self.target = feat
89             else:
90                 self.input_features.append(feat)
```

We try to infer the type of each feature. Sometimes this can be wrong, (e.g., when the numbers are really categorical) and may need to be set explicitly.

learnProblem.py — (continued)

```
92     def infer_type(self,domain):
93         """Infers the type of a feature with domain
94         """
95         if all(v in {True,False} for v in domain):
96             return "boolean"
```

```
97          if all(isinstance(v,(float,int)) for v in domain):
98              return "numeric"
99          else:
100             return "categorical"
```

7.1.1 Creating Boolean Conditions from Features

Some of the algorithms require Boolean input features or features with range $\{0, 1\}$. In order to be able to use these algorithms on datasets that allow for arbitrary domains of input variables, we construct Boolean conditions from the attributes.

There are 3 cases:

- When the range only has two values, we designate one to be the "true" value.
- When the values are all numeric, we assume they are ordered (as opposed to just being some classes that happen to be labelled with numbers) and construct Boolean features for splits of the data. That is, the feature is $e[ind] < cut$ for some value cut. We choose a number of cut values, up to a maximum number of cuts, given by max_num_cuts.
- When the values are not all numeric, we create an indicator function for each value. An indicator function for a value returns true when that value is given and false otherwise. Note that we can't create an indicator function for values that appear in the test set but not in the training set because we haven't seen the test set. For the examples in the test set with a value that doesn't appear in the training set for that feature, the indicator functions all return false.

There is also an option `categorical_only` to create only Boolean features for categorical input features, and not to make cuts for numerical values.

learnProblem.py — (continued)

```
102     def conditions(self, max_num_cuts=8, categorical_only = False):
103         """returns a set of boolean conditions from the input features
104         max_num_cuts is the maximum number of cute for numeric features
105         categorical_only is true if only categorical features are made
                binary
106         """
107         if (max_num_cuts, categorical_only) in self.conditions_cache:
108             return self.conditions_cache[(max_num_cuts, categorical_only)]
109         conds = []
110         for ind,frange in enumerate(self.domains):
111             if ind != self.target_index and len(frange)>1:
112                 if len(frange) == 2:
113                     # two values, the feature is equality to one of them.
114                     true_val = list(frange)[1] # choose one as true
```

```
                    def feat(e, i=ind, tv=true_val):
                        return e[i]==tv
                    if self.header:
                        feat.__doc__ = f"{self.header[ind]}=={true_val}"
                    else:
                        feat.__doc__ = f"e[{ind}]=={true_val}"
                    feat.frange = boolean
                    feat.ftype = "boolean"
                    conds.append(feat)
                elif all(isinstance(val,(int,float)) for val in frange):
                    if categorical_only: # numeric, don't make cuts
                        def feat(e, i=ind):
                            return e[i]
                        feat.__doc__ = f"e[{ind}]"
                        conds.append(feat)
                    else:
                        # all numeric, create cuts of the data
                        sorted_frange = sorted(frange)
                        num_cuts = min(max_num_cuts,len(frange))
                        cut_positions = [len(frange)*i//num_cuts for i in
                            range(1,num_cuts)]
                        for cut in cut_positions:
                            cutat = sorted_frange[cut]
                            def feat(e, ind_=ind, cutat=cutat):
                                return e[ind_] < cutat

                            if self.header:
                                feat.__doc__ = self.header[ind]+"<"+str(cutat)
                            else:
                                feat.__doc__ = "e["+str(ind)+"]<"+str(cutat)
                            feat.frange = boolean
                            feat.ftype = "boolean"
                            conds.append(feat)
                else:
                    # create an indicator function for every value
                    for val in frange:
                        def feat(e, ind_=ind, val_=val):
                            return e[ind_] == val_
                        if self.header:
                            feat.__doc__ = self.header[ind]+"=="+str(val)
                        else:
                            feat.__doc__= "e["+str(ind)+"]=="+str(val)
                        feat.frange = boolean
                        feat.ftype = "boolean"
                        conds.append(feat)
        self.conditions_cache[(max_num_cuts, categorical_only)] = conds
        return conds
```

Exercise 7.1 Change the code so that it splits using $e[ind] \leq cut$ instead of $e[ind] < cut$. Check boundary cases, such as 3 elements with 2 cuts. As a test case, make sure that when the range is the 30 integers from 100 to 129, and you want 2 cuts,

the resulting Boolean features should be $e[ind] \leq 109$ and $e[ind] \leq 119$ to make sure that each of the resulting domains is of equal size.

Exercise 7.2 This splits on whether the feature is less than one of the values in the training set. Sam suggested it might be better to split between the values in the training set, and suggested using

$$cutat = (sorted\_frange[cut] + sorted\_frange[cut-1])/2$$

Why might Sam have suggested this? Does this work better? (Try it on a few datasets).

7.1.2 Evaluating Predictions

A **predictor** is a function that takes an example and makes a prediction on the values of the target features.

A **loss** takes a prediction and the actual value and returns a non-negative real number; lower is better. The **error** for a dataset is either the mean loss, or sometimes the sum of the losses. When reporting results the mean is usually used. When it is the sum, this will be made explicit.

The function *evaluate_dataset* returns the average error for each example, where the error for each example depends on the evaluation criteria. Here we consider three evaluation criteria, the squared error (average of the square of the difference between the actual and predicted values), absolute errors (average of the absolute difference between the actual and predicted values) and the log loss (the average negative log-likelihood, which can be interpreted as the number of bits to describe an example using a code based on the prediction treated as a probability).

learnProblem.py — (continued)

```
162     def evaluate_dataset(self, data, predictor, error_measure):
163         """Evaluates predictor on data according to the error_measure
164         predictor is a function that takes an example and returns a
165                 prediction for the target features.
166         error_measure(prediction,actual) -> non-negative real
167         """
168         if data:
169             try:
170                 value = statistics.mean(error_measure(predictor(e),
                        self.target(e))
171                         for e in data)
172             except ValueError: # if error_measure gives an error
173                 return float("inf") # infinity
174             return value
175         else:
176             return math.nan # not a number
```

The following evaluation criteria are defined. This is defined using a class, `Evaluate` but no instances will be created. Just use `Evaluate.squared_loss` etc.

(Please keep the __doc__ strings a consistent length as they are used in tables.) The prediction is either a real value or a $\{value : probability\}$ dictionary or a list. The actual is either a real number or a key of the prediction.

learnProblem.py — (continued)

```
class Evaluate(object):
    """A container for the evaluation measures"""

    def squared_loss(prediction, actual):
        "squared loss "
        if isinstance(prediction, (list,dict)):
            return (1-prediction[actual])**2 # the correct value is 1
        else:
            return (prediction-actual)**2

    def absolute_loss(prediction, actual):
        "absolute loss "
        if isinstance(prediction, (list,dict)):
            return abs(1-prediction[actual]) # the correct value is 1
        else:
            return abs(prediction-actual)

    def log_loss(prediction, actual):
        "log loss (bits)"
        try:
            if isinstance(prediction, (list,dict)):
                return -math.log2(prediction[actual])
            else:
                return -math.log2(prediction) if actual==1 else
                    -math.log2(1-prediction)
        except ValueError:
            return float("inf") # infinity

    def accuracy(prediction, actual):
        "accuracy    "
        if isinstance(prediction, dict):
            prev_val = prediction[actual]
            return 1 if all(prev_val >= v for v in prediction.values())
                else 0
        if isinstance(prediction, list):
            prev_val = prediction[actual]
            return 1 if all(prev_val >= v for v in prediction) else 0
        else:
            return 1 if abs(actual-prediction) <= 0.5 else 0

    all_criteria = [accuracy, absolute_loss, squared_loss, log_loss]
```

7.1.3 Creating Test and Training Sets

The following method partitions the data into a training set and a test set. Note that this does not guarantee that the test set will contain exactly a proportion of the data equal to *prob_test*.

[An alternative is to use *random.sample*() which can guarantee that the test set will contain exactly a particular proportion of the data. However this would require knowing how many elements are in the dataset, which we may not know, as *data* may just be a generator of the data (e.g., when reading the data from a file).]

learnProblem.py — (continued)

```
218 def partition_data(data, prob_test=0.30):
219     """partitions the data into a training set and a test set, where
220     prob_test is the probability of each example being in the test set.
221     """
222     train = []
223     test = []
224     for example in data:
225         if random.random() < prob_test:
226             test.append(example)
227         else:
228             train.append(example)
229     return train, test
```

7.1.4 Importing Data From File

A dataset is typically loaded from a file. The default here is that it loaded from a CSV (comma separated values) file, although the separator can be changed. This assumes that all lines that contain the separator are valid data (so we only include those data items that contain more than one element). This allows for blank lines and comment lines that do not contain the separator. However, it means that this method is not suitable for cases where there is only one feature.

Note that *data_all* and *data_tuples* are generators. *data_all* is a generator of a list of list of strings. This version assumes that CSV files are simple. The standard *csv* package, that allows quoted arguments, can be used by uncommenting the line for *data_all* and commenting out the following line. *data_tuples* contains only those lines that contain the delimiter (others lines are assumed to be empty or comments), and tries to convert the elements to numbers whenever possible.

This allows for some of the columns to be included; specified by *include_only*. Note that if *include_only* is specified, the target index is the index for the included columns, not the original columns.

learnProblem.py — (continued)

```
231 class Data_from_file(Data_set):
232     def __init__(self, file_name, separator=',', num_train=None,
        prob_test=0.3,
```

```
            has_header=False, target_index=0, boolean_features=True,
            categorical=[], target_type= None, include_only=None,
                seed=None): #seed=12345):
        """create a dataset from a file
        separator is the character that separates the attributes
        num_train is a number specifying the first num_train tuples are
            training, or None
        prob_test is the probability an example should in the test set (if
            num_train is None)
        has_header is True if the first line of file is a header
        target_index specifies which feature is the target
        boolean_features specifies whether we want to create Boolean
            features
            (if False, it uses the original features).
        categorical is a set (or list) of features that should be treated
            as categorical
        target_type is either None for automatic detection of target type
             or one of "numeric", "boolean", "categorical"
        include_only is a list or set of indexes of columns to include
        """
        self.boolean_features = boolean_features
        with open(file_name,'r',newline='') as csvfile:
            self.display(1,"Loading",file_name)
            # data_all = csv.reader(csvfile,delimiter=separator) # for more
                complicated CSV files
            data_all = (line.strip().split(separator) for line in csvfile)
            if include_only is not None:
                data_all = ([v for (i,v) in enumerate(line) if i in
                    include_only]
                            for line in data_all)
            if has_header:
                header = next(data_all)
            else:
                header = None
            data_tuples = (interpret_elements(d) for d in data_all if
                len(d)>1)
            if num_train is not None:
                # training set is divided into training then text examples
                # the file is only read once, and the data is placed in
                    appropriate list
                train = []
                for i in range(num_train):  # will give an error if
                    insufficient examples
                    train.append(next(data_tuples))
                test = list(data_tuples)
                Data_set.__init__(self,train, test=test,
                    target_index=target_index,header=header)
            else:    # randomly assign training and test examples
                Data_set.__init__(self,data_tuples, test=None,
                    prob_test=prob_test,
```

```
271                                 target_index=target_index, header=header,
                                        seed=seed, target_type=target_type)
```

The following class is used for datasets where the training and test are in different files

learnProblem.py — (continued)

```
273 class Data_from_files(Data_set):
274     def __init__(self, train_file_name, test_file_name, separator=',',
275                  has_header=False, target_index=0, boolean_features=True,
276                  categorical=[], target_type= None, include_only=None):
277         """create a dataset from separate training and file
278         separator is the character that separates the attributes
279         num_train is a number specifying the first num_train tuples are
                training, or None
280         prob_test is the probability an example should in the test set (if
                num_train is None)
281         has_header is True if the first line of file is a header
282         target_index specifies which feature is the target
283         boolean_features specifies whether we want to create Boolean
                features
284            (if False, it uses the original features).
285         categorical is a set (or list) of features that should be treated
                as categorical
286         target_type is either None for automatic detection of target type
287             or one of "numeric", "boolean", "categorical"
288         include_only is a list or set of indexes of columns to include
289         """
290         self.boolean_features = boolean_features
291         with open(train_file_name,'r',newline='') as train_file:
292           with open(test_file_name,'r',newline='') as test_file:
293             # data_all = csv.reader(csvfile,delimiter=separator) # for more
                    complicated CSV files
294             train_data = (line.strip().split(separator) for line in
                    train_file)
295             test_data = (line.strip().split(separator) for line in
                    test_file)
296             if include_only is not None:
297                 train_data = ([v for (i,v) in enumerate(line) if i in
                        include_only]
298                               for line in train_data)
299                 test_data = ([v for (i,v) in enumerate(line) if i in
                        include_only]
300                               for line in test_data)
301             if has_header: # this assumes the training file has a header
                    and the test file doesn't
302                 header = next(train_data)
303             else:
304                 header = None
305             train_tuples = [interpret_elements(d) for d in train_data if
                    len(d)>1]
```

```
306             test_tuples = [interpret_elements(d) for d in test_data if
                    len(d)>1]
307             Data_set.__init__(self,train_tuples, test_tuples,
308                               target_index=target_index, header=header)
```

When reading from a file all of the values are strings. This next method tries to convert each value into a number (an int or a float) or Boolean, if it is possible.

learnProblem.py — (continued)

```
310 def interpret_elements(str_list):
311     """make the elements of string list str_list numeric if possible.
312     Otherwise remove initial and trailing spaces.
313     """
314     res = []
315     for e in str_list:
316         try:
317             res.append(int(e))
318         except ValueError:
319             try:
320                 res.append(float(e))
321             except ValueError:
322                 se = e.strip()
323                 if se in ["True","true","TRUE"]:
324                     res.append(True)
325                 elif se in ["False","false","FALSE"]:
326                     res.append(False)
327                 else:
328                     res.append(e.strip())
329     return res
```

7.1.5 Augmented Features

Sometimes we want to augment the features with new features computed from the old features (e.g., the product of features). Here we allow the creation of a new dataset from an old dataset but with new features. Note that special cases of these are **kernel**s; mapping the original feature space into a new space, which allow a neat way to do learning in the augmented space for many mappings (the "kernel trick"). This is beyond the scope of AIPython; those interested should read about support vector machines.

A feature is a function of examples. A unary feature constructor takes a feature and returns a new feature. A binary feature combiner takes two features and returns a new feature.

learnProblem.py — (continued)

```
331 class Data_set_augmented(Data_set):
332     def __init__(self, dataset, unary_functions=[], binary_functions=[],
            include_orig=True):
333         """creates a dataset like dataset but with new features
```

```
334         unary_function is a list of unary feature constructors
335         binary_functions is a list of binary feature combiners.
336         include_orig specifies whether the original features should be
                included
337         """
338         self.orig_dataset = dataset
339         self.unary_functions = unary_functions
340         self.binary_functions = binary_functions
341         self.include_orig = include_orig
342         self.target = dataset.target
343         Data_set.__init__(self,dataset.train, test=dataset.test,
344                           target_index = dataset.target_index)
345
346     def create_features(self):
347         if self.include_orig:
348             self.input_features = self.orig_dataset.input_features.copy()
349         else:
350             self.input_features = []
351         for u in self.unary_functions:
352             for f in self.orig_dataset.input_features:
353                 self.input_features.append(u(f))
354         for b in self.binary_functions:
355             for f1 in self.orig_dataset.input_features:
356                 for f2 in self.orig_dataset.input_features:
357                     if f1 != f2:
358                         self.input_features.append(b(f1,f2))
```

The following are useful unary feature constructors and binary feature combiner.

learnProblem.py — (continued)

```
360 def square(f):
361     """a unary feature constructor to construct the square of a feature
362     """
363     def sq(e):
364         return f(e)**2
365     sq.__doc__ = f.__doc__+"**2"
366     return sq
367
368 def power_feat(n):
369     """given n returns a unary feature constructor to construct the nth
            power of a feature.
370     e.g., power_feat(2) is the same as square, defined above
371     """
372     def fn(f,n=n):
373         def pow(e,n=n):
374             return f(e)**n
375         pow.__doc__ = f.__doc__+"**"+str(n)
376         return pow
377     return fn
378
```

```
def prod_feat(f1,f2):
    """a new feature that is the product of features f1 and f2
    """
    def feat(e):
        return f1(e)*f2(e)
    feat.__doc__ = f1.__doc__+"*"+f2.__doc__
    return feat

def eq_feat(f1,f2):
    """a new feature that is 1 if f1 and f2 give same value
    """
    def feat(e):
        return 1 if f1(e)==f2(e) else 0
    feat.__doc__ = f1.__doc__+"=="+f2.__doc__
    return feat

def neq_feat(f1,f2):
    """a new feature that is 1 if f1 and f2 give different values
    """
    def feat(e):
        return 1 if f1(e)!=f2(e) else 0
    feat.__doc__ = f1.__doc__+"!="+f2.__doc__
    return feat
```

Example:

learnProblem.py — (continued)

```
# from learnProblem import Data_set_augmented,prod_feat
# data = Data_from_file('data/holiday.csv', has_header=True, num_train=19,
    target_index=-1)
# data = Data_from_file('data/iris.data', prob_test=1/3, target_index=-1)
## Data = Data_from_file('data/SPECT.csv', prob_test=0.5, target_index=0)
# dataplus = Data_set_augmented(data,[],[prod_feat])
# dataplus = Data_set_augmented(data,[],[prod_feat,neq_feat])
```

Exercise 7.3 For symmetric properties, such as product, we don't need both $f1 * f2$ as well as $f2 * f1$ as extra properties. Allow the user to be able to declare feature constructors as symmetric (by associating a Boolean feature with them). Change *construct_features* so that it does not create both versions for symmetric combiners.

7.2 Generic Learner Interface

A **learner** takes a dataset (and possibly other arguments specific to the method). To get it to learn, we call the *learn*() method. This implements *Displayable* so that we can display traces at multiple levels of detail (perhaps with a GUI).

learnProblem.py — (continued)

```
from display import Displayable
```

```
410
411 class Learner(Displayable):
412     def __init__(self, dataset):
413         raise NotImplementedError("Learner.__init__") # abstract method
414
415     def learn(self):
416         """returns a predictor, a function from a tuple to a value for the
                target feature
417         """
418         raise NotImplementedError("learn") # abstract method
```

7.3 Learning With No Input Features

If we make the same prediction for each example, what prediction should we make? This can be used as a naive baseline; if a more sophisticated method does not do better than this, it is not useful. This also provides the base case for some methods, such as decision-tree learning.

> To run demo to compare different prediction methods on various evaluation criteria, in folder "`aipython`", load "`learnNoInputs.py`", using e.g., `ipython -i learnNoInputs.py`, and it prints some test results.

There are a few alternatives as to what could be allowed in a prediction:

- a point prediction, where we are only allowed to predict one of the values of the feature. For example, if the values of the feature are $\{0, 1\}$ we are only allowed to predict 0 or 1 or of the values are ratings in $\{1, 2, 3, 4, 5\}$, we can only predict one of these integers.
- a point prediction, where we are allowed to predict any value. For example, if the values of the feature are $\{0, 1\}$ we may be allowed to predict 0.3, 1, or even 1.7. For all of the criteria we can imagine, there is no point in predicting a value greater than 1 or less that zero (but that doesn't mean we can't), but it is often useful to predict a value between 0 and 1. If the values are ratings in $\{1, 2, 3, 4, 5\}$, we may want to predict 3.4.
- a probability distribution over the values of the feature. For each value v, we predict a non-negative number p_v, such that the sum over all predictions is 1.

For regression, we do the first of these. For classification, we do the second. The third can be implemented by having multiple indicator functions for the target.

Here are some prediction functions that take in an enumeration of values, a domain, and returns a value or dictionary of $\{value : prediction\}$. Note that `cmedian` returns one of the middle values when there are an even number of

examples, whereas median gives the average of them (and so cmedian is applicable for ordinals that cannot be considered cardinal values). Similarly, cmode picks one of the values when more than one value has the maximum number of elements.

learnNoInputs.py — Learning ignoring all input features

```
from learnProblem import Evaluate
import math, random, collections, statistics
import utilities # argmax for (element,value) pairs

class Predict(object):
    """The class of prediction methods for a list of values.
    Please make the doc strings the same length, because they are used in tables.
    Note that we don't need self argument, as we are creating Predict objects,
    To use call Predict.laplace(data) etc."""

    ### The following return a distribution over values (for classification)
    def empirical(data, domain=[0,1], icount=0):
        "empirical dist "
        # returns a distribution over values
        counts = {v:icount for v in domain}
        for e in data:
            counts[e] += 1
        s = sum(counts.values())
        return {k:v/s for (k,v) in counts.items()}

    def bounded_empirical(data, domain=[0,1], bound=0.01):
        "bounded empirical"
        return {k:min(max(v,bound),1-bound) for (k,v) in
            Predict.empirical(data, domain).items()}

    def laplace(data, domain=[0,1]):
        "Laplace       "  # for categorical data
        return Predict.empirical(data, domain, icount=1)

    def cmode(data, domain=[0,1]):
        "mode          " # for categorical data
        md = statistics.mode(data)
        return {v: 1 if v==md else 0 for v in domain}

    def cmedian(data, domain=[0,1]):
        "median        " # for categorical data
        md = statistics.median_low(data) # always return one of the values
        return {v: 1 if v==md else 0 for v in domain}

    ### The following return a single prediction (for regression). domain is ignored.

```

```
    def mean(data, domain=[0,1]):
        "mean          "
        # returns a real number
        return statistics.mean(data)

    def rmean(data, domain=[0,1], mean0=0, pseudo_count=1):
        "regularized mean"
        # returns a real number.
        # mean0 is the mean to be used for 0 data points
        # With mean0=0.5, pseudo_count=2, same as laplace for [0,1] data
        # this works for enumerations as well as lists
        sum = mean0 * pseudo_count
        count = pseudo_count
        for e in data:
            sum += e
            count += 1
        return sum/count

    def mode(data, domain=[0,1]):
        "mode          "
        return statistics.mode(data)

    def median(data, domain=[0,1]):
        "median        "
        return statistics.median(data)

    all = [empirical, mean, rmean, bounded_empirical, laplace, cmode, mode,
        median,cmedian]

    # The following suggests appropriate predictions as a function of the
        target type
    select = {"boolean": [empirical, bounded_empirical, laplace, cmode,
        cmedian],
             "categorical": [empirical, bounded_empirical, laplace, cmode,
                 cmedian],
             "numeric": [mean, rmean, mode, median]}
```

7.3.1 Evaluation

To evaluate a point prediction, we first generate some data from a simple (Bernoulli) distribution, where there are two possible values, 0 and 1 for the target feature. Given *prob*, a number in the range $[0, 1]$, this generate some training and test data where *prob* is the probability of each example being 1. To generate a 1 with probability *prob*, we generate a random number in range [0,1] and return 1 if that number is less than *prob*. A prediction is computed by applying the predictor to the training data, which is evaluated on the test set. This is repeated `num_samples` times.

Let's evaluate the predictions of the possible selections according to the different evaluation criteria, for various training sizes.

learnNoInputs.py — (continued)

```
def test_no_inputs(error_measures = Evaluate.all_criteria,
    num_samples=10000,
                   test_size=10, training_sizes=
                       [1,2,3,4,5,10,20,100,1000]):
    for train_size in training_sizes:
        results = {predictor: {error_measure: 0 for error_measure in
            error_measures}
                   for predictor in Predict.all}
        for sample in range(num_samples):
            prob = random.random()
            training = [1 if random.random()<prob else 0 for i in
                range(train_size)]
            test = [1 if random.random()<prob else 0 for i in
                range(test_size)]
            for predictor in Predict.all:
                prediction = predictor(training)
                for error_measure in error_measures:
                    results[predictor][error_measure] += sum(
                        error_measure(prediction,actual) for actual in
                        test)/test_size
        print(f"For training size {train_size}:")
        print("  Predictor\t","\t".join(error_measure.__doc__ for
                                        error_measure in
                                            error_measures),sep="\t")
        for predictor in Predict.all:
            print(f"  {predictor.__doc__}",
                  "\t".join("{:.7f}".format(results[predictor][error_measure]/num_samples)
                                for error_measure in
                                    error_measures),sep="\t")

if __name__ == "__main__":
    test_no_inputs()
```

Exercise 7.4 Which predictor works best for low counts when the error is

(a) Squared error

(b) Absolute error

(c) Log loss

You may need to try this a few times to make sure your answer is supported by the evidence. Does the difference from the other methods get more or less as the number of examples grow?

Exercise 7.5 Suggest some other predictions that only take the training data. Does your method do better than the given methods? A simple way to get other predictors is to vary the threshold of bounded average, or to change the pseodo-counts of the Laplace method (use other numbers instead of 1 and 2).

7.4 Decision Tree Learning

To run the decision tree learning demo, in folder "aipython", load "learnDT.py", using e.g., ipython -i learnDT.py, and it prints some test results. To try more examples, copy and paste the commented-out commands at the bottom of that file. This requires Python 3 with matplotlib.

The decision tree algorithm does binary splits, and assumes that all input features are binary functions of the examples. It stops splitting if there are no input features, the number of examples is less than a specified number of examples or all of the examples agree on the target feature.

learnDT.py — Learning a binary decision tree

```
from learnProblem import Learner, Evaluate
from learnNoInputs import Predict
import math

class DT_learner(Learner):
    def __init__(self,
                 dataset,
                 split_to_optimize=Evaluate.log_loss, # to minimize for at each split
                 leaf_prediction=Predict.empirical, # what to use for value at leaves
                 train=None,                  # used for cross validation
                 max_num_cuts=8, # maximum number of conditions to split a numeric feature into
                 gamma=1e-7 , # minimum improvement needed to expand a node
                 min_child_weight=10):
        self.dataset = dataset
        self.target = dataset.target
        self.split_to_optimize = split_to_optimize
        self.leaf_prediction = leaf_prediction
        self.max_num_cuts = max_num_cuts
        self.gamma = gamma
        self.min_child_weight = min_child_weight
        if train is None:
            self.train = self.dataset.train
        else:
            self.train = train

    def learn(self, max_num_cuts=8):
        """learn a decision tree"""
        return self.learn_tree(self.dataset.conditions(self.max_num_cuts), self.train)
```

The main recursive algorithm, takes in a set of input features and a set of training data. It first decides whether to split. If it doesn't split, it makes a point prediction, ignoring the input features.

It only splits if the best split increases the error by at least gamma. This implies it does not split when:

- there are no more input features
- there are fewer examples than *min_number_examples*,
- all the examples agree on the value of the target, or
- the best split puts all examples in the same partition.

If it splits, it selects the best split according to the evaluation criterion (assuming that is the only split it gets to do), and returns the condition to split on (in the variable *split*) and the corresponding partition of the examples.

learnDT.py — (continued)

```
40      def learn_tree(self, conditions, data_subset):
41          """returns a decision tree
42          conditions is a set of possible conditions
43          data_subset is a subset of the data used to build this (sub)tree
44
45          where a decision tree is a function that takes an example and
46          makes a prediction on the target feature
47          """
48          self.display(2,f"learn_tree with {len(conditions)} features and
                {len(data_subset)} examples")
49          split, partn = self.select_split(conditions, data_subset)
50          if split is None: # no split; return a point prediction
51              prediction = self.leaf_value(data_subset, self.target.frange)
52              self.display(2,f"leaf prediction for {len(data_subset)}
                    examples is {prediction}")
53              def leaf_fun(e):
54                  return prediction
55              leaf_fun.__doc__ = str(prediction)
56              leaf_fun.num_leaves = 1
57              return leaf_fun
58          else:  # a split succeeded
59              false_examples, true_examples = partn
60              rem_features = [fe for fe in conditions if fe != split]
61              self.display(2,"Splitting on",split.__doc__,"with examples
                    split",
62                           len(true_examples),":",len(false_examples))
63              true_tree = self.learn_tree(rem_features,true_examples)
64              false_tree = self.learn_tree(rem_features,false_examples)
65              def fun(e):
66                  if split(e):
67                      return true_tree(e)
68                  else:
69                      return false_tree(e)
70              #fun = lambda e: true_tree(e) if split(e) else false_tree(e)
71              fun.__doc__ = (f"(if {split.__doc__} then {true_tree.__doc__}"
```

```
                        f" else {false_tree.__doc__})")
            fun.num_leaves = true_tree.num_leaves + false_tree.num_leaves
            return fun
```

learnDT.py — (continued)

```
    def leaf_value(self, egs, domain):
        return self.leaf_prediction((self.target(e) for e in egs), domain)

    def select_split(self, conditions, data_subset):
        """finds best feature to split on.

        conditions is a non-empty list of features.
        returns feature, partition
        where feature is an input feature with the smallest error as
              judged by split_to_optimize or
              feature==None if there are no splits that improve the error
        partition is a pair (false_examples, true_examples) if feature is
            not None
        """
        best_feat = None # best feature
        # best_error = float("inf") # infinity - more than any error
        best_error = self.sum_losses(data_subset) - self.gamma
        self.display(3," no split has
            error=",best_error,"with",len(conditions),"conditions")
        best_partition = None
        for feat in conditions:
            false_examples, true_examples = partition(data_subset,feat)
            if
                min(len(false_examples),len(true_examples))>=self.min_child_weight:
                err = (self.sum_losses(false_examples)
                       + self.sum_losses(true_examples))
                self.display(3," split on",feat.__doc__,"has error=",err,
                         "splits
                             into",len(true_examples),":",len(false_examples),"gamma=",self.gamma)
                if err < best_error:
                    best_feat = feat
                    best_error=err
                    best_partition = false_examples, true_examples
        self.display(2,"best split is on",best_feat.__doc__,
                             "with err=",best_error)
        return best_feat, best_partition

    def sum_losses(self, data_subset):
        """returns sum of losses for dataset (with no more splits)
        There a single prediction for all leaves using leaf_prediction
        It is evaluated using split_to_optimize
        """
        prediction = self.leaf_value(data_subset, self.target.frange)
        error = sum(self.split_to_optimize(prediction, self.target(e))
                    for e in data_subset)
```

```
        return error

def partition(data_subset,feature):
    """partitions the data_subset by the feature"""
    true_examples = []
    false_examples = []
    for example in data_subset:
        if feature(example):
            true_examples.append(example)
        else:
            false_examples.append(example)
    return false_examples, true_examples
```

Test cases:

learnDT.py — (continued)

```
from learnProblem import Data_set, Data_from_file

def testDT(data, print_tree=True, selections = None, **tree_args):
    """Prints errors and the trees for various evaluation criteria and ways
        to select leaves.
    """
    if selections == None: # use selections suitable for target type
        selections = Predict.select[data.target.ftype]
    evaluation_criteria = Evaluate.all_criteria
    print("Split Choice","Leaf Choice\t","#leaves",'\t'.join(ecrit.__doc__
                                            for ecrit in
                                                evaluation_criteria),sep="\t")
    for crit in evaluation_criteria:
        for leaf in selections:
            tree = DT_learner(data, split_to_optimize=crit,
                leaf_prediction=leaf,
                                **tree_args).learn()
            print(crit.__doc__, leaf.__doc__, tree.num_leaves,
                   "\t".join("{:.7f}".format(data.evaluate_dataset(data.test,
                       tree, ecrit))
                                for ecrit in evaluation_criteria),sep="\t")
            if print_tree:
                print(tree.__doc__)

#DT_learner.max_display_level = 4
if __name__ == "__main__":
    # Choose one of the data files
    #data=Data_from_file('data/SPECT.csv', target_index=0);
        print("SPECT.csv")
    #data=Data_from_file('data/iris.data', target_index=-1);
        print("iris.data")
    data = Data_from_file('data/carbool.csv', target_index=-1, seed=123)
    #data = Data_from_file('data/mail_reading.csv', target_index=-1);
        print("mail_reading.csv")
```

```
158    #data = Data_from_file('data/holiday.csv', has_header=True,
           num_train=19, target_index=-1); print("holiday.csv")
159    testDT(data, print_tree=False)
```

Note that different runs may provide different values as they split the training and test sets differently. So if you have a hypothesis about what works better, make sure it is true for different runs.

Exercise 7.6 The current algorithm does not have a very sophisticated stopping criterion. What is the current stopping criterion? (Hint: you need to look at both *learn_tree* and *select_split*.)

Exercise 7.7 Extend the current algorithm to include in the stopping criterion

(a) A minimum child size; don't use a split if one of the children has fewer elements that this.

(b) A depth-bound on the depth of the tree.

(c) An improvement bound such that a split is only carried out if error with the split is better than the error without the split by at least the improvement bound.

Which values for these parameters make the prediction errors on the test set the smallest? Try it on more than one dataset.

Exercise 7.8 Without any input features, it is often better to include a pseudo-count that is added to the counts from the training data. Modify the code so that it includes a pseudo-count for the predictions. When evaluating a split, including pseudo counts can make the split worse than no split. Does pruning with an improvement bound and pseudo-counts make the algorithm work better than with an improvement bound by itself?

Exercise 7.9 Some people have suggested using information gain (which is equivalent to greedy optimization of log loss) as the measure of improvement when building the tree, even in they want to have non-probabilistic predictions in the final tree. Does this work better than myopically choosing the split that is best for the evaluation criteria we will use to judge the final prediction?

7.5 Cross Validation and Parameter Tuning

To run the cross validation demo, in folder "aipython", load "learnCrossValidation.py", using e.g., `ipython -i learnCrossValidation.py`. Run the examples at the end to produce a graph like Figure 7.15. Note that different runs will produce different graphs, so your graph will not look like the one in the textbook. To try more examples, copy and paste the commented-out commands at the bottom of that file. This requires Python 3 with `matplotlib`.

The above decision tree overfits the data. One way to determine whether the prediction is overfitting is by cross validation. The code below implements k-fold cross validation, which can be used to choose the value of parameters to best fit the training data. If we want to use parameter tuning to improve predictions on a particular dataset, we can only use the training data (and not the test data) to tune the parameter.

In k-fold cross validation, we partition the training set into k approximately equal-sized folds (each fold is an enumeration of examples). For each fold, we train on the other examples, and determine the error of the prediction on that fold. For example, if there are 10 folds, we train on 90% of the data, and then test on remaining 10% of the data. We do this 10 times, so that each example gets used as a test set once, and in the training set 9 times.

The code below creates one copy of the data, and multiple views of the data. For each fold, *fold* enumerates the examples in the fold, and *fold_complement* enumerates the examples not in the fold.

learnCrossValidation.py — Cross Validation for Parameter Tuning

```
from learnProblem import Data_set, Data_from_file, Evaluate
from learnNoInputs import Predict
from learnDT import DT_learner
import matplotlib.pyplot as plt
import random

class K_fold_dataset(object):
    def __init__(self, training_set, num_folds):
        self.data = training_set.train.copy()
        self.target = training_set.target
        self.input_features = training_set.input_features
        self.num_folds = num_folds
        self.conditions = training_set.conditions

        random.shuffle(self.data)
        self.fold_boundaries = [(len(self.data)*i)//num_folds
                                for i in range(0,num_folds+1)]

    def fold(self, fold_num):
        for i in range(self.fold_boundaries[fold_num],
                       self.fold_boundaries[fold_num+1]):
            yield self.data[i]

    def fold_complement(self, fold_num):
        for i in range(0,self.fold_boundaries[fold_num]):
            yield self.data[i]
        for i in range(self.fold_boundaries[fold_num+1],len(self.data)):
            yield self.data[i]
```

The validation error is the average error for each example, where we test on each fold, and learn on the other folds.

learnCrossValidation.py — (continued)

```
40      def validation_error(self, learner, error_measure, **other_params):
41          error = 0
42          try:
43              for i in range(self.num_folds):
44                  predictor = learner(self,
                        train=list(self.fold_complement(i)),
45                                  **other_params).learn()
46                  error += sum( error_measure(predictor(e), self.target(e))
47                              for e in self.fold(i))
48          except ValueError:
49              return float("inf") #infinity
50          return error/len(self.data)
```

The *plot_error* method plots the average error as a function of the minimum number of examples in decision-tree search, both for the validation set and for the test set. The error on the validation set can be used to tune the parameter — choose the value of the parameter that minimizes the error. The error on the test set cannot be used to tune the parameters; if it were to be used this way it could not be used to test how well the method works on unseen examples.

learnCrossValidation.py — (continued)

```
52  def plot_error(data, criterion=Evaluate.squared_loss,
        leaf_prediction=Predict.empirical,
53                  num_folds=5, maxx=None, xscale='linear'):
54      """Plots the error on the validation set and the test set
55      with respect to settings of the minimum number of examples.
56      xscale should be 'log' or 'linear'
57      """
58      plt.ion()
59      plt.xscale(xscale) # change between log and linear scale
60      plt.xlabel("min_child_weight")
61      plt.ylabel("average "+criterion.__doc__)
62      folded_data = K_fold_dataset(data, num_folds)
63      if maxx == None:
64          maxx = len(data.train)//2+1
65      verrors = [] # validation errors
66      terrors = [] # test set errors
67      for mcw in range(1,maxx):
68          verrors.append(folded_data.validation_error(DT_learner,criterion,leaf_prediction=leaf_predi
69                                                      min_child_weight=mcw))
70          tree = DT_learner(data, criterion, leaf_prediction=leaf_prediction,
              min_child_weight=mcw).learn()
71          terrors.append(data.evaluate_dataset(data.test,tree,criterion))
72      plt.plot(range(1,maxx), verrors, ls='-',color='k',
73                  label="validation for "+criterion.__doc__)
74      plt.plot(range(1,maxx), terrors, ls='--',color='k',
75                  label="test set for "+criterion.__doc__)
76      plt.legend()
77      plt.draw()
78
```

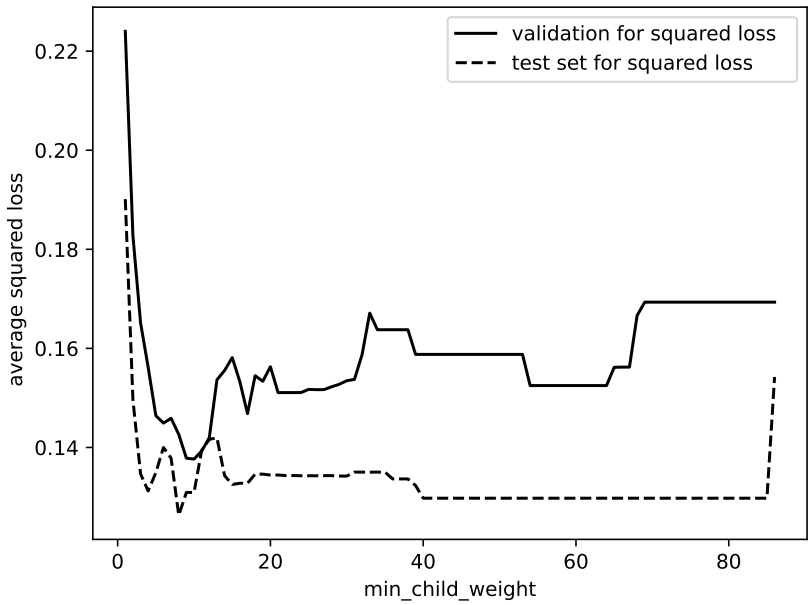


Figure 7.2: plot_error for SPECT dataset

```
79 # The following produces the graphs of Figure 7.18 of Poole and Mackworth
       [2023]
80 # data = Data_from_file('data/SPECT.csv',target_index=0, seed=123)
81 # plot_error(data, criterion=Evaluate.log_loss,
       leaf_prediction=Predict.laplace)
82
83 #also try:
84 # plot_error(data)
85 # data = Data_from_file('data/carbool.csv', target_index=-1, seed=123)
```

Figure 7.2 shows the average squared loss in the validation and test sets as a function of the min_child_weight in the decision-tree learning algorithm. (SPECT data with seed 12345 followed by plot_error(data)). Different seeds will produce different graphs. The assumption behind cross validation is that the parameter that minimizes the loss on the validation set, will be a good parameter for the test set.

Note that different runs for the same data will have the same test error, but different validation error. If you rerun the Data_from_file, with a different seed, you will get the new test and training sets, and so the graph will change.

Exercise 7.10 Change the error plot so that it can evaluate the stopping criteria of the exercise of Section 7.6. Which criteria makes the most difference?

7.6 Linear Regression and Classification

Here is a stochastic gradient descent searcher for linear regression and classification.

learnLinear.py — Linear Regression and Classification

```
from learnProblem import Learner
import random, math

class Linear_learner(Learner):
    def __init__(self, dataset, train=None,
                 learning_rate=0.1, max_init = 0.2,
                 squashed=True, batch_size=10):
        """Creates a gradient descent searcher for a linear classifier.
        The main learning is carried out by learn()

        dataset provides the target and the input features
        train provides a subset of the training data to use
        number_iterations is the default number of steps of gradient descent
        learning_rate is the gradient descent step size
        max_init is the maximum absolute value of the initial weights
        squashed specifies whether the output is a squashed linear function
        """
        self.dataset = dataset
        self.target = dataset.target
        if train==None:
            self.train = self.dataset.train
        else:
            self.train = train
        self.learning_rate = learning_rate
        self.squashed = squashed
        self.batch_size = batch_size
        self.input_features = [one]+dataset.input_features # one is defined
            below
        self.weights = {feat:random.uniform(-max_init,max_init)
                        for feat in self.input_features}
```

predictor predicts the value of an example from the current parameter settings. *predictor_string* gives a string representation of the predictor.

learnLinear.py — (continued)

```

    def predictor(self,e):
        """returns the prediction of the learner on example e"""
        linpred = sum(w*f(e) for f,w in self.weights.items())
        if self.squashed:
            return sigmoid(linpred)
        else:
            return linpred

    def predictor_string(self, sig_dig=3):
```

```
        """returns the doc string for the current prediction function
        sig_dig is the number of significant digits in the numbers"""
        doc = "+".join(str(round(val,sig_dig))+"*"+feat.__doc__
                       for feat,val in self.weights.items())
        if self.squashed:
            return "sigmoid("+ doc+")"
        else:
            return doc
```

learn is the main algorithm of the learner. It does *num_iter* steps of stochastic gradient descent. Only the number of iterations is specified; the other parameters it gets from the class.

learnLinear.py — (continued)

```
    def learn(self,num_iter=100):
        batch_size = min(self.batch_size, len(self.train))
        d = {feat:0 for feat in self.weights}
        for it in range(num_iter):
            self.display(2,"prediction=",self.predictor_string())
            for e in random.sample(self.train, batch_size):
                error = self.predictor(e) - self.target(e)
                update = self.learning_rate*error
                for feat in self.weights:
                    d[feat] += update*feat(e)
            for feat in self.weights:
                self.weights[feat] -= d[feat]
                d[feat]=0
        return self.predictor
```

one is a function that always returns 1. This is used for one of the input properties.

learnLinear.py — (continued)

```
def one(e):
    "1"
    return 1
```

$sigmoid(x)$ is the function

$$\frac{1}{1+e^{-x}}$$

The inverse of *sigmoid* is the *logit* function

learnLinear.py — (continued)

```
def sigmoid(x):
    return 1/(1+math.exp(-x))

def logit(x):
    return -math.log(1/x-1)
```

$sigmoid([x_0, v_2, \ldots])$ returns $[v_0, v_2, \ldots]$ where

$$v_i = \frac{exp(x_i)}{\sum_j exp(x_j)}$$

The inverse of *sigmoid* is the *logit* function

learnLinear.py — (continued)

```
def softmax(xs,domain=None):
    """xs is a list of values, and
    domain is the domain (a list) or None if the list should be returned
    returns a distribution over the domain (a dict)
    """
    m = max(xs) # use of m prevents overflow (and all values underflowing)
    exps = [math.exp(x-m) for x in xs]
    s = sum(exps)
    if domain:
        return {d:v/s for (d,v) in zip(domain,exps)}
    else:
        return [v/s for v in exps]

def indicator(v, domain):
    return [1 if v==dv else 0 for dv in domain]
```

The following tests the learner on a datasets. Uncomment the other datasets for different examples.

learnLinear.py — (continued)

```
from learnProblem import Data_set, Data_from_file, Evaluate
from learnProblem import Evaluate
import matplotlib.pyplot as plt

def test(**args):
    data = Data_from_file('data/SPECT.csv', target_index=0)
    # data = Data_from_file('data/mail_reading.csv', target_index=-1)
    # data = Data_from_file('data/carbool.csv', target_index=-1)
    learner = Linear_learner(data,**args)
    learner.learn()
    print("function learned is", learner.predictor_string())
    for ecrit in Evaluate.all_criteria:
        test_error = data.evaluate_dataset(data.test, learner.predictor,
            ecrit)
        print("   Average", ecrit.__doc__, "is", test_error)
```

The following plots the errors on the training and test sets as a function of the number of steps of gradient descent.

learnLinear.py — (continued)

```
def plot_steps(learner=None,
               data = None,
               criterion=Evaluate.squared_loss,
```

```
            step=1,
            num_steps=1000,
            log_scale=True,
            legend_label=""):
    """
    plots the training and test error for a learner.
    data is the
    learner_class is the class of the learning algorithm
    criterion gives the evaluation criterion plotted on the y-axis
    step specifies how many steps are run for each point on the plot
    num_steps is the number of points to plot

    """
    if legend_label != "": legend_label+=" "
    plt.ion()
    plt.xlabel("step")
    plt.ylabel("Average "+criterion.__doc__)
    if log_scale:
        plt.xscale('log') #plt.semilogx() #Makes a log scale
    else:
        plt.xscale('linear')
    if data is None:
        data = Data_from_file('data/holiday.csv', has_header=True,
            num_train=19, target_index=-1)
        #data = Data_from_file('data/SPECT.csv', target_index=0)
        # data = Data_from_file('data/mail_reading.csv', target_index=-1)
        # data = Data_from_file('data/carbool.csv', target_index=-1)
    #random.seed(None)  # reset seed
    if learner is None:
        learner = Linear_learner(data)
    train_errors = []
    test_errors = []
    for i in range(1,num_steps+1,step):
        test_errors.append(data.evaluate_dataset(data.test,
            learner.predictor, criterion))
        train_errors.append(data.evaluate_dataset(data.train,
            learner.predictor, criterion))
        learner.display(2, "Train error:",train_errors[-1],
                        "Test error:",test_errors[-1])
        learner.learn(num_iter=step)
    plt.plot(range(1,num_steps+1,step),train_errors,ls='-',label=legend_label+"training")
    plt.plot(range(1,num_steps+1,step),test_errors,ls='--',label=legend_label+"test")
    plt.legend()
    plt.draw()
    learner.display(1, "Train error:",train_errors[-1],
                        "Test error:",test_errors[-1])

if __name__ == "__main__":
    test()

```

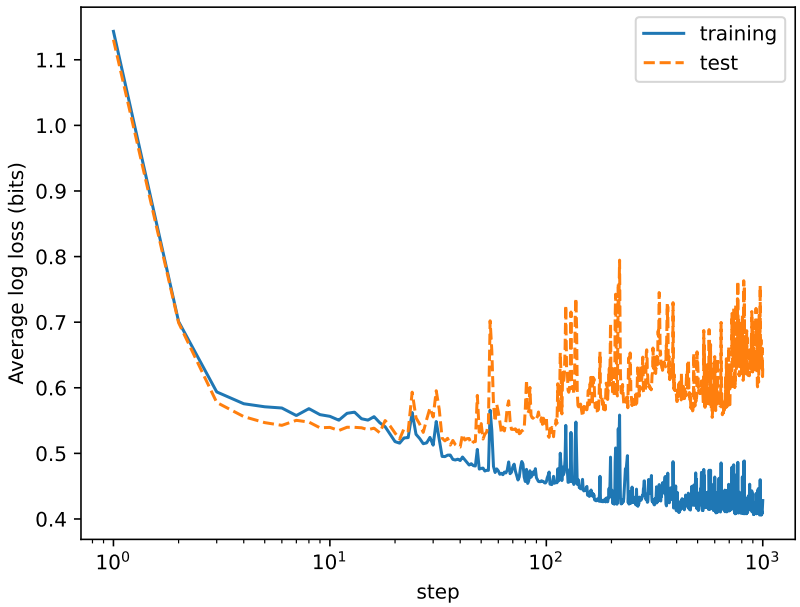


Figure 7.3: plot_steps for SPECT dataset

```
# This generates the figure
# from learnProblem import Data_set_augmented, prod_feat
# data = Data_from_file('data/SPECT.csv', prob_test=0.5, target_index=0,
    seed=123)
# dataplus = Data_set_augmented(data, [], [prod_feat])
# plot_steps(data=data, num_steps=1000)
# plot_steps(data=dataplus, num_steps=1000) # warning very slow
```

Figure 7.3 shows the result of `plot_steps(data=data, num_steps=1000)` in the code above. What would you expect to happen with the augmented data (with extra features)? Hint: think about underfitting and overfitting.

Exercise 7.11 The squashed learner only makes predictions in the range $(0,1)$. If the output values are $\{1,2,3,4\}$ there is no use predicting less than 1 or greater than 4. Change the squashed learner so that it can learn values in the range $(1,4)$. Test it on the file `'data/car.csv'`.

The following plots the prediction as a function of the number of steps of gradient descent. We first define a version of *range* that allows for real numbers (integers and floats).

learnLinear.py — (continued)

```
def arange(start,stop,step):
    """returns enumeration of values in the range [start,stop) separated by
        step.
```

```
174     like the built-in range(start,stop,step) but allows for integers and
            floats.
175     Note that rounding errors are expected with real numbers. (or use
            numpy.arange)
176     """
177     while start<stop:
178         yield start
179         start += step
180 
181 def plot_prediction(data,
182                 learner = None,
183                 minx = 0,
184                 maxx = 5,
185                 step_size = 0.01,  # for plotting
186                 label = "function"):
187     plt.ion()
188     plt.xlabel("x")
189     plt.ylabel("y")
190     if learner is None:
191         learner = Linear_learner(data, squashed=False)
192     learner.learning_rate=0.001
193     learner.learn(100)
194     learner.learning_rate=0.0001
195     learner.learn(1000)
196     learner.learning_rate=0.00001
197     learner.learn(10000)
198     learner.display(1,"function learned is", learner.predictor_string(),
199                 "error=",data.evaluate_dataset(data.train, learner.predictor,
                        Evaluate.squared_loss))
200     plt.plot([e[0] for e in data.train],[e[-1] for e in
            data.train],"bo",label="data")
201     plt.plot(list(arange(minx,maxx,step_size)),[learner.predictor([x])
202                                         for x in
                                                arange(minx,maxx,step_size)],
203                                       label=label)
204     plt.legend()
205     plt.draw()
```

learnLinear.py — (continued)

```
207 from learnProblem import Data_set_augmented, power_feat
208 def plot_polynomials(data,
209                 learner_class = Linear_learner,
210                 max_degree = 5,
211                 minx = 0,
212                 maxx = 5,
213                 num_iter = 1000000,
214                 learning_rate = 0.00001,
215                 step_size = 0.01, # for plotting
216                 ):
217     plt.ion()
```

```
218     plt.xlabel("x")
219     plt.ylabel("y")
220     plt.plot([e[0] for e in data.train],[e[-1] for e in
            data.train],"ko",label="data")
221     x_values = list(arange(minx,maxx,step_size))
222     line_styles = ['-','--','-.',':']
223     colors = ['0.5','k','k','k','k']
224     for degree in range(max_degree):
225         data_aug = Data_set_augmented(data,[power_feat(n) for n in
                range(1,degree+1)],
226                                       include_orig=False)
227         learner = learner_class(data_aug,squashed=False)
228         learner.learning_rate = learning_rate
229         learner.learn(num_iter)
230         learner.display(1,"For degree",degree,
231                         "function learned is", learner.predictor_string(),
232                         "error=",data.evaluate_dataset(data.train,
                            learner.predictor, Evaluate.squared_loss))
233         ls = line_styles[degree % len(line_styles)]
234         col = colors[degree % len(colors)]
235         plt.plot(x_values,[learner.predictor([x]) for x in x_values],
                linestyle=ls, color=col,
236                          label="degree="+str(degree))
237         plt.legend(loc='upper left')
238         plt.draw()
239
240 # Try:
241 # data0 = Data_from_file('data/simp_regr.csv', prob_test=0,
        boolean_features=False, target_index=-1)
242 # plot_prediction(data0)
243 # plot_polynomials(data0)
244 # What if the step size was bigger?
245 #datam = Data_from_file('data/mail_reading.csv', target_index=-1)
246 #plot_prediction(datam)
```

7.7 Boosting

The following code implements functional gradient boosting for regression.

A Boosted dataset is created from a base dataset by subtracting the prediction of the offset function from each example. This does not save the new dataset, but generates it as needed. The amount of space used is constant, independent on the size of the dataset.

learnBoosting.py — Functional Gradient Boosting

```
11 from learnProblem import Data_set, Learner, Evaluate
12 from learnNoInputs import Predict
13 from learnLinear import sigmoid
14 import statistics
15 import random
```

```
16
17 class Boosted_dataset(Data_set):
18     def __init__(self, base_dataset, offset_fun, subsample=1.0):
19         """new dataset which is like base_dataset,
20            but offset_fun(e) is subtracted from the target of each example e
21         """
22         self.base_dataset = base_dataset
23         self.offset_fun = offset_fun
24         self.train =
               random.sample(base_dataset.train,int(subsample*len(base_dataset.train)))
25         self.test = base_dataset.test
26         #Data_set.__init__(self, base_dataset.train, base_dataset.test,
27         #                  base_dataset.prob_test, base_dataset.target_index)
28
29         #def create_features(self):
30         """creates new features - called at end of Data_set.init()
31         defines a new target
32         """
33         self.input_features = self.base_dataset.input_features
34         def newout(e):
35             return self.base_dataset.target(e) - self.offset_fun(e)
36         newout.frange = self.base_dataset.target.frange
37         newout.ftype = self.infer_type(newout.frange)
38         self.target = newout
39
40     def conditions(self, *args, colsample_bytree=0.5, **nargs):
41         conds = self.base_dataset.conditions(*args, **nargs)
42         return random.sample(conds, int(colsample_bytree*len(conds)))
```

A boosting learner takes in a dataset and a base learner, and returns a new predictor. The base learner, takes a dataset, and returns a Learner object.

learnBoosting.py — (continued)

```
44 class Boosting_learner(Learner):
45     def __init__(self, dataset, base_learner_class, subsample=0.8):
46         self.dataset = dataset
47         self.base_learner_class = base_learner_class
48         self.subsample = subsample
49         mean = sum(self.dataset.target(e)
50                    for e in self.dataset.train)/len(self.dataset.train)
51         self.predictor = lambda e:mean  # function that returns mean for
               each example
52         self.predictor.__doc__ = "lambda e:"+str(mean)
53         self.offsets = [self.predictor] # list of base learners
54         self.predictors = [self.predictor] # list of predictors
55         self.errors = [data.evaluate_dataset(data.test, self.predictor,
               Evaluate.squared_loss)]
56         self.display(1,"Predict mean test set mean squared loss=",
               self.errors[0] )
57
58
```

```
59      def learn(self, num_ensembles=10):
60          """adds num_ensemble learners to the ensemble.
61          returns a new predictor.
62          """
63          for i in range(num_ensembles):
64              train_subset = Boosted_dataset(self.dataset, self.predictor,
                    subsample=self.subsample)
65              learner = self.base_learner_class(train_subset)
66              new_offset = learner.learn()
67              self.offsets.append(new_offset)
68              def new_pred(e, old_pred=self.predictor, off=new_offset):
69                  return old_pred(e)+off(e)
70              self.predictor = new_pred
71              self.predictors.append(new_pred)
72              self.errors.append(data.evaluate_dataset(data.test,
                    self.predictor, Evaluate.squared_loss))
73              self.display(1,f"Iteration {len(self.offsets)-1},treesize =
                    {new_offset.num_leaves}. mean squared
                    loss={self.errors[-1]}")
74          return self.predictor
```

For testing, *sp_DT_learner* returns a learner that predicts the mean at the leaves and is evaluated using squared loss. It can also take arguments to change the default arguments for the trees.

learnBoosting.py — (continued)

```
76  # Testing
77
78  from learnDT import DT_learner
79  from learnProblem import Data_set, Data_from_file
80
81  def sp_DT_learner(split_to_optimize=Evaluate.squared_loss,
82                            leaf_prediction=Predict.mean,**nargs):
83      """Creates a learner with different default arguments replaced by
            **nargs
84      """
85      def new_learner(dataset):
86          return DT_learner(dataset,split_to_optimize=split_to_optimize,
87                                leaf_prediction=leaf_prediction, **nargs)
88      return new_learner
89
90  #data = Data_from_file('data/car.csv', target_index=-1) regression
91  data = Data_from_file('data/student/student-mat-nq.csv',
        separator=';',has_header=True,target_index=-1,seed=13,include_only=list(range(30))+[32])
        #2.0537973790924946
92  #data = Data_from_file('data/SPECT.csv', target_index=0, seed=62) #123)
93  #data = Data_from_file('data/mail_reading.csv', target_index=-1)
94  #data = Data_from_file('data/holiday.csv', has_header=True, num_train=19,
        target_index=-1)
95  #learner10 = Boosting_learner(data,
        sp_DT_learner(split_to_optimize=Evaluate.squared_loss,
```

```
    leaf_prediction=Predict.mean, min_child_weight=10))
#learner7 = Boosting_learner(data, sp_DT_learner(0.7))
#learner5 = Boosting_learner(data, sp_DT_learner(0.5))
#predictor9 =learner9.learn(10)
#for i in learner9.offsets: print(i.__doc__)
import matplotlib.pyplot as plt

def plot_boosting_trees(data, steps=10, mcws=[30,20,20,10], gammas=
    [100,200,300,500]):
    # to reduce clutter uncomment one of following two lines
    #mcws=[10]
    #gammas=[200]
    learners = [(mcw, gamma, Boosting_learner(data,
        sp_DT_learner(min_child_weight=mcw, gamma=gamma)))
                for gamma in gammas for mcw in mcws
                ]
    plt.ion()
    plt.xscale('linear') # change between log and linear scale
    plt.xlabel("number of trees")
    plt.ylabel("mean squared loss")
    markers = (m+c for c in ['k','g','r','b','m','c','y'] for m in
        ['-','--','-.',':'])
    for (mcw,gamma,learner) in learners:
        data.display(1,f"min_child_weight={mcw}, gamma={gamma}")
        learner.learn(steps)
        plt.plot(range(steps+1), learner.errors, next(markers),
                 label=f"min_child_weight={mcw}, gamma={gamma}")
    plt.legend()
    plt.draw()

# plot_boosting_trees(data)
```

7.7.1 Gradient Tree Boosting

The following implements gradient Boosted trees for classification. If you want to use this gradient tree boosting for a real problem, we recommend using **XGBoost** [Chen and Guestrin, 2016] or **LightGBM** [Ke, Meng, Finley, Wang, Chen, Ma, Ye, and Liu, 2017].

`GTB_learner` subclasses `DT_learner`. The method `learn_tree` is used unchanged. `DT_learner` assumes that the value at the leaf is the prediction of the leaf, thus `leaf_value` needs to be overridden. It also assumes that all nodes at a leaf have the same prediction, but in GBT the elements of a leaf can have different values, depending on the previous trees. Thus `sum_losses` also needs to be overridden.

learnBoosting.py — (continued)

```
class GTB_learner(DT_learner):
    def __init__(self, dataset, number_trees, lambda_reg=1, gamma=0,
        **dtargs):
```

```
126         DT_learner.__init__(self, dataset,
                split_to_optimize=Evaluate.log_loss, **dtargs)
127         self.number_trees = number_trees
128         self.lambda_reg = lambda_reg
129         self.gamma = gamma
130         self.trees = []
131
132     def learn(self):
133         for i in range(self.number_trees):
134             tree =
                    self.learn_tree(self.dataset.conditions(self.max_num_cuts),
                    self.train)
135             self.trees.append(tree)
136             self.display(1,f"""Iteration {i} treesize = {tree.num_leaves}
                    train logloss={
137                 self.dataset.evaluate_dataset(self.dataset.train,
                        self.gtb_predictor, Evaluate.log_loss)
138                     } test logloss={
139                 self.dataset.evaluate_dataset(self.dataset.test,
                        self.gtb_predictor, Evaluate.log_loss)}""")
140         return self.gtb_predictor
141
142     def gtb_predictor(self, example, extra=0):
143         """prediction for example,
144         extras is an extra contribution for this example being considered
145         """
146         return sigmoid(sum(t(example) for t in self.trees)+extra)
147
148     def leaf_value(self, egs, domain=[0,1]):
149         """value at the leaves for examples egs
150         domain argument is ignored"""
151         pred_acts = [(self.gtb_predictor(e),self.target(e)) for e in egs]
152         return sum(a-p for (p,a) in pred_acts) /(sum(p*(1-p) for (p,a) in
                pred_acts)+self.lambda_reg)
153
154
155     def sum_losses(self, data_subset):
156         """returns sum of losses for dataset (assuming a leaf is formed
                with no more splits)
157         """
158         leaf_val = self.leaf_value(data_subset)
159         error = sum(Evaluate.log_loss(self.gtb_predictor(e,leaf_val),
                    self.target(e))
160                     for e in data_subset) + self.gamma
161         return error
```

Testing

learnBoosting.py — (continued)

```
163 # data = Data_from_file('data/carbool.csv', target_index=-1, seed=123)
164 # gtb_learner = GTB_learner(data, 10)
```

```
# gtb_learner.learn()
```

Chapter 8

Neural Networks and Deep Learning

Warning: this is not meant to be an efficient implementation of deep learning. If you want to do serious machine learning on meduim-sized or large data, we recommend Keras (`https://keras.io`) [Chollet, 2021] or PyTorch (`https://pytorch.org`), which are very efficient, particularly on GPUs. They are, however, black boxes. The AIPython neural network code should be seen like a car engine made of glass; you can see exactly how it works, even if it is not fast.

The parameters that are the same as in Keras have the same names. In AIPython, actication functions are treated as separate layers, which makes them more modular and readable.

8.1 Layers

A neural network is built from layers.

This provides a modular implementation of layers. Layers can easily be stacked in many configurations. A layer needs to implement a function to compute the output values from the inputs, a way to back-propagate the error, and perhaps update its parameters.

learnNN.py — Neural Network Learning

```
11 from learnProblem import Learner, Data_set, Data_from_file,
       Data_from_files, Evaluate
12 from learnLinear import sigmoid, one, softmax, indicator
13 import random, math, time
14
15 class Layer(object):
16     def __init__(self, nn, num_outputs=None):
```

```
        """Given a list of inputs, outputs will produce a list of length
            num_outputs.
        nn is the neural network this layer is part of
        num outputs is the number of outputs for this layer.
        """
        self.nn = nn
        self.num_inputs = nn.num_outputs # output of nn is the input to
            this layer
        if num_outputs:
            self.num_outputs = num_outputs
        else:
            self.num_outputs = nn.num_outputs # same as the inputs

    def output_values(self,input_values, training=False):
        """Return the outputs for this layer for the given input values.
        input_values is a list of the inputs to this layer (of length
            num_inputs)
        returns a list of length self.num_outputs.
        It can act differently when training and when predicting.
        """
        raise NotImplementedError("output_values") # abstract method

    def backprop(self,errors):
        """Backpropagate the errors on the outputs
        errors is a list of errors for the outputs (of length
            self.num_outputs).
        Returns the errors for the inputs to this layer (of length
            self.num_inputs).

        You can assume that this is only called after corresponding
            output_values,
           which can remember information information required for the
               back-propagation.
        """
        raise NotImplementedError("backprop") # abstract method

    def update(self):
        """updates parameters after a batch.
        overridden by layers that have parameters
        """
        pass
```

8.1.1 Linear Layer

A linear layer maintains an array of weights. $self.weights[o][i]$ is the weight between input i and output o. A 1 is added to the end of the inputs. The default initialization is the Glorot uniform initializer [Glorot and Bengio, 2010], which is the default in Keras. An alternative is to provide a limit, in which case the

values are selected uniformly in the range $[-limit, limit]$. Keras treats the bias separately, and by default initialzes to zero.

learnNN.py — (continued)

```
class Linear_complete_layer(Layer):
    """a completely connected layer"""
    def __init__(self, nn, num_outputs, limit=None):
        """A completely connected linear layer.
        nn is a neural network that the inputs come from
        num_outputs is the number of outputs
        the random initialization of parameters is in range [-limit,limit]
        """
        Layer.__init__(self, nn, num_outputs)
        if limit is None:
            limit =math.sqrt(6/(self.num_inputs+self.num_outputs))
        # self.weights[o][i] is the weight between input i and output o
        self.weights = [[random.uniform(-limit, limit) if inf <
            self.num_inputs else 0
                         for inf in range(self.num_inputs+1)]
                        for outf in range(self.num_outputs)]
        self.delta = [[0 for inf in range(self.num_inputs+1)]
                        for outf in range(self.num_outputs)]

    def output_values(self,input_values, training=False):
        """Returns the outputs for the input values.
        It remembers the values for the backprop.

        Note in self.weights there is a weight list for every output,
        so wts in self.weights loops over the outputs.
        The bias is the *last* value of each list in self.weights.
        """
        self.inputs = input_values + [1]
        return [sum(w*val for (w,val) in zip(wts,self.inputs))
                    for wts in self.weights]

    def backprop(self,errors):
        """Backpropagate the errors, updating the weights and returning the
            error in its inputs.
        """
        input_errors = [0]*(self.num_inputs+1)
        for out in range(self.num_outputs):
            for inp in range(self.num_inputs+1):
                input_errors[inp] += self.weights[out][inp] * errors[out]
                self.delta[out][inp] += self.inputs[inp] * errors[out]
        return input_errors[:-1] # remove the error for the "1"

    def update(self):
        """updates parameters after a batch"""
        batch_step_size = self.nn.learning_rate / self.nn.batch_size
        for out in range(self.num_outputs):
            for inp in range(self.num_inputs+1):
```

```
97                self.weights[out][inp] -= batch_step_size *
                      self.delta[out][inp]
98                self.delta[out][inp] = 0
```

8.1.2 ReLU Layer

The standard activation function for hidden nodes is the **ReLU**.

learnNN.py — (continued)

```
100 class ReLU_layer(Layer):
101     """Rectified linear unit (ReLU) f(z) = max(0, z).
102     The number of outputs is equal to the number of inputs.
103     """
104     def __init__(self, nn):
105         Layer.__init__(self, nn)
106 
107     def output_values(self, input_values, training=False):
108         """Returns the outputs for the input values.
109         It remembers the input values for the backprop.
110         """
111         self.input_values = input_values
112         self.outputs= [max(0,inp) for inp in input_values]
113         return self.outputs
114 
115     def backprop(self,errors):
116         """Returns the derivative of the errors"""
117         return [e if inp>0 else 0 for e,inp in zip(errors,
                    self.input_values)]
```

8.1.3 Sigmoid Layer

One of the old standards for the activation function for hidden layers is the sigmoid. It is included here to experiment with.

learnNN.py — (continued)

```
119 class Sigmoid_layer(Layer):
120     """sigmoids of the inputs.
121     The number of outputs is equal to the number of inputs.
122     Each output is the sigmoid of its corresponding input.
123     """
124     def __init__(self, nn):
125         Layer.__init__(self, nn)
126 
127     def output_values(self, input_values, training=False):
128         """Returns the outputs for the input values.
129         It remembers the output values for the backprop.
130         """
131         self.outputs= [sigmoid(inp) for inp in input_values]
132         return self.outputs
```

```
133
134     def backprop(self,errors):
135         """Returns the derivative of the errors"""
136         return [e*out*(1-out) for e,out in zip(errors, self.outputs)]
```

8.2 Feedforward Networks

learnNN.py — (continued)

```
138 class NN(Learner):
139     def __init__(self, dataset, validation_proportion = 0.1,
            learning_rate=0.001):
140         """Creates a neural network for a dataset,
141         layers is the list of layers
142         """
143         self.dataset = dataset
144         self.output_type = dataset.target.ftype
145         self.learning_rate = learning_rate
146         self.input_features = dataset.input_features
147         self.num_outputs = len(self.input_features)
148         validation_num = int(len(self.dataset.train)*validation_proportion)
149         if validation_num > 0:
150             random.shuffle(self.dataset.train)
151             self.validation_set = self.dataset.train[-validation_num:]
152             self.training_set = self.dataset.train[:-validation_num]
153         else:
154             self.validation_set = []
155             self.training_set = self.dataset.train
156         self.layers = []
157         self.bn = 0 # number of batches run
158
159     def add_layer(self,layer):
160         """add a layer to the network.
161         Each layer gets number of inputs from the previous layers outputs.
162         """
163         self.layers.append(layer)
164         self.num_outputs = layer.num_outputs
165
166     def predictor(self,ex):
167         """Predicts the value of the first output for example ex.
168         """
169         values = [f(ex) for f in self.input_features]
170         for layer in self.layers:
171             values = layer.output_values(values)
172         return sigmoid(values[0]) if self.output_type =="boolean" \
173                else softmax(values, self.dataset.target.frange) if
                       self.output_type == "categorical" \
174                else values[0]
175
```

```
176     def predictor_string(self):
177         return "not implemented"
```

The *learn* method learns a network.

learnNN.py — (continued)

```
179     def learn(self, epochs=5, batch_size=32, num_iter = None,
            report_each=10):
180         """Learns parameters for a neural network using stochastic gradient
                decent.
181         epochs is the number of times through the data (on average)
182         batch_size is the maximum size of each batch
183         num_iter is the number of iterations over the batches
184             - overrides epochs if provided (allows for fractions of epochs)
185         report_each means give the errors after each multiple of that
                iterations
186         """
187         self.batch_size = min(batch_size, len(self.training_set)) # don't
                have batches bigger than training size
188         if num_iter is None:
189             num_iter = (epochs * len(self.training_set)) // self.batch_size
190         #self.display(0,"Batch\t","\t".join(criterion.__doc__ for criterion
                in Evaluate.all_criteria))
191         for i in range(num_iter):
192             batch = random.sample(self.training_set, self.batch_size)
193             for e in batch:
194                 # compute all outputs
195                 values = [f(e) for f in self.input_features]
196                 for layer in self.layers:
197                     values = layer.output_values(values, training=True)
198                 # backpropagate
199                 predicted = [sigmoid(v) for v in values] if self.output_type
                        == "boolean"\
200                         else softmax(values) if self.output_type ==
                            "categorical"\
201                         else values
202                 actuals = indicator(self.dataset.target(e),
                        self.dataset.target.frange) \
203                         if self.output_type == "categorical"\
204                         else [self.dataset.target(e)]
205                 errors = [pred-obsd for (obsd,pred) in
                        zip(actuals,predicted)]
206                 for layer in reversed(self.layers):
207                     errors = layer.backprop(errors)
208             # Update all parameters in batch
209             for layer in self.layers:
210                 layer.update()
211             self.bn+=1
212             if (i+1)%report_each==0:
213                 self.display(0,self.bn,"\t",
214                              "\t\t".join("{:.4f}".format(
```

```
                                self.dataset.evaluate_dataset(self.validation_set,
                                    self.predictor, criterion))
                               for criterion in Evaluate.all_criteria),
                                   sep="")
```

8.3 Improved Optimization

8.3.1 Momentum

learnNN.py — (continued)

```
class Linear_complete_layer_momentum(Linear_complete_layer):
    """a completely connected layer"""
    def __init__(self, nn, num_outputs, limit=None, alpha=0.9, epsilon =
        1e-07, vel0=0):
        """A completely connected linear layer.
        nn is a neural network that the inputs come from
        num_outputs is the number of outputs
        max_init is the maximum value for random initialization of
            parameters
        vel0 is the initial velocity for each parameter
        """
        Linear_complete_layer.__init__(self, nn, num_outputs, limit=limit)
        # self.weights[o][i] is the weight between input i and output o
        self.velocity = [[vel0 for inf in range(self.num_inputs+1)]
                         for outf in range(self.num_outputs)]
        self.alpha = alpha
        self.epsilon = epsilon

    def update(self):
        """updates parameters after a batch"""
        batch_step_size = self.nn.learning_rate / self.nn.batch_size
        for out in range(self.num_outputs):
            for inp in range(self.num_inputs+1):
                self.velocity[out][inp] = self.alpha*self.velocity[out][inp]
                    - batch_step_size * self.delta[out][inp]
                self.weights[out][inp] += self.velocity[out][inp]
                self.delta[out][inp] = 0
```

8.3.2 RMS-Prop

learnNN.py — (continued)

```
class Linear_complete_layer_RMS_Prop(Linear_complete_layer):
    """a completely connected layer"""
    def __init__(self, nn, num_outputs, limit=None, rho=0.9, epsilon =
        1e-07):
        """A completely connected linear layer.
        nn is a neural network that the inputs come from
        num_outputs is the number of outputs
```

```
        max_init is the maximum value for random initialization of
            parameters
        """
        Linear_complete_layer.__init__(self, nn, num_outputs, limit=limit)
        # self.weights[o][i] is the weight between input i and output o
        self.ms = [[0 for inf in range(self.num_inputs+1)]
                      for outf in range(self.num_outputs)]
        self.rho = rho
        self.epsilon = epsilon

    def update(self):
        """updates parameters after a batch"""
        for out in range(self.num_outputs):
            for inp in range(self.num_inputs+1):
                gradient = self.delta[out][inp] / self.nn.batch_size
                self.ms[out][inp] = self.rho*self.ms[out][inp]+ (1-self.rho)
                    * gradient**2
                self.weights[out][inp] -=
                    self.nn.learning_rate/(self.ms[out][inp]+self.epsilon)**0.5
                    * gradient
                self.delta[out][inp] = 0
```

8.4 Dropout

Dropout is implemented as a layer.

learnNN.py — (continued)

```
from utilities import flip
class Dropout_layer(Layer):
    """Dropout layer
    """

    def __init__(self, nn, rate=0):
        """
        rate is fraction of the input units to drop. 0 =< rate < 1
        """
        self.rate = rate
        Layer.__init__(self, nn)

    def output_values(self, input_values, training=False):
        """Returns the outputs for the input values.
        It remembers the input values for the backprop.
        """
        if training:
            scaling = 1/(1-self.rate)
            self.mask = [0 if flip(self.rate) else 1
                         for _ in input_values]
            return [x*y*scaling for (x,y) in zip(input_values, self.mask)]
        else:
            return input_values
```

```
290
291     def backprop(self,errors):
292         """Returns the derivative of the errors"""
293         return [x*y for (x,y) in zip(errors, self.mask)]
294
295 class Dropout_layer_0(Layer):
296     """Dropout layer
297     """
298
299     def __init__(self, nn, rate=0):
300         """
301         rate is fraction of the input units to drop. 0 =< rate < 1
302         """
303         self.rate = rate
304         Layer.__init__(self, nn)
305
306     def output_values(self, input_values, training=False):
307         """Returns the outputs for the input values.
308         It remembers the input values for the backprop.
309         """
310         if training:
311             scaling = 1/(1-self.rate)
312             self.outputs= [0 if flip(self.rate) else inp*scaling # make 0
                    with probability rate
313                            for inp in input_values]
314             return self.outputs
315         else:
316             return input_values
317
318     def backprop(self,errors):
319         """Returns the derivative of the errors"""
320         return errors
```

8.4.1 Examples

The following constructs a neural network with one hidden layer. The output is assumed to be Boolean or Real. If it is categorical, the final layer should have the same number of outputs as the number of cetegories (so it can use a softmax).

learnNN.py — (continued)

```
322 #data = Data_from_file('data/mail_reading.csv', target_index=-1)
323 #data = Data_from_file('data/mail_reading_consis.csv', target_index=-1)
324 data = Data_from_file('data/SPECT.csv', prob_test=0.3, target_index=0,
        seed=12345)
325 #data = Data_from_file('data/iris.data', prob_test=0.2, target_index=-1) #
        150 examples approx 120 test + 30 test
326 #data = Data_from_file('data/if_x_then_y_else_z.csv', num_train=8,
        target_index=-1) # not linearly sep
```

```
#data = Data_from_file('data/holiday.csv', target_index=-1) #,
    num_train=19)
#data = Data_from_file('data/processed.cleveland.data', target_index=-1)
#random.seed(None)

# nn3 is has a single hidden layer of width 3
nn3 = NN(data, validation_proportion = 0)
nn3.add_layer(Linear_complete_layer(nn3,3))
#nn3.add_layer(Sigmoid_layer(nn3))
nn3.add_layer(ReLU_layer(nn3))
nn3.add_layer(Linear_complete_layer(nn3,1)) # when using
    output_type="boolean"
#nn3.learn(epochs = 100)

# nn3do is like nn3 but with dropout on the hidden layer
nn3do = NN(data, validation_proportion = 0)
nn3do.add_layer(Linear_complete_layer(nn3do,3))
#nn3.add_layer(Sigmoid_layer(nn3)) # comment this or the next
nn3do.add_layer(ReLU_layer(nn3do))
nn3do.add_layer(Dropout_layer(nn3do, rate=0.5))
nn3do.add_layer(Linear_complete_layer(nn3do,1))
#nn3do.learn(epochs = 100)

# nn3_rmsp is like nn3 but uses RMS prop
nn3_rmsp = NN(data, validation_proportion = 0)
nn3_rmsp.add_layer(Linear_complete_layer_RMS_Prop(nn3_rmsp,3))
#nn3_rmsp.add_layer(Sigmoid_layer(nn3_rmsp)) # comment this or the next
nn3_rmsp.add_layer(ReLU_layer(nn3_rmsp))
nn3_rmsp.add_layer(Linear_complete_layer_RMS_Prop(nn3_rmsp,1))
#nn3_rmsp.learn(epochs = 100)

# nn3_m is like nn3 but uses momentum
mm1_m = NN(data, validation_proportion = 0)
mm1_m.add_layer(Linear_complete_layer_momentum(mm1_m,3))
#mm1_m.add_layer(Sigmoid_layer(mm1_m)) # comment this or the next
mm1_m.add_layer(ReLU_layer(mm1_m))
mm1_m.add_layer(Linear_complete_layer_momentum(mm1_m,1))
#mm1_m.learn(epochs = 100)

# nn2 has a single a hidden layer of width 2
nn2 = NN(data, validation_proportion = 0)
nn2.add_layer(Linear_complete_layer_RMS_Prop(nn2,2))
nn2.add_layer(ReLU_layer(nn2))
nn2.add_layer(Linear_complete_layer_RMS_Prop(nn2,1))

# nn5 is has a single hidden layer of width 5
nn5 = NN(data, validation_proportion = 0)
nn5.add_layer(Linear_complete_layer_RMS_Prop(nn5,5))
nn5.add_layer(ReLU_layer(nn5))
nn5.add_layer(Linear_complete_layer_RMS_Prop(nn5,1))
```

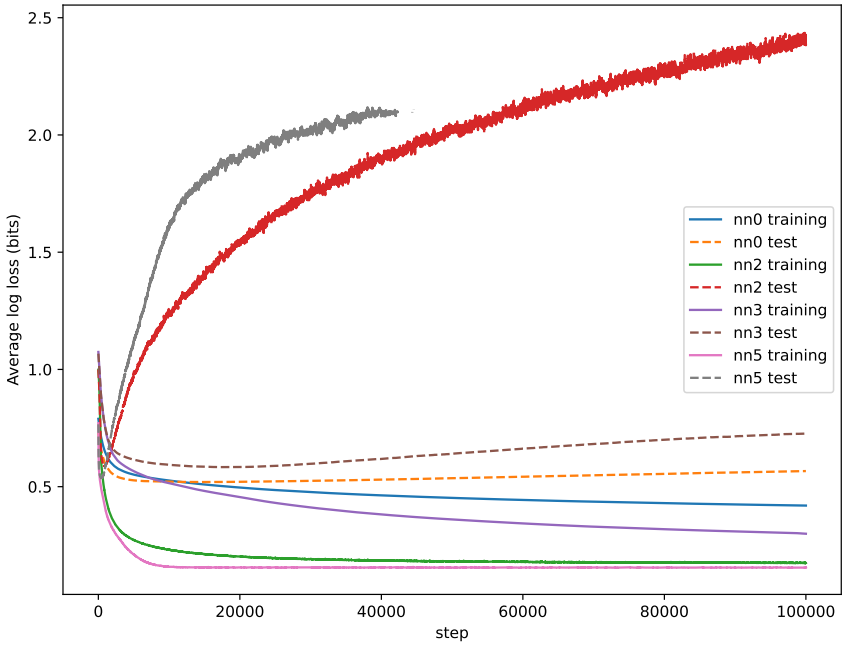


Figure 8.1: Plotting train and test log loss for various algorithms on SPECT dataset

```
375
376 # nn0 has no hidden layers, and so is just logistic regression:
377 nn0 = NN(data, validation_proportion = 0) #learning_rate=0.05)
378 nn0.add_layer(Linear_complete_layer(nn0,1))
379 # Or try this for RMS-Prop:
380 #nn0.add_layer(Linear_complete_layer_RMS_Prop(nn0,1))
```

Plotting. Figure 8.1 shows the training and test performance on the SPECT dataset for the architectures above. Note the `nn5` test has infinite log loss on the test set after about 45,000 steps. The noisyness of the predictions might indicate that the step size is too big. This was produced by the code below:

learnNN.py — (continued)

```
382 from learnLinear import plot_steps
383 from learnProblem import Evaluate
384
385 # To show plots first choose a criterion to use
386 # crit = Evaluate.log_loss
387 # crit = Evaluate.accuracy
388 # plot_steps(learner = nn0, data = data, criterion=crit, num_steps=10000,
    log_scale=False, legend_label="nn0")
```

```
# plot_steps(learner = nn2, data = data, criterion=crit, num_steps=10000, log_scale=False, legend_label="nn2")
# plot_steps(learner = nn3, data = data, criterion=crit, num_steps=10000, log_scale=False, legend_label="nn3")
# plot_steps(learner = nn5, data = data, criterion=crit, num_steps=10000, log_scale=False, legend_label="nn5")

# for (nn,nname) in [(nn0,"nn0"),(nn2,"nn2"),(nn3,"nn3"),(nn5,"nn5")]:
    plot_steps(learner = nn, data = data, criterion=crit,
    num_steps=100000, log_scale=False, legend_label=nname)

# Print some training examples
#for eg in random.sample(data.train,10): print(eg,nn3.predictor(eg))

# Print some test examples
#for eg in random.sample(data.test,10): print(eg,nn3.predictor(eg))

# To see the weights learned in linear layers
# nn3.layers[0].weights
# nn3.layers[2].weights

# Print test:
# for e in data.train: print(e,nn0.predictor(e))

def test(data, hidden_widths = [5], epochs=100,
            optimizers = [Linear_complete_layer,
                      Linear_complete_layer_momentum,
                          Linear_complete_layer_RMS_Prop]):
    data.display(0,"Batch\t","\t".join(criterion.__doc__ for criterion in Evaluate.all_criteria))
    for optimizer in optimizers:
        nn = NN(data)
        for width in hidden_widths:
            nn.add_layer(optimizer(nn,width))
            nn.add_layer(ReLU_layer(nn))
        if data.target.ftype == "boolean":
            nn.add_layer(optimizer(nn,1))
        else:
            error(f"Not implemented: {data.output_type}")
        nn.learn(epochs)
```

The following tests on MNIST. The original files are from `http://yann.lecun.com/exdb/mnist/`. This code assumes you use the csv files from `https://pjreddie.com/projects/mnist-in-csv/`, and put them in the directory `../MNIST/`. Note that this is **very** inefficient; you would be better to use Keras or Pytorch. There are $28 * 28 = 784$ input units and 512 hidden units, which makes 401,408 parameters for the lowest linear layer. So don't be surprised when it takes many hours in AIPython (even if it only takes a few seconds in Keras).

learnNN.py — (continued)

```
# Simplified version: (6000 training instances)
# data_mnist = Data_from_file('../MNIST/mnist_train.csv', prob_test=0.9,
    target_index=0, boolean_features=False, target_type="categorical")

# Full version:
# data_mnist = Data_from_files('../MNIST/mnist_train.csv',
    '../MNIST/mnist_test.csv', target_index=0, boolean_features=False,
    target_type="categorical")

# nn_mnist = NN(data_mnist, validation_proportion = 0.02,
    learning_rate=0.001) #validation set = 1200
# nn_mnist.add_layer(Linear_complete_layer_RMS_Prop(nn_mnist,512));
    nn_mnist.add_layer(ReLU_layer(nn_mnist));
    nn_mnist.add_layer(Linear_complete_layer_RMS_Prop(nn_mnist,10))
# start_time = time.perf_counter();nn_mnist.learn(epochs=1,
    batch_size=128);end_time = time.perf_counter();print("Time:", end_time
    - start_time,"seconds") #1 epoch
# determine test error:
# data_mnist.evaluate_dataset(data_mnist.test, nn_mnist.predictor,
    Evaluate.accuracy)
# Print some random predictions:
# for eg in random.sample(data_mnist.test,10):
    print(data_mnist.target(eg),nn_mnist.predictor(eg),nn_mnist.predictor(eg)[data_mnist.target(eg
```

Exercise 8.1 In the definition of *nn*3 above, for each of the following, first hypothesize what will happen, then test your hypothesis, then explain whether you testing confirms your hypothesis or not. Test it for more than one data set, and use more than one run for each data set.

(a) Which fits the data better, having a sigmoid layer or a ReLU layer after the first linear layer?

(b) Which is faster, having a sigmoid layer or a ReLU layer after the first linear layer?

(c) What happens if you have both the sigmoid layer and then a ReLU layer after the first linear layer and before the second linear layer?

(d) What happens if you have a ReLU layer then a sigmoid layer after the first linear layer and before the second linear layer?

(e) What happens if you have neither the sigmoid layer nor a ReLU layer after the first linear layer?

Exercise 8.2 Do some

Chapter 9

Reasoning with Uncertainty

9.1 Representing Probabilistic Models

A probabilisitic model uses the same definition of a variable as a CSP (Section 4.1.1, page 69). A variable consists of a name, a domain and an optional (x,y) position (for displaying). The domain of a variable is a list or a tuple, as the ordering will matter in the representation of factors.

9.2 Representing Factors

A **factor** is, mathematically, a function from variables into a number; that is given a value for each of its variable, it gives a number. Factors are used for conditional probabilities, utilities in the next chapter, and are explicitly constructed by some algorithms (in particular variable elimination).

A variable assignment, or just **assignment**, is represented as a $\{variable : value\}$ dictionary. A factor can be evaluated when all of its variables are assigned. The method `get_value` evaluates the factor for an assignment. The assignment can include extra variables not in the factor. This method needs to be defined for every subclass.

probFactors.py — Factors for graphical models

```
11 from display import Displayable
12 import math
13
14 class Factor(Displayable):
15     nextid=0 # each factor has a unique identifier; for printing
16
17     def __init__(self, variables, name=None):
18         self.variables = variables # list of variables
```

```
        if name:
            self.name = name
        else:
            self.name = f"f{Factor.nextid}"
            Factor.nextid += 1

    def can_evaluate(self,assignment):
        """True when the factor can be evaluated in the assignment
        assignment is a {variable:value} dict
        """
        return all(v in assignment for v in self.variables)

    def get_value(self,assignment):
        """Returns the value of the factor given the assignment of values
            to variables.
        Needs to be defined for each subclass.
        """
        assert self.can_evaluate(assignment)
        raise NotImplementedError("get_value") # abstract method
```

The method `__str__` returns a brief definition (like "`f7(X,Y,Z)`").The method `to_table` returns string representations of a table showing all of the assignments of values to variables, and the corresponding value.

probFactors.py — (continued)

```
    def __str__(self):
        """returns a string representing a summary of the factor"""
        return f"{self.name}({','.join(str(var) for var in
            self.variables)})"

    def to_table(self, variables=None, given={}):
        """returns a string representation of the factor.
        Allows for an arbitrary variable ordering.
        variables is a list of the variables in the factor
        (can contain other variables)"""
        if variables==None:
            variables = [v for v in self.variables if v not in given]
        else: #enforce ordering and allow for extra variables in ordering
            variables = [v for v in variables if v in self.variables and v
                not in given]
        head = "\t".join(str(v) for v in variables)+"\t"+self.name
        return head+"\n"+self.ass_to_str(variables, given, variables)

    def ass_to_str(self, vars, asst, allvars):
        #print(f"ass_to_str({vars}, {asst}, {allvars})")
        if vars:
            return "\n".join(self.ass_to_str(vars[1:], {**asst,
                vars[0]:val}, allvars)
                             for val in vars[0].domain)
        else:
            val = self.get_value(asst)
```

```
61            val_st = "{:.6f}".format(val) if isinstance(val,float) else
                  str(val)
62            return ("\t".join(str(asst[var]) for var in allvars)
63                      + "\t"+val_st)
64
65    __repr__ = __str__
```

9.3 Conditional Probability Distributions

A **conditional probability distribution (CPD)** is a type of factor that represents a conditional probability. A CPD representing $P(X \mid Y_1 \ldots Y_k)$ is a type of factor, where given values for X and each Y_i returns a number.

probFactors.py — (continued)

```
67 class CPD(Factor):
68     def __init__(self, child, parents):
69         """represents P(variable | parents)
70         """
71         self.parents = parents
72         self.child = child
73         Factor.__init__(self, parents+[child], name=f"Probability")
74
75     def __str__(self):
76         """A brief description of a factor using in tracing"""
77         if self.parents:
78             return f"P({self.child}|{','.join(str(p) for p in
                   self.parents)})"
79         else:
80             return f"P({self.child})"
81
82     __repr__ = __str__
```

A constant CPD has no parents, and has probability 1 when the variable has the value specified, and 0 when the variable has a different value.

probFactors.py — (continued)

```
84 class ConstantCPD(CPD):
85     def __init__(self, variable, value):
86         CPD.__init__(self, variable, [])
87         self.value = value
88     def get_value(self, assignment):
89         return 1 if self.value==assignment[self.child] else 0
```

9.3.1 Logistic Regression

A **logistic regression** CPD, for Boolean variable X represents $P(X{=}True \mid Y_1 \ldots Y_k)$, using $k+1$ real-values weights so

$$P(X{=}True \mid Y_1 \ldots Y_k) = sigmoid(w_0 + \sum_i w_i Y_i)$$

where for Boolean Y_i, True is represented as 1 and False as 0.

probFactors.py — (continued)

```
from learnLinear import sigmoid, logit

class LogisticRegression(CPD):
    def __init__(self, child, parents, weights):
        """A logistic regression representation of a conditional
            probability.
        child is the Boolean (or 0/1) variable whose CPD is being defined
        parents is the list of parents
        weights is list of parameters, such that weights[i+1] is the weight
            for parents[i]
        """
        assert len(weights) == 1+len(parents)
        CPD.__init__(self, child, parents)
        self.weights = weights

    def get_value(self,assignment):
        assert self.can_evaluate(assignment)
        prob = sigmoid(self.weights[0]
                       + sum(self.weights[i+1]*assignment[self.parents[i]]
                             for i in range(len(self.parents))))
        if assignment[self.child]: #child is true
            return prob
        else:
            return (1-prob)
```

9.3.2 Noisy-or

A **noisy-or**, for Boolean variable X with Boolean parents $Y_1 \ldots Y_k$ is parametrized by $k+1$ parameters $p_0, p_1, \ldots, p_k$, where each $0 \leq p_i \leq 1$. The semantics is defined as though there are $k+1$ hidden variables $Z_0, Z_1 \ldots Z_k$, where $P(Z_0) = p_0$ and $P(Z_i \mid Y_i) = p_i$ for $i \geq 1$, and where X is true if and only if $Z_0 \vee Z_1 \vee \cdots \vee Z_k$ (where $\vee$ is "or"). Thus X is false if all of the Z_i are false. Intuitively, Z_0 is the probability of X when all Y_i are false and each Z_i is a noisy (probabilistic) measure that Y_i makes X true, and X only needs one to make it true.

probFactors.py — (continued)

```
class NoisyOR(CPD):
    def __init__(self, child, parents, weights):
        """A noisy representation of a conditional probability.
        variable is the Boolean (or 0/1) child variable whose CPD is being
            defined
        parents is the list of Boolean (or 0/1) parents
        weights is list of parameters, such that weights[i+1] is the weight
            for parents[i]
        """
        assert len(weights) == 1+len(parents)
```

```
122         CPD.__init__(self, child, parents)
123         self.weights = weights
124 
125     def get_value(self,assignment):
126         assert self.can_evaluate(assignment)
127         probfalse = (1-self.weights[0])*math.prod(1-self.weights[i+1]
128                                                   for i in
                                                      range(len(self.parents))
129                                                   if
                                                      assignment[self.parents[i]])
130         if assignment[self.child]:
131             return 1-probfalse
132         else:
133             return probfalse
```

9.3.3 Tabular Factors and Prob

A **tabular factor** is a factor that represents each assignment of values to variables separately. It is represented by a Python array (or python dict). If the variables are $V_1, V_2, \ldots, V_k$, the value of $f(V_1 = v_1, V_2 = v_1, \ldots, V_k = v_k)$ is stored in $f[v_1][v_2]\ldots[v_k]$.

If the domain of V_i is $[0, \ldots, n_i - 1]$ this can be represented as an array. Otherwise we can use a dictionary. Python is nice in that it doesn't care, whether an array or dict is used **except when enumerating the values**; enumerating a dict gives the keys (the variables) but enumerating an array gives the values. So we have to be careful not to do this.

probFactors.py — (continued)

```
135 class TabFactor(Factor):
136 
137     def __init__(self, variables, values, name=None):
138         Factor.__init__(self, variables, name=name)
139         self.values = values
140 
141     def get_value(self, assignment):
142         return self.get_val_rec(self.values, self.variables, assignment)
143 
144     def get_val_rec(self, value, variables, assignment):
145         if variables == []:
146            return value
147         else:
148             return self.get_val_rec(value[assignment[variables[0]]],
149                                     variables[1:],assignment)
```

Prob is a factor that represents a conditional probability by enumerating all of the values.

probFactors.py — (continued)

```
151 class Prob(CPD,TabFactor):
```

```
152     """A factor defined by a conditional probability table"""
153     def __init__(self, var, pars, cpt, name=None):
154         """Creates a factor from a conditional probability table, cpt
155         The cpt values are assumed to be for the ordering par+[var]
156         """
157         TabFactor.__init__(self, pars+[var], cpt, name)
158         self.child = var
159         self.parents = pars
```

9.3.4 Decision Tree Representations of Factors

A decision tree representation of a conditional probability is either:

- `IFeq(var, val, true_cond, false_cond)` where `true_cond` and `false_cond` are decision trees. `true_cond` is used if variable `var` has value `val` in an assignment; `false_cond` is used if `var` has a different value.
- a distribution over the child variable

Note that not all parents needs to be assigned to evaluate the decision tree; you only need the branch down the tree that gives the distribition.

probFactors.py — (continued)

```
161 class ProbDT(CPD):
162     def __init__(self, child, parents, dt):
163         CPD.__init__(self, child, parents)
164         self.dt = dt
165 
166     def get_value(self, assignment):
167         return self.dt.get_value(assignment, self.child)
168 
169     def can_evaluate(self, assignment):
170         return self.child in assignment and self.dt.can_evaluate(assignment)
```

Decison trees are made up of conditons; here equality equality of a value and a variable:

probFactors.py — (continued)

```
172 class IFeq:
173     def __init__(self, var, val, true_cond, false_cond):
174         self.var = var
175         self.val = val
176         self.true_cond = true_cond
177         self.false_cond = false_cond
178 
179     def get_value(self, assignment, child):
180         if assignment[self.var] == self.val:
181             return self.true_cond.get_value(assignment, child)
182         else:
183             return self.false_cond.get_value(assignment,child)
```

```
184
185     def can_evaluate(self, assignment):
186         if self.var not in assignment:
187             return False
188         elif assignment[self.var] == self.val:
189             return self.true_cond.can_evaluate(assignment)
190         else:
191             return self.false_cond.can_evaluate(assignment)
```

At the leaves are distribitions over the child variable.

probFactors.py — (continued)

```
193 class Dist:
194     def __init__(self, dist):
195         """Dist is an arror or dictionary indexed by value of current
                child"""
196         self.dist = dist
197
198     def get_value(self, assignment, child):
199         return self.dist[assignment[child]]
200
201     def can_evaluate(self, assignment):
202         return True
```

The following shows a decision representation of the Example 9.18 of Poole and Mackworth [2023]. When the `Action` is to go out, the probability is a function of `rain`; otherwise it is a function of `full`.

probFactors.py — (continued)

```
204 ##### A decision tree representation Example 9.18 of AIFCA 3e
205 from variable import Variable
206
207 boolean = [False, True]
208
209 action = Variable('Action', ['go_out', 'get_coffee'], position=(0.5,0.8))
210 rain = Variable('Rain', boolean, position=(0.2,0.8))
211 full = Variable('Cup Full', boolean, position=(0.8,0.8))
212
213 wet = Variable('Wet', boolean, position=(0.5,0.2))
214 p_wet = ProbDT(wet,[action,rain,full],
215                 IFeq(action, 'go_out',
216                         IFeq(rain, True, Dist([0.2,0.8]),
                                Dist([0.9,0.1])),
217                         IFeq(full, True, Dist([0.4,0.6]),
                                Dist([0.7,0.3]))))
218
219 # See probRC for wetBN which expands this example to a complete network
```

9.4 Graphical Models

A graphical model consists of a set of variables and a set of factors. A belief network is a graphical model where all of the factors represent conditional probabilities. There are some operations (such as pruning variables) which are applicable to belief networks, but are not applicable to more general models. At the moment, we will treat them as the same.

probGraphicalModels.py — Graphical Models and Belief Networks

```
from display import Displayable
from probFactors import CPD
import matplotlib.pyplot as plt

class GraphicalModel(Displayable):
    """The class of graphical models.
    A graphical model consists of a title, a set of variables and a set of
        factors.

    vars is a set of variables
    factors is a set of factors
    """
    def __init__(self, title, variables=None, factors=None):
        self.title = title
        self.variables = variables
        self.factors = factors
```

A **belief network** (also known as a **Bayesian network**) is a graphical model where all of the factors are conditional probabilities, and every variable has a conditional probability of it given its parents. This only checks the first condition, and builds some useful data structures.

probGraphicalModels.py — (continued)

```
class BeliefNetwork(GraphicalModel):
    """The class of belief networks."""

    def __init__(self, title, variables, factors):
        """vars is a set of variables
        factors is a set of factors. All of the factors are instances of
            CPD (e.g., Prob).
        """
        GraphicalModel.__init__(self, title, variables, factors)
        assert all(isinstance(f,CPD) for f in factors), factors
        self.var2cpt = {f.child:f for f in factors}
        self.var2parents = {f.child:f.parents for f in factors}
        self.children = {n:[] for n in self.variables}
        for v in self.var2parents:
            for par in self.var2parents[v]:
                self.children[par].append(v)
        self.topological_sort_saved = None
```

The following creates a topological sort of the nodes, where the parents of a node come before the node in the resulting order. This is based on Kahn's algorithm from 1962.

probGraphicalModels.py — (continued)

```
    def topological_sort(self):
        """creates a topological ordering of variables such that the
            parents of
        a node are before the node.
        """
        if self.topological_sort_saved:
            return self.topological_sort_saved
        next_vars = {n for n in self.var2parents if not self.var2parents[n]
            }
        self.display(3,'topological_sort: next_vars',next_vars)
        top_order=[]
        while next_vars:
            var = next_vars.pop()
            self.display(3,'select variable',var)
            top_order.append(var)
            next_vars |= {ch for ch in self.children[var]
                          if all(p in top_order for p in
                              self.var2parents[ch])}
            self.display(3,'var_with_no_parents_left',next_vars)
        self.display(3,"top_order",top_order)
        assert
            set(top_order)==set(self.var2parents),(top_order,self.var2parents)
        self.topologicalsort_saved=top_order
        return top_order
```

9.4.1 Showing Belief Networks

The **show** method uses matplotlib to show the graphical structure of a belief network.

probGraphicalModels.py — (continued)

```
    def show(self, fontsize=10, facecolor='orange'):
        plt.ion()  # interactive
        ax = plt.figure().gca()
        ax.set_axis_off()
        plt.title(self.title, fontsize=fontsize)
        bbox =
            dict(boxstyle="round4,pad=1.0,rounding_size=0.5",facecolor=facecolor)
        for var in self.variables: #reversed(self.topological_sort()):
            for par in self.var2parents[var]:
                  ax.annotate(var.name, par.position, xytext=var.position,
                              arrowprops={'arrowstyle':'<-'},bbox=bbox,
                              ha='center', va='center',
                                  fontsize=fontsize)
```

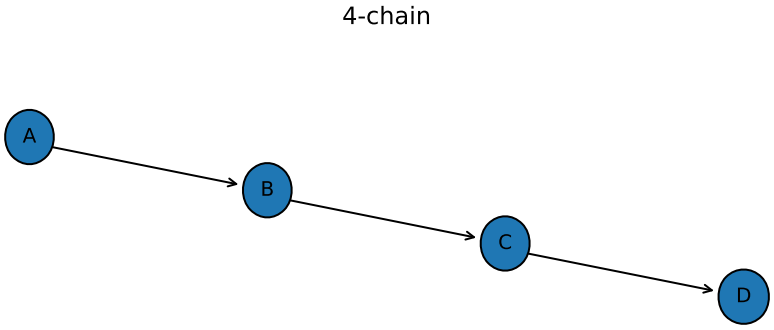


Figure 9.1: bn_4ch.show()

```
76          for var in self.variables:
77                  x,y = var.position
78                  plt.text(x,y,var.name,bbox=bbox,ha='center', va='center',
                        fontsize=fontsize)
```

9.4.2 Example Belief Networks

A Chain of 4 Variables

The first example belief network is a simple chain $A \longrightarrow B \longrightarrow C \longrightarrow D$, shown in Figure 9.1.

Please do not change this, as it is the example used for testing.

probExamples.py — Example belief networks

```
11 from variable import Variable
12 from probFactors import CPD, Prob, LogisticRegression, NoisyOR, ConstantCPD
13 from probGraphicalModels import BeliefNetwork
14
15 #### Simple Example Used for Unit Tests ####
16 boolean = [False, True]
17 A = Variable("A", boolean, position=(0,0.8))
18 B = Variable("B", boolean, position=(0.333,0.7))
19 C = Variable("C", boolean, position=(0.666,0.6))
20 D = Variable("D", boolean, position=(1,0.5))
21
22 f_a = Prob(A,[],[0.4,0.6])
23 f_b = Prob(B,[A],[[0.9,0.1],[0.2,0.8]])
24 f_c = Prob(C,[B],[[0.6,0.4],[0.3,0.7]])
25 f_d = Prob(D,[C],[[0.1,0.9],[0.75,0.25]])
26
27 bn_4ch = BeliefNetwork("4-chain", {A,B,C,D}, {f_a,f_b,f_c,f_d})
```

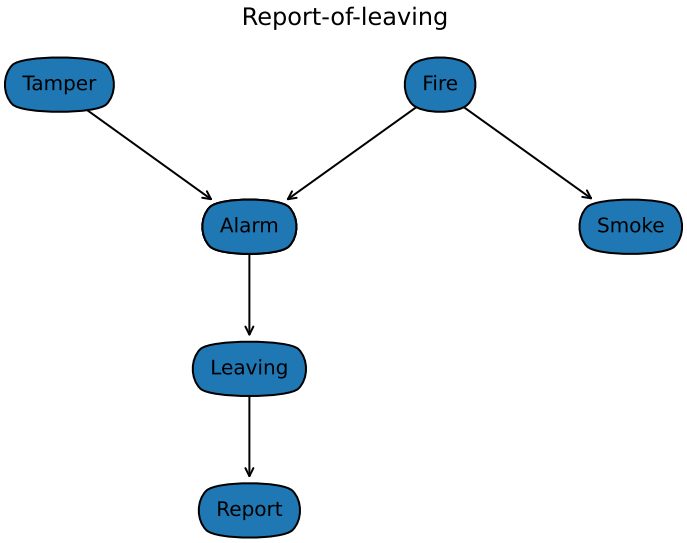


Figure 9.2: The report-of-leaving belief network

Report-of-Leaving Example

The second belief network, `bn_report`, is Example 9.13 of Poole and Mackworth [2023] (`http://artint.info`). The output of `bn_report.show()` is shown in Figure 9.2 of this document.

probExamples.py — (continued)

```
# Belief network report-of-leaving example (Example 9.13 shown in Figure 9.3) of
# Poole and Mackworth, Artificial Intelligence, 2023 http://artint.info
boolean = [False, True]

Alarm =  Variable("Alarm", boolean, position=(0.366,0.5))
Fire =   Variable("Fire",  boolean, position=(0.633,0.75))
Leaving = Variable("Leaving", boolean, position=(0.366,0.25))
Report = Variable("Report", boolean, position=(0.366,0.0))
Smoke =  Variable("Smoke", boolean, position=(0.9,0.5))
Tamper = Variable("Tamper", boolean, position=(0.1,0.75))

f_ta = Prob(Tamper,[],[0.98,0.02])
f_fi = Prob(Fire,[],[0.99,0.01])
f_sm = Prob(Smoke,[Fire],[[0.99,0.01],[0.1,0.9]])
f_al = Prob(Alarm,[Fire,Tamper],[[[0.9999, 0.0001], [0.15, 0.85]], [[0.01, 0.99], [0.5, 0.5]]])
f_lv = Prob(Leaving,[Alarm],[[0.999, 0.001], [0.12, 0.88]])
```

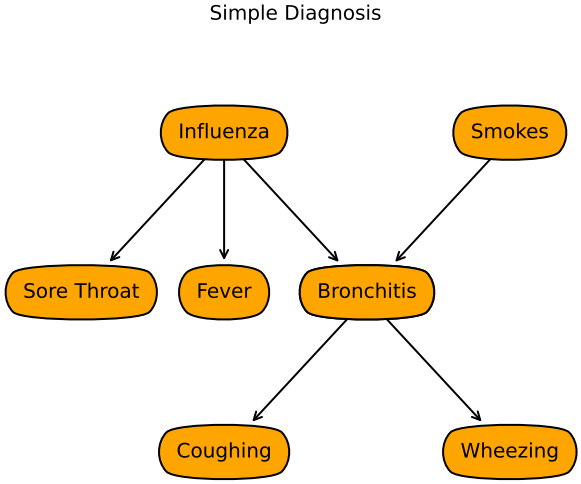


Figure 9.3: Simple diagnosis example; `simple_diagnosis.show()`

```
f_re = Prob(Report,[Leaving],[[0.99, 0.01], [0.25, 0.75]])

bn_report = BeliefNetwork("Report-of-leaving",
    {Tamper,Fire,Smoke,Alarm,Leaving,Report},
                           {f_ta,f_fi,f_sm,f_al,f_lv,f_re})
```

Simple Diagnostic Example

This is the "simple diagnostic example" of Exercise 9.1 of Poole and Mackworth [2023], reproduced here as Figure 9.3

probExamples.py — (continued)

```
# Belief network simple-diagnostic example (Exercise 9.3 shown in Figure
    9.39) of
# Poole and Mackworth, Artificial Intelligence, 2023 http://artint.info

Influenza =  Variable("Influenza", boolean, position=(0.4,0.8))
Smokes =     Variable("Smokes", boolean, position=(0.8,0.8))
SoreThroat = Variable("Sore Throat", boolean, position=(0.2,0.5))
HasFever =      Variable("Fever", boolean, position=(0.4,0.5))
Bronchitis = Variable("Bronchitis", boolean, position=(0.6,0.5))
Coughing =   Variable("Coughing", boolean, position=(0.4,0.2))
Wheezing =   Variable("Wheezing", boolean, position=(0.8,0.2))

p_infl =   Prob(Influenza,[],[0.95,0.05])
p_smokes = Prob(Smokes,[],[0.8,0.2])
p_sth =    Prob(SoreThroat,[Influenza],[[0.999,0.001],[0.7,0.3]])
p_fever =  Prob(HasFever,[Influenza],[[0.99,0.05],[0.9,0.1]])
p_bronc = Prob(Bronchitis,[Influenza,Smokes],[[[0.9999, 0.0001], [0.3,
    0.7]], [[0.1, 0.9], [0.01, 0.99]]])
```

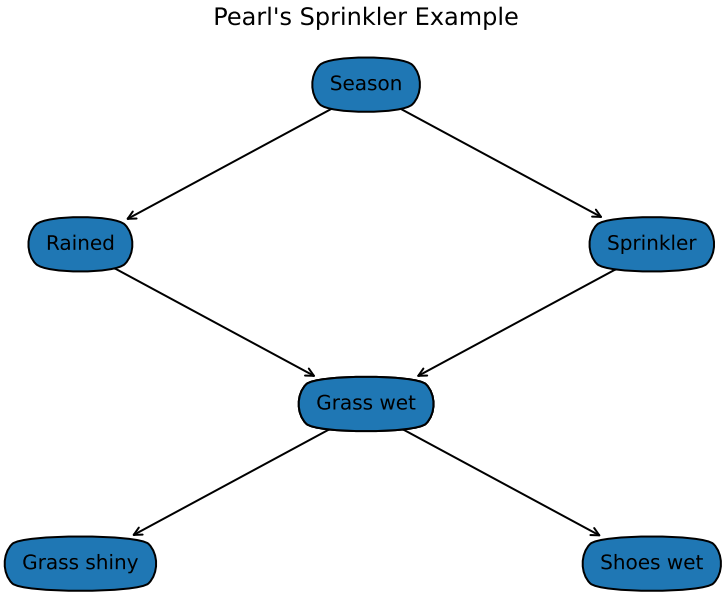


Figure 9.4: The sprinkler belief network

```
66 p_cough =  Prob(Coughing,[Bronchitis],[[0.93,0.07],[0.2,0.8]])
67 p_wheeze =  Prob(Wheezing,[Bronchitis],[[0.999,0.001],[0.4,0.6]])
68
69 simple_diagnosis = BeliefNetwork("Simple Diagnosis",
70                   {Influenza, Smokes, SoreThroat, HasFever, Bronchitis,
                        Coughing, Wheezing},
71                   {p_infl, p_smokes, p_sth, p_fever, p_bronc, p_cough,
                        p_wheeze})
```

Sprinkler Example

The third belief network is the sprinkler example from Pearl. The output of `bn_sprinkler.show()` is shown in Figure 9.4 of this document.

probExamples.py — (continued)

```
73 Season = Variable("Season", ["dry_season","wet_season"],
       position=(0.5,0.9))
74 Sprinkler = Variable("Sprinkler", ["on","off"], position=(0.9,0.6))
75 Rained = Variable("Rained", boolean, position=(0.1,0.6))
76 Grass_wet = Variable("Grass wet", boolean, position=(0.5,0.3))
77 Grass_shiny = Variable("Grass shiny", boolean, position=(0.1,0))
78 Shoes_wet = Variable("Shoes wet", boolean, position=(0.9,0))
79
80 f_season = Prob(Season,[],{'dry_season':0.5, 'wet_season':0.5})
```

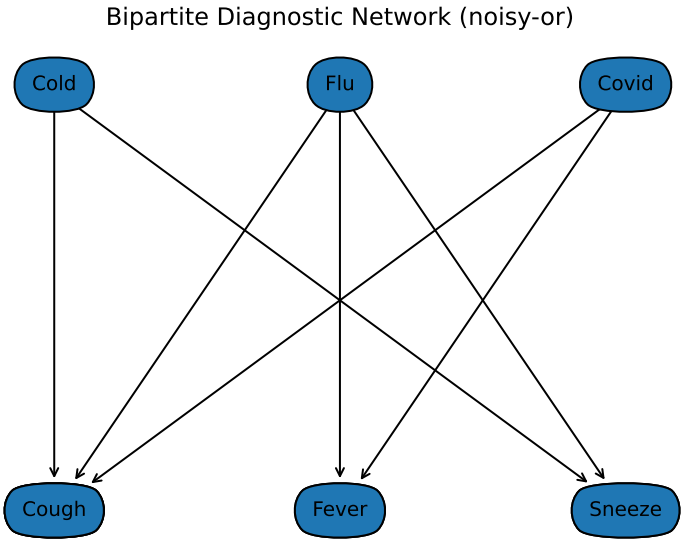


Figure 9.5: A bipartite diagnostic network

```
f_sprinkler = Prob(Sprinkler,[Season],{'dry_season':{'on':0.4,'off':0.6},
                                       'wet_season':{'on':0.01,'off':0.99}})
f_rained = Prob(Rained,[Season],{'dry_season':[0.9,0.1], 'wet_season':
    [0.2,0.8]})
f_wet = Prob(Grass_wet,[Sprinkler,Rained], {'on': [[0.1,0.9],[0.01,0.99]],
                                            'off':[[0.99,0.01],[0.3,0.7]]})
f_shiny = Prob(Grass_shiny, [Grass_wet], [[0.95,0.05], [0.3,0.7]])
f_shoes = Prob(Shoes_wet, [Grass_wet], [[0.98,0.02], [0.35,0.65]])

bn_sprinkler = BeliefNetwork("Pearl's Sprinkler Example",
                        {Season, Sprinkler, Rained, Grass_wet, Grass_shiny,
                            Shoes_wet},
                        {f_season, f_sprinkler, f_rained, f_wet, f_shiny,
                            f_shoes})
```

Bipartite Diagnostic Model with Noisy-or

The belief network `bn_no1` is a bipartite diagnostic model, with independent diseases, and the symptoms depend on the diseases, where the CPDs are defined using noisy-or. Bipartite means it is in two parts; the diseases are only connected to the symptoms and the symptoms are only connected to the diseases. The output of `bn_no1.show()` is shown in Figure 9.5 of this document.

probExamples.py — (continued)

```
#### Bipartite Diagnostic Network ###
Cough = Variable("Cough", boolean, (0.1,0.1))
Fever = Variable("Fever", boolean, (0.5,0.1))
Sneeze = Variable("Sneeze", boolean, (0.9,0.1))
Cold = Variable("Cold",boolean, (0.1,0.9))
Flu = Variable("Flu",boolean, (0.5,0.9))
Covid = Variable("Covid",boolean, (0.9,0.9))

p_cold_no = Prob(Cold,[],[0.9,0.1])
p_flu_no = Prob(Flu,[],[0.95,0.05])
p_covid_no = Prob(Covid,[],[0.99,0.01])

p_cough_no = NoisyOR(Cough, [Cold,Flu,Covid], [0.1, 0.3, 0.2, 0.7])
p_fever_no = NoisyOR(Fever, [   Flu,Covid], [0.01,    0.6,  0.7])
p_sneeze_no = NoisyOR(Sneeze, [Cold,Flu  ], [0.05, 0.5,  0.2   ])

bn_no1 = BeliefNetwork("Bipartite Diagnostic Network (noisy-or)",
                       {Cough, Fever, Sneeze, Cold, Flu, Covid},
                        {p_cold_no, p_flu_no, p_covid_no, p_cough_no,
                            p_fever_no, p_sneeze_no})

# to see the conditional probability of Noisy-or do:
# print(p_cough_no.to_table())

# example from box "Noisy-or compared to logistic regression"
# X = Variable("X",boolean)
# w0 = 0.01
# print(NoisyOR(X,[A,B,C,D],[w0, 1-(1-0.05)/(1-w0), 1-(1-0.1)/(1-w0),
    1-(1-0.2)/(1-w0), 1-(1-0.2)/(1-w0), ]).to_table(given={X:True}))
```

Bipartite Diagnostic Model with Logistic Regression

The belief network `bn_lr1` is a bipartite diagnostic model, with independent diseases, and the symptoms depend on the diseases, where the CPDs are defined using logistic regression. It has the same graphical structure as the previous example (see Figure 9.5). This has the (approximately) the same conditional probabilities as the previous example when zero or one diseases are present. Note that $sigmoid(-2.2) \approx 0.1$

probExamples.py — (continued)

```

p_cold_lr = Prob(Cold,[],[0.9,0.1])
p_flu_lr = Prob(Flu,[],[0.95,0.05])
p_covid_lr = Prob(Covid,[],[0.99,0.01])

p_cough_lr = LogisticRegression(Cough, [Cold,Flu,Covid], [-2.2, 1.67,
    1.26, 3.19])
```

```
p_fever_lr = LogisticRegression(Fever, [  Flu,Covid], [-4.6,      5.02,
    5.46])
p_sneeze_lr = LogisticRegression(Sneeze, [Cold,Flu ], [-2.94, 3.04, 1.79
    ])

bn_lr1 = BeliefNetwork("Bipartite Diagnostic Network - logistic
    regression",
                       {Cough, Fever, Sneeze, Cold, Flu, Covid},
                        {p_cold_lr, p_flu_lr, p_covid_lr, p_cough_lr,
                            p_fever_lr, p_sneeze_lr})

# to see the conditional probability of Noisy-or do:
#print(p_cough_lr.to_table())

# example from box "Noisy-or compared to logistic regression"
# from learnLinear import sigmoid, logit
# w0=logit(0.01)
# X = Variable("X",boolean)
# print(LogisticRegression(X,[A,B,C,D],[w0, logit(0.05)-w0, logit(0.1)-w0,
    logit(0.2)-w0, logit(0.2)-w0]).to_table(given={X:True}))
# try to predict what would happen (and then test) if we had
# w0=logit(0.01)
```

9.5 Inference Methods

Each of the inference methods implements the query method that computes the posterior probability of a variable given a dictionary of $\{variable : value\}$ observations. The methods are Displayable because they implement the *display* method which is currently text-based.

probGraphicalModels.py — (continued)

```
from display import Displayable
from probExamples import bn_4ch, B, D

class InferenceMethod(Displayable):
    """The abstract class of graphical model inference methods"""
    method_name = "unnamed" # each method should have a method name

    def __init__(self,gm=None):
        self.gm = gm

    def query(self, qvar, obs={}):
        """returns a {value:prob} dictionary for the query variable"""
        raise NotImplementedError("InferenceMethod query") # abstract method
```

We use bn_4ch as the test case, in particular $P(B \mid D = true)$. This needs an error threshold, particularly for the approximate methods, where the default threshold is much too accurate.

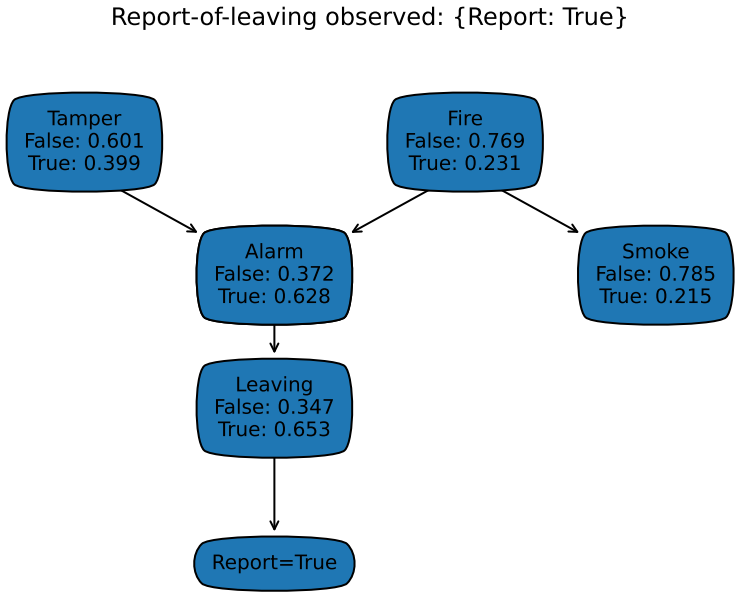


Figure 9.6: The report-of-leaving belief network with posterior distributions

probGraphicalModels.py — (continued)

```
94      def testIM(self, threshold=0.0000000001):
95          solver = self(bn_4ch)
96          res = solver.query(B,{D:True})
97          correct_answer = 0.429632380245
98          assert correct_answer-threshold < res[True] <
                correct_answer+threshold, \
99                  f"value {res[True]} not in desired range for
                        {self.method_name}"
100         print(f"Unit test passed for {self.method_name}.")
```

9.5.1 Showing Posterior Distributions

The `show_post` method draws the posterior distribution of all variables. Figure 9.6 shows the result of `bn_reportRC.show_post({Report:True})` when run after loading `probRC.py` (see below).

probGraphicalModels.py — (continued)

```
102     def show_post(self, obs={}, num_format="{:.3f}", fontsize=10,
            facecolor='orange'):
103         """draws the graphical model conditioned on observations obs
104           num_format is number format (allows for more or less precision)
105           fontsize gives size of the text
```

```
106             facecolor gives the color of the nodes
107         """
108         plt.ion()  # interactive
109         ax = plt.figure().gca()
110         ax.set_axis_off()
111         plt.title(self.gm.title+" observed: "+str(obs), fontsize=fontsize)
112         bbox = dict(boxstyle="round4,pad=1.0,rounding_size=0.5",
                facecolor=facecolor)
113         vartext = {} # variable:text dictionary
114         for var in self.gm.variables: #reversed(self.gm.topological_sort()):
115             if var in obs:
116                 text = var.name + "=" + str(obs[var])
117             else:
118                 distn = self.query(var, obs=obs)
119 
120                 text = var.name + "\n" + "\n".join(str(d)+":
                        "+num_format.format(v) for (d,v) in distn.items())
121             vartext[var] = text
122             # Draw arcs
123             for par in self.gm.var2parents[var]:
124                    ax.annotate(text, par.position, xytext=var.position,
125                                arrowprops={'arrowstyle':'<-'},bbox=bbox,
126                                ha='center', va='center',
                                    fontsize=fontsize)
127         for var in self.gm.variables:
128             x,y = var.position
129             plt.text(x,y,vartext[var], bbox=bbox, ha='center', va='center',
                    fontsize=fontsize)
```

9.6 Naive Search

An instance of a *ProbSearch* object takes in a graphical model. The query method uses naive search to compute the probability of a query variable given observations on other variables. See Figure 9.9 of Poole and Mackworth [2023].

probRC.py — Recursive Conditioning for Graphical Models

```
11 import math
12 from probGraphicalModels import GraphicalModel, InferenceMethod
13 from probFactors import Factor
14 
15 class ProbSearch(InferenceMethod):
16     """The class that queries graphical models using recursive conditioning
17 
18     gm is graphical model to query
19     """
20     method_name = "naive search"
21 
22     def __init__(self,gm=None):
23         InferenceMethod.__init__(self, gm)
```

```
24          ## self.max_display_level = 3
25
26      def query(self, qvar, obs={}, split_order=None):
27          """computes P(qvar | obs) where
28          qvar is the query variable
29          obs is a variable:value dictionary
30          split_order is a list of the non-observed non-query variables in gm
31          """
32          if qvar in obs:
33              return {val:(1 if val == obs[qvar] else 0)
34                          for val in qvar.domain}
35          else:
36             if split_order == None:
37                 split_order = [v for v in self.gm.variables
38                                if (v not in obs) and v != qvar]
39             unnorm = [self.prob_search({qvar:val}|obs, self.gm.factors,
                           split_order)
40                       for val in qvar.domain]
41             p_obs = sum(unnorm)
42             return {val:pr/p_obs for val,pr in zip(qvar.domain, unnorm)}
```

The following is the naive search-based algorithm. It is exponential in the number of variables, so is not very useful. However, it is simple, and useful to understand before looking at the more complicated algorithm used in the subclass.

probRC.py — (continued)

```
44      def prob_search(self, context, factors, split_order):
45          """simple search algorithm
46          context: a variable:value dictionary
47          factors: a set of factors
48          split_order: list of variables not assigned in context
49          returns sum over variable assignments to variables in split order
                of product of factors """
50          self.display(2,"calling prob_search,",(context,factors,split_order))
51          if not factors:
52              return 1
53          elif to_eval := {fac for fac in factors
54                           if fac.can_evaluate(context)}:
55              # evaluate factors when all variables are assigned
56              self.display(3,"prob_search evaluating factors",to_eval)
57              val = math.prod(fac.get_value(context) for fac in to_eval)
58              return val * self.prob_search(context, factors-to_eval,
                    split_order)
59          else:
60              total = 0
61              var = split_order[0]
62              self.display(3, "prob_search branching on", var)
63              for val in var.domain:
64                  total += self.prob_search({var:val}|context, factors,
                        split_order[1:])
```

```
65            self.display(3, "prob_search branching on", var,"returning",
                  total)
66            return total
```

9.7 Recursive Conditioning

The **recursive conditioning** algorithm adds forgetting and caching and recognizing disconnected components to the naive search. We do this by adding a cache and redefining the recursive search algorithm. In inherits the query method. See Figure 9.12 of Poole and Mackworth [2023].

probRC.py — (continued)

```
68  class ProbRC(ProbSearch):
69      method_name = "recursive conditioning"
70
71      def __init__(self,gm=None):
72          self.cache = {(frozenset(), frozenset()):1}
73          ProbSearch.__init__(self,gm)
74
75      def prob_search(self, context, factors, split_order):
76          """ returns \sum_{split_order} \prod_{factors} given assignment in
                context
77          context is a variable:value dictionary
78          factors is a set of factors
79          split_order: list of variables in factors that are not in context
80          """
81          self.display(3,"calling rc,",(context,factors))
82          ce = (frozenset(context.items()), frozenset(factors)) # key for the
                cache entry
83          if ce in self.cache:
84              self.display(3,"rc cache lookup",(context,factors))
85              return self.cache[ce]
86  #       if not factors: #no factors; not needed with forgetting and caching
87  #           return 1
88          elif vars_not_in_factors := {var for var in context
89                                       if not any(var in fac.variables
90                                                  for fac in factors)}:
91              # forget variables not in any factor
92              self.display(3,"rc forgetting variables", vars_not_in_factors)
93              return self.prob_search({key:val for (key,val) in
                    context.items()
94                                       if key not in vars_not_in_factors},
95                                      factors, split_order)
96          elif to_eval := {fac for fac in factors
97                           if fac.can_evaluate(context)}:
98              # evaluate factors when all variables are assigned
99              self.display(3,"rc evaluating factors",to_eval)
100             val = math.prod(fac.get_value(context) for fac in to_eval)
101             if val == 0:
```

```
            return 0
        else:
         return val * self.prob_search(context,
                                       {fac for fac in factors
                                                if fac not in to_eval},
                                       split_order)
    elif len(comp := connected_components(context, factors,
        split_order)) > 1:
        # there are disconnected components
        self.display(3,"splitting into connected components",comp,"in
            context",context)
        return(math.prod(self.prob_search(context,f,eo) for (f,eo) in
            comp))
    else:
        assert split_order, "split_order should not be empty to get
            here"
        total = 0
        var = split_order[0]
        self.display(3, "rc branching on", var)
        for val in var.domain:
            total += self.prob_search({var:val}|context, factors,
                split_order[1:])
        self.cache[ce] = total
        self.display(2, "rc branching on", var,"returning", total)
        return total
```

`connected_components` returns a list of connected components, where a connected component is a set of factors and a set of variables, where the graph that connects variables and factors that involve them is connected. The connected components are built one at a time; with a current connected component. At all times `factors` is partitioned into 3 disjoint sets:

- `component_factors` containing factors in the current connected component where all factors that share a variable are already in the component
- `factors_to_check` containing factors in the current connected component where potentially some factors that share a variable are not in the component; these need to be checked
- `other_factors` the other factors that are not (yet) in the connected component

probRC.py — (continued)

```
def connected_components(context, factors, split_order):
    """returns a list of (f,e) where f is a subset of factors and e is a
        subset of split_order
    such that each element shares the same variables that are disjoint from
        other elements.
    """
    other_factors = set(factors) #copies factors
```

```
    factors_to_check = {other_factors.pop()} # factors in connected component still to be checked
    component_factors = set() # factors in first connected component already checked
    component_variables = set() # variables in first connected component
    while factors_to_check:
        next_fac = factors_to_check.pop()
        component_factors.add(next_fac)
        new_vars = set(next_fac.variables) - component_variables - context.keys()
        component_variables |= new_vars
        for var in new_vars:
            factors_to_check |= {f for f in other_factors
                                 if var in f.variables}
            other_factors -= factors_to_check # set difference
    if other_factors:
        return ( [(component_factors,[e for e in split_order
                                      if e in component_variables])]
                 + connected_components(context, other_factors,
                                        [e for e in split_order
                                          if e not in component_variables]) )
    else:
        return [(component_factors, split_order)]
```

Testing:

probRC.py — (continued)

```
from probExamples import bn_4ch, A,B,C,D,f_a,f_b,f_c,f_d
bn_4chv = ProbRC(bn_4ch)
## bn_4chv.query(A,{})
## bn_4chv.query(D,{})
## InferenceMethod.max_display_level = 3 # show more detail in displaying
## InferenceMethod.max_display_level = 1 # show less detail in displaying
## bn_4chv.query(A,{D:True},[C,B])
## bn_4chv.query(B,{A:True,D:False})

from probExamples import bn_report,Alarm,Fire,Leaving,Report,Smoke,Tamper
bn_reportRC = ProbRC(bn_report) # answers queries using recursive conditioning
## bn_reportRC.query(Tamper,{})
## InferenceMethod.max_display_level = 0 # show no detail in displaying
## bn_reportRC.query(Leaving,{})
## bn_reportRC.query(Tamper,{},
    split_order=[Smoke,Fire,Alarm,Leaving,Report])
## bn_reportRC.query(Tamper,{Report:True})
## bn_reportRC.query(Tamper,{Report:True,Smoke:False})

## To display resulting posteriors try:
# bn_reportRC.show_post({})
# bn_reportRC.show_post({Smoke:False})
# bn_reportRC.show_post({Report:True})
```

```
# bn_reportRC.show_post({Report:True, Smoke:False})

## Note what happens to the cache when these are called in turn:
## bn_reportRC.query(Tamper,{Report:True},
    split_order=[Smoke,Fire,Alarm,Leaving])
## bn_reportRC.query(Smoke,{Report:True},
    split_order=[Tamper,Fire,Alarm,Leaving])

from probExamples import bn_sprinkler, Season, Sprinkler, Rained,
    Grass_wet, Grass_shiny, Shoes_wet
bn_sprinklerv = ProbRC(bn_sprinkler)
## bn_sprinklerv.query(Shoes_wet,{})
## bn_sprinklerv.query(Shoes_wet,{Rained:True})
## bn_sprinklerv.query(Shoes_wet,{Grass_shiny:True})
## bn_sprinklerv.query(Shoes_wet,{Grass_shiny:False,Rained:True})

from probExamples import bn_no1, bn_lr1, Cough, Fever, Sneeze, Cold, Flu,
    Covid
bn_no1v = ProbRC(bn_no1)
bn_lr1v = ProbRC(bn_lr1)
## bn_no1v.query(Flu, {Fever:1, Sneeze:1})
## bn_lr1v.query(Flu, {Fever:1, Sneeze:1})
## bn_lr1v.query(Cough,{})
## bn_lr1v.query(Cold,{Cough:1,Sneeze:0,Fever:1})
## bn_lr1v.query(Flu,{Cough:0,Sneeze:1,Fever:1})
## bn_lr1v.query(Covid,{Cough:1,Sneeze:0,Fever:1})
## bn_lr1v.query(Covid,{Cough:1,Sneeze:0,Fever:1,Flu:0})
## bn_lr1v.query(Covid,{Cough:1,Sneeze:0,Fever:1,Flu:1})

if __name__ == "__main__":
    InferenceMethod.testIM(ProbSearch)
    InferenceMethod.testIM(ProbRC)
```

The following example uses the decision tree representation of Section 9.3.4 (page 207). Does recursive conditioning split on variable `full` for the query commented out below? What can be done to guarantee that it does?

probRC.py — (continued)

```
from probFactors import Prob, action, rain, full, wet, p_wet
from probGraphicalModels import BeliefNetwork
p_action = Prob(action,[],{'go_out':0.3, 'get_coffee':0.7})
p_rain = Prob(rain,[],[0.4,0.6])
p_full = Prob(full,[],[0.1,0.9])

wetBN = BeliefNetwork("Wet (decision tree CPD)", {action, rain, full, wet},
                      {p_action, p_rain, p_full, p_wet})
wetRC = ProbRC(wetBN)
# wetRC.query(wet, {action:'go_out', rain:True})
# wetRC.show_post({action:'go_out', rain:True})
# wetRC.show_post({action:'go_out', wet:True})
```

9.8 Variable Elimination

An instance of a *VE* object takes in a graphical model. The query method uses variable elimination to compute the probability of a variable given observations on some other variables.

probVE.py — Variable Elimination for Graphical Models

```
from probFactors import Factor, FactorObserved, FactorSum, factor_times
from probGraphicalModels import GraphicalModel, InferenceMethod

class VE(InferenceMethod):
    """The class that queries Graphical Models using variable elimination.

    gm is graphical model to query
    """
    method_name = "variable elimination"

    def __init__(self,gm=None):
        InferenceMethod.__init__(self, gm)

    def query(self,var,obs={},elim_order=None):
        """computes P(var|obs) where
        var is a variable
        obs is a {variable:value} dictionary"""
        if var in obs:
            return {var:1 if val == obs[var] else 0 for val in var.domain}
        else:
            if elim_order == None:
                elim_order = self.gm.variables
            projFactors = [self.project_observations(fact,obs)
                           for fact in self.gm.factors]
            for v in elim_order:
                if v != var and v not in obs:
                    projFactors = self.eliminate_var(projFactors,v)
            unnorm = factor_times(var,projFactors)
            p_obs=sum(unnorm)
            self.display(1,"Unnormalized probs:",unnorm,"Prob obs:",p_obs)
            return {val:pr/p_obs for val,pr in zip(var.domain, unnorm)}
```

A *FactorObserved* is a factor that is the result of some observations on another factor. We don't store the values in a list; we just look them up as needed. The observations can include variables that are not in the list, but should have some intersection with the variables in the factor.

probFactors.py — (continued)

```
class FactorObserved(Factor):
    def __init__(self,factor,obs):
        Factor.__init__(self, [v for v in factor.variables if v not in obs])
        self.observed = obs
        self.orig_factor = factor

```

```
227     def get_value(self,assignment):
228         return self.orig_factor.get_value(assignment|self.observed)
```

A *FactorSum* is a factor that is the result of summing out a variable from the product of other factors. I.e., it constructs a representation of:

$$\sum_{var} \prod_{f \in factors} f.$$

We store the values in a list in a lazy manner; if they are already computed, we used the stored values. If they are not already computed we can compute and store them.

probFactors.py — (continued)

```
230 class FactorSum(Factor):
231     def __init__(self,var,factors):
232         self.var_summed_out = var
233         self.factors = factors
234         vars = list({v for fac in factors
235                      for v in fac.variables if v is not var})
236         #for fac in factors:
237         #    for v in fac.variables:
238         #        if v is not var and v not in vars:
239         #            vars.append(v)
240         Factor.__init__(self,vars)
241         self.values = {}
242 
243     def get_value(self,assignment):
244         """lazy implementation: if not saved, compute it. Return saved
                value"""
245         asst = frozenset(assignment.items())
246         if asst in self.values:
247             return self.values[asst]
248         else:
249             total = 0
250             new_asst = assignment.copy()
251             for val in self.var_summed_out.domain:
252                 new_asst[self.var_summed_out] = val
253                 total += math.prod(fac.get_value(new_asst) for fac in
                        self.factors)
254             self.values[asst] = total
255             return total
```

The method *factor_times* multiples a set of factors that are all factors on the same variable (or on no variables). This is the last step in variable elimination before normalizing. It returns an array giving the product for each value of *variable*.

probFactors.py — (continued)

```
257 def factor_times(variable, factors):
258     """when factors are factors just on variable (or on no variables)"""
259     prods = []
```

```
260     facs = [f for f in factors if variable in f.variables]
261     for val in variable.domain:
262         ast = {variable:val}
263         prods.append(math.prod(f.get_value(ast) for f in facs))
264     return prods
```

To project observations onto a factor, for each variable that is observed in the factor, we construct a new factor that is the factor projected onto that variable. *Factor_observed* creates a new factor that is the result is assigning a value to a single variable.

probVE.py — (continued)

```
43     def project_observations(self,factor,obs):
44         """Returns the resulting factor after observing obs
45 
46         obs is a dictionary of {variable:value} pairs.
47         """
48         if any((var in obs) for var in factor.variables):
49             # a variable in factor is observed
50             return FactorObserved(factor,obs)
51         else:
52             return factor
53 
54     def eliminate_var(self,factors,var):
55         """Eliminate a variable var from a list of factors.
56         Returns a new set of factors that has var summed out.
57         """
58         self.display(2,"eliminating ",str(var))
59         contains_var = []
60         not_contains_var = []
61         for fac in factors:
62             if var in fac.variables:
63                 contains_var.append(fac)
64             else:
65                 not_contains_var.append(fac)
66         if contains_var == []:
67             return factors
68         else:
69             newFactor = FactorSum(var,contains_var)
70             self.display(2,"Multiplying:",[str(f) for f in contains_var])
71             self.display(2,"Creating factor:", newFactor)
72             self.display(3, newFactor.to_table()) # factor in detail
73             not_contains_var.append(newFactor)
74             return not_contains_var
75 
76 from probExamples import bn_4ch, A,B,C,D
77 bn_4chv = VE(bn_4ch)
78 ## bn_4chv.query(A,{})
79 ## bn_4chv.query(D,{})
80 ## InferenceMethod.max_display_level = 3 # show more detail in displaying
81 ## InferenceMethod.max_display_level = 1 # show less detail in displaying
```

```
## bn_4chv.query(A,{D:True})
## bn_4chv.query(B,{A:True,D:False})

from probExamples import bn_report,Alarm,Fire,Leaving,Report,Smoke,Tamper
bn_reportv = VE(bn_report) # answers queries using variable elimination
## bn_reportv.query(Tamper,{})
## InferenceMethod.max_display_level = 0 # show no detail in displaying
## bn_reportv.query(Leaving,{})
## bn_reportv.query(Tamper,{},elim_order=[Smoke,Report,Leaving,Alarm,Fire])
## bn_reportv.query(Tamper,{Report:True})
## bn_reportv.query(Tamper,{Report:True,Smoke:False})

from probExamples import bn_sprinkler, Season, Sprinkler, Rained,
    Grass_wet, Grass_shiny, Shoes_wet
bn_sprinklerv = VE(bn_sprinkler)
## bn_sprinklerv.query(Shoes_wet,{})
## bn_sprinklerv.query(Shoes_wet,{Rained:True})
## bn_sprinklerv.query(Shoes_wet,{Grass_shiny:True})
## bn_sprinklerv.query(Shoes_wet,{Grass_shiny:False,Rained:True})

from probExamples import bn_lr1, Cough, Fever, Sneeze, Cold, Flu, Covid
vediag = VE(bn_lr1)
## vediag.query(Cough,{})
## vediag.query(Cold,{Cough:1,Sneeze:0,Fever:1})
## vediag.query(Flu,{Cough:0,Sneeze:1,Fever:1})
## vediag.query(Covid,{Cough:1,Sneeze:0,Fever:1})
## vediag.query(Covid,{Cough:1,Sneeze:0,Fever:1,Flu:0})
## vediag.query(Covid,{Cough:1,Sneeze:0,Fever:1,Flu:1})

if __name__ == "__main__":
    InferenceMethod.testIM(VE)
```

9.9 Stochastic Simulation

9.9.1 Sampling from a discrete distribution

The method *sample_one* generates a single sample from a (possible unnormalized) distribution. *dist* is a $\{value : weight\}$ dictionary, where $weight \geq 0$. This returns a value with probability in proportion to its weight.

probStochSim.py — Probabilistic inference using stochastic simulation

```
import random
from probGraphicalModels import InferenceMethod

def sample_one(dist):
    """returns the index of a single sample from normalized distribution
        dist."""
    rand = random.random()*sum(dist.values())
    cum = 0    # cumulative weights
```

```
18      for v in dist:
19          cum += dist[v]
20          if cum > rand:
21              return v
```

If we want to generate multiple samples, repeatedly calling *sample_one* may not be efficient. If we want to generate n samples, and the distribution is over m values, *sample_one* takes time $O(mn)$. If m and n are of the same order of magnitude, we can do better.

The method *sample_multiple* generates multiple samples from a distribution defined by *dist*, where *dist* is a $\{value : weight\}$ dictionary, where $weight \geq 0$ and the weights cannot all be zero. This returns a list of values, of length *num_samples*, where each sample is selected with a probability proportional to its weight.

The method generates all of the random numbers, sorts them, and then goes through the distribution once, saving the selected samples.

probStochSim.py — (continued)

```
23  def sample_multiple(dist, num_samples):
24      """returns a list of num_samples values selected using distribution
            dist.
25      dist is a {value:weight} dictionary that does not need to be normalized
26      """
27      total = sum(dist.values())
28      rands = sorted(random.random()*total for i in range(num_samples))
29      result = []
30      dist_items = list(dist.items())
31      cum = dist_items[0][1] # cumulative sum
32      index = 0
33      for r in rands:
34          while r>cum:
35              index += 1
36              cum += dist_items[index][1]
37          result.append(dist_items[index][0])
38      return result
```

Exercise 9.1

What is the time and space complexity the following 4 methods to generate n samples, where m is the length of *dist*:

(a) n calls to *sample_one*

(b) *sample_multiple*

(c) Create the cumulative distribution (choose how this is represented) and, for each random number, do a binary search to determine the sample associated with the random number.

(d) Choose a random number in the range $[i/n, (i+1)/n)$ for each $i \in range(n)$, where n is the number of samples. Use these as the random numbers to select the particles. (Does this give random samples?)

For each method suggest when it might be the best method.

The *test_sampling* method can be used to generate the statistics from a number of samples. It is useful to see the variability as a function of the number of samples. Try it for few samples and also for many samples.

probStochSim.py — (continued)

```
def test_sampling(dist, num_samples):
    """Given a distribution, dist, draw num_samples samples
    and return the resulting counts
    """
    result = {v:0 for v in dist}
    for v in sample_multiple(dist, num_samples):
        result[v] += 1
    return result

# try the following queries a number of times each:
# test_sampling({1:1,2:2,3:3,4:4}, 100)
# test_sampling({1:1,2:2,3:3,4:4}, 100000)
```

9.9.2 Sampling Methods for Belief Network Inference

A *SamplingInferenceMethod* is an *InferenceMethod,* but the query method also takes arguments for the number of samples and the sample-order (which is an ordering of factors). The first methods assume a belief network (and not an undirected graphical model).

probStochSim.py — (continued)

```
class SamplingInferenceMethod(InferenceMethod):
    """The abstract class of sampling-based belief network inference
        methods"""

    def __init__(self,gm=None):
        InferenceMethod.__init__(self, gm)

    def query(self,qvar,obs={},number_samples=1000,sample_order=None):
        raise NotImplementedError("SamplingInferenceMethod query") #
            abstract
```

9.9.3 Rejection Sampling

probStochSim.py — (continued)

```
class RejectionSampling(SamplingInferenceMethod):
    """The class that queries Graphical Models using Rejection Sampling.

    gm is a belief network to query
    """
    method_name = "rejection sampling"
```

```
68
69      def __init__(self, gm=None):
70          SamplingInferenceMethod.__init__(self, gm)
71
72      def query(self, qvar, obs={}, number_samples=1000, sample_order=None):
73          """computes P(qvar | obs) where
74          qvar is a variable.
75          obs is a {variable:value} dictionary.
76          sample_order is a list of variables where the parents
77            come before the variable.
78          """
79          if sample_order is None:
80              sample_order = self.gm.topological_sort()
81          self.display(2,*sample_order,sep="\t")
82          counts = {val:0 for val in qvar.domain}
83          for i in range(number_samples):
84              rejected = False
85              sample = {}
86              for nvar in sample_order:
87                  fac = self.gm.var2cpt[nvar]  #factor with nvar as child
88                  val = sample_one({v:fac.get_value({**sample, nvar:v}) for v
                        in nvar.domain})
89                  self.display(2,val,end="\t")
90                  if nvar in obs and obs[nvar] != val:
91                      rejected = True
92                      self.display(2,"Rejected")
93                      break
94                  sample[nvar] = val
95              if not rejected:
96                  counts[sample[qvar]] += 1
97                  self.display(2,"Accepted")
98          tot = sum(counts.values())
99          # As well as the distribution we also include raw counts
100         dist = {c:v/tot if tot>0 else 1/len(qvar.domain) for (c,v) in
                counts.items()}
101         dist["raw_counts"] = counts
102         return dist
```

9.9.4 Likelihood Weighting

Likelihood weighting includes a weight for each sample. Instead of rejecting samples based on observations, likelihood weighting changes the weights of the sample in proportion with the probability of the observation. The weight then becomes the probability that the variable would have been rejected.

probStochSim.py — (continued)

```
104 class LikelihoodWeighting(SamplingInferenceMethod):
105     """The class that queries Graphical Models using Likelihood weighting.
106
107     gm is a belief network to query
108     """
```

```
    method_name = "likelihood weighting"

    def __init__(self, gm=None):
        SamplingInferenceMethod.__init__(self, gm)

    def query(self,qvar,obs={},number_samples=1000,sample_order=None):
        """computes P(qvar | obs) where
        qvar is a variable.
        obs is a {variable:value} dictionary.
        sample_order is a list of factors where factors defining the parents
          come before the factors for the child.
        """
        if sample_order is None:
            sample_order = self.gm.topological_sort()
        self.display(2,*[v for v in sample_order
                         if v not in obs],sep="\t")
        counts = {val:0 for val in qvar.domain}
        for i in range(number_samples):
            sample = {}
            weight = 1.0
            for nvar in sample_order:
                fac = self.gm.var2cpt[nvar]
                if nvar in obs:
                    sample[nvar] = obs[nvar]
                    weight *= fac.get_value(sample)
                else:
                    val = sample_one({v:fac.get_value({**sample,nvar:v}) for
                        v in nvar.domain})
                    self.display(2,val,end="\t")
                    sample[nvar] = val
            counts[sample[qvar]] += weight
            self.display(2,weight)
        tot = sum(counts.values())
        # as well as the distribution we also include the raw counts
        dist = {c:v/tot for (c,v) in counts.items()}
        dist["raw_counts"] = counts
        return dist
```

Exercise 9.2 Change this algorithm so that it does **importance sampling** using a proposal distribution. It needs *sample_one* using a different distribution and then update the weight of the current sample. For testing, use a proposal distribution that only specifies probabilities for some of the variables (and the algorithm uses the probabilities for the network in other cases).

9.9.5 Particle Filtering

In this implementation, a particle is a $\{variable : value\}$ dictionary. Because adding a new value to dictionary involves a side effect, the dictionaries need to be copied during resampling.

probStochSim.py — (continued)

```
class ParticleFiltering(SamplingInferenceMethod):
    """The class that queries Graphical Models using Particle Filtering.

    gm is a belief network to query
    """
    method_name = "particle filtering"

    def __init__(self, gm=None):
        SamplingInferenceMethod.__init__(self, gm)

    def query(self, qvar, obs={}, number_samples=1000, sample_order=None):
        """computes P(qvar | obs) where
        qvar is a variable.
        obs is a {variable:value} dictionary.
        sample_order is a list of factors where factors defining the parents
          come before the factors for the child.
        """
        if sample_order is None:
            sample_order = self.gm.topological_sort()
        self.display(2,*[v for v in sample_order
                         if v not in obs],sep="\t")
        particles = [{} for i in range(number_samples)]
        for nvar in sample_order:
            fac = self.gm.var2cpt[nvar]
            if nvar in obs:
                weights = [fac.get_value({**part, nvar:obs[nvar]})
                           for part in particles]
                particles = [{**p, nvar:obs[nvar]}
                             for p in resample(particles, weights,
                                 number_samples)]
            else:
                for part in particles:
                    part[nvar] = sample_one({v:fac.get_value({**part,
                        nvar:v})
                                             for v in nvar.domain})
                self.display(2,part[nvar],end="\t")
        counts = {val:0 for val in qvar.domain}
        for part in particles:
            counts[part[qvar]] += 1
        tot = sum(counts.values())
        # as well as the distribution we also include the raw counts
        dist = {c:v/tot for (c,v) in counts.items()}
        dist["raw_counts"] = counts
        return dist
```

Resampling

Resample is based on *sample_multiple* but works with an array of particles. (Aside: Python doesn't let us use *sample_multiple* directly as it uses a dictionary,

and particles, represented as dictionaries can't be the key of dictionaries).

probStochSim.py — (continued)

```
def resample(particles, weights, num_samples):
    """returns num_samples copies of particles resampled according to
        weights.
    particles is a list of particles
    weights is a list of positive numbers, of same length as particles
    num_samples is n integer
    """
    total = sum(weights)
    rands = sorted(random.random()*total for i in range(num_samples))
    result = []
    cum = weights[0]   # cumulative sum
    index = 0
    for r in rands:
        while r>cum:
            index += 1
            cum += weights[index]
        result.append(particles[index])
    return result
```

9.9.6 Examples

probStochSim.py — (continued)

```
from probExamples import bn_4ch, A,B,C,D
bn_4chr = RejectionSampling(bn_4ch)
bn_4chL = LikelihoodWeighting(bn_4ch)
## InferenceMethod.max_display_level = 2 # detailed tracing for all
    inference methods
## bn_4chr.query(A,{})
## bn_4chr.query(C,{})
## bn_4chr.query(A,{C:True})
## bn_4chr.query(B,{A:True,C:False})

from probExamples import bn_report,Alarm,Fire,Leaving,Report,Smoke,Tamper
bn_reportr = RejectionSampling(bn_report) # answers queries using
    rejection sampling
bn_reportL = LikelihoodWeighting(bn_report) # answers queries using
    likelihood weighting
bn_reportp = ParticleFiltering(bn_report) # answers queries using particle
    filtering
## bn_reportr.query(Tamper,{})
## bn_reportr.query(Tamper,{})
## bn_reportr.query(Tamper,{Report:True})
## InferenceMethod.max_display_level = 0 # no detailed tracing for all
    inference methods
## bn_reportr.query(Tamper,{Report:True},number_samples=100000)
## bn_reportr.query(Tamper,{Report:True,Smoke:False})
```

```
## bn_reportr.query(Tamper,{Report:True,Smoke:False},number_samples=100)

## bn_reportL.query(Tamper,{Report:True,Smoke:False},number_samples=100)
## bn_reportL.query(Tamper,{Report:True,Smoke:False},number_samples=100)

from probExamples import bn_sprinkler,Season, Sprinkler
from probExamples import Rained, Grass_wet, Grass_shiny, Shoes_wet
bn_sprinklerr = RejectionSampling(bn_sprinkler) # answers queries using
    rejection sampling
bn_sprinklerL = LikelihoodWeighting(bn_sprinkler) # answers queries using
    rejection sampling
bn_sprinklerp = ParticleFiltering(bn_sprinkler) # answers queries using
    particle filtering
#bn_sprinklerr.query(Shoes_wet,{Grass_shiny:True,Rained:True})
#bn_sprinklerL.query(Shoes_wet,{Grass_shiny:True,Rained:True})
#bn_sprinklerp.query(Shoes_wet,{Grass_shiny:True,Rained:True})

if __name__ == "__main__":
    InferenceMethod.testIM(RejectionSampling, threshold=0.1)
    InferenceMethod.testIM(LikelihoodWeighting, threshold=0.1)
    InferenceMethod.testIM(ParticleFiltering, threshold=0.1)
```

Exercise 9.3 This code keeps regenerating the distribution of a variable given its parents. Implement one or both of the following, and compare them to the original. Make *cond_dist* return a slice that corresponds to the distribution, and then use the slice instead of the dictionary (a list slice does not generate new data structures). Make *cond_dist* remember values it has already computed, and only return these.

9.9.7 Gibbs Sampling

The following implements **Gibbs sampling**, a form of **Markov Chain Monte Carlo** MCMC.

probStochSim.py — (continued)

```
#import random
#from probGraphicalModels import InferenceMethod

#from probStochSim import sample_one, SamplingInferenceMethod

class GibbsSampling(SamplingInferenceMethod):
    """The class that queries Graphical Models using Gibbs Sampling.

    bn is a graphical model (e.g., a belief network) to query
    """
    method_name = "Gibbs sampling"

    def __init__(self, gm=None):
        SamplingInferenceMethod.__init__(self, gm)
        self.gm = gm
```

```
260
261     def query(self, qvar, obs={}, number_samples=1000, burn_in=100,
            sample_order=None):
262         """computes P(qvar | obs) where
263         qvar is a variable.
264         obs is a {variable:value} dictionary.
265         sample_order is a list of non-observed variables in order, or
266         if sample_order None, an arbitrary ordering is used
267         """
268         counts = {val:0 for val in qvar.domain}
269         if sample_order is not None:
270             variables = sample_order
271         else:
272             variables = [v for v in self.gm.variables if v not in obs]
273             random.shuffle(variables)
274         var_to_factors = {v:set() for v in self.gm.variables}
275         for fac in self.gm.factors:
276             for var in fac.variables:
277                 var_to_factors[var].add(fac)
278         sample = {var:random.choice(var.domain) for var in variables}
279         self.display(3,"Sample:",sample)
280         sample.update(obs)
281         for i in range(burn_in + number_samples):
282             for var in variables:
283                 # get unnormalized probability distribution of var given its
                        neighbors
284                 vardist = {val:1 for val in var.domain}
285                 for val in var.domain:
286                     sample[var] = val
287                     for fac in var_to_factors[var]: # Markov blanket
288                         vardist[val] *= fac.get_value(sample)
289                 sample[var] = sample_one(vardist)
290             if i >= burn_in:
291                 counts[sample[qvar]] +=1
292                 self.display(3,"     ",sample)
293         tot = sum(counts.values())
294         # as well as the computed distribution, we also include raw counts
295         dist = {c:v/tot for (c,v) in counts.items()}
296         dist["raw_counts"] = counts
297         self.display(2, f"Gibbs sampling P({qvar}|{obs}) = {dist}")
298         return dist
299
300 #from probExamples import bn_4ch, A,B,C,D
301 bn_4chg = GibbsSampling(bn_4ch)
302 ## InferenceMethod.max_display_level = 2 # detailed tracing for all
        inference methods
303 bn_4chg.query(A,{})
304 ## bn_4chg.query(D,{})
305 ## bn_4chg.query(B,{D:True})
306 ## bn_4chg.query(B,{A:True,C:False})
```

```
307
308 from probExamples import bn_report,Alarm,Fire,Leaving,Report,Smoke,Tamper
309 bn_reportg = GibbsSampling(bn_report)
310 ## bn_reportg.query(Tamper,{Report:True},number_samples=1000)
311
312 if __name__ == "__main__":
313     InferenceMethod.testIM(GibbsSampling, threshold=0.1)
```

Exercise 9.4 Change the code so that it can have multiple query variables. Make the list of query variable be an input to the algorithm, so that the default value is the list of all non-observed variables.

Exercise 9.5 In this algorithm, explain where it computes the probability of a variable given its Markov blanket. Instead of returning the average of the samples for the query variable, it is possible to return the average estimate of the probability of the query variable given its Markov blanket. Does this converge to the same answer as the given code? Does it converge faster, slower, or the same?

9.9.8 Plotting Behavior of Stochastic Simulators

The stochastic simulation runs can give different answers each time they are run. For the algorithms that give the same answer in the limit as the number of samples approaches infinity (as do all of these algorithms), the algorithms can be compared by comparing the accuracy for multiple runs. Summary statistics like the variance may provide some information, but the assumptions behind the variance being appropriate (namely that the distribution is approximately Gaussian) may not hold for cases where the predictions are bounded and often skewed.

It is more appropriate to plot the distribution of predictions over multiple runs. The *plot_stats* method plots the prediction of a particular variable (or for the partition function) for a number of runs of the same algorithm. On the x-axis, is the prediction of the algorithm. On the y-axis is the number of runs with prediction less than or equal to the x value. Thus this is like a cumulative distribution over the predictions, but with counts on the y-axis.

Note that for runs where there are no samples that are consistent with the observations (as can happen with rejection sampling), the prediction of probability is 1.0 (as a convention for $0/0$).

That variable *what* contains the query variable, or *what* is "*prob_ev*", the probability of evidence.

probStochSim.py — (continued)

```
315 import matplotlib.pyplot as plt
316
317 def plot_stats(method, qvar, qval, obs, number_runs=1000, **queryargs):
318     """Plots a cumulative distribution of the prediction of the model.
319     method is a InferenceMethod (that implements appropriate query(.))
320     plots P(qvar=qval | obs)
321     qvar is the query variable, qval is corresponding value
```

```
322     obs is the {variable:value} dictionary representing the observations
323     number_iterations is the number of runs that are plotted
324     **queryargs is the arguments to query (often number_samples for
            sampling methods)
325     """
326     plt.ion()
327     plt.xlabel("value")
328     plt.ylabel("Cumulative Number")
329     method.max_display_level, prev_mdl = 0, method.max_display_level #no
            display
330     answers = [method.query(qvar,obs,**queryargs)
331                for i in range(number_runs)]
332     values = [ans[qval] for ans in answers]
333     label = f"""{method.method_name}
            P({qvar}={qval}|{','.join(f'{var}={val}'
334                                                     for (var,val) in
                                                        obs.items())})"""
335     values.sort()
336     plt.plot(values,range(number_runs),label=label)
337     plt.legend() #loc="upper left")
338     plt.draw()
339     method.max_display_level = prev_mdl # restore display level
340
341 # Try:
342 # plot_stats(bn_reportr,Tamper,True,{Report:True,Smoke:True},
        number_samples=1000, number_runs=1000)
343 # plot_stats(bn_reportL,Tamper,True,{Report:True,Smoke:True},
        number_samples=1000, number_runs=1000)
344 # plot_stats(bn_reportp,Tamper,True,{Report:True,Smoke:True},
        number_samples=1000, number_runs=1000)
345 # plot_stats(bn_reportr,Tamper,True,{Report:True,Smoke:True},
        number_samples=100, number_runs=1000)
346 # plot_stats(bn_reportL,Tamper,True,{Report:True,Smoke:True},
        number_samples=100, number_runs=1000)
347 # plot_stats(bn_reportg,Tamper,True,{Report:True,Smoke:True},
        number_samples=1000, number_runs=1000)
348
349 def plot_mult(methods, example, qvar, qval, obs, number_samples=1000,
        number_runs=1000):
350     for method in methods:
351         solver = method(example)
352         if isinstance(method,SamplingInferenceMethod):
353             plot_stats(solver, qvar, qval, obs,
                    number_samples=number_samples, number_runs=number_runs)
354         else:
355             plot_stats(solver, qvar, qval, obs, number_runs=number_runs)
356
357 from probRC import ProbRC
358 # Try following (but it takes a while..)
359 methods =
```

```
    [ProbRC,RejectionSampling,LikelihoodWeighting,ParticleFiltering,GibbsSampling]
#plot_mult(methods,bn_report,Tamper,True,{Report:True,Smoke:False},number_samples=100,
    number_runs=1000)
#
    plot_mult(methods,bn_report,Tamper,True,{Report:False,Smoke:True},number_samples=100,
    number_runs=1000)

# Sprinkler Example:
#
    plot_stats(bn_sprinklerr,Shoes_wet,True,{Grass_shiny:True,Rained:True},number_samples=1000)
#
    plot_stats(bn_sprinklerL,Shoes_wet,True,{Grass_shiny:True,Rained:True},number_samples=1000)
```

9.10 Hidden Markov Models

This code for hidden Markov models is independent of the graphical models code, to keep it simple. Section 9.11 gives code that models hidden Markov models, and more generally, dynamic belief networks, using the graphical models code.

This HMM code assumes there are multiple Boolean observation variables that depend on the current state and are independent of each other given the state.

probHMM.py — Hidden Markov Model

```
import random
from probStochSim import sample_one, sample_multiple

class HMM(object):
    def __init__(self, states, obsvars, pobs, trans, indist):
        """A hidden Markov model.
        states - set of states
        obsvars - set of observation variables
        pobs - probability of observations, pobs[i][s] is P(Obs_i=True |
            State=s)
        trans - transition probability - trans[i][j] gives P(State=j |
            State=i)
        indist - initial distribution - indist[s] is P(State_0 = s)
        """
        self.states = states
        self.obsvars = obsvars
        self.pobs = pobs
        self.trans = trans
        self.indist = indist
```

Consider the following example. Suppose you want to unobtrusively keep track of an animal in a triangular enclosure using sound. Suppose you have 3 microphones that provide unreliable (noisy) binary information at each time

step. The animal is either close to one of the 3 points of the triangle or in the middle of the triangle.

probHMM.py — (continued)

```
# state
#       0=middle, 1,2,3 are corners
states1 = {'middle', 'c1', 'c2', 'c3'} # states
obs1 = {'m1','m2','m3'} # microphones
```

The observation model is as follows. If the animal is in a corner, it will be detected by the microphone at that corner with probability 0.6, and will be independently detected by each of the other microphones with a probability of 0.1. If the animal is in the middle, it will be detected by each microphone with a probability of 0.4.

probHMM.py — (continued)

```
# pobs gives the observation model:
#pobs[mi][state] is P(mi=on | state)
closeMic=0.6; farMic=0.1; midMic=0.4
pobs1 = {'m1':{'middle':midMic, 'c1':closeMic, 'c2':farMic, 'c3':farMic},
    # mic 1
        'm2':{'middle':midMic, 'c1':farMic, 'c2':closeMic, 'c3':farMic}, #
            mic 2
        'm3':{'middle':midMic, 'c1':farMic, 'c2':farMic, 'c3':closeMic}} #
            mic 3
```

The transition model is as follows: If the animal is in a corner it stays in the same corner with probability 0.80, goes to the middle with probability 0.1 or goes to one of the other corners with probability 0.05 each. If it is in the middle, it stays in the middle with probability 0.7, otherwise it moves to one the corners, each with probability 0.1.

probHMM.py — (continued)

```
# trans specifies the dynamics
# trans[i] is the distribution over states resulting from state i
# trans[i][j] gives P(S=j | S=i)
sm=0.7; mmc=0.1             # transition probabilities when in middle
sc=0.8; mcm=0.1; mcc=0.05   # transition probabilities when in a corner
trans1 = {'middle':{'middle':sm, 'c1':mmc, 'c2':mmc, 'c3':mmc}, # was in
    middle
         'c1':{'middle':mcm, 'c1':sc, 'c2':mcc, 'c3':mcc}, # was in corner
             1
         'c2':{'middle':mcm, 'c1':mcc, 'c2':sc, 'c3':mcc}, # was in corner
             2
         'c3':{'middle':mcm, 'c1':mcc, 'c2':mcc, 'c3':sc}} # was in corner
             3
```

Initially the animal is in one of the four states, with equal probability.

probHMM.py — (continued)

```
# initially we have a uniform distribution over the animal's state
```

```
indist1 = {st:1.0/len(states1) for st in states1}

hmm1 = HMM(states1, obs1, pobs1, trans1, indist1)
```

9.10.1 Exact Filtering for HMMs

A *HMMVEfilter* has a current state distribution which can be updated by observing or by advancing to the next time.

probHMM.py — (continued)

```
from display import Displayable

class HMMVEfilter(Displayable):
    def __init__(self,hmm):
        self.hmm = hmm
        self.state_dist = hmm.indist

    def filter(self, obsseq):
        """updates and returns the state distribution following the
            sequence of
        observations in obsseq using variable elimination.

        Note that it first advances time.
        This is what is required if it is called sequentially.
        If that is not what is wanted initially, do an observe first.
        """
        for obs in obsseq:
            self.advance()    # advance time
            self.observe(obs) # observe
        return self.state_dist

    def observe(self, obs):
        """updates state conditioned on observations.
        obs is a list of values for each observation variable"""
        for i in self.hmm.obsvars:
            self.state_dist = {st:self.state_dist[st]*(self.hmm.pobs[i][st]
                                                       if obs[i] else
                                                           (1-self.hmm.pobs[i][st]))
                               for st in self.hmm.states}
        norm = sum(self.state_dist.values()) # normalizing constant
        self.state_dist = {st:self.state_dist[st]/norm for st in
            self.hmm.states}
        self.display(2,"After observing",obs,"state
            distribution:",self.state_dist)

    def advance(self):
        """advance to the next time"""
        nextstate = {st:0.0 for st in self.hmm.states}  # distribution over
            next states
        for j in self.hmm.states:     # j ranges over next states
```

```
            for i in self.hmm.states: # i ranges over previous states
                nextstate[j] += self.hmm.trans[i][j]*self.state_dist[i]
        self.state_dist = nextstate
        self.display(2,"After advancing state
            distribution:",self.state_dist)
```

The following are some queries for *hmm*1.

probHMM.py — (continued)

```
hmm1f1 = HMMVEfilter(hmm1)
# hmm1f1.filter([{'m1':0, 'm2':1, 'm3':1}, {'m1':1, 'm2':0, 'm3':1}])
## HMMVEfilter.max_display_level = 2 # show more detail in displaying
# hmm1f2 = HMMVEfilter(hmm1)
# hmm1f2.filter([{'m1':1, 'm2':0, 'm3':0}, {'m1':0, 'm2':1, 'm3':0},
    {'m1':1, 'm2':0, 'm3':0},
#               {'m1':0, 'm2':0, 'm3':0}, {'m1':0, 'm2':0, 'm3':0},
    {'m1':0, 'm2':0, 'm3':0},
#               {'m1':0, 'm2':0, 'm3':0}, {'m1':0, 'm2':0, 'm3':1},
    {'m1':0, 'm2':0, 'm3':1},
#               {'m1':0, 'm2':0, 'm3':1}])
# hmm1f3 = HMMVEfilter(hmm1)
# hmm1f3.filter([{'m1':1, 'm2':0, 'm3':0}, {'m1':0, 'm2':0, 'm3':0},
    {'m1':1, 'm2':0, 'm3':0}, {'m1':1, 'm2':0, 'm3':1}])

# How do the following differ in the resulting state distribution?
# Note they start the same, but have different initial observations.
## HMMVEfilter.max_display_level = 1 # show less detail in displaying
# for i in range(100): hmm1f1.advance()
# hmm1f1.state_dist
# for i in range(100): hmm1f3.advance()
# hmm1f3.state_dist
```

Exercise 9.6 The representation assumes that there are a list of Boolean observations. Extend the representation so that the each observation variable can have multiple discrete values. You need to choose a representation for the model, and change the algorithm.

9.10.2 Localization

The localization example in the book is a controlled HMM, where there is a given action at each time and the transition depends on the action.

probLocalization.py — Controlled HMM and Localization example

```
from probHMM import HMMVEfilter, HMM
from display import Displayable
import matplotlib.pyplot as plt
from matplotlib.widgets import Button, CheckButtons

class HMM_Controlled(HMM):
    """A controlled HMM, where the transition probability depends on the
        action.
```

```
18        Instead of the transition probability, it has a function act2trans
19        from action to transition probability.
20        Any algorithms need to select the transition probability according
              to the action.
21      """
22      def __init__(self, states, obsvars, pobs, act2trans, indist):
23          self.act2trans = act2trans
24          HMM.__init__(self, states, obsvars, pobs, None, indist)
25
26
27  local_states = list(range(16))
28  door_positions = {2,4,7,11}
29  def prob_door(loc): return 0.8 if loc in door_positions else 0.1
30  local_obs = {'door':[prob_door(i) for i in range(16)]}
31  act2trans = {'right': [[0.1 if next == current
32                          else 0.8 if next == (current+1)%16
33                          else 0.074 if next == (current+2)%16
34                          else 0.002 for next in range(16)]
35                             for current in range(16)],
36               'left': [[0.1 if next == current
37                          else 0.8 if next == (current-1)%16
38                          else 0.074 if next == (current-2)%16
39                          else 0.002 for next in range(16)]
40                            for current in range(16)]}
41  hmm_16pos = HMM_Controlled(local_states, {'door'}, local_obs,
42                             act2trans, [1/16 for i in range(16)])
```

To change the VE localization code to allow for controlled HMMs, notice that the action selects which transition probability to us.

probLocalization.py — (continued)

```
43  class HMM_Local(HMMVEfilter):
44      """VE filter for controlled HMMs
45      """
46      def __init__(self, hmm):
47          HMMVEfilter.__init__(self, hmm)
48
49      def go(self, action):
50          self.hmm.trans = self.hmm.act2trans[action]
51          self.advance()
52
53  loc_filt = HMM_Local(hmm_16pos)
54  # loc_filt.observe({'door':True}); loc_filt.go("right");
        loc_filt.observe({'door':False}); loc_filt.go("right");
        loc_filt.observe({'door':True})
55  # loc_filt.state_dist
```

The following lets us interactively move the agent and provide observations. It shows the distribution over locations.

probLocalization.py — (continued)

```
57  class Show_Localization(Displayable):
```

```
    def __init__(self,hmm, fontsize=10):
        self.hmm = hmm
        self.fontsize = fontsize
        self.loc_filt = HMM_Local(hmm)
        fig,(self.ax) = plt.subplots()
        plt.subplots_adjust(bottom=0.2)
        ## Set up buttons:
        left_butt = Button(plt.axes([0.05,0.02,0.1,0.05]), "left")
        left_butt.label.set_fontsize(self.fontsize)
        left_butt.on_clicked(self.left)
        right_butt = Button(plt.axes([0.25,0.02,0.1,0.05]), "right")
        right_butt.label.set_fontsize(self.fontsize)
        right_butt.on_clicked(self.right)
        door_butt = Button(plt.axes([0.45,0.02,0.1,0.05]), "door")
        door_butt.label.set_fontsize(self.fontsize)
        door_butt.on_clicked(self.door)
        nodoor_butt = Button(plt.axes([0.65,0.02,0.1,0.05]), "no door")
        nodoor_butt.label.set_fontsize(self.fontsize)
        nodoor_butt.on_clicked(self.nodoor)
        reset_butt = Button(plt.axes([0.85,0.02,0.1,0.05]), "reset")
        reset_butt.label.set_fontsize(self.fontsize)
        reset_butt.on_clicked(self.reset)
        ## draw the distribution
        plt.subplot(1, 1, 1)
        self.draw_dist()
        plt.show()

    def draw_dist(self):
        self.ax.clear()
        plt.ylim(0,1)
        plt.ylabel("Probability", fontsize=self.fontsize)
        plt.xlabel("Location", fontsize=self.fontsize)
        plt.title("Location Probability Distribution",
            fontsize=self.fontsize)
        plt.xticks(self.hmm.states,fontsize=self.fontsize)
        plt.yticks(fontsize=self.fontsize)
        vals = [self.loc_filt.state_dist[i] for i in self.hmm.states]
        self.bars = self.ax.bar(self.hmm.states, vals, color='black')
        self.ax.bar_label(self.bars,["{v:.2f}".format(v=v) for v in vals],
            padding = 1, fontsize=self.fontsize)
        plt.draw()

    def left(self,event):
        self.loc_filt.go("left")
        self.draw_dist()
    def right(self,event):
        self.loc_filt.go("right")
        self.draw_dist()
    def door(self,event):
        self.loc_filt.observe({'door':True})
```

```
106         self.draw_dist()
107     def nodoor(self,event):
108         self.loc_filt.observe({'door':False})
109         self.draw_dist()
110     def reset(self,event):
111         self.loc_filt.state_dist = {i:1/16 for i in range(16)}
112         self.draw_dist()
113 
114 # sl = Show_Localization(hmm_16pos)
115 # sl = Show_Localization(hmm_16pos, fontsize=15) # for demos - enlarge
      window
```

9.10.3 Particle Filtering for HMMs

In this implementation a particle is just a state. If you want to do some form of smoothing, a particle should probably be a history of states. This maintains, *particles*, an array of states, *weights* an array of (non-negative) real numbers, such that $weights[i]$ is the weight of $particles[i]$.

probHMM.py — (continued)

```
114 from display import Displayable
115 from probStochSim import resample
116 
117 class HMMparticleFilter(Displayable):
118     def __init__(self,hmm,number_particles=1000):
119         self.hmm = hmm
120         self.particles = [sample_one(hmm.indist)
121                           for i in range(number_particles)]
122         self.weights = [1 for i in range(number_particles)]
123 
124     def filter(self, obsseq):
125         """returns the state distribution following the sequence of
126         observations in obsseq using particle filtering.
127 
128         Note that it first advances time.
129         This is what is required if it is called after previous filtering.
130         If that is not what is wanted initially, do an observe first.
131         """
132         for obs in obsseq:
133             self.advance()   # advance time
134             self.observe(obs) # observe
135             self.resample_particles()
136             self.display(2,"After observing", str(obs),
137                          "state distribution:",
                                 self.histogram(self.particles))
138         self.display(1,"Final state distribution:",
                self.histogram(self.particles))
139         return self.histogram(self.particles)
140 
```

```
    def advance(self):
        """advance to the next time.
        This assumes that all of the weights are 1."""
        self.particles = [sample_one(self.hmm.trans[st])
                          for st in self.particles]

    def observe(self, obs):
        """reweighs the particles to incorporate observations obs"""
        for i in range(len(self.particles)):
            for obv in obs:
                if obs[obv]:
                    self.weights[i] *= self.hmm.pobs[obv][self.particles[i]]
                else:
                    self.weights[i] *=
                        1-self.hmm.pobs[obv][self.particles[i]]

    def histogram(self, particles):
        """returns list of the probability of each state as represented by
        the particles"""
        tot=0
        hist = {st: 0.0 for st in self.hmm.states}
        for (st,wt) in zip(self.particles,self.weights):
            hist[st]+=wt
            tot += wt
        return {st:hist[st]/tot for st in hist}

    def resample_particles(self):
        """resamples to give a new set of particles."""
        self.particles = resample(self.particles, self.weights,
            len(self.particles))
        self.weights = [1] * len(self.particles)
```

The following are some queries for *hmm*1.

probHMM.py — (continued)

```
hmm1pf1 = HMMparticleFilter(hmm1)
# HMMparticleFilter.max_display_level = 2 # show each step
# hmm1pf1.filter([{'m1':0, 'm2':1, 'm3':1}, {'m1':1, 'm2':0, 'm3':1}])
# hmm1pf2 = HMMparticleFilter(hmm1)
# hmm1pf2.filter([{'m1':1, 'm2':0, 'm3':0}, {'m1':0, 'm2':1, 'm3':0},
    {'m1':1, 'm2':0, 'm3':0},
#               {'m1':0, 'm2':0, 'm3':0}, {'m1':0, 'm2':0, 'm3':0},
    {'m1':0, 'm2':0, 'm3':0},
#               {'m1':0, 'm2':0, 'm3':0}, {'m1':0, 'm2':0, 'm3':1},
    {'m1':0, 'm2':0, 'm3':1},
#               {'m1':0, 'm2':0, 'm3':1}])
# hmm1pf3 = HMMparticleFilter(hmm1)
# hmm1pf3.filter([{'m1':1, 'm2':0, 'm3':0}, {'m1':0, 'm2':0, 'm3':0},
    {'m1':1, 'm2':0, 'm3':0}, {'m1':1, 'm2':0, 'm3':1}])
```

Exercise 9.7 A form of importance sampling can be obtained by not resampling.

Is it better or worse than particle filtering? Hint: you need to think about how they can be compared. Is the comparison different if there are more states than particles?

Exercise 9.8 Extend the particle filtering code to continuous variables and observations. In particular, suppose the state transition is a linear function with Gaussian noise of the previous state, and the observations are linear functions with Gaussian noise of the state. You may need to research how to sample from a Gaussian distribution.

9.10.4 Generating Examples

The following code is useful for generating examples.

probHMM.py — (continued)

```
182 def simulate(hmm,horizon):
183     """returns a pair of (state sequence, observation sequence) of length
            horizon.
184     for each time t, the agent is in state_sequence[t] and
185     observes observation_sequence[t]
186     """
187     state = sample_one(hmm.indist)
188     obsseq=[]
189     stateseq=[]
190     for time in range(horizon):
191         stateseq.append(state)
192         newobs =
                {obs:sample_one({0:1-hmm.pobs[obs][state],1:hmm.pobs[obs][state]})
193                  for obs in hmm.obsvars}
194         obsseq.append(newobs)
195         state = sample_one(hmm.trans[state])
196     return stateseq,obsseq
197
198 def simobs(hmm,stateseq):
199     """returns observation sequence for the state sequence"""
200     obsseq=[]
201     for state in stateseq:
202         newobs =
                {obs:sample_one({0:1-hmm.pobs[obs][state],1:hmm.pobs[obs][state]})
203                  for obs in hmm.obsvars}
204         obsseq.append(newobs)
205     return obsseq
206
207 def create_eg(hmm,n):
208     """Create an annotated example for horizon n"""
209     seq,obs = simulate(hmm,n)
210     print("True state sequence:",seq)
211     print("Sequence of observations:\n",obs)
212     hmmfilter = HMMVEfilter(hmm)
213     dist = hmmfilter.filter(obs)
```

```
214     print("Resulting distribution over states:\n",dist)
```

9.11 Dynamic Belief Networks

A **dynamic belief network (DBN)** is a belief network that extends in time.

There are a number of ways that reasoning can be carried out in a DBN, including:

- Rolling out the DBN for some time period, and using standard belief network inference. The latest time that needs to be in the rolled out network is the time of the latest observation or the time of a query (whichever is later). This allows us to observe any variables at any time and query any variables at any time. This is covered in Section 9.11.2.
- An unrolled belief network may be very large, and we might only be interested in asking about "now". In this case we can just representing the variables "now". In this approach we can observe and query the current variables. We can them move to the next time. This does not allow for arbitrary historical queries (about the past or the future), but can be much simpler. This is covered in Section 9.11.3.

9.11.1 Representing Dynamic Belief Networks

To specify a DBN, think about the distribution *now*. *Now* will be represented as time 1. Each variable will have a corresponding previous variable; these will be created together.

A dynamic belief network consists of:

- A set of features. A variable is a feature-time pair.
- An initial distribution over the features "now" (time 1). This is a belief network with all variables being time 1 variables.
- A specification of the dynamics. We define the how the variables *now* (time 1) depend on variables *now* and the previous time (time 0), in such a way that the graph is acyclic.

probDBN.py — Dynamic belief networks

```
11  from variable import Variable
12  from probGraphicalModels import GraphicalModel, BeliefNetwork
13  from probFactors import Prob, Factor, CPD
14  from probVE import VE
15  from display import Displayable
16
17  class DBNvariable(Variable):
18      """A random variable that incorporates the stage (time)
```

```
19
20      A variable can have both a name and an index. The index defaults to 1.
21      """
22      def __init__(self,name,domain=[False,True],index=1):
23          Variable.__init__(self,f"{name}_{index}",domain)
24          self.basename = name
25          self.domain = domain
26          self.index = index
27          self.previous = None
28
29      def __lt__(self,other):
30          if self.name != other.name:
31              return self.name<other.name
32          else:
33              return self.index<other.index
34
35      def __gt__(self,other):
36          return other<self
37
38  def variable_pair(name,domain=[False,True]):
39      """returns a variable and its predecessor. This is used to define
            2-stage DBNs
40
41      If the name is X, it returns the pair of variables X_prev,X_now"""
42      var_now = DBNvariable(name,domain,index='now')
43      var_prev = DBNvariable(name,domain,index='prev')
44      var_now.previous = var_prev
45      return var_prev, var_now
```

A *FactorRename* is a factor that is the result renaming the variables in the factor. It takes a factor, *fac,* and a $\{new : old\}$ dictionary, where *new* is the name of a variable in the resulting factor and *old* is the corresponding name in *fac*. This assumes that the all variables are renamed.

probDBN.py — (continued)

```
47  class FactorRename(Factor):
48      def __init__(self,fac,renaming):
49          """A renamed factor.
50          fac is a factor
51          renaming is a dictionary of the form {new:old} where old and new
                var variables,
52             where the variables in fac appear exactly once in the renaming
53          """
54          Factor.__init__(self,[n for (n,o) in renaming.items() if o in
                fac.variables])
55          self.orig_fac = fac
56          self.renaming = renaming
57
58      def get_value(self,assignment):
59          return self.orig_fac.get_value({self.renaming[var]:val
60                                          for (var,val) in assignment.items()
```

```
61                                  if var in self.variables})
```

The following class renames the variables of a conditional probability distribution. It is used for template models (e.g., dynamic decision networks or relational models)

probDBN.py — (continued)

```
63 class CPDrename(FactorRename, CPD):
64     def __init__(self, cpd, renaming):
65         renaming_inverse = {old:new for (new,old) in renaming.items()}
66         CPD.__init__(self,renaming_inverse[cpd.child],[renaming_inverse[p]
               for p in cpd.parents])
67         self.orig_fac = cpd
68         self.renaming = renaming
```

probDBN.py — (continued)

```
70 class DBN(Displayable):
71     """The class of stationary Dynamic Belief networks.
72     * name is the DBN name
73     * vars_now is a list of current variables (each must have
74     previous variable).
75     * transition_factors is a list of factors for P(X|parents) where X
76     is a current variable and parents is a list of current or previous
           variables.
77     * init_factors is a list of factors for P(X|parents) where X is a
78     current variable and parents can only include current variables
79     The graph of transition factors + init factors must be acyclic.
80
81     """
82     def __init__(self, title, vars_now, transition_factors=None,
           init_factors=None):
83         self.title = title
84         self.vars_now = vars_now
85         self.vars_prev = [v.previous for v in vars_now]
86         self.transition_factors = transition_factors
87         self.init_factors = init_factors
88         self.var_index = {}    # var_index[v] is the index of variable v
89         for i,v in enumerate(vars_now):
90             self.var_index[v]=i
```

Here is a 3 variable DBN:

probDBN.py — (continued)

```
92 A0,A1 = variable_pair("A", domain=[False,True])
93 B0,B1 = variable_pair("B", domain=[False,True])
94 C0,C1 = variable_pair("C", domain=[False,True])
95
96 # dynamics
97 pc = Prob(C1,[B1,C0],[[[0.03,0.97],[0.38,0.62]],[[0.23,0.77],[0.78,0.22]]])
98 pb = Prob(B1,[A0,A1],[[[0.5,0.5],[0.77,0.23]],[[0.4,0.6],[0.83,0.17]]])
99 pa = Prob(A1,[A0,B0],[[[0.1,0.9],[0.65,0.35]],[[0.3,0.7],[0.8,0.2]]])
```

```
100
101 # initial distribution
102 pa0 = Prob(A1,[],[0.9,0.1])
103 pb0 = Prob(B1,[A1],[[0.3,0.7],[0.8,0.2]])
104 pc0 = Prob(C1,[],[0.2,0.8])
105
106 dbn1 = DBN("Simple DBN",[A1,B1,C1],[pa,pb,pc],[pa0,pb0,pc0])
```

Here is the animal example

probDBN.py — (continued)

```
108 from probHMM import closeMic, farMic, midMic, sm, mmc, sc, mcm, mcc
109
110 Pos_0,Pos_1 = variable_pair("Position",domain=[0,1,2,3])
111 Mic1_0,Mic1_1 = variable_pair("Mic1")
112 Mic2_0,Mic2_1 = variable_pair("Mic2")
113 Mic3_0,Mic3_1 = variable_pair("Mic3")
114
115 # conditional probabilities - see hmm for the values of sm,mmc, etc
116 ppos = Prob(Pos_1, [Pos_0],
117            [[sm, mmc, mmc, mmc], #was in middle
118             [mcm, sc, mcc, mcc], #was in corner 1
119             [mcm, mcc, sc, mcc], #was in corner 2
120             [mcm, mcc, mcc, sc]]) #was in corner 3
121 pm1 = Prob(Mic1_1, [Pos_1], [[1-midMic, midMic], [1-closeMic, closeMic],
122                          [1-farMic, farMic], [1-farMic, farMic]])
123 pm2 = Prob(Mic2_1, [Pos_1], [[1-midMic, midMic], [1-farMic, farMic],
124                          [1-closeMic, closeMic], [1-farMic, farMic]])
125 pm3 = Prob(Mic3_1, [Pos_1], [[1-midMic, midMic], [1-farMic, farMic],
126                          [1-farMic, farMic], [1-closeMic, closeMic]])
127 ipos = Prob(Pos_1,[], [0.25, 0.25, 0.25, 0.25])
128 dbn_an =DBN("Animal DBN",[Pos_1,Mic1_1,Mic2_1,Mic3_1],
129            [ppos, pm1, pm2, pm3],
130            [ipos, pm1, pm2, pm3])
```

9.11.2 Unrolling DBNs

probDBN.py — (continued)

```
132 class BNfromDBN(BeliefNetwork):
133     """Belief Network unrolled from a dynamic belief network
134     """
135
136     def __init__(self,dbn,horizon):
137         """dbn is the dynamic belief network being unrolled
138         horizon>0 is the number of steps (so there will be horizon+1
                variables for each DBN variable.
139         """
140         self.name2var = {var.basename:
                [DBNvariable(var.basename,var.domain,index) for index in
                range(horizon+1)]
```

```
                        for var in dbn.vars_now}
        self.display(1,f"name2var={self.name2var}")
        variables = {v for vs in self.name2var.values() for v in vs}
        self.display(1,f"variables={variables}")
        bnfactors = {CPDrename(fac,{self.name2var[var.basename][0]:var
                                    for var in fac.variables})
                     for fac in dbn.init_factors}
        bnfactors |= {CPDrename(fac,{self.name2var[var.basename][i]:var
                                    for var in fac.variables if
                                        var.index=='prev'}
                                  | {self.name2var[var.basename][i+1]:var
                                    for var in fac.variables if
                                        var.index=='now'})
                     for fac in dbn.transition_factors
                        for i in range(horizon)}
        self.display(1,f"bnfactors={bnfactors}")
        BeliefNetwork.__init__(self, dbn.title, variables, bnfactors)
```

Here are two examples. Note that we need to use `bn.name2var['B'][2]` to get the variable B2 (B at time 2).

probDBN.py — (continued)

```
# Try
#from probRC import ProbRC
#bn = BNfromDBN(dbn1,2) # construct belief network
#drc = ProbRC(bn)            # initialize recursive conditioning
#B2 = bn.name2var['B'][2]
#drc.query(B2) #P(B2)
#drc.query(bn.name2var['B'][1],{bn.name2var['B'][0]:True,bn.name2var['C'][1]:False})
    #P(B1|B0,C1)
```

9.11.3 DBN Filtering

If we only wanted to ask questions about the current state, we can save space by forgetting the history variables.

probDBN.py — (continued)

```
class DBNVEfilter(VE):
    def __init__(self,dbn):
        self.dbn = dbn
        self.current_factors = dbn.init_factors
        self.current_obs = {}

    def observe(self, obs):
        """updates the current observations with obs.
        obs is a variable:value dictionary where variable is a current
        variable.
        """
        assert all(self.current_obs[var]==obs[var] for var in obs
                   if var in self.current_obs),"inconsistent current
                       observations"
```

```
177         self.current_obs.update(obs) # note 'update' is a dict method
178
179     def query(self,var):
180         """returns the posterior probability of current variable var"""
181         return
                VE(GraphicalModel(self.dbn.title,self.dbn.vars_now,self.current_factors)).query(var,self.c
182
183     def advance(self):
184         """advance to the next time"""
185         prev_factors = [self.make_previous(fac) for fac in
                self.current_factors]
186         prev_obs = {var.previous:val for var,val in
                self.current_obs.items()}
187         two_stage_factors = prev_factors + self.dbn.transition_factors
188         self.current_factors =
                self.elim_vars(two_stage_factors,self.dbn.vars_prev,prev_obs)
189         self.current_obs = {}
190
191     def make_previous(self,fac):
192          """Creates new factor from fac where the current variables in fac
193          are renamed to previous variables.
194          """
195          return FactorRename(fac, {var.previous:var for var in
                 fac.variables})
196
197     def elim_vars(self,factors, vars, obs):
198         for var in vars:
199             if var in obs:
200                 factors = [self.project_observations(fac,obs) for fac in
                        factors]
201             else:
202                 factors = self.eliminate_var(factors, var)
203         return factors
```

Example queries:

probDBN.py — (continued)

```
205 #df = DBNVEfilter(dbn1)
206 #df.observe({B1:True}); df.advance(); df.observe({C1:False})
207 #df.query(B1) #P(B1|B0,C1)
208 #df.advance(); df.query(B1)
209 #dfa = DBNVEfilter(dbn_an)
210 # dfa.observe({Mic1_1:0, Mic2_1:1, Mic3_1:1})
211 # dfa.advance()
212 # dfa.observe({Mic1_1:1, Mic2_1:0, Mic3_1:1})
213 # dfa.query(Pos_1)
```

Chapter 10

Learning with Uncertainty

10.1 Bayesian Learning

The section contains two implementations of the (discretized) beta distribution. The first represents Bayesian learning as a belief network. The second is an interactive tool to understand the beta distribution.

The following uses a belief network representation from the previous chapter to learn (discretized) probabilities. Figure 10.1 shows the output after observing *heads, heads, tails*. Notice the prediction of future tosses.

learnBayesian.py — Bayesian Learning

```
from variable import Variable
from probFactors import Prob
from probGraphicalModels import BeliefNetwork
from probRC import ProbRC

#### Coin Toss ###
# multiple coin tosses:
toss = ['tails','heads']
tosses = [ Variable(f"Toss#{i}", toss,
                    (0.8, 0.9-i/10) if i<10 else (0.4,0.2))
               for i in range(11)]

def coinTossBN(num_bins = 10):
    prob_bins = [x/num_bins for x in range(num_bins+1)]
    PH = Variable("P_heads", prob_bins, (0.1,0.9))
    p_PH = Prob(PH,[],{x:0.5/num_bins if x in [0,1] else 1/num_bins for x
        in prob_bins})
    p_tosses = [ Prob(tosses[i],[PH], {x:{'tails':1-x,'heads':x} for x in
        prob_bins})
             for i in range(11)]
```

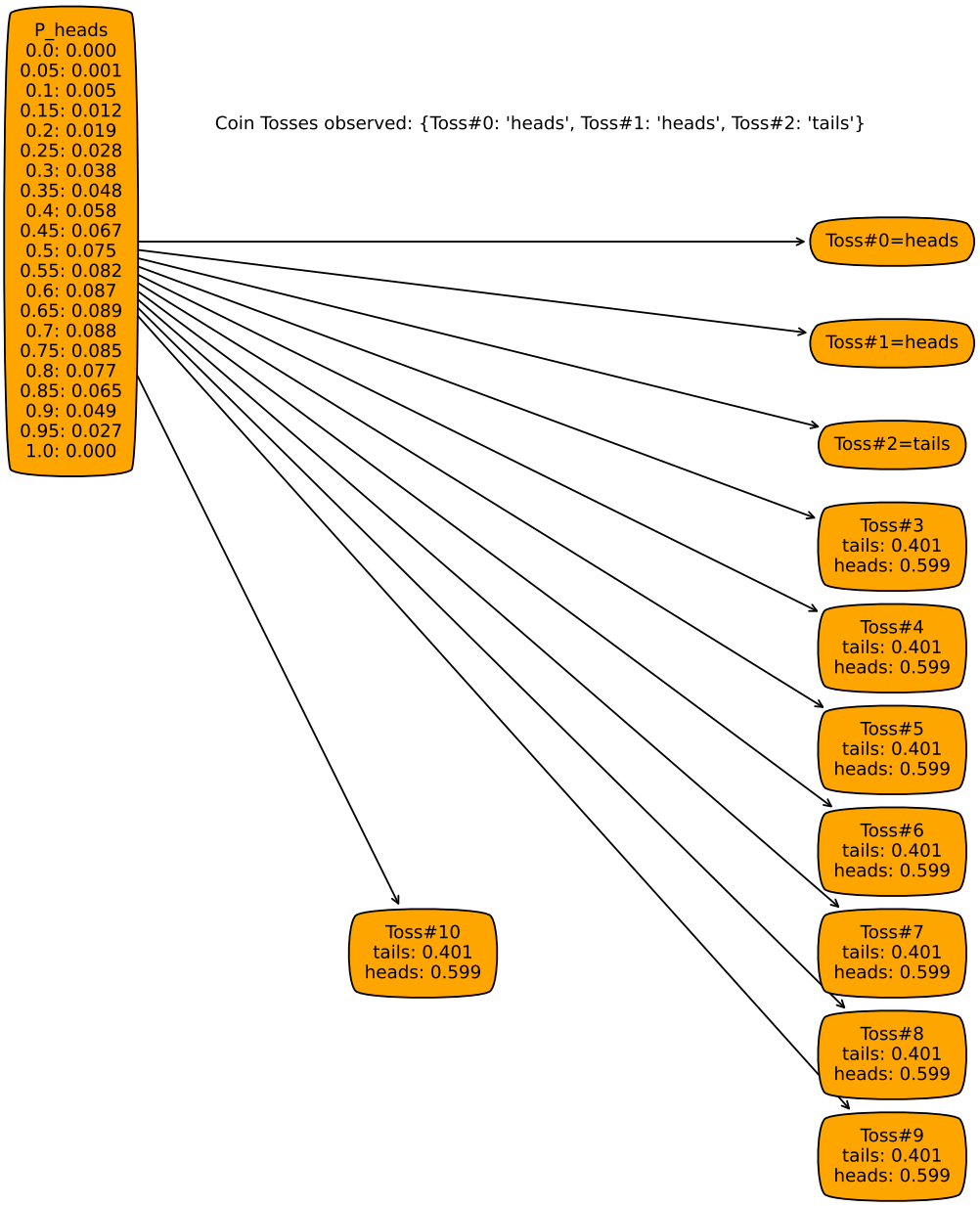


Figure 10.1: coinTossBN after observing heads, heads, tails

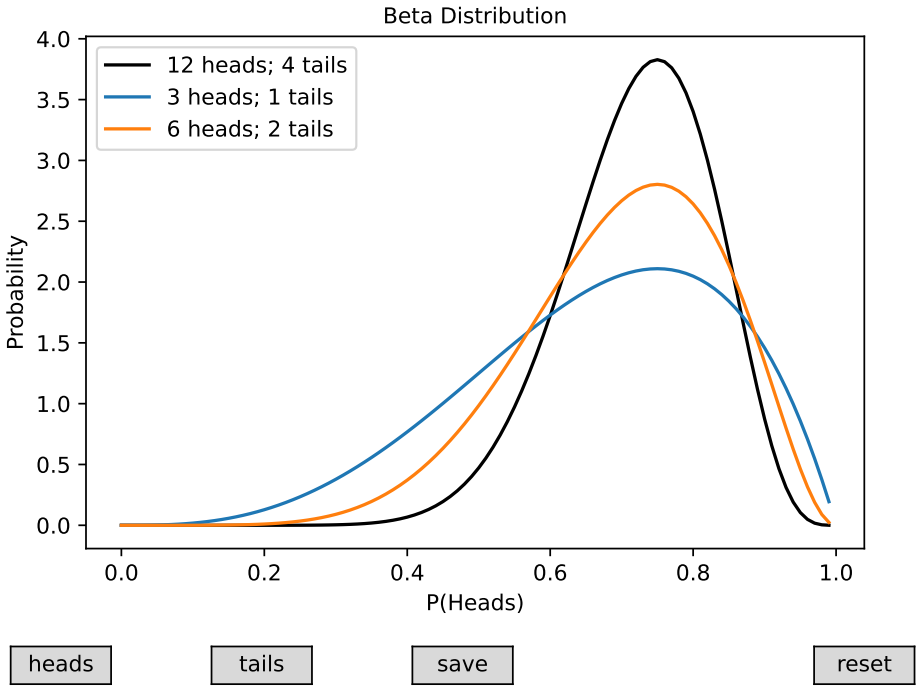


Figure 10.2: Beta distribution after some observations

```
    return BelifNetwork("Coin Tosses",
                        [PH]+tosses,
                        [p_PH]+p_tosses)


#
# coinRC = ProbRC(coinTossBN(20))
# coinRC.query(tosses[10],{tosses[0]:'heads'})
# coinRC.show_post({})
# coinRC.show_post({tosses[0]:'heads'})
# coinRC.show_post({tosses[0]:'heads',tosses[1]:'heads'})
# coinRC.show_post({tosses[0]:'heads',tosses[1]:'heads',tosses[2]:'tails'})
```

Figure 10.2 shows a plot of the Beta distribution (the *P_head* variable in the previous belief network) given some sets of observations.

This is a plot that is produced by the following interactive tool.

learnBayesian.py — (continued)

```
from display import Displayable
import matplotlib.pyplot as plt
from matplotlib.widgets import Button, CheckButtons

class Show_Beta(Displayable):
```

```
    def __init__(self,num=100, fontsize=10):
        self.num = num
        self.dist = [1 for i in range(num)]
        self.vals = [i/num for i in range(num)]
        self.fontsize = fontsize
        self.saves = []
        self.num_heads = 0
        self.num_tails = 0
        plt.ioff()
        fig,(self.ax) = plt.subplots()
        plt.subplots_adjust(bottom=0.2)
        ## Set up buttons:
        heads_butt = Button(plt.axes([0.05,0.02,0.1,0.05]), "heads")
        heads_butt.label.set_fontsize(self.fontsize)
        heads_butt.on_clicked(self.heads)
        tails_butt = Button(plt.axes([0.25,0.02,0.1,0.05]), "tails")
        tails_butt.label.set_fontsize(self.fontsize)
        tails_butt.on_clicked(self.tails)
        save_butt = Button(plt.axes([0.45,0.02,0.1,0.05]), "save")
        save_butt.label.set_fontsize(self.fontsize)
        save_butt.on_clicked(self.save)
        reset_butt = Button(plt.axes([0.85,0.02,0.1,0.05]), "reset")
        reset_butt.label.set_fontsize(self.fontsize)
        reset_butt.on_clicked(self.reset)
        ## draw the distribution
        plt.subplot(1, 1, 1)
        self.draw_dist()
        plt.show()

    def draw_dist(self):
        sv = self.num/sum(self.dist)
        self.dist = [v*sv for v in self.dist]
        #print(self.dist)
        self.ax.clear()
        plt.ylabel("Probability", fontsize=self.fontsize)
        plt.xlabel("P(Heads)", fontsize=self.fontsize)
        plt.title("Beta Distribution", fontsize=self.fontsize)
        plt.xticks(fontsize=self.fontsize)
        plt.yticks(fontsize=self.fontsize)
        self.ax.plot(self.vals, self.dist, color='black', label =
            f"{self.num_heads} heads; {self.num_tails} tails")
        for (nh,nt,d) in self.saves:
            self.ax.plot(self.vals, d, label = f"{nh} heads; {nt} tails")
        self.ax.legend()
        plt.draw()

    def heads(self,event):
        self.num_heads += 1
        self.dist = [self.dist[i]*self.vals[i] for i in range(self.num)]
        self.draw_dist()
```

```
    def tails(self,event):
        self.num_tails += 1
        self.dist = [self.dist[i]*(1-self.vals[i]) for i in range(self.num)]
        self.draw_dist()
    def save(self,event):
        self.saves.append((self.num_heads,self.num_tails,self.dist))
        self.draw_dist()
    def reset(self,event):
        self.num_tails = 0
        self.num_heads = 0
        self.dist = [1/self.num for i in range(self.num)]
        self.draw_dist()

# s1 = Show_Beta(100)
# sl = Show_Beta(100, fontsize=15) # for demos - enlarge window
```

10.2 K-means

The k-means learner maintains two lists that suffice as sufficient statistics to classify examples, and to learn the classification:

- *class_counts* is a list such that $class\_counts[c]$ is the number of examples in the training set with $class = c$.
- *feature_sum* is a list such that $feature\_sum[i][c]$ is sum of the values for the i'th feature i for members of class c. The average value of the ith feature in class i is

$$\frac{feature\_sum[i][c]}{class\_counts[c]}$$

The class is initialized by randomly assigning examples to classes, and updating the statistics for *class_counts* and *feature_sum*.

learnKMeans.py — k-means learning

```
from learnProblem import Data_set, Learner, Data_from_file
import random
import matplotlib.pyplot as plt

class K_means_learner(Learner):
    def __init__(self,dataset, num_classes):
        self.dataset = dataset
        self.num_classes = num_classes
        self.random_initialize()

    def random_initialize(self):
        # class_counts[c] is the number of examples with class=c
        self.class_counts = [0]*self.num_classes
```

```
24        # feature_sum[i][c] is the sum of the values of feature i for class
              c
25        self.feature_sum = [[0]*self.num_classes
26                            for feat in self.dataset.input_features]
27        for eg in self.dataset.train:
28            cl = random.randrange(self.num_classes) # assign eg to random
                  class
29            self.class_counts[cl] += 1
30            for (ind,feat) in enumerate(self.dataset.input_features):
31                self.feature_sum[ind][cl] += feat(eg)
32        self.num_iterations = 0
33        self.display(1,"Initial class counts: ",self.class_counts)
```

The distance from (the mean of) a class to an example is the sum, over all features, of the sum-of-squares differences of the class mean and the example value.

learnKMeans.py — (continued)

```
35    def distance(self,cl,eg):
36        """distance of the eg from the mean of the class"""
37        return sum( (self.class_prediction(ind,cl)-feat(eg))**2
38                        for (ind,feat) in
39                            enumerate(self.dataset.input_features))
40
41    def class_prediction(self,feat_ind,cl):
42        """prediction of the class cl on the feature with index feat_ind"""
43        if self.class_counts[cl] == 0:
44            return 0 # there are no examples so we can choose any value
45        else:
46            return self.feature_sum[feat_ind][cl]/self.class_counts[cl]
47
48    def class_of_eg(self,eg):
49        """class to which eg is assigned"""
50        return (min((self.distance(cl,eg),cl)
51                        for cl in range(self.num_classes)))[1]
52              # second element of tuple, which is a class with minimum
53                  distance
```

One step of k-means updates the *class_counts* and *feature_sum*. It uses the old values to determine the classes, and so the new values for *class_counts* and *feature_sum*. At the end it determines whether the values of these have changes, and then replaces the old ones with the new ones. It returns an indicator of whether the values are stable (have not changed).

learnKMeans.py — (continued)

```
53    def k_means_step(self):
54        """Updates the model with one step of k-means.
55        Returns whether the assignment is stable.
56        """
57        new_class_counts = [0]*self.num_classes
```

```
        # feature_sum[i][c] is the sum of the values of feature i for class c
        new_feature_sum = [[0]*self.num_classes
                           for feat in self.dataset.input_features]
        for eg in self.dataset.train:
            cl = self.class_of_eg(eg)
            new_class_counts[cl] += 1
            for (ind,feat) in enumerate(self.dataset.input_features):
                new_feature_sum[ind][cl] += feat(eg)
        stable = (new_class_counts == self.class_counts) and (self.feature_sum == new_feature_sum)
        self.class_counts = new_class_counts
        self.feature_sum = new_feature_sum
        self.num_iterations += 1
        return stable


    def learn(self,n=100):
        """do n steps of k-means, or until convergence"""
        i=0
        stable = False
        while i<n and not stable:
            stable = self.k_means_step()
            i += 1
            self.display(1,"Iteration",self.num_iterations,
                           "class counts: ",self.class_counts,"
                               Stable=",stable)
        return stable

    def show_classes(self):
        """sorts the data by the class and prints in order.
        For visualizing small data sets
        """
        class_examples = [[] for i in range(self.num_classes)]
        for eg in self.dataset.train:
            class_examples[self.class_of_eg(eg)].append(eg)
        print("Class","Example",sep='\t')
        for cl in range(self.num_classes):
            for eg in class_examples[cl]:
                print(cl,*eg,sep='\t')

    def plot_error(self, maxstep=20):
        """Plots the sum-of-squares error as a function of the number of steps"""
        plt.ion()
        plt.xlabel("step")
        plt.ylabel("Ave sum-of-squares error")
        train_errors = []
        if self.dataset.test:
            test_errors = []
```

```
104         for i in range(maxstep):
105             self.learn(1)
106             train_errors.append( sum(self.distance(self.class_of_eg(eg),eg)
107                                      for eg in self.dataset.train)
108                                  /len(self.dataset.train))
109             if self.dataset.test:
110                 test_errors.append(
                        sum(self.distance(self.class_of_eg(eg),eg)
111                                       for eg in self.dataset.test)
112                                   /len(self.dataset.test))
113         plt.plot(range(1,maxstep+1),train_errors,
114                  label=str(self.num_classes)+" classes. Training set")
115         if self.dataset.test:
116             plt.plot(range(1,maxstep+1),test_errors,
117                      label=str(self.num_classes)+" classes. Test set")
118         plt.legend()
119         plt.draw()
120
121 %data = Data_from_file('data/emdata1.csv', num_train=10,
        target_index=2000) % trivial example
122 data = Data_from_file('data/emdata2.csv', num_train=10, target_index=2000)
123 %data = Data_from_file('data/emdata0.csv', num_train=14,
        target_index=2000) % example from textbook
124 kml = K_means_learner(data,2)
125 num_iter=4
126 print("Class assignment after",num_iter,"iterations:")
127 kml.learn(num_iter); kml.show_classes()
128
129 # Plot the error
130 # km2=K_means_learner(data,2); km2.plot_error(20) # 2 classes
131 # km3=K_means_learner(data,3); km3.plot_error(20) # 3 classes
132 # km13=K_means_learner(data,13); km13.plot_error(20) # 13 classes
133
134 # data = Data_from_file('data/carbool.csv',
        target_index=2000,boolean_features=True)
135 # kml = K_means_learner(data,3)
136 # kml.learn(20); kml.show_classes()
137 # km3=K_means_learner(data,3); km3.plot_error(20) # 3 classes
138 # km3=K_means_learner(data,30); km3.plot_error(20) # 30 classes
```

Exercise 10.1 Change *boolean_features = True* flag to allow for numerical features. K-means assumes the features are numerical, so we want to make non-numerical features into numerical features (using characteristic functions) but we probably don't want to change numerical features into Boolean.

Exercise 10.2 If there are many classes, some of the classes can become empty (e.g., try 100 classes with carbool.csv). Implement a way to put some examples into a class, if possible. Two ideas are:

(a) Initialize the classes with actual examples, so that the classes will not start empty. (Do the classes become empty?)

(b) In *class_prediction*, we test whether the code is empty, and make a prediction of 0 for an empty class. It is possible to make a different prediction to "steal" an example (but you should make sure that a class has a consistent value for each feature in a loop).

Make your own suggestions, and compare it with the original, and whichever of these you think may work better.

10.3 EM

In the following definition, a class, c, is a integer in range $[0, num\_classes)$. i is an index of a feature, so $feat[i]$ is the ith feature, and a feature is a function from tuples to values. val is a value of a feature.

A model consists of 2 lists, which form the sufficient statistics:

- $class\_counts$ is a list such that $class\_counts[c]$ is the number of tuples with $class = c$, where each tuple is weighted by its probability, i.e.,

$$class\_counts[c] = \sum_{t:class(t)=c} P(t)$$

- $feature\_counts$ is a list such that $feature\_counts[i][val][c]$ is the weighted count of the number of tuples t with $feat[i](t) = val$ and $class(t) = c$, each tuple is weighted by its probability, i.e.,

$$feature\_counts[i][val][c] = \sum_{t:feat[i](t)=val \text{ and } class(t)=c} P(t)$$

learnEM.py — EM Learning

```
from learnProblem import Data_set, Learner, Data_from_file
import random
import math
import matplotlib.pyplot as plt

class EM_learner(Learner):
    def __init__(self,dataset, num_classes):
        self.dataset = dataset
        self.num_classes = num_classes
        self.class_counts = None
        self.feature_counts = None
```

The function *em_step* goes though the training examples, and updates these counts. The first time it is run, when there is no model, it uses random distributions.

learnEM.py — (continued)

```
    def em_step(self, orig_class_counts, orig_feature_counts):
```

```
24        """updates the model."""
25        class_counts = [0]*self.num_classes
26        feature_counts = [{val:[0]*self.num_classes
27                           for val in feat.frange}
28                           for feat in self.dataset.input_features]
29        for tple in self.dataset.train:
30            if orig_class_counts: # a model exists
31                tpl_class_dist = self.prob(tple, orig_class_counts,
                      orig_feature_counts)
32            else:               # initially, with no model, return a random
                  distribution
33                tpl_class_dist = random_dist(self.num_classes)
34            for cl in range(self.num_classes):
35                class_counts[cl] += tpl_class_dist[cl]
36                for (ind,feat) in enumerate(self.dataset.input_features):
37                    feature_counts[ind][feat(tple)][cl] += tpl_class_dist[cl]
38        return class_counts, feature_counts
```

prob computes the probability of a class c for a tuple *tpl*, given the current statistics.

$$\begin{aligned} P(c \mid tple) &\propto P(c) * \prod_i P(X_i{=}tple(i) \mid c) \\ &= \frac{class\_counts[c]}{len(self.dataset)} * \prod_i \frac{feature\_counts[i][feat_i(tple)][c]}{class\_counts[c]} \\ &\propto \frac{\prod_i feature\_counts[i][feat_i(tple)][c]}{class\_counts[c]^{|feats|-1}} \end{aligned}$$

The last step is because $len(self.dataset)$ is a constant (independent of c). $class\_counts[c]$ can be taken out of the product, but needs to be raised to the power of the number of features, and one of them cancels.

learnEM.py — (continued)

```
40    def prob(self, tple, class_counts, feature_counts):
41        """returns a distribution over the classes for tuple tple in the
              model defined by the counts
42        """
43        feats = self.dataset.input_features
44        unnorm = [prod(feature_counts[i][feat(tple)][c]
45                       for (i,feat) in enumerate(feats))
46                    /(class_counts[c]**(len(feats)-1))
47                  for c in range(self.num_classes)]
48        thesum = sum(unnorm)
49        return [un/thesum for un in unnorm]
```

learn does n steps of EM:

learnEM.py — (continued)

```
51    def learn(self,n):
52        """do n steps of em"""
```

```
53          for i in range(n):
54              self.class_counts,self.feature_counts =
                    self.em_step(self.class_counts,
55                                                      self.feature_counts)
```

The following is for visualizing the classes. It prints the dataset ordered by the probability of class c.

learnEM.py — (continued)

```
57      def show_class(self,c):
58          """sorts the data by the class and prints in order.
59          For visualizing small data sets
60          """
61          sorted_data =
                sorted((self.prob(tpl,self.class_counts,self.feature_counts)[c],
62                              ind,  # preserve ordering for equal
                                    probabilities
63                              tpl)
64                             for (ind,tpl) in enumerate(self.dataset.train))
65          for cc,r,tpl in sorted_data:
66              print(cc,*tpl,sep='\t')
```

The following are for evaluating the classes.

The probability of a tuple can be evaluated by marginalizing over the classes:

$$\begin{aligned} P(tple) &= \sum_c P(c) * \prod_i P(X_i{=}tple(i) \mid c) \\ &= \sum_c \frac{cc[c]}{len(self.dataset)} * \prod_i \frac{fc[i][feat_i(tple)][c]}{cc[c]} \end{aligned}$$

where cc is the class count and fc is feature count. $len(self.dataset)$ can be distributed out of the sum, and $cc[c]$ can be taken out of the product:

$$= \frac{1}{len(self.dataset)} \sum_c \frac{1}{cc[c]^{\#feats-1}} * \prod_i fc[i][feat_i(tple)][c]$$

Given the probability of each tuple, we can evaluate the logloss, as the negative of the log probability:

learnEM.py — (continued)

```
68      def logloss(self,tple):
69          """returns the logloss of the prediction on tple, which is
                -log(P(tple))
70          based on the current class counts and feature counts
71          """
72          feats = self.dataset.input_features
73          res = 0
74          cc = self.class_counts
75          fc = self.feature_counts
76          for c in range(self.num_classes):
77              res += prod(fc[i][feat(tple)][c]
```

```
                    for (i,feat) in
                        enumerate(feats))/(cc[c]**(len(feats)-1))
        if res>0:
            return -math.log2(res/len(self.dataset.train))
        else:
            return float("inf") #infinity

    def plot_error(self, maxstep=20):
        """Plots the logloss error as a function of the number of steps"""
        plt.ion()
        plt.xlabel("step")
        plt.ylabel("Ave Logloss (bits)")
        train_errors = []
        if self.dataset.test:
            test_errors = []
        for i in range(maxstep):
            self.learn(1)
            train_errors.append( sum(self.logloss(tple) for tple in
                self.dataset.train)
                            /len(self.dataset.train))
            if self.dataset.test:
                test_errors.append( sum(self.logloss(tple) for tple in
                    self.dataset.test)
                                /len(self.dataset.test))
        plt.plot(range(1,maxstep+1),train_errors,
                 label=str(self.num_classes)+" classes. Training set")
        if self.dataset.test:
            plt.plot(range(1,maxstep+1),test_errors,
                     label=str(self.num_classes)+" classes. Test set")
        plt.legend()
        plt.draw()

def prod(L):
    """returns the product of the elements of L"""
    res = 1
    for e in L:
        res *= e
    return res

def random_dist(k):
    """generate k random numbers that sum to 1"""
    res = [random.random() for i in range(k)]
    s = sum(res)
    return [v/s for v in res]

data = Data_from_file('data/emdata2.csv', num_train=10, target_index=2000)
eml = EM_learner(data,2)
num_iter=2
print("Class assignment after",num_iter,"iterations:")
eml.learn(num_iter); eml.show_class(0)
```

```
125
126 # Plot the error
127 # em2=EM_learner(data,2); em2.plot_error(40) # 2 classes
128 # em3=EM_learner(data,3); em3.plot_error(40) # 3 classes
129 # em13=EM_learner(data,13); em13.plot_error(40) # 13 classes
130
131 # data = Data_from_file('data/carbool.csv',
        target_index=2000,boolean_features=False)
132 # [f.frange for f in data.input_features]
133 # eml = EM_learner(data,3)
134 # eml.learn(20); eml.show_class(0)
135 # em3=EM_learner(data,3); em3.plot_error(60) # 3 classes
136 # em3=EM_learner(data,30); em3.plot_error(60) # 30 classes
```

Exercise 10.3 For the EM data, where there are naturally 2 classes, 3 classes does better on the training set after a while than 2 classes, but worse on the test set. Explain why. Hint: look what the 3 classes are. Use "em3.show_class(i)" for each of the classes $i \in [0,3)$.

Exercise 10.4 Write code to plot the logloss as a function of the number of classes (from 1 to say 15) for a fixed number of iterations. (From the experience with the existing code, think about how many iterations is appropriate.)

Chapter 11

Causality

11.1 Do Questions

A causal model can answer "do" questions.

The intervene function takes a belief network and a variable:value dictionary specifying what to "do", and returns a belief network resulting from intervening to set each variable in the dictionary to its value specified. It replaces the CPD of each intervened variable with an constant CPD.

probDo.py — Probabilistic inference with the do operator

```
11 from probGraphicalModels import InferenceMethod, BeliefNetwork
12 from probFactors import CPD, ConstantCPD
13
14 def intervene(bn, do={}):
15     assert isinstance(bn, BeliefNetwork), f"Do only applies to belief
           networks ({bn.title})"
16     if do=={}:
17         return bn
18     else:
19         newfacs = ({f for (ch,f) in bn.var2cpt.items() if ch not in do} |
20                    {ConstantCPD(v,c) for (v,c) in do.items()})
21         return BeliefNetwork(f"{bn.title}(do={do})", bn.variables, newfacs)
```

The following adds the queryDo method to the InferenceMethod class, so it can be used with any inference method. It replaces the graphical model with the modified one, runs the inference algorithm, and restores the initial belief network.

probDo.py — (continued)

```
23 def queryDo(self, qvar, obs={}, do={}):
24     """Extends query method to also allow for interventions.
25     """
```

```
26      oldBN, self.gm = self.gm, intervene(self.gm, do)
27      result = self.query(qvar, obs)
28      self.gm = oldBN # restore original
29      return result
30
31  # make queryDo available for all inference methods
32  InferenceMethod.queryDo = queryDo
```

Test cases:

probDo.py — (continued)

```
34  from probRC import ProbRC
35
36  from probExamples import bn_sprinkler, Season, Sprinkler, Rained,
        Grass_wet, Grass_shiny, Shoes_wet
37  bn_sprinklerv = ProbRC(bn_sprinkler)
38  ## bn_sprinklerv.queryDo(Shoes_wet)
39  ## bn_sprinklerv.queryDo(Shoes_wet,obs={Sprinkler:"on"})
40  ## bn_sprinklerv.queryDo(Shoes_wet,do={Sprinkler:"on"})
41  ## bn_sprinklerv.queryDo(Season, obs={Sprinkler:"on"})
42  ## bn_sprinklerv.queryDo(Season, do={Sprinkler:"on"})
43
44  ### Showing posterior distributions:
45  # bn_sprinklerv.show_post({})
46  # bn_sprinklerv.show_post({Sprinkler:"on"})
47  # spon = intervene(bn_sprinkler, do={Sprinkler:"on"})
48  # ProbRC(spon).show_post({})
```

The following is a representation of a possible model where marijuana is a gateway drug to harder drugs (or not). Try the queries at the end.

probDo.py — (continued)

```
50  from variable import Variable
51  from probFactors import Prob
52  from probGraphicalModels import BeliefNetwork
53  boolean = [False, True]
54
55  Drug_Prone = Variable("Drug_Prone", boolean, position=(0.1,0.5)) #
        (0.5,0.9))
56  Side_Effects = Variable("Side_Effects", boolean, position=(0.1,0.5)) #
        (0.5,0.1))
57  Takes_Marijuana = Variable("\nTakes_Marijuana\n", boolean,
        position=(0.1,0.5))
58  Takes_Hard_Drugs = Variable("Takes_Hard_Drugs", boolean,
        position=(0.9,0.5))
59
60  p_dp = Prob(Drug_Prone, [], [0.8, 0.2])
61  p_be = Prob(Side_Effects, [Takes_Marijuana], [[1, 0], [0.4, 0.6]])
62  p_tm = Prob(Takes_Marijuana, [Drug_Prone], [[0.98, 0.02], [0.2, 0.8]])
63  p_thd = Prob(Takes_Hard_Drugs, [Side_Effects, Drug_Prone],
64                 # Drug_Prone=False  Drug_Prone=True
65                 [[[0.999, 0.001],   [0.6, 0.4]], # Side_Effects=False
```

```
                [[0.99999, 0.00001], [0.995, 0.005]]]) # Side_Effects=True

drugs = BeliefNetwork("Gateway Drug?",
                  [Drug_Prone,Side_Effects, Takes_Marijuana,
                      Takes_Hard_Drugs],
                  [p_tm, p_dp, p_be, p_thd])

drugsq = ProbRC(drugs)
# drugsq.queryDo(Takes_Hard_Drugs)
# drugsq.queryDo(Takes_Hard_Drugs, obs = {Takes_Marijuana: True})
# drugsq.queryDo(Takes_Hard_Drugs, obs = {Takes_Marijuana: False})
# drugsq.queryDo(Takes_Hard_Drugs, do = {Takes_Marijuana: True})
# drugsq.queryDo(Takes_Hard_Drugs, do = {Takes_Marijuana: False})


# ProbRC(drugs).show_post({})
# ProbRC(drugs).show_post({Takes_Marijuana: True})
# ProbRC(drugs).show_post({Takes_Marijuana: False})
# ProbRC(intervene(drugs, do={Takes_Marijuana: True})).show_post({})
# ProbRC(intervene(drugs, do={Takes_Marijuana: False})).show_post({})
# Why was that? Try the following then repeat:
# Drug_Prone.position=(0.5,0.9); Side_Effects.position=(0.5,0.1)
```

11.2 Counterfactual Example

Consider chain $A \rightarrow B \rightarrow C$ where you want "what if A was True" or "what if A was False". For example, suppose A=*true*, C=*true* is observed and you want the probability of C if A were false. See Figure 11.1.

probCounterfactual.py — Counterfactual Query Example

```
from variable import Variable
from probFactors import Prob, ProbDT, IFeq, Dist
from probGraphicalModels import BeliefNetwork
from probRC import ProbRC
from probDo import queryDo

boolean = [False, True]
```

The following is a simple chain $Ap \rightarrow Bp \rightarrow Cp$. According to the probabilities, Bp is independent of Ap. This does not usually cause a problem; often parents are included because they might be relevant.

probCounterfactual.py — (continued)

```
# without a deterministic system
Ap = Variable("Ap", boolean, position=(0.2,0.8))
Bp = Variable("Bp", boolean, position=(0.2,0.4))
Cp = Variable("Cp", boolean, position=(0.2,0.0))
p_Ap = Prob(Ap, [], [0.5,0.5])
p_Bp = Prob(Bp, [Ap], [[0.6,0.4], [0.6,0.4]]) # does not depend on A!
```

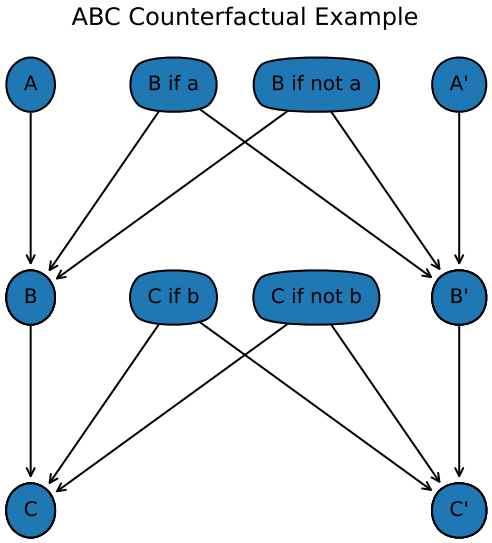


Figure 11.1: $A \rightarrow B \rightarrow C$ belief network for "what if A"

```
p_Cp = Prob(Cp, [Bp], [[0.2,0.8], [0.9,0.1]])
abcSimple = BeliefNetwork("ABC Simple",
                      [Ap,Bp,Cp],
                      [p_Ap, p_Bp, p_Cp])
ABCsimpq = ProbRC(abcSimple)
# ABCsimpq.show_post(obs = {Ap:True, Cp:True})
```

Here is the same network as a a deterministic system with noise variables. The Primed variables correspond to "what if A were True" or "what if A were False". In this scenario, `Aprime` should be conditioned on. Conditioning on `Aprime` should not affect the non-primed variables. (You should check this).

probCounterfactual.py — (continued)

```
# as a deterministic system with independent noise
A = Variable("A", boolean, position=(0.2,0.8))
B = Variable("B", boolean, position=(0.2,0.4))
C = Variable("C", boolean, position=(0.2,0.0))
Aprime = Variable("A'", boolean, position=(0.8,0.8))
Bprime = Variable("B'", boolean, position=(0.8,0.4))
Cprime = Variable("C'", boolean, position=(0.8,0.0))
BifA = Variable("B if a", boolean, position=(0.4,0.8))
BifnA = Variable("B if not a", boolean, position=(0.6,0.8))
CifB = Variable("C if b", boolean, position=(0.4,0.4))
CifnB = Variable("C if not b", boolean, position=(0.6,0.4))
```

The following uses a tabular representation of the if-then-else structure that arises with a single Boolean parent when converted to a deterministic system with noise. The deterministic probability $P(p \mid c, n_1, n_0)$ where n_i is the noise variable that is used when $c = i$ is given by $if1then2else3[c][n_1][n_0][p]$. For example $P(p{=}1 \mid c{=}0, n_1{=}0, n_0{=}1)$ is $if1then2else3[0][0][1][1]$ has value 1 because p takes the value of n_0 when $c{=}0$.

probCounterfactual.py — (continued)

```
# if1then2else3 is a probability table
# if1then2else3[x][y][z] is the deterministic probability that
#  is the value of y if x is 1 otherwise it is the value of z
if1then2else3 = [[[[1,0],[0,1]],[[1,0],[0,1]]],
                 [[[1,0],[1,0]],[[0,1],[0,1]]]]


p_A = Prob(A, [], [0.5,0.5])
p_B = Prob(B, [A, BifA, BifnA], if1then2else3)
p_C = Prob(C, [B, CifB, CifnB], if1then2else3)
p_Aprime = Prob(Aprime,[], [0.5,0.5])
p_Bprime = Prob(Bprime, [Aprime, BifA, BifnA], if1then2else3)
p_Cprime = Prob(Cprime, [Bprime, CifB, CifnB], if1then2else3)
p_bifa = Prob(BifA, [], [0.6,0.4])
p_bifna = Prob(BifnA, [], [0.6,0.4])
p_cifb = Prob(CifB, [], [0.9,0.1])
p_cifnb = Prob(CifnB, [], [0.2,0.8])

abcCounter = BeliefNetwork("ABC Counterfactual Example",
                   [A,B,C,Aprime,Bprime,Cprime,BifA, BifnA, CifB, CifnB],
                   [p_A,p_B,p_C,p_Aprime,p_Bprime, p_Cprime, p_bifa,
                       p_bifna, p_cifb, p_cifnb])
```

Here are some queries you might like to try. The show_post queries might be most useful if you have the space to show multiple queries.

probCounterfactual.py — (continued)

```
abcq = ProbRC(abcCounter)
# abcq.queryDo(Cprime, obs = {Aprime:False, A:True})
# abcq.queryDo(Cprime, obs = {C:True, Aprime:False})
# abcq.queryDo(Cprime, obs = {A:True, C:True, Aprime:False})
# abcq.queryDo(Cprime, obs = {A:True, C:True, Aprime:False})
# abcq.queryDo(Cprime, obs = {A:False, C:True, Aprime:False})
# abcq.queryDo(CifB, obs = {C:True,Aprime:False})
# abcq.queryDo(CifnB, obs = {C:True,Aprime:False})

# abcq.show_post(obs = {})
# abcq.show_post(obs = {Aprime:False, A:True})
# abcq.show_post(obs = {A:True, C:True, Aprime:False})
# abcq.show_post(obs = {A:True, C:True, Aprime:True})
```

Exercise 11.1 Is the above reasonable? What is surprising about this?

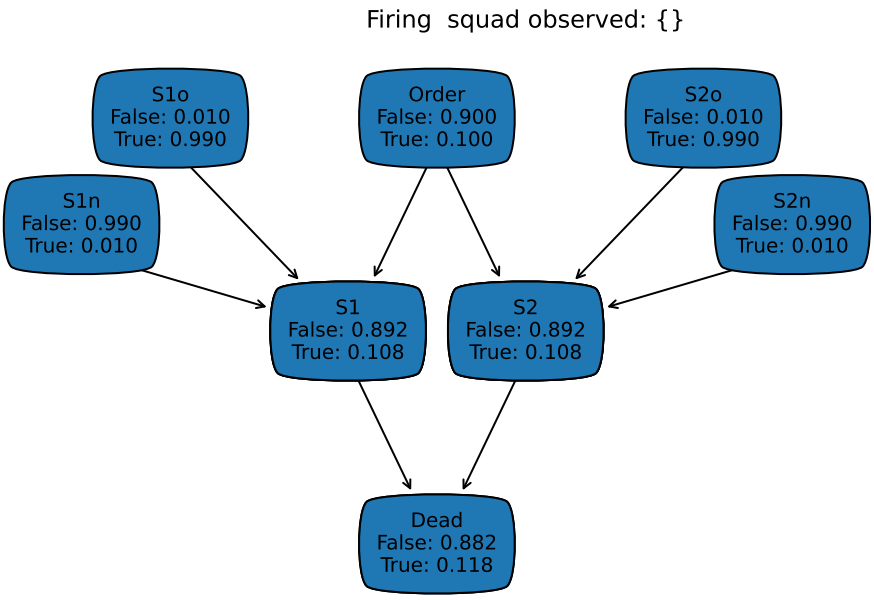


Figure 11.2: Firing squad belief network

What if B did depend on A, but not by very much (e.g. $P(B \mid A) = 0.41$). Is the answer then reasonable?

Are there guidelines as to when a reasonable counterfactual probability is to be expected?

11.2.1 Firing Squad Example

The following is the firing squad example of Pearl as a deterministic system. See Figure 11.2.

probCounterfactual.py — (continued)

```
80 Order = Variable("Order", boolean, position=(0.4,0.8))
81 S1 = Variable("S1", boolean, position=(0.3,0.4))
82 S1o = Variable("S1o", boolean, position=(0.1,0.8))
83 S1n = Variable("S1n", boolean, position=(0.0,0.6))
84 S2 = Variable("S2", boolean, position=(0.5,0.4))
85 S2o = Variable("S2o", boolean, position=(0.7,0.8))
86 S2n = Variable("S2n", boolean, position=(0.8,0.6))
87 Dead = Variable("Dead", boolean, position=(0.4,0.0))
```

Instead of the tabular representation of the if-then-else structure used for the $A \rightarrow B \rightarrow C$ network above, the following uses the decision tree representation of conditional probabilities of Section 9.3.4.

probCounterfactual.py — (continued)

```
def eqto(var):
    return IFeq(var,True,Dist([0,1]), Dist([1,0]))

p_S1 = ProbDT(S1, [Order, S1o, S1n],
                  IFeq(Order,True, eqto(S1o), eqto(S1n)))
p_S2 = ProbDT(S2, [Order, S2o, S2n],
                  IFeq(Order,True, eqto(S2o), eqto(S2n)))
p_dead = Prob(Dead, [S1,S2], [[[1,0],[0,1]],[[0,1],[0,1]]])
p_order = Prob(Order, [], [0.9, 0.1])
p_s1o = Prob(S1o, [], [0.01, 0.99])
p_s1n = Prob(S1n, [], [0.99, 0.01])
p_s2o = Prob(S2o, [], [0.01, 0.99])
p_s2n = Prob(S2n, [], [0.99, 0.01])

firing_squad = BeliefNetwork("Firing squad",
                         [Order, S1, S1o, S1n, S2, S2o, S2n, Dead],
                         [p_order, p_dead, p_S1, p_s1o, p_s1n, p_S2, p_s2o,
                             p_s2n])
fsq = ProbRC(firing_squad)
# fsq.queryDo(Dead)
# fsq.queryDo(Order, obs={Dead:True})
# fsq.queryDo(Dead, obs={Order:True})
# fsq.show_post({})
# fsq.show_post({Dead:True})
# fsq.show_post({Order:True})
```

Exercise 11.2 Create the network for "what if shooter 2 did or did not shoot". Give the probabilities of the following counterfactuals:

(a) The prisoner is dead; what is the probability that the prisoner would be dead if shooter 2 did not shoot?

(b) Shooter 2 shot; what is the probability that the prisoner would be dead if shooter 2 did not shoot?

(c) No order was given, but the prisoner is dead; what is the probability that the prisoner would be dead if shooter 2 did not shoot?

Exercise 11.3 Create the network for "what if the order was or was not given". Give the probabilities of the following counterfactuals:

(a) The prisoner is dead; what is the probability that the prisoner would be dead if the order was not given?

(b) The prisoner is not dead; what is the probability that the prisoner would be dead if the order was not given? (Is this different from the prior that the prisoner is dead, or the posterior that the prisoner was dead given the order was not given).

(c) Shooter 2 shot; what is the probability that the prisoner would be dead if the order was not given?

(d) Shooter 2 did not shoot; what is the probability that the prisoner would be dead if the order was given? (Is this different from the probability that the the prisoner would be dead if the order was given without the counterfactual observation)?

Chapter 12

Planning with Uncertainty

12.1 Decision Networks

The decision network code builds on the representation for belief networks of Chapter 9.

We first allow for factors that define the utility. Here the **utility** is a function of the variables in *vars*. In a **utility table** the utility is defined in terms of a tabular factor – a list that enumerates the values – as in Section 9.3.3.

decnNetworks.py — Representations for Decision Networks

```
from probGraphicalModels import GraphicalModel, BeliefNetwork
from probFactors import Factor, CPD, TabFactor, factor_times, Prob
from variable import Variable
import matplotlib.pyplot as plt

class Utility(Factor):
    """A factor defining a utility"""
    pass

class UtilityTable(TabFactor, Utility):
    """A factor defining a utility using a table"""
    def __init__(self, vars, table, position=None):
        """Creates a factor on vars from the table.
        The table is ordered according to vars.
        """
        TabFactor.__init__(self,vars,table, name="Utility")
        self.position = position
```

A **decision variable** is a like a random variable with a string name, and a domain, which is a list of possible values. The decision variable also includes the parents, a list of the variables whose value will be known when the decision is made. It also includes a potion, which is only used for plotting.

decnNetworks.py — (continued)

```
class DecisionVariable(Variable):
    def __init__(self, name, domain, parents, position=None):
        Variable.__init__(self, name, domain, position)
        self.parents = parents
        self.all_vars = set(parents) | {self}
```

A decision network is a graphical model where the variables can be random variables or decision variables. Among the factors we assume there is one utility factor.

decnNetworks.py — (continued)

```
class DecisionNetwork(BeliefNetwork):
    def __init__(self, title, vars, factors):
        """vars is a list of variables
        factors is a list of factors (instances of CPD and Utility)
        """
        GraphicalModel.__init__(self, title, vars, factors) # don't call
            init for BeliefNetwork
        self.var2parents = ({v : v.parents for v in vars if
            isinstance(v,DecisionVariable)}
                | {f.child:f.parents for f in factors if
                    isinstance(f,CPD)})
        self.children = {n:[] for n in self.variables}
        for v in self.var2parents:
            for par in self.var2parents[v]:
                self.children[par].append(v)
        self.utility_factor = [f for f in factors if
            isinstance(f,Utility)][0]
        self.topological_sort_saved = None

    def __str__(self):
        return self.title
```

The split order ensures that the parents of a decision node are split before the decision node, and no other variables (if that is possible).

decnNetworks.py — (continued)

```
    def split_order(self):
        so = []
        tops = self.topological_sort()
        for v in tops:
            if isinstance(v,DecisionVariable):
                so += [p for p in v.parents if p not in so]
                so.append(v)
        so += [v for v in tops if v not in so]
        return so
```

decnNetworks.py — (continued)

```
    def show(self, fontsize=10,
```

```
                colors={'utility':'red', 'decision':'lime',
                    'random':'orange'} ):
        plt.ion()  # interactive
        ax = plt.figure().gca()
        ax.set_axis_off()
        plt.title(self.title, fontsize=fontsize)
        for par in self.utility_factor.variables:
            ax.annotate("Utility", par.position,
                xytext=self.utility_factor.position,
                                arrowprops={'arrowstyle':'<-'},
                                bbox=dict(boxstyle="sawtooth,pad=1.0",color=colors['utility']),
                                ha='center', va='center',
                                    fontsize=fontsize)
        for var in reversed(self.topological_sort()):
            if isinstance(var,DecisionVariable):
                bbox =
                    dict(boxstyle="square,pad=1.0",color=colors['decision'])
            else:
               bbox =
                   dict(boxstyle="round4,pad=1.0,rounding_size=0.5",color=colors['random'])
            if self.var2parents[var]:
                for par in self.var2parents[var]:
                    ax.annotate(var.name, par.position, xytext=var.position,
                                arrowprops={'arrowstyle':'<-'},bbox=bbox,
                                ha='center', va='center',
                                    fontsize=fontsize)
            else:
                x,y = var.position
                plt.text(x,y,var.name,bbox=bbox,ha='center', va='center',
                    fontsize=fontsize)
```

12.1.1 Example Decision Networks

Umbrella Decision Network

Here is a simple "umbrella" decision network. The output of `umbrella_dn.show()` is shown in Figure 12.1.

decnNetworks.py — (continued)

```
Weather = Variable("Weather", ["NoRain", "Rain"], position=(0.5,0.8))
Forecast = Variable("Forecast", ["Sunny", "Cloudy", "Rainy"],
    position=(0,0.4))
# Each variant uses one of the following:
Umbrella = DecisionVariable("Umbrella", ["Take", "Leave"], {Forecast},
    position=(0.5,0))

p_weather = Prob(Weather, [], {"NoRain":0.7, "Rain":0.3})
p_forecast = Prob(Forecast, [Weather], {"NoRain":{"Sunny":0.7,
    "Cloudy":0.2, "Rainy":0.1},
```

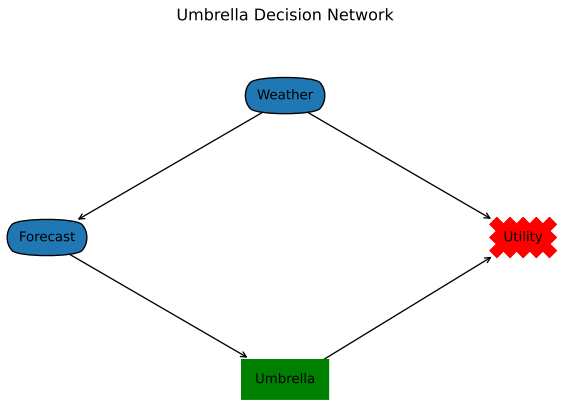


Figure 12.1: The umbrella decision network

```
                                        "Rain":{"Sunny":0.15,
                                            "Cloudy":0.25, "Rainy":0.6}})
umb_utility = UtilityTable([Weather, Umbrella], {"NoRain":{"Take":20,
    "Leave":100},
                                                 "Rain":{"Take":70,
                                                     "Leave":0}},
                                                     position=(1,0.4))

umbrella_dn = DecisionNetwork("Umbrella Decision Network",
                              {Weather, Forecast, Umbrella},
                              {p_weather, p_forecast, umb_utility})

# umbrella_dn.show()
# umbrella_dn.show(fontsize=15)
```

The following is a variant with the umbrella decision having 2 parents; nothing else has changed. This is interesting because one of the parents is not needed; if the agent knows the weather, it can ignore the forecast.

decnNetworks.py — (continued)

```
Umbrella2p = DecisionVariable("Umbrella", ["Take", "Leave"], {Forecast,
    Weather}, position=(0.5,0))
umb_utility2p = UtilityTable([Weather, Umbrella2p], {"NoRain":{"Take":20,
    "Leave":100},
                                                 "Rain":{"Take":70,
                                                     "Leave":0}},
                                                     position=(1,0.4))
umbrella_dn2p = DecisionNetwork("Umbrella Decision Network (extra arc)",
                              {Weather, Forecast, Umbrella2p},
                              {p_weather, p_forecast, umb_utility2p})

# umbrella_dn2p.show()
```

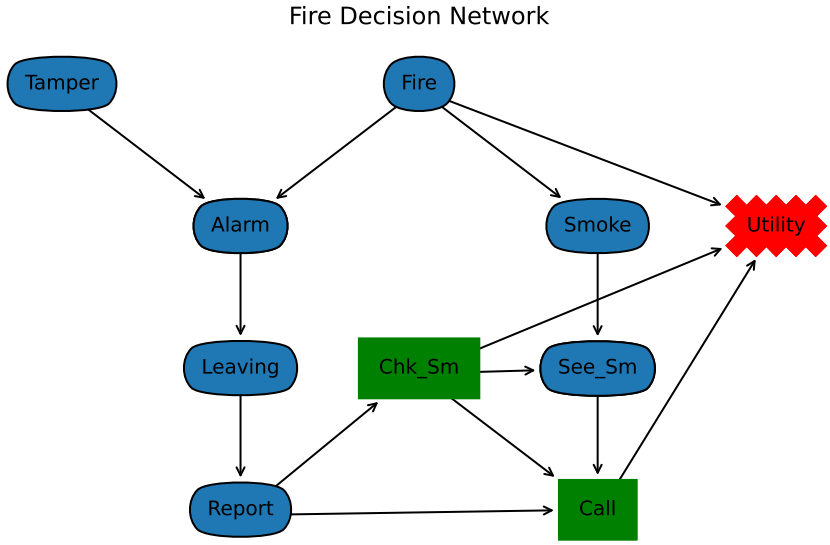


Figure 12.2: Fire Decision Network

```
114 # umbrella_dn2p.show(fontsize=15)
```

Fire Decision Network

The fire decision network of Figure 12.2 (showing the result of `fire_dn.show()`) is represented as:

decnNetworks.py — (continued)

```
116 boolean = [False, True]
117 Alarm = Variable("Alarm", boolean, position=(0.25,0.633))
118 Fire = Variable("Fire", boolean, position=(0.5,0.9))
119 Leaving = Variable("Leaving", boolean, position=(0.25,0.366))
120 Report = Variable("Report", boolean, position=(0.25,0.1))
121 Smoke = Variable("Smoke", boolean, position=(0.75,0.633))
122 Tamper = Variable("Tamper", boolean, position=(0,0.9))
123
124 See_Sm = Variable("See_Sm", boolean, position=(0.75,0.366) )
125 Chk_Sm = DecisionVariable("Chk_Sm", boolean, {Report}, position=(0.5,
        0.366))
126 Call = DecisionVariable("Call", boolean,{See_Sm,Chk_Sm,Report},
        position=(0.75,0.1))
127
128 f_ta = Prob(Tamper,[],[0.98,0.02])
```

```
f_fi = Prob(Fire,[],[0.99,0.01])
f_sm = Prob(Smoke,[Fire],[[0.99,0.01],[0.1,0.9]])
f_al = Prob(Alarm,[Fire,Tamper],[[[0.9999, 0.0001], [0.15, 0.85]], [[0.01,
    0.99], [0.5, 0.5]]])
f_lv = Prob(Leaving,[Alarm],[[0.999, 0.001], [0.12, 0.88]])
f_re = Prob(Report,[Leaving],[[0.99, 0.01], [0.25, 0.75]])
f_ss = Prob(See_Sm,[Chk_Sm,Smoke],[[[1,0],[1,0]],[[1,0],[0,1]]])

ut = UtilityTable([Chk_Sm,Fire,Call],
                  [[[0,-200],[-5000,-200]],[[-20,-220],[-5020,-220]]],
                  position=(1,0.633))

fire_dn = DecisionNetwork("Fire Decision Network",
                      {Tamper,Fire,Alarm,Leaving,Smoke,Call,See_Sm,Chk_Sm,Report},
                      {f_ta,f_fi,f_sm,f_al,f_lv,f_re,f_ss,ut})

# print(ut.to_table())
# fire_dn.show()
# fire_dn.show(fontsize=15)
```

Cheating Decision Network

The following is the representation of the cheating decision of Figure 12.3. Note that we keep the names of the variables short (less than 8 characters) so that the tables look good when printed.

decnNetworks.py — (continued)

```
grades = ['A','B','C','F']
Watched = Variable("Watched", boolean, position=(0,0.9))
Caught1 = Variable("Caught1", boolean, position=(0.2,0.7))
Caught2 = Variable("Caught2", boolean, position=(0.6,0.7))
Punish = Variable("Punish", ["None","Suspension","Recorded"],
    position=(0.8,0.9))
Grade_1 = Variable("Grade_1", grades, position=(0.2,0.3))
Grade_2 = Variable("Grade_2", grades, position=(0.6,0.3))
Fin_Grd = Variable("Fin_Grd", grades, position=(0.8,0.1))
Cheat_1 = DecisionVariable("Cheat_1", boolean, set(), position=(0,0.5))
    #no parents
Cheat_2 = DecisionVariable("Cheat_2", boolean, {Cheat_1,Caught1},
    position=(0.4,0.5))

p_wa = Prob(Watched,[],[0.7, 0.3])
p_cc1 = Prob(Caught1,[Watched,Cheat_1],[[[1.0, 0.0], [0.9, 0.1]], [[1.0,
    0.0], [0.5, 0.5]]])
p_cc2 = Prob(Caught2,[Watched,Cheat_2],[[[1.0, 0.0], [0.9, 0.1]], [[1.0,
    0.0], [0.5, 0.5]]])
p_pun = Prob(Punish,[Caught1,Caught2],
               [[{"None":0,"Suspension":0,"Recorded":0},
                 {"None":0.5,"Suspension":0.4,"Recorded":0.1}],
                [{"None":0.6,"Suspension":0.2,"Recorded":0.2},
```

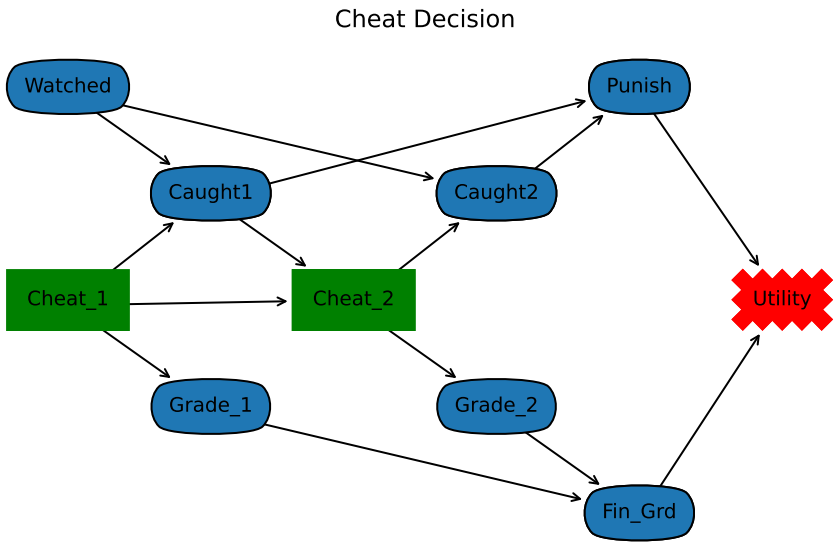


Figure 12.3: Cheating Decision Network

```
                    {"None":0.2,"Suspension":0.3,"Recorded":0.3}]])
p_gr1 = Prob(Grade_1,[Cheat_1], [{'A':0.2, 'B':0.3, 'C':0.3, 'F': 0.2},
                                 {'A':0.5, 'B':0.3, 'C':0.2, 'F':0.0}])
p_gr2 = Prob(Grade_2,[Cheat_2], [{'A':0.2, 'B':0.3, 'C':0.3, 'F': 0.2},
                                 {'A':0.5, 'B':0.3, 'C':0.2, 'F':0.0}])
p_fg = Prob(Fin_Grd,[Grade_1,Grade_2],
       {'A':{'A':{'A':1.0, 'B':0.0, 'C': 0.0, 'F':0.0},
             'B': {'A':0.5, 'B':0.5, 'C': 0.0, 'F':0.0},
             'C':{'A':0.25, 'B':0.5, 'C': 0.25, 'F':0.0},
             'F':{'A':0.25, 'B':0.25, 'C': 0.25, 'F':0.25}},
        'B':{'A':{'A':0.5, 'B':0.5, 'C': 0.0, 'F':0.0},
             'B': {'A':0.0, 'B':1, 'C': 0.0, 'F':0.0},
             'C':{'A':0.0, 'B':0.5, 'C': 0.5, 'F':0.0},
             'F':{'A':0.0, 'B':0.25, 'C': 0.5, 'F':0.25}},
        'C':{'A':{'A':0.25, 'B':0.5, 'C': 0.25, 'F':0.0},
             'B': {'A':0.0, 'B':0.5, 'C': 0.5, 'F':0.0},
             'C':{'A':0.0, 'B':0.0, 'C': 1, 'F':0.0},
             'F':{'A':0.0, 'B':0.0, 'C': 0.5, 'F':0.5}},
        'F':{'A':{'A':0.25, 'B':0.25, 'C': 0.25, 'F':0.25},
             'B': {'A':0.0, 'B':0.25, 'C': 0.5, 'F':0.25},
             'C':{'A':0.0, 'B':0.0, 'C': 0.5, 'F':0.5},
             'F':{'A':0.0, 'B':0.0, 'C': 0, 'F':1.0}}})

utc = UtilityTable([Punish,Fin_Grd],
```

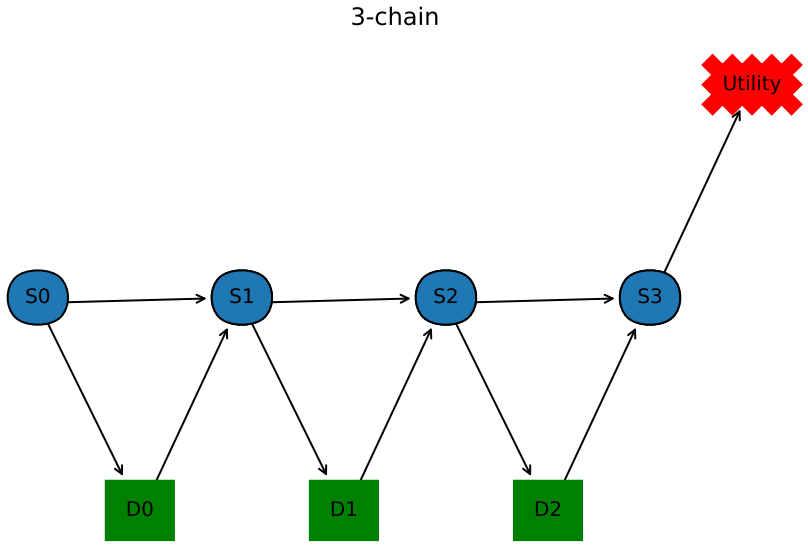


Figure 12.4: A decision network that is a chain of 3 decisions

```
                    {'None':{'A':100, 'B':90, 'C': 70, 'F':50},
                     'Suspension':{'A':40, 'B':20, 'C': 10, 'F':0},
                     'Recorded':{'A':70, 'B':60, 'C': 40, 'F':20}},
                    position=(1,0.5))

cheating_dn = DecisionNetwork("Cheating Decision Network",
               {Punish,Caught2,Watched,Fin_Grd,Grade_2,Grade_1,Cheat_2,Caught1,Cheat_1},
               {p_wa, p_cc1, p_cc2, p_pun, p_gr1, p_gr2,p_fg,utc})

# cheating_dn.show()
# cheating_dn.show(fontsize=15)
```

Chain of 3 decisions

The following example is a finite-stage fully-observable Markov decision process with a single reward (utility) at the end. It is interesting because the parents do not include all predecessors. The methods we use will work without change on this, even though the agent does not condition on all of its previous observations and actions. The output of `ch3.show()` is shown in Figure 12.4.

decnNetworks.py — (continued)

```
S0 = Variable('S0', boolean, position=(0,0.5))
D0 = DecisionVariable('D0', boolean, {S0}, position=(1/7,0.1))
S1 = Variable('S1', boolean, position=(2/7,0.5))
```

```
D1 = DecisionVariable('D1', boolean, {S1}, position=(3/7,0.1))
S2 = Variable('S2', boolean, position=(4/7,0.5))
D2 = DecisionVariable('D2', boolean, {S2}, position=(5/7,0.1))
S3 = Variable('S3', boolean, position=(6/7,0.5))

p_s0 = Prob(S0, [], [0.5,0.5])
tr = [[[0.1, 0.9], [0.9, 0.1]], [[0.2, 0.8], [0.8, 0.2]]] # 0 is flip, 1 is keep value
p_s1 = Prob(S1, [D0,S0], tr)
p_s2 = Prob(S2, [D1,S1], tr)
p_s3 = Prob(S3, [D2,S2], tr)

ch3U = UtilityTable([S3],[0,1], position=(7/7,0.9))

ch3 = DecisionNetwork("3-chain",
    {S0,D0,S1,D1,S2,D2,S3},{p_s0,p_s1,p_s2,p_s3,ch3U})

# ch3.show()
# ch3.show(fontsize=15)
```

12.1.2 Decision Functions

The output of an optimation function is an optimal policy, a list of decision functions, and the expected value of the optimal policy. A decision function is the action for each decision variable as a function of its parents.

decnNetworks.py — (continued)

```
class DictFactor(Factor):
    """A factor the represents the values using a dicionary"""
    def __init__(self, *pargs, **kwargs):
        self.values = {}
        Factor.__init__(self, *pargs, **kwargs)

    def assign(self, assignment, value):
        self.values[frozenset(assignment.items())] = value

    def get_value(self, assignment):
        ass = frozenset(assignment.items())
        assert ass in self.values, f"assignment {assignment} cannot be evaluated"
        return self.values[ass]

class DecisionFunction(DictFactor):
    def __init__(self, decision, parents):
        """ A decision function
        decision is a decision variable
        parents is a set of variables
        """
        self.decision = decision
        self.parent = parents
```

```
245         DictFactor.__init__(self, parents, name=decision.name)
```

12.1.3 Recursive Conditioning for decision networks

An instance of a RC_DN object takes in a decision network. The query method uses recursive conditioning to compute the expected utility of the optimal policy. self.opt_policy becomes the optimal policy.

decnNetworks.py — (continued)

```
247 import math
248 from probGraphicalModels import GraphicalModel, InferenceMethod
249 from probFactors import Factor
250 from probRC import connected_components
251
252 class RC_DN(InferenceMethod):
253     """The class that queries graphical models using recursive conditioning
254
255     gm is graphical model to query
256     """
257
258     def __init__(self,gm=None):
259         self.gm = gm
260         self.cache = {(frozenset(), frozenset()):1}
261         ## self.max_display_level = 3
262
263     def optimize(self, split_order=None, algorithm=None):
264         """computes expected utility, and creates optimal decision
                functions, where
265         elim_order is a list of the non-observed non-query variables in gm
266         algorithm is the (search algortithm to use). Default is self.rc
267         """
268         if algorithm is None:
269             algorithm = self.rc
270         if split_order == None:
271             split_order = self.gm.split_order()
272         self.opt_policy = {v:DecisionFunction(v, v.parents)
273                            for v in self.gm.variables
274                            if isinstance(v,DecisionVariable)}
275         return algorithm({}, self.gm.factors, split_order)
276
277     def show_policy(self):
278         print('\n'.join(df.to_table() for df in self.opt_policy.values()))
```

The following us the simplest search-based algorithm. It is exponential in the number of variables, so is not very useful. However, it is simple, and useful to understand before looking at the more complicated algorithm. Note that the above code does not call rc0; you will need to change the self.rc to self.rc0 in above code to use it.

decnNetworks.py — (continued)

```
280     def rc0(self, context, factors, split_order):
281         """simplest search algorithm
282         context is a variable:value dictionary
283         factors is a set of factors
284         split_order is a list of variables in factors that are not in
                context
285         """
286         self.display(3,"calling rc0,",(context,factors),"with
                SO",split_order)
287         if not factors:
288             return 1
289         elif to_eval := {fac for fac in factors if
                fac.can_evaluate(context)}:
290             self.display(3,"rc0 evaluating factors",to_eval)
291             val = math.prod(fac.get_value(context) for fac in to_eval)
292             return val * self.rc0(context, factors-to_eval, split_order)
293         else:
294             var = split_order[0]
295             self.display(3, "rc0 branching on", var)
296             if isinstance(var,DecisionVariable):
297                 assert set(context) <= set(var.parents), f"cannot optimize
                        {var} in context {context}"
298                 maxres = -math.inf
299                 for val in var.domain:
300                     self.display(3,"In rc0, branching on",var,"=",val)
301                     newres = self.rc0({var:val}|context, factors,
                            split_order[1:])
302                     if newres > maxres:
303                         maxres = newres
304                         theval = val
305                 self.opt_policy[var].assign(context,theval)
306                 return maxres
307             else:
308                 total = 0
309                 for val in var.domain:
310                     total += self.rc0({var:val}|context, factors,
                            split_order[1:])
311                 self.display(3, "rc0 branching on", var,"returning", total)
312                 return total
```

We can combine the optimization for decision networks above, with the improvements of recursive conditioning used for graphical models (Section 9.7, page 220).

decnNetworks.py — (continued)

```
314     def rc(self, context, factors, split_order):
315         """ returns the number \sum_{split_order} \prod_{factors} given
                assignments in context
316         context is a variable:value dictionary
317         factors is a set of factors
```

```
318         split_order is a list of variables in factors that are not in
                context
319         """
320         self.display(3,"calling rc,",(context,factors))
321         ce = (frozenset(context.items()), frozenset(factors)) # key for the
                cache entry
322         if ce in self.cache:
323             self.display(2,"rc cache lookup",(context,factors))
324             return self.cache[ce]
325 #       if not factors: # no factors; needed if you don't have forgetting
        and caching
326 #           return 1
327         elif vars_not_in_factors := {var for var in context
328                                      if not any(var in fac.variables for
                                           fac in factors)}:
329             # forget variables not in any factor
330             self.display(3,"rc forgetting variables", vars_not_in_factors)
331             return self.rc({key:val for (key,val) in context.items()
332                             if key not in vars_not_in_factors},
333                            factors, split_order)
334         elif to_eval := {fac for fac in factors if
                fac.can_evaluate(context)}:
335             # evaluate factors when all variables are assigned
336             self.display(3,"rc evaluating factors",to_eval)
337             val = math.prod(fac.get_value(context) for fac in to_eval)
338             if val == 0:
339                 return 0
340             else:
341                 return val * self.rc(context, {fac for fac in factors if fac
                        not in to_eval}, split_order)
342         elif len(comp := connected_components(context, factors,
                split_order)) > 1:
343             # there are disconnected components
344             self.display(2,"splitting into connected components",comp)
345             return(math.prod(self.rc(context,f,eo) for (f,eo) in comp))
346         else:
347             assert split_order, f"split_order empty rc({context},{factors})"
348             var = split_order[0]
349             self.display(3, "rc branching on", var)
350             if isinstance(var,DecisionVariable):
351                 assert set(context) <= set(var.parents), f"cannot optimize
                        {var} in context {context}"
352                 maxres = -math.inf
353                 for val in var.domain:
354                     self.display(3,"In rc, branching on",var,"=",val)
355                     newres = self.rc({var:val}|context, factors,
                            split_order[1:])
356                     if newres > maxres:
357                         maxres = newres
358                         theval = val
```

```
359                 self.opt_policy[var].assign(context,theval)
360                 self.cache[ce] = maxres
361                 return maxres
362             else:
363                 total = 0
364                 for val in var.domain:
365                     total += self.rc({var:val}|context, factors,
                            split_order[1:])
366                 self.display(3, "rc branching on", var,"returning", total)
367                 self.cache[ce] = total
368                 return total
```

Here is how to run the optimize the example decision networks:

decnNetworks.py — (continued)

```
370 # Umbrella decision network
371 #urc = RC_DN(umbrella_dn)
372 #urc.optimize(algorithm=urc.rc0) #RC0
373 #urc.optimize()  #RC
374 #urc.show_policy()
375
376 #rc_fire = RC_DN(fire_dn)
377 #rc_fire.optimize()
378 #rc_fire.show_policy()
379
380 #rc_cheat = RC_DN(cheating_dn)
381 #rc_cheat.optimize()
382 #rc_cheat.show_policy()
383
384 #rc_ch3 = RC_DN(ch3)
385 #rc_ch3.optimize()
386 #rc_ch3.show_policy()
387 # rc_ch3.optimize(algorithm=rc_ch3.rc0) # why does that happen?
```

12.1.4 Variable elimination for decision networks

VE_DN is variable elimination for decision networks. The method *optimize* is used to optimize all the decisions. Note that *optimize* requires a legal elimination ordering of the random and decision variables, otherwise it will give an exception. (A decision node can only be maximized if the variables that are not its parents have already been eliminated.)

decnNetworks.py — (continued)

```
389 from probVE import VE
390
391 class VE_DN(VE):
392     """Variable Elimination for Decision Networks"""
393     def __init__(self,dn=None):
394         """dn is a decision network"""
395         VE.__init__(self,dn)
```

```
396         self.dn = dn
397
398     def optimize(self,elim_order=None,obs={}):
399         if elim_order == None:
400                 elim_order = reversed(self.gm.split_order())
401         self.opt_policy = {}
402         proj_factors = [self.project_observations(fact,obs)
403                         for fact in self.dn.factors]
404         for v in elim_order:
405             if isinstance(v,DecisionVariable):
406                 to_max = [fac for fac in proj_factors
407                           if v in fac.variables and set(fac.variables) <=
                              v.all_vars]
408                 assert len(to_max)==1, "illegal variable order
                      "+str(elim_order)+" at "+str(v)
409                 newFac = FactorMax(v, to_max[0])
410                 self.opt_policy[v]=newFac.decision_fun
411                 proj_factors = [fac for fac in proj_factors if fac is not
                      to_max[0]]+[newFac]
412                 self.display(2,"maximizing",v )
413                 self.display(3,newFac)
414             else:
415                 proj_factors = self.eliminate_var(proj_factors, v)
416         assert len(proj_factors)==1,"Should there be only one element of
                proj_factors?"
417         return proj_factors[0].get_value({})
418
419     def show_policy(self):
420         print('\n'.join(df.to_table() for df in self.opt_policy.values()))
```

decnNetworks.py — (continued)

```
422 class FactorMax(TabFactor):
423     """A factor obtained by maximizing a variable in a factor.
424     Also builds a decision_function. This is based on FactorSum.
425     """
426
427     def __init__(self, dvar, factor):
428         """dvar is a decision variable.
429         factor is a factor that contains dvar and only parents of dvar
430         """
431         self.dvar = dvar
432         self.factor = factor
433         vars = [v for v in factor.variables if v is not dvar]
434         Factor.__init__(self,vars)
435         self.values = {}
436         self.decision_fun = DecisionFunction(dvar, dvar.parents)
437
438     def get_value(self,assignment):
439         """lazy implementation: if saved, return saved value, else compute
              it"""
```

```
        new_asst = {x:v for (x,v) in assignment.items() if x in
            self.variables}
        asst = frozenset(new_asst.items())
        if asst in self.values:
            return self.values[asst]
        else:
            max_val = float("-inf") # -infinity
            for elt in self.dvar.domain:
                fac_val = self.factor.get_value(assignment|{self.dvar:elt})
                if fac_val>max_val:
                    max_val = fac_val
                    best_elt = elt
            self.values[asst] = max_val
            self.decision_fun.assign(assignment, best_elt)
            return max_val
```

Here are some example queries:

decnNetworks.py — (continued)

```
# Example queries:
# vf = VE_DN(fire_dn)
# vf.optimize()
# vf.show_policy()

# VE_DN.max_display_level = 3 # if you want to show lots of detail
# vc = VE_DN(cheating_dn)
# vc.optimize()
# vc.show_policy()


def test(dn):
    rc0dn = RC_DN(dn)
    rc0v = rc0dn.optimize(algorithm=rc0dn.rc0)
    rcdn = RC_DN(dn)
    rcv = rcdn.optimize()
    assert abs(rc0v-rcv)<1e-10, f"rc0 produces {rc0v}; rc produces {rcv}"
    vedn = VE_DN(dn)
    vev = vedn.optimize()
    assert abs(vev-rcv)<1e-10, f"VE_DN produces {vev}; RC produces {rcv}"
    print(f"passed unit test. rc0, rc and VE gave same result for {dn}")

if __name__ == "__main__":
    test(fire_dn)
```

12.2 Markov Decision Processes

The following represent a **Markov decision process** (**MDP**) directly, rather than using the recursive conditioning or variable elimination code, as was dome for decision networks.

mdpProblem.py — Representations for Markov Decision Processes

```
import random
from display import Displayable
from utilities import argmaxd

class MDP(Displayable):
    """A Markov Decision Process. Must define:
    title a string that gives the title of the MDP
    states the set (or list) of states
    actions the set (or list) of actions
    discount a real-valued discount
    """

    def __init__(self, title, states, actions, discount, init=0):
        self.title = title
        self.states = states
        self.actions = actions
        self.discount = discount
        self.initv = self.V = {s:init for s in self.states}
        self.initq = self.Q = {s: {a: init for a in self.actions} for s in
            self.states}

    def P(self,s,a):
        """Transition probability function
        returns a dictionary of {s1:p1} such that P(s1 | s,a)=p1. Other
            probabilities are zero.
        """
        raise NotImplementedError("P") # abstract method

    def R(self,s,a):
        """Reward function R(s,a)
        returns the expected reward for doing a in state s.
        """
        raise NotImplementedError("R") # abstract method
```

Two state partying example (Example 12.29 in Poole and Mackworth [2023]):

mdpExamples.py — MDP Examples

```
from mdpProblem import MDP, ProblemDomain, distribution
from mdpGUI import GridDomain
import matplotlib.pyplot as plt

class partyMDP(MDP):
    """Simple 2-state, 2-Action Partying MDP Example"""
    def __init__(self, discount=0.9):
        states = {'healthy','sick'}
        actions = {'relax', 'party'}
        MDP.__init__(self, "party MDP", states, actions, discount)

    def R(self,s,a):
```

```
        "R(s,a)"
        return { 'healthy': {'relax': 7, 'party': 10},
                 'sick':   {'relax': 0, 'party': 2 }}[s][a]

    def P(self,s,a):
        "returns a dictionary of {s1:p1} such that P(s1 | s,a)=p1. Other
            probabilities are zero."
        phealthy = { # P('healthy' | s, a)
                    'healthy': {'relax': 0.95, 'party': 0.7},
                    'sick': {'relax': 0.5, 'party': 0.1 }}[s][a]
        return {'healthy':phealthy, 'sick':1-phealthy}
```

The `distribution` class is used to represent distibutions as they are being created. Probability distributions are represented as item:value dictionaries. When being constructed, adding an item:value to the dictionary has to act differently when the item is already in the dictionary and when it isn't. The `add_prob` method works whether the item is in the dictionary or not.

mdpProblem.py — (continued)

```
class distribution(dict):
    """A distribution is an item:prob dictionary.
    The only new part is when a new item:pr is added, and item is already
        there, the values are summed
    """
    def __init__(self,d):
        dict.__init__(self,d)

    def add_prob(self, item, pr):
        if item in self:
            self[item] += pr
        else:
            self[item] = pr
        return self
```

12.2.1 Problem Domains

An MDP does not contain enough information to simulate a domain, because

(a) the rewards and resulting state can be correlated (e.g., in the grid domains below, crashing into a wall results in both a negative reward and the agent not moving), and

(b) it represents the *expected* reward (e.g., a reward of 1 is has the same expected value as as a reward of 100 with probability 1/100 and 0 otherwise, but these are different in a simulation).

A problem domain represents a problem as a function `result` from states and actions into a distribution of $(state, reward)$ pairs. This can be a subclass of `MDP` because it implements `R` and `P`. A problem domain also specifies an initial state and coordinate information used by the graphical user interfaces.

mdpProblem.py — (continued)

```
class ProblemDomain(MDP):
    """A ProblemDomain implements
    self.result(state, action) -> {(reward, state):probability}.
    Other pairs have probability are zero.
    The probabilities must sum to 1.
    """
    def __init__(self, title, states, actions, discount,
                 initial_state=None, x_dim=0, y_dim = 0,
                 vinit=0, offsets={}):
        """A problem domain
        * title is list of titles
        * states is the list of states
        * actions is the list of actions
        * discount is the discount factor
        * initial_state is the state the agent starts at (for simulation)
            if known
        * x_dim and y_dim are the dimensions used by the GUI to show the
            states in 2-dimensions
        * vinit is the initial value
        * offsets is a {action:(x,y)} map which specifies how actions are
            displayed in GUI
        """
        MDP.__init__(self, title, states, actions, discount)
        if initial_state is not None:
            self.state = initial_state
        else:
            self.state = random.choice(states)
        self.vinit = vinit # value to reset v,q to
        # The following are for the GUI:
        self.x_dim = x_dim
        self.y_dim = y_dim
        self.offsets = offsets

    def state2pos(self,state):
        """When displaying as a grid, this specifies how the state is
            mapped to (x,y) position.
        The default is for domains where the (x,y) position is the state
        """
        return state

    def state2goal(self,state):
        """When displaying as a grid, this specifies how the state is
            mapped to goal position.
        The default is for domains where there is no goal
        """
        return None

    def pos2state(self,pos):
        """When displaying as a grid, this specifies how the state is
```

```
            mapped to (x,y) position.
        The default is for domains where the (x,y) position is the state
        """
        return pos

    def P(self, state, action):
        """Transition probability function
        returns a dictionary of {s1:p1} such that P(s1 | state,action)=p1.
        Other probabilities are zero.
        """
        res = self.result(state, action)
        acc = 1e-6 # accuracy for test of equality
        assert 1-acc<sum(res.values())<1+acc, f"result({state},{action})
            not a distribution, sum={sum(res.values())}"
        dist = distribution({})
        for ((r,s),p) in res.items():
            dist.add_prob(s,p)
        return dist

    def R(self, state, action):
        """Reward function R(s,a)
        returns the expected reward for doing a in state s.
        """
        return sum(r*p for ((r,s),p) in self.result(state, action).items())
```

Tiny Game

The next example is the tiny game from Example 13.1 and Figure 13.1 of Poole and Mackworth [2023] The state is represented as (x, y) where x counts from zero from the left, and y counts from zero upwards, so the state $(0, 0)$ is on the bottom-left state. The actions are `upC` for up-careful, `upR` for up-risky, `left`, and `left`. (Note that `GridDomain` means that it can be shown with the MDP GUI in Section 12.2.3).

mdpExamples.py — (continued)

```
class MDPtiny(ProblemDomain, GridDomain):
    def __init__(self, discount=0.9):
        x_dim = 2  # x-dimension
        y_dim = 3
        ProblemDomain.__init__(self,
            "Tiny MDP", # title
            [(x,y) for x in range(x_dim) for y in range(y_dim)], #states
            ['right', 'upC', 'left', 'upR'], #actions
            discount,
            x_dim=x_dim, y_dim = y_dim,
            offsets = {'right':(0.25,0), 'upC':(0,-0.25), 'left':(-0.25,0),
                'upR':(0,0.25)}
            )

```

```
47      def result(self, state, action):
48          """return a dictionary of {(r,s):p} where p is the probability of
                reward r, state s
49          a state is an (x,y) pair
50          """
51          (x,y) = state
52          right = (-x,(1,y)) # reward is -1 if x was 1
53          left = (0,(0,y)) if x==1 else [(-1,(0,0)), (-100,(0,1)),
                (10,(0,0))][y]
54          up = (0,(x,y+1)) if y<2 else (-1,(x,y))
55          if action == 'right':
56              return {right:1}
57          elif action == 'upC':
58              (r,s) = up
59              return {(r-1,s):1}
60          elif action == 'left':
61             return {left:1}
62          elif action == 'upR':
63              return distribution({left:
                    0.1}).add_prob(right,0.1).add_prob(up,0.8)
64              # Exercise: what is wrong with return {left: 0.1, right:0.1,
                    up:0.8}
65
66  # To show GUI do
67  #  MDPtiny().viGUI()
```

Grid World

Here is the domain of Example 12.30 of Poole and Mackworth [2023], shown here in Figure 12.5. A state is represented as (x, y) where x counts from zero from the left, and y counts from zero upwards, so the state $(0, 0)$ is on the bottom-left.

mdpExamples.py — (continued)

```
69  class grid(ProblemDomain, GridDomain):
70      """ x_dim * y_dim grid with rewarding states"""
71      def __init__(self, discount=0.9, x_dim=10, y_dim=10):
72          ProblemDomain.__init__(self,
73              "Grid World",
74              [(x,y) for x in range(y_dim) for y in range(y_dim)], #states
75              ['up', 'down', 'right', 'left'], #actions
76              discount,
77              x_dim = x_dim, y_dim = y_dim,
78              offsets = {'right':(0.25,0), 'up':(0,0.25), 'left':(-0.25,0),
                    'down':(0,-0.25)})
79          self.rewarding_states = {(3,2):-10, (3,5):-5, (8,2):10, (7,7):3 }
80          self.fling_states = {(8,2), (7,7)} # assumed a subset of
                rewarding_states
81
82      def intended_next(self,s,a):
```

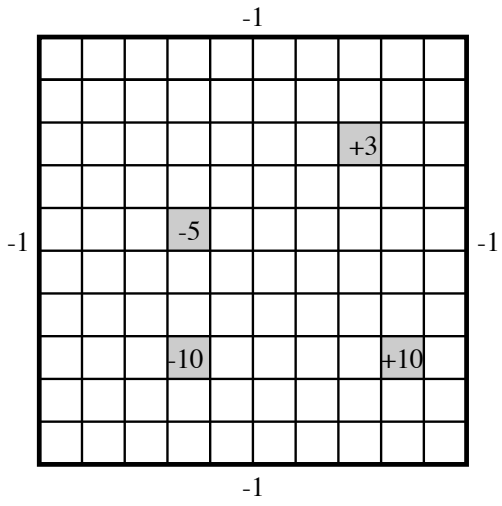


Figure 12.5: Grid world

```
83          """returns the (reward, state) in the direction a.
84          This is where the agent will end up if to goes in its
                intended_direction
85           (which it does with probability 0.7).
86          """
87          (x,y) = s
88          if a=='up':
89              return (0, (x,y+1)) if y+1 < self.y_dim else (-1, (x,y))
90          if a=='down':
91              return (0, (x,y-1)) if y > 0 else (-1, (x,y))
92          if a=='right':
93              return (0, (x+1,y)) if x+1 < self.x_dim else (-1, (x,y))
94          if a=='left':
95              return (0, (x-1,y)) if x > 0 else (-1, (x,y))
96
97      def result(self,s,a):
98          """return a dictionary of {(r,s):p} where p is the probability of
                reward r, state s.
99          a state is an (x,y) pair
100         """
101         r0 = self.rewarding_states[s] if s in self.rewarding_states else 0
102         if s in self.fling_states:
103             return {(r0,(0,0)): 0.25, (r0,(self.x_dim-1,0)):0.25,
104                     (r0,(0,self.y_dim-1)):0.25,
                            (r0,(self.x_dim-1,self.y_dim-1)):0.25}
105         dist = distribution({})
106         for a1 in self.actions:
107             (r1,s1) = self.intended_next(s,a1)
108             rs = (r1+r0, s1)
109             p = 0.7 if a1==a else 0.1
```

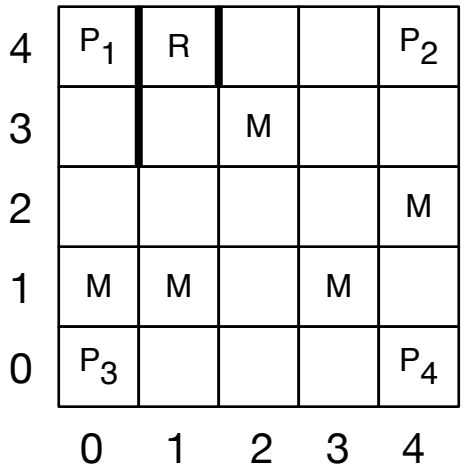


Figure 12.6: Monster game

```
            dist.add_prob(rs,p)
        return dist
```

Monster Game

This is for the game depicted in Figure 13.1 (Example 13.2 of Poole and Mackworth [2023]).

mdpExamples.py — (continued)

```
class Monster_game(ProblemDomain, GridDomain):

    vwalls = [(0,3), (0,4), (1,4)] # vertical walls right of these locations
    crash_reward = -1

    prize_locs = [(0,0), (0,4), (4,0), (4,4)]
    prize_apears_prob = 0.3
    prize_reward = 10

    monster_locs = [(0,1), (1,1), (2,3), (3,1), (4,2)]
    monster_appears_prob = 0.4
    monster_reward_when_damaged = -10
    repair_stations = [(1,4)]

    def __init__(self, discount=0.9):
        x_dim = 5
        y_dim = 5
            # which damaged and prize to show
        ProblemDomain.__init__(self,
            "Monster Game",
            [(x,y,damaged,prize)
```

```
                for x in range(x_dim)
                for y in range(y_dim)
                for damaged in [False,True]
                for prize in [None]+self.prize_locs], #states
            ['up', 'down', 'right', 'left'], #actions
            discount,
            x_dim = x_dim, y_dim = y_dim,
            offsets = {'right':(0.25,0), 'up':(0,0.25), 'left':(-0.25,0),
                'down':(0,-0.25)})
        self.state = (2,2,False,None)

    def intended_next(self,xy,a):
        """returns the (reward, (x,y)) in the direction a.
        This is where the agent will end up if to goes in its
            intended_direction
             (which it does with probability 0.7).
        """
        (x,y) = xy # original x-y position
        if a=='up':
            return (0, (x,y+1)) if y+1 < self.y_dim else
                (self.crash_reward, (x,y))
        if a=='down':
            return (0, (x,y-1)) if y > 0 else (self.crash_reward, (x,y))
        if a=='right':
            if (x,y) in self.vwalls or x+1==self.x_dim: # hit wall
                return (self.crash_reward, (x,y))
            else:
                return (0, (x+1,y))
        if a=='left':
            if (x-1,y) in self.vwalls or x==0: # hit wall
                            return (self.crash_reward, (x,y))
            else:
                return (0, (x-1,y))

    def result(self,s,a):
        """return a dictionary of {(r,s):p} where p is the probability of
            reward r, state s.
        a state is an (x,y) pair
        """
        (x,y,damaged,prize) = s
        dist = distribution({})
        for a1 in self.actions: # possible results
            mp = 0.7 if a1==a else 0.1
            mr,(xn,yn) = self.intended_next((x,y),a1)
            if (xn,yn) in self.monster_locs:
                if damaged:
                    dist.add_prob((mr+self.monster_reward_when_damaged,(xn,yn,True,prize)),
                        mp*self.monster_appears_prob)
                    dist.add_prob((mr,(xn,yn,True,prize)),
                        mp*(1-self.monster_appears_prob))
```

```
178                 else:
179                     dist.add_prob((mr,(xn,yn,True,prize)),
                            mp*self.monster_appears_prob)
180                     dist.add_prob((mr,(xn,yn,False,prize)),
                            mp*(1-self.monster_appears_prob))
181             elif (xn,yn) == prize:
182                 dist.add_prob((mr+self.prize_reward,(xn,yn,damaged,None)),
                        mp)
183             elif (xn,yn) in self.repair_stations:
184                 dist.add_prob((mr,(xn,yn,False,prize)), mp)
185             else:
186                 dist.add_prob((mr,(xn,yn,damaged,prize)), mp)
187         if prize is None:
188             res = distribution({})
189             for (r,(x2,y2,d,p2)),p in dist.items():
190                 res.add_prob((r,(x2,y2,d,None)),
                        p*(1-self.prize_apears_prob))
191                 for pz in self.prize_locs:
192                     res.add_prob((r,(x2,y2,d,pz)),
                            p*self.prize_apears_prob/len(self.prize_locs))
193             return res
194         else:
195             return dist
196
197     def state2pos(self, state):
198         """When displaying as a grid, this specifies how the state is
                mapped to (x,y) position.
199         The default is for domains where the (x,y) position is the state
200         """
201         (x,y,d,p) = state
202         return (x,y)
203
204     def pos2state(self, pos):
205         """When displaying as a grid, this specifies how the state is
                mapped to (x,y) position.
206         """
207         (x,y) = pos
208         (xs, ys, damaged, prize) = self.state
209         return (x, y, damaged, prize)
210
211     def state2goal(self,state):
212         """the (x,y) position for the goal
213         """
214         (x, y, damaged, prize) = state
215         return prize
216
217 # To see value iterations:
218 # mg = Monster_game()
219 # mg.viGUI() # then run vi a few times
220 # to see other states, exit the GUI
```