Towards Cohesion-Fairness Harmony: Contrastive Regularization in Individual Fair Graph Clustering

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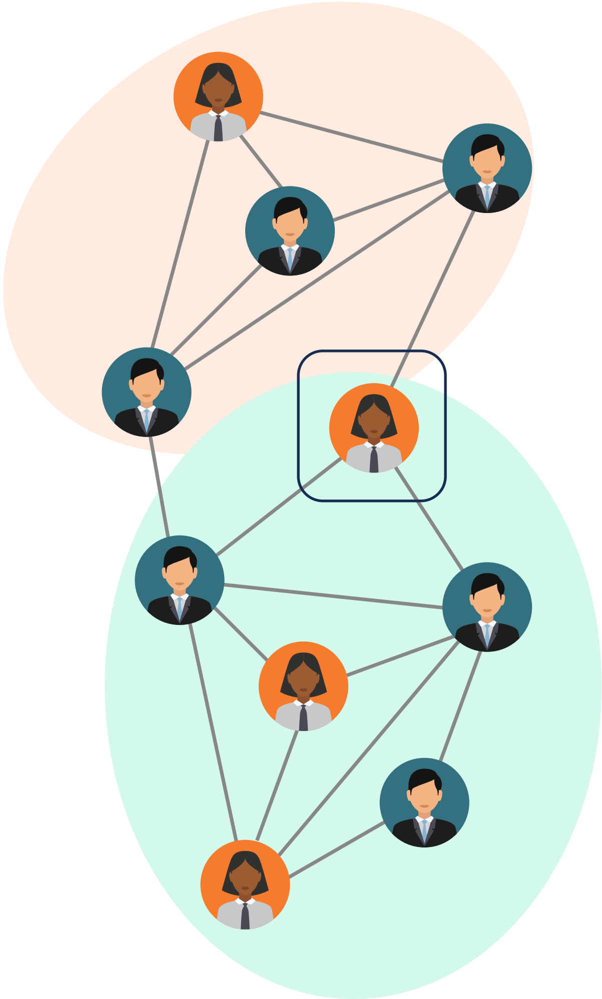
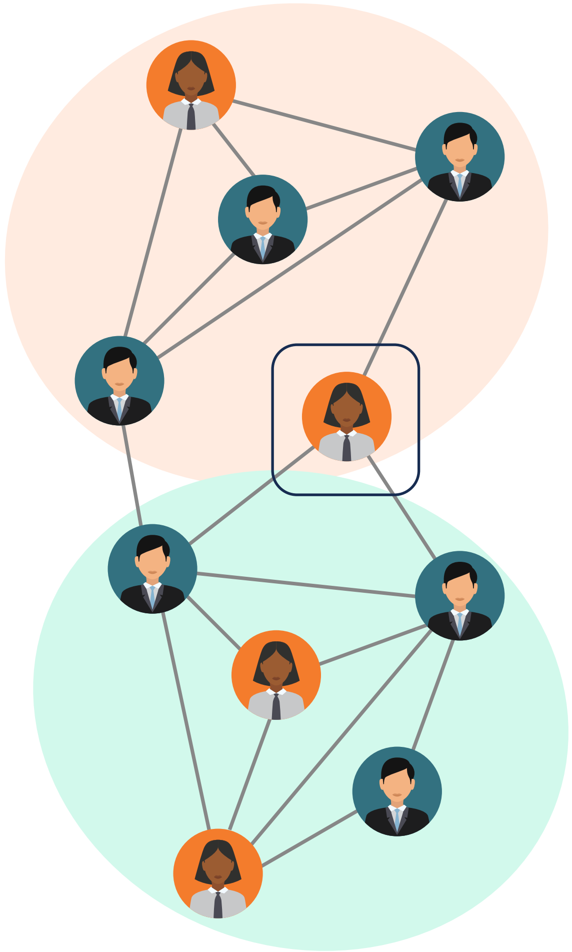
Example

- Teacher with 10 students, 4 female 6 male.
- First day of school.
- You know only the gender and friendship of students from their last year records.
- How to divide: 2 teams (clusters) for classroom assignments?

Clustering #1



Clustering #2



Example contd.

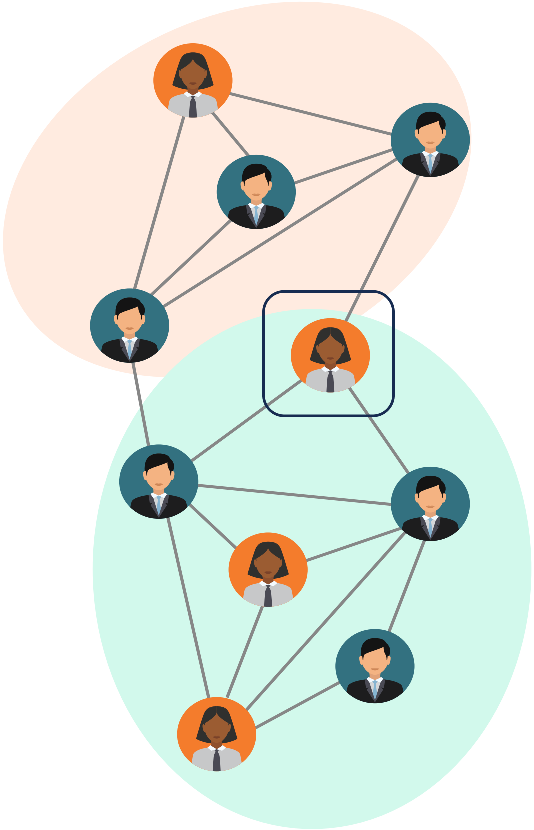
Avg ind fairness score $\bar{\delta} = 0.141$

Group fairness score $B = 0.33$

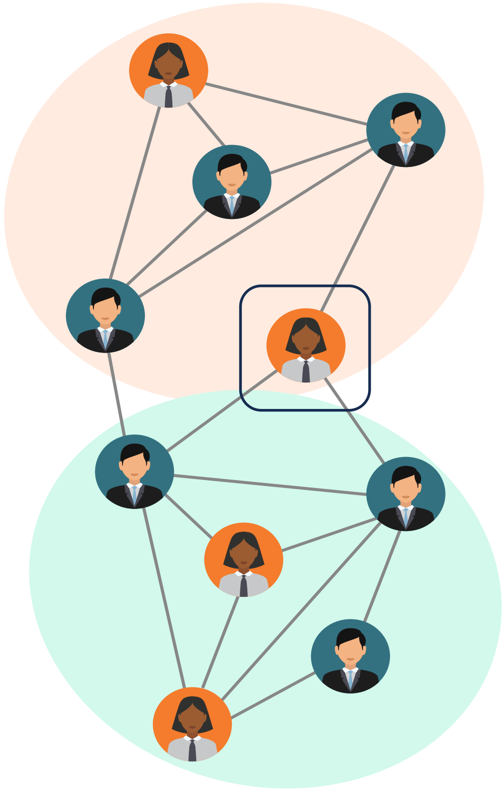
B Group well-being ?

$\bar{\delta}$ Individual links ?

Clustering #1



Clustering #2



$\bar{\delta} = 0.174 \uparrow$

$B = 0.66 \uparrow$

- ❑ What about clustering quality? C#1 is better or C#2 ?
- ❑ What should be the **clustering quality /fairness equilibrium**?

Problem Formulation

Inputs

- **Undirected graph** $\mathcal{G} = (V, E)$ where $V = \{v_1, v_2, \ldots, v_n\}$
- **No self-loops** $E \subseteq V \times V$ is a binary set of edges
- **Adjacency matrix** $\boldsymbol{A} \in \mathbb{R}^{n \times n}$ encodes edge information
- **Sensitive attribute** V_s $V = \dot{\cup}_{s \in [m]} V_s$

Output

- **Non-overlapping clustering** V into $k \geq 2$ $\longrightarrow$ $V = C_1 \dot{\cup} \ldots \dot{\cup} C_k$

Fair graph clustering: Individual Fairness

identically independently distributed data

Individual fairness (in i.i.d. data literature)

- Pair-wise node distances in the input-output space → Lipschitz continuity condition [2, 3].

$$D(f(v_i), f(v_j)) \leq L \cdot d(v_i, v_j)$$

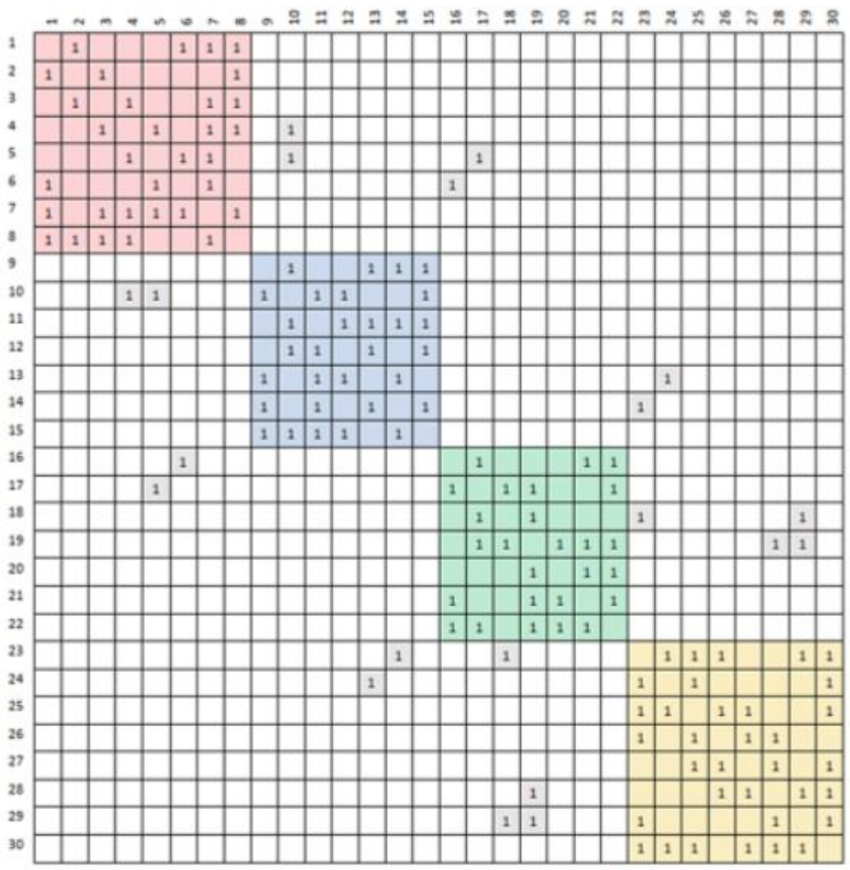
Individual fairness in graph clustering [4]

- Distribute representation (one-hop neighbors) of each node within clusters (max-min)

$$\frac{|\mathcal{C}_k \cap \mathcal{N}_{\mathcal{R}}(i)|}{|\mathcal{C}_k|} = \frac{|\mathcal{N}_{\mathcal{R}}(i)|}{N}, \quad \forall k \in [K], \ \forall i \in [N].$$

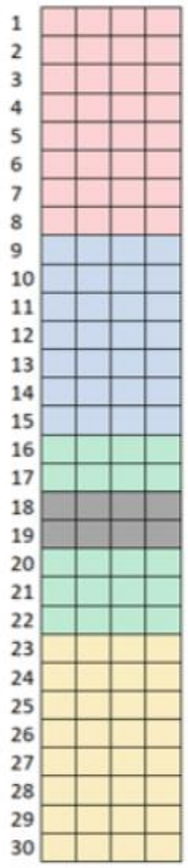
Symmetric Non-negative Matrix Factorization (SNMF) [7]

$$\min_{\boldsymbol{H} \geq 0} \|\boldsymbol{A} - \boldsymbol{H}\boldsymbol{H}^\top\|_F^2, \qquad \boldsymbol{H} \in \mathbb{R}^{n \times k}$$



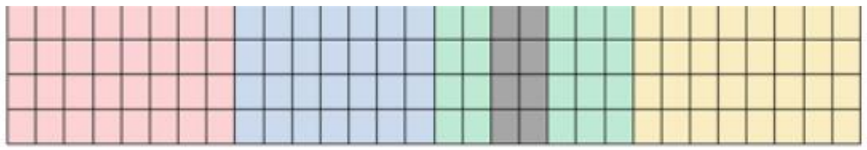
$\boldsymbol{A}$

$\approx$



$\boldsymbol{H}$

$\times$



$\boldsymbol{H}^\top$

$$(A_{ij} > 0) \rightarrow (\boldsymbol{h}_i \boldsymbol{h}_j^\top > 0)$$

$$(A_{ij} = 0) \rightarrow (\boldsymbol{h}_i \boldsymbol{h}_j^\top = 0)$$

Symmetric Non-negative Matrix Factorization (SNMF) [7]

$$\min_{\boldsymbol{H} \geq 0} \|\boldsymbol{A} - \boldsymbol{H}\boldsymbol{H}^\top\|_F^2,$$

$$(A_{ij} > 0) \longrightarrow (\boldsymbol{h}_i \boldsymbol{h}_j^\top > 0)$$

$$(A_{ij} = 0) \longrightarrow (\boldsymbol{h}_i \boldsymbol{h}_j^\top = 0)$$

$$\boldsymbol{A} \in \mathbb{R}^{n \times n}$$

$$\boldsymbol{H} \in \mathbb{R}^{n \times k}$$

Non-negative Matrix Tri-Factorization (NMTF) [8]

$$\min_{\boldsymbol{H}, \boldsymbol{W} \geq 0} \|\boldsymbol{A} - \boldsymbol{H}\boldsymbol{W}\boldsymbol{H}^\top\|_F^2,$$

$$A_{ij} \approx \boldsymbol{h}^{(i)} \boldsymbol{W} \boldsymbol{h}^{(j)^\top}$$

$$\boxed{\boldsymbol{W} \in \mathbb{R}^{k \times k}} \longrightarrow$$ **Interpretability Factor**

iFairNMTF

NMTF → iFair

$$\min_{\boldsymbol{H},\boldsymbol{W}\geq 0} \|\boldsymbol{A} - \boldsymbol{H}\boldsymbol{W}\boldsymbol{H}^\top\|_F^2 + \lambda \mathcal{R}_{\boldsymbol{C}}(\boldsymbol{H}),$$

$$\boldsymbol{C} = \boldsymbol{P} - \boldsymbol{N}$$

$$\mathcal{P}_{i,j} = \begin{cases} 1, & \text{if } g_i \neq g_j \\ 0, & \text{otherwise.} \end{cases}$$

$$P_{ij} = \mathcal{P}_{ij} / \sum_{r=1}^{n} \mathcal{P}_{ir},$$

$$\mathcal{N}_{i,j} = \begin{cases} 1, & \text{if } g_i = g_j \\ 0, & \text{otherwise.} \end{cases}$$

$$N_{ij} = \mathcal{N}_{ij} / \sum_{r=1}^{n} \mathcal{N}_{ir},$$

$$\min_{\boldsymbol{H}} \mathcal{R}_{\boldsymbol{C}} = \sum_{i=1}^{n}\sum_{j=1}^{n} \|\boldsymbol{h}^{(i)} - \boldsymbol{h}^{(j)}\|^2 C_{ij} = \mathrm{Tr}(\boldsymbol{H}^\top \boldsymbol{L}\boldsymbol{H}).$$

$$\text{iFairNMTF} \longrightarrow \mathcal{L} = \mathcal{L}_{\mathcal{F}} + \lambda \mathcal{R}_{\boldsymbol{C}}$$

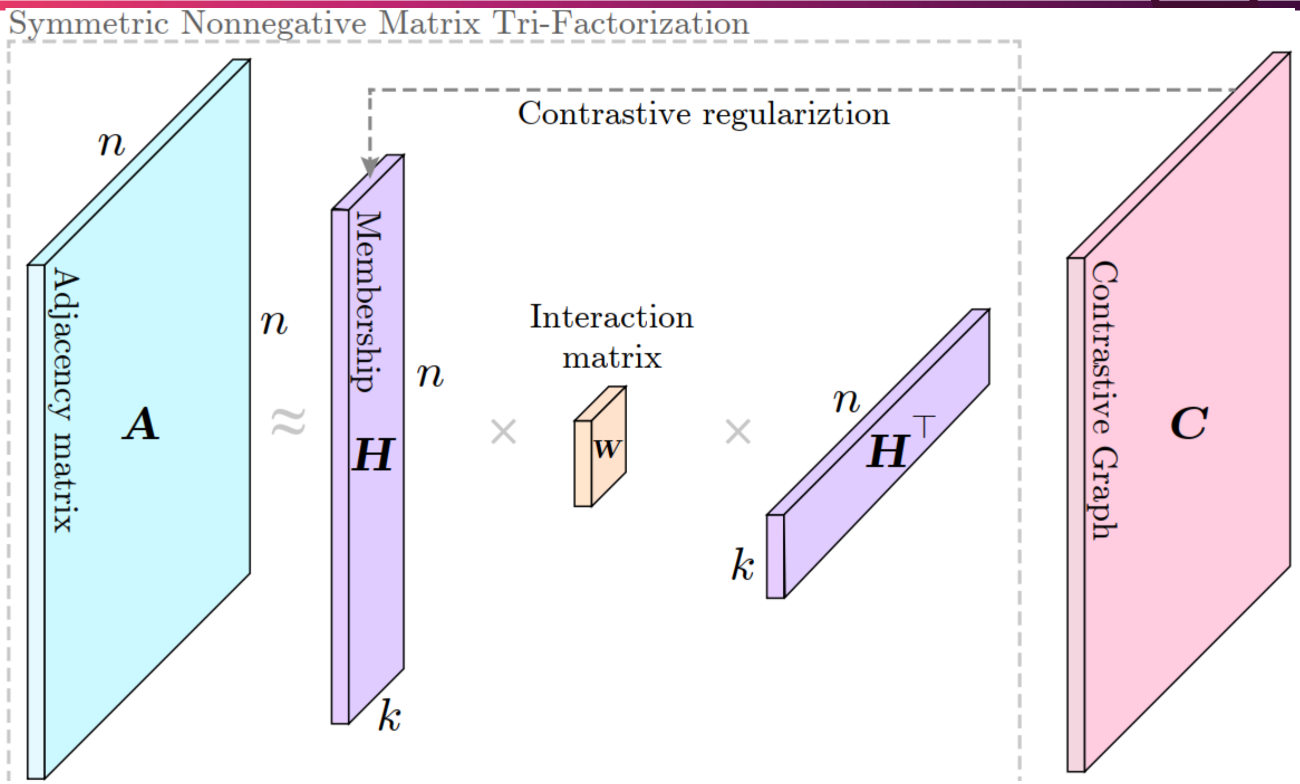
$$\boxed{\min_{\boldsymbol{H},\boldsymbol{W}\geq 0} \|\boldsymbol{A} - \boldsymbol{H}\boldsymbol{W}\boldsymbol{H}^\top\|_F^2 + \lambda \mathrm{Tr}(\boldsymbol{H}^\top \boldsymbol{L}\boldsymbol{H})}$$

iFairNMTF

$$\min_{\boldsymbol{H},\boldsymbol{W}\geq 0} \|\boldsymbol{A} - \boldsymbol{H}\boldsymbol{W}\boldsymbol{H}^\top\|_F^2 + \lambda \mathrm{Tr}(\boldsymbol{H}^\top \boldsymbol{L}\boldsymbol{H}),$$

$$\boldsymbol{H} \leftarrow \boldsymbol{H} \odot \left(\frac{\boldsymbol{A}^\top \boldsymbol{H}\boldsymbol{W} + \boldsymbol{A}\boldsymbol{H}\boldsymbol{W}^\top + \lambda \boldsymbol{L}^- \boldsymbol{H}}{\boldsymbol{H}\boldsymbol{W}^\top \boldsymbol{H}^\top \boldsymbol{H}\boldsymbol{W} + \boldsymbol{H}\boldsymbol{W}\boldsymbol{H}^\top \boldsymbol{H}\boldsymbol{W}^\top + \lambda \boldsymbol{L}^+ \boldsymbol{H}} \right)^{\frac{1}{4}}$$

$$\boldsymbol{W} \leftarrow \boldsymbol{W} \odot \frac{\boldsymbol{H}^\top \boldsymbol{A}\boldsymbol{H}}{\boldsymbol{H}^\top \boldsymbol{H}\boldsymbol{W}\boldsymbol{H}^\top \boldsymbol{H}}$$



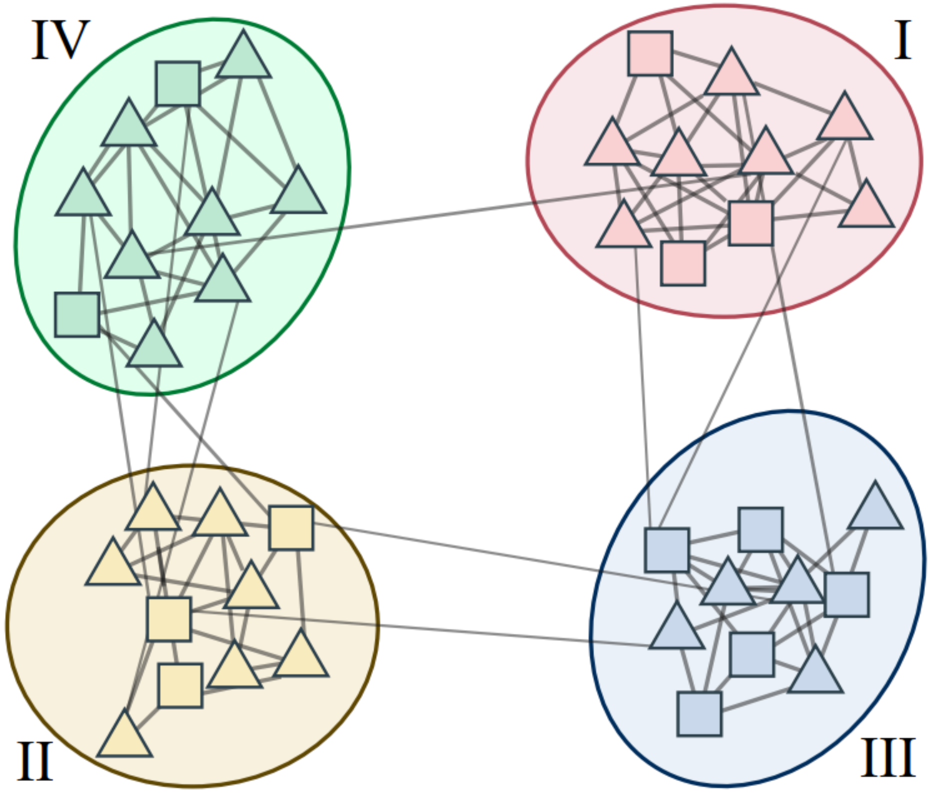
Algorithm 1 Individual Fair Nonnegative Matrix Tri-Factorization (iFairNMTF)

Input: adjacency matrix $\boldsymbol{A}$, group set g, latent factor k, trade-off parameter λ;
Output: cluster assignment M;

1: Construct the contrastive graph $\boldsymbol{C}$ according to (7);
2: **while** convergence not reached **do**
3: Update cluster-membership matrix $\boldsymbol{H}$ according to (13);
4: Update cluster-interaction matrix $\boldsymbol{W}$ according to (16);
5: **end while**
6: Calculate cluster assignment $M_i \leftarrow \arg\max(\boldsymbol{h}^{(i)}), \forall i \in \{1, \ldots, n\}$
7: **return** cluster-membership matrix $\boldsymbol{H}$ and cluster-interaction matrix $\boldsymbol{W}$;

iFairNMTF (Interpretability)

$$\min_{\boldsymbol{H},\boldsymbol{W}\geq 0} \|\boldsymbol{A} - \boldsymbol{H}\boxed{\boldsymbol{W}}\boldsymbol{H}^\top\|_F^2 + \lambda \mathrm{Tr}(\boldsymbol{H}^\top \boldsymbol{L}\boldsymbol{H})$$



	I	II	III	IV
I	2.9	0	4E-12	7E-13
II	0	1.10	5E-08	5E-04
III	4E-12	5E-08	2.74	0
IV	7E-13	5E-04	0	1.91

$\boldsymbol{W}$

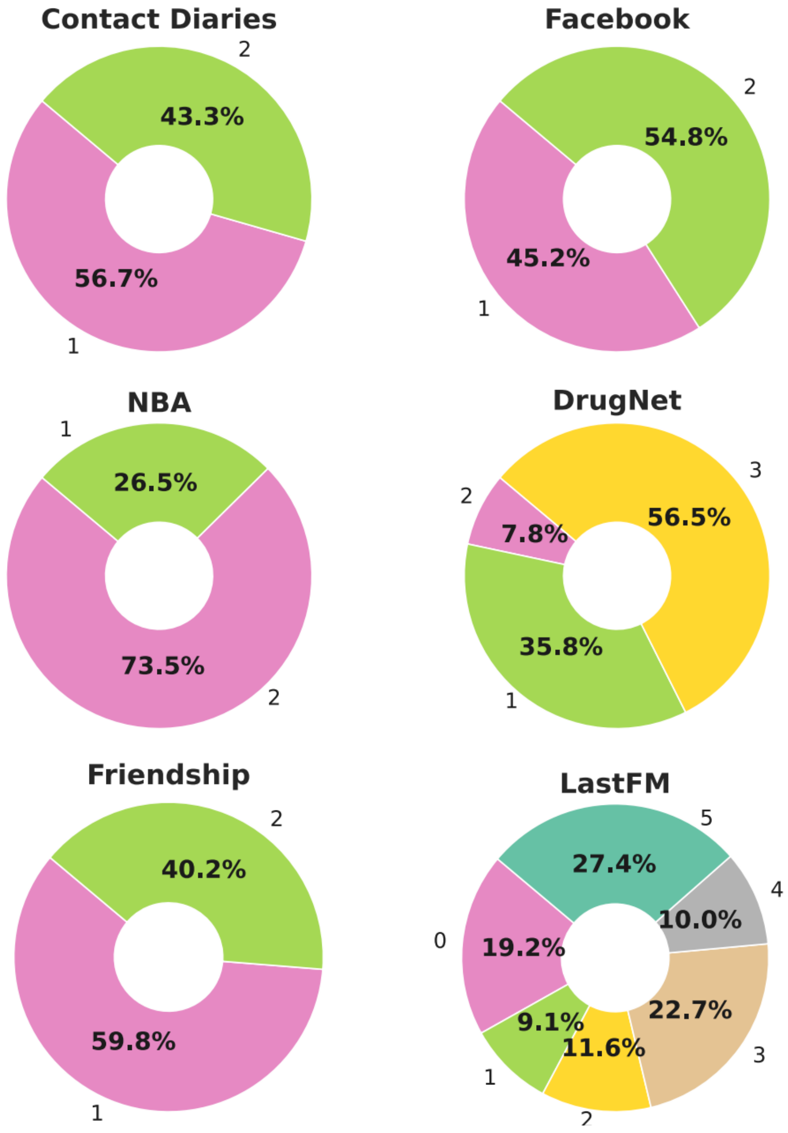
Fig. 4: Interpretability of $\boldsymbol{W}$ factor for a 40-node graph divided to 4 clusters. Shapes indicate groups.

Dataset Statistics

Table 1: Dataset statistics. $|V|$, $|E|$, $|g|$, indicate the number of nodes, edges, and groups.

Network	$\|V\|$	$\|E\|$	Sensitive Attribute	$\|g\|$	Edge Density	Homophily
SBM	2,000	267,430	attribute	2	0.133	0.82
	5,000	978,959	attribute	2	0.078	0.82
	10,000	2,603,190	attribute	2	0.052	0.82
Diaries	120	348	gender	2	0.048	0.61
Friendship	134	406	gender	2	0.049	0.60
Facebook	156	1,437	gender	2	0.120	0.57
DrugNet	293	284	ethnicity	3	0.014	0.88
NBA	403	8,285	nationality	2	0.102	0.72
LastFM	7,624	27,806	country	6	0.001	0.92

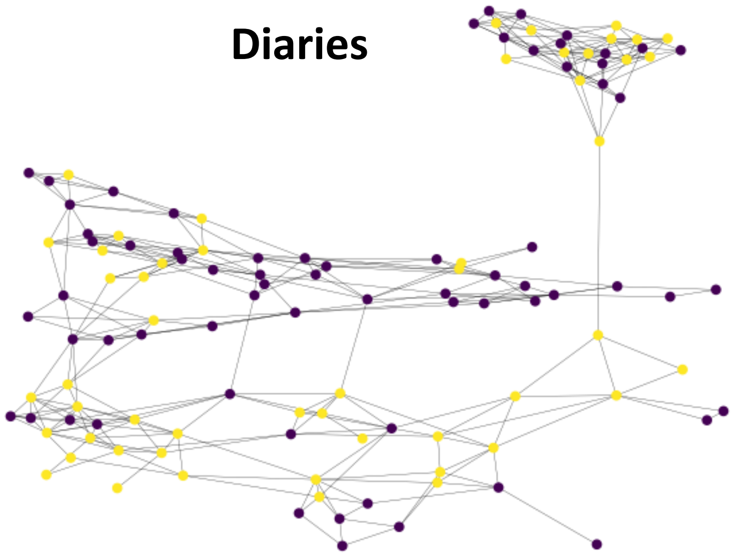
$$\rho = \frac{2|E|}{|V|(|V|-1)}$$



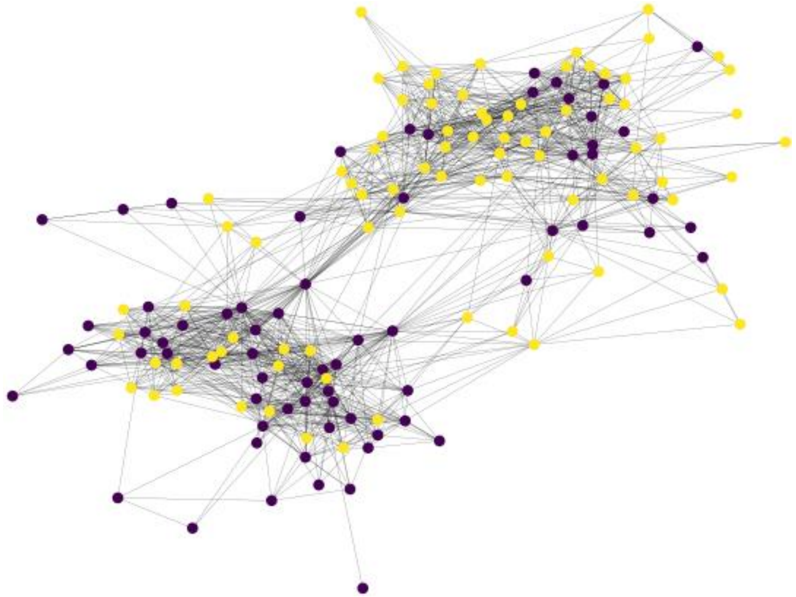
Group Imbalance Ratio as per sensitive-attribute

Datasets: Group Imbalance Ratio

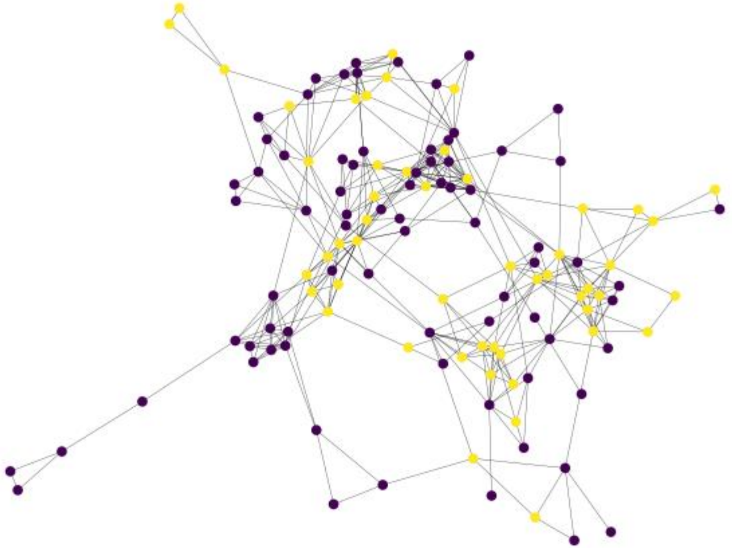
Diaries

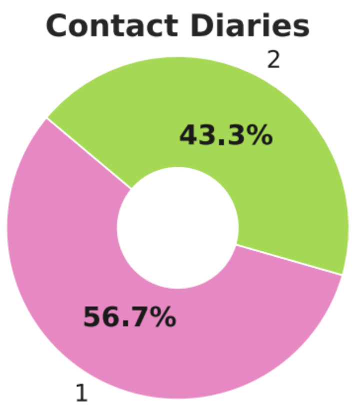


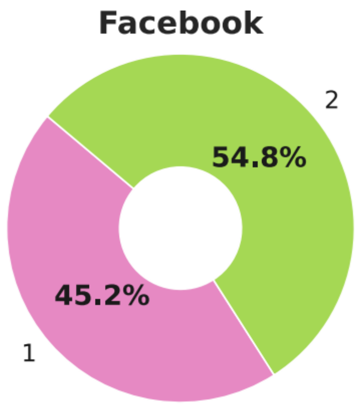
Facebook

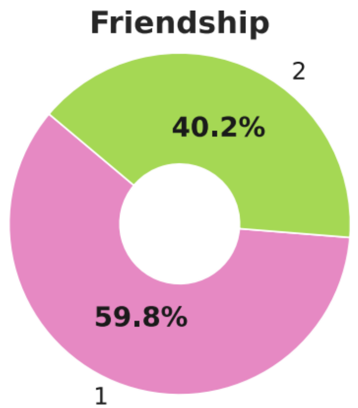


Friendship



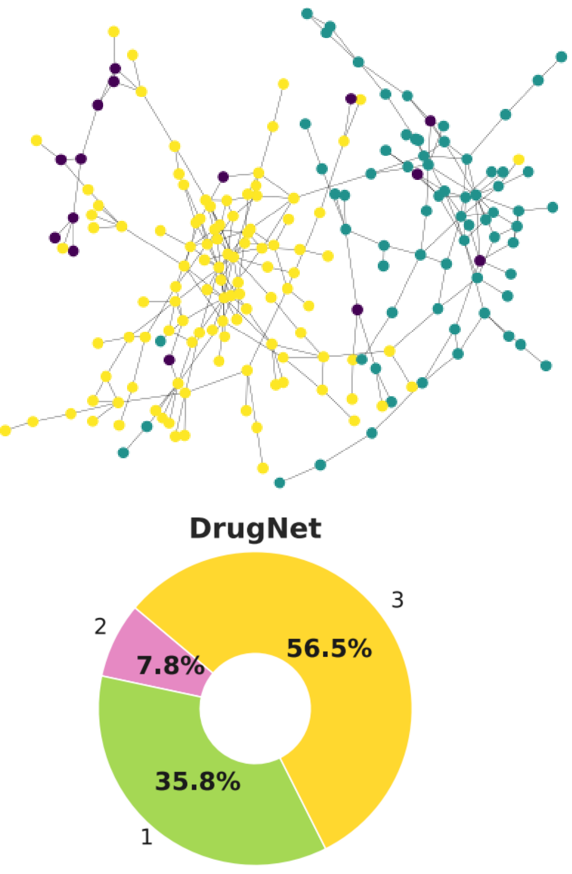




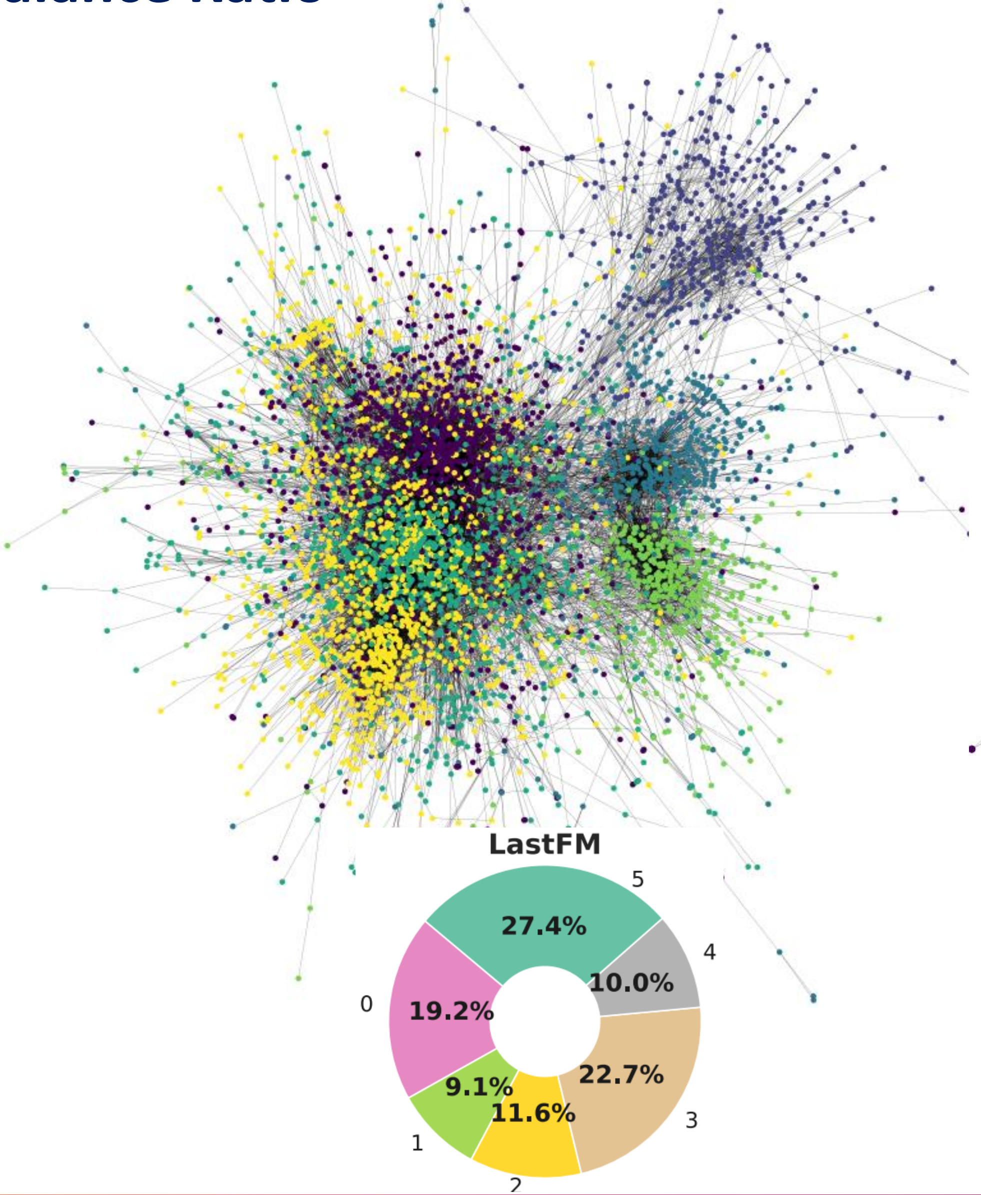


Datasets: Group Imbalance Ratio

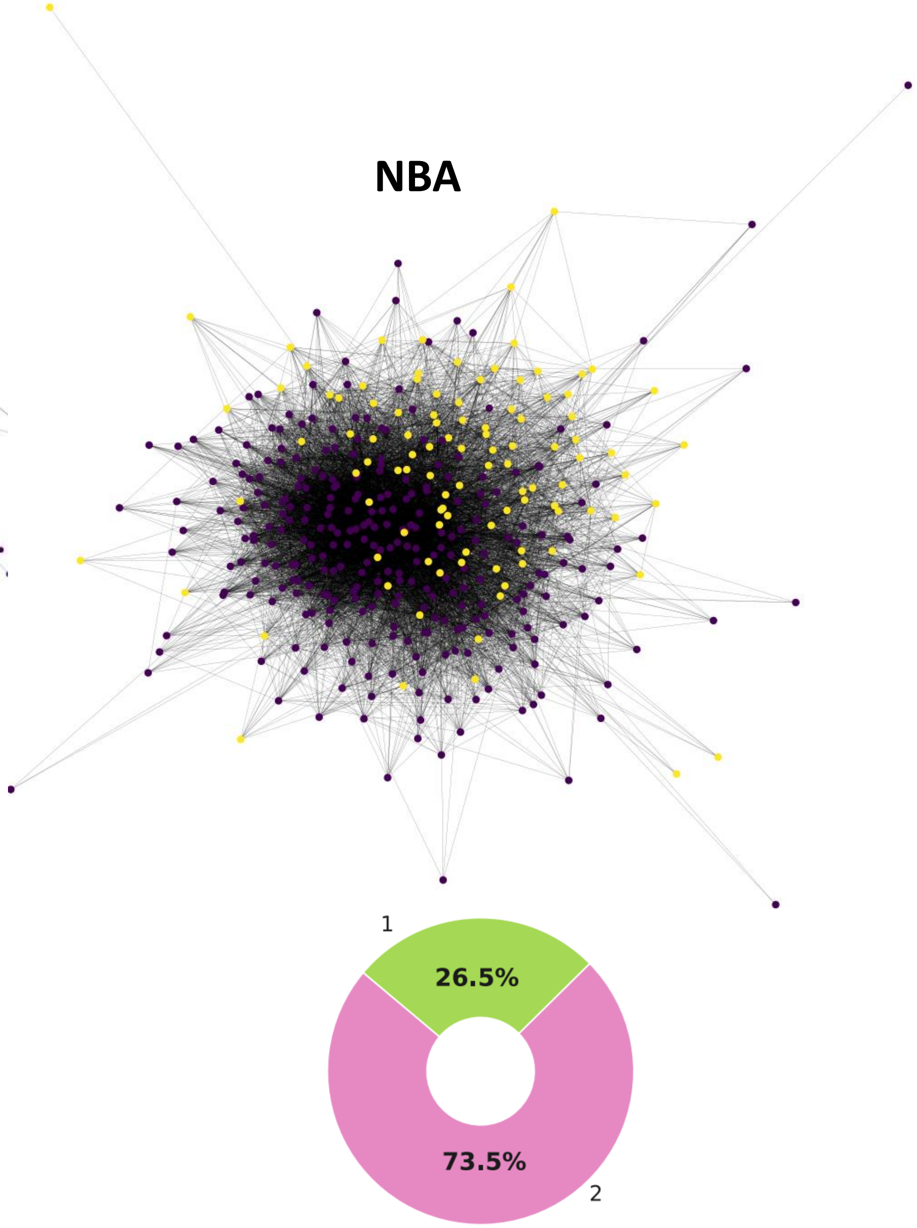
DrugNet



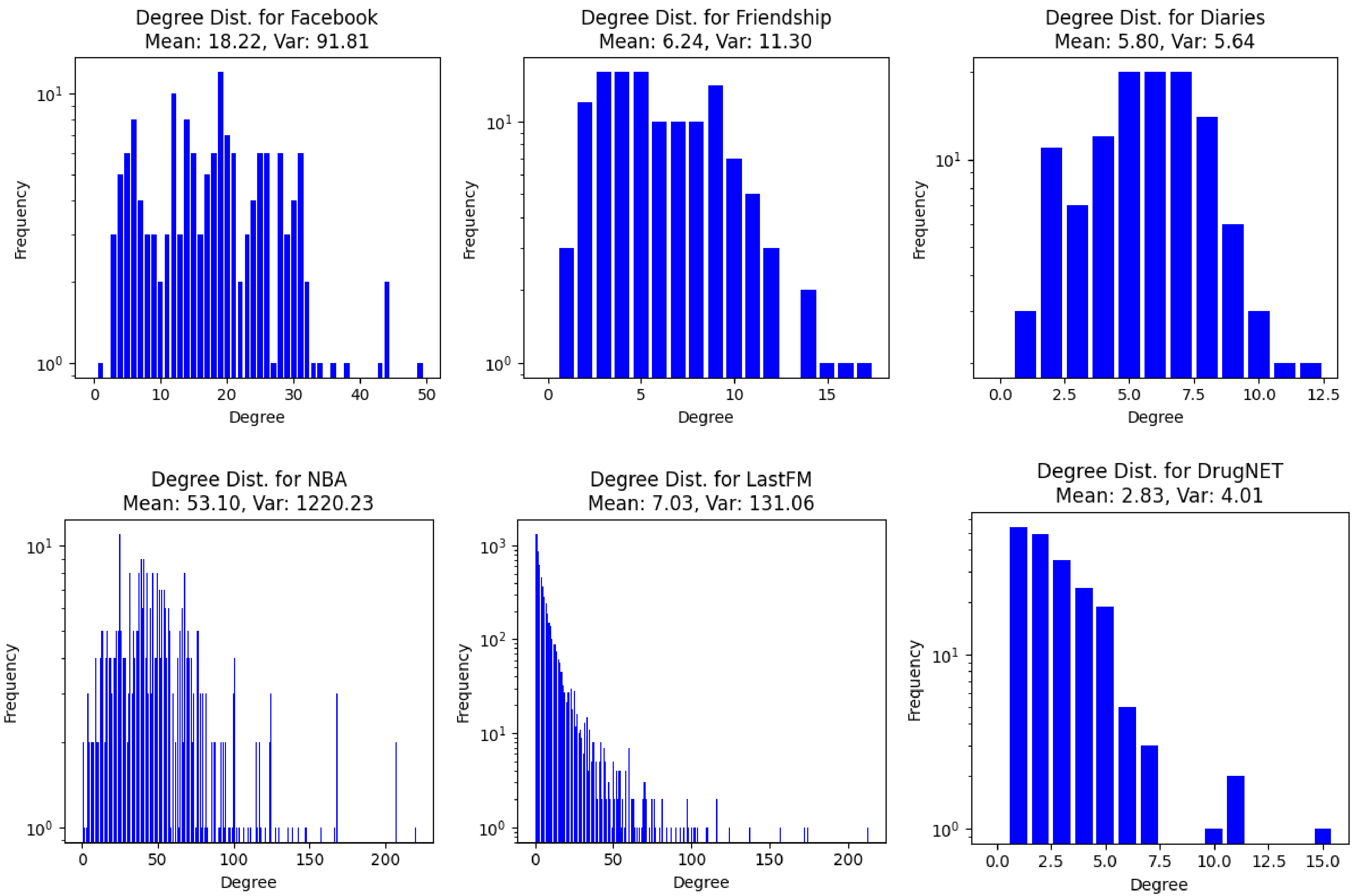
LastFM



NBA



Datasets: Node degree distributions



Metrics

Average Individual Balance [4, 9]

$$\bar{\delta} = \frac{1}{n}\sum_{i=1}^{n}\delta_i$$

$$\delta_i = min_{k,l\in\{1,\dots,K\}}\frac{|C_k : \cap N_{v_i}|}{|C_l : \cap N_{v_i}|}$$

Average Group Balance [1, 9]

$$B = \frac{1}{k}\sum_{l=1}^{k} Balance(C_l),$$

$$Balance(C_l) = \min_{s\neq s'\in[m]}\frac{|V_s \cap C_l|}{|V_{s'} \cap C_l|}$$

Cohesion (Modularity) [10]

$$Q = \frac{1}{|E|}\sum_{i,j}\left(A_{ij} - \frac{deg(i)deg(j)}{|E|}\right)\delta(c_i, c_j)$$

i(individual)-Fairness

Individual balance [4, 9]

$$\overline{\delta} = \frac{1}{n}\sum_{i=1}^{n}\delta_i \longrightarrow \delta_i = min_{k,l\in\{1,\ldots,K\}} \frac{|C_k : \cap N_{v_i}|}{|C_l : \cap N_{v_i}|}$$

- Fixed number of clusters k=5.
- Best value of λ for our model according to slide 19.

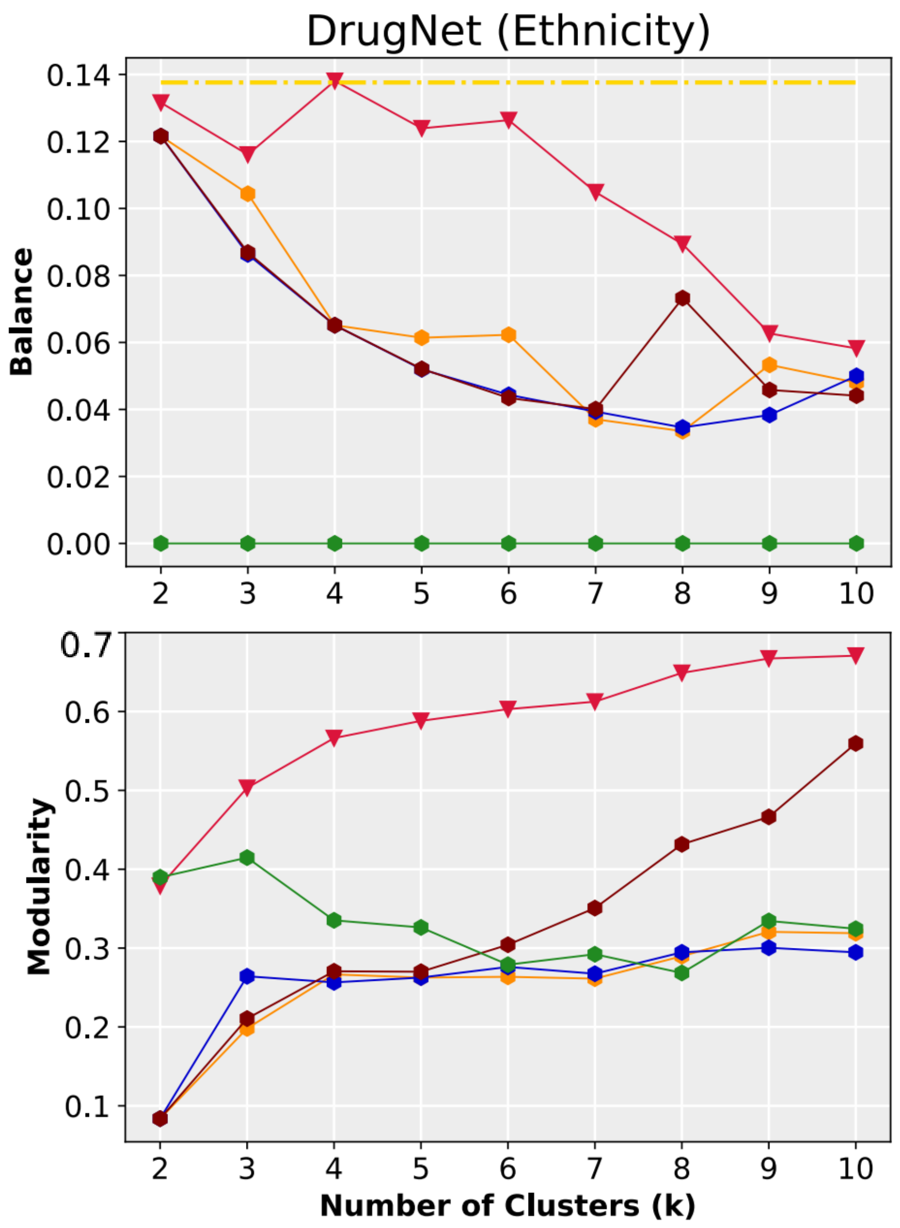
$\overline{\delta}$ individual balance score				
Network	**FairSC**	**SC**	**iFairSC**	**iFairNMTF**
Contact Diaries	0.123	0.347	0.166	**0.426**
Facebook	0.011	0.010	0.000	**0.348**
Friendship	0.028	0.033	0.031	**0.519**
DrugNet	0.000	0.016	0.000	**0.339**
NBA	0.000	0.000	0.000	**0.323**
LastFM	0.000	0.001	0.000	**0.020**

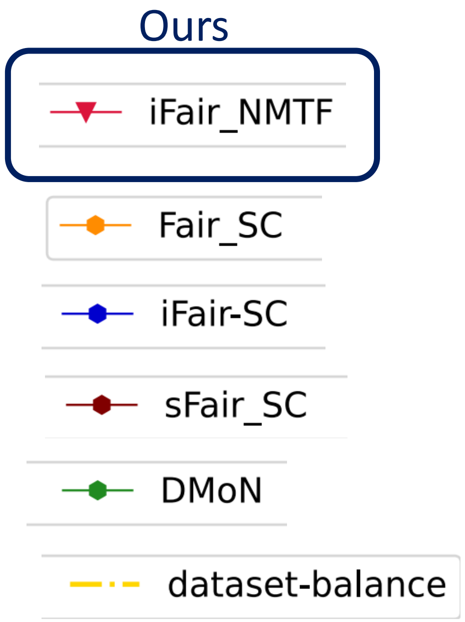
Fairness-Cohesion

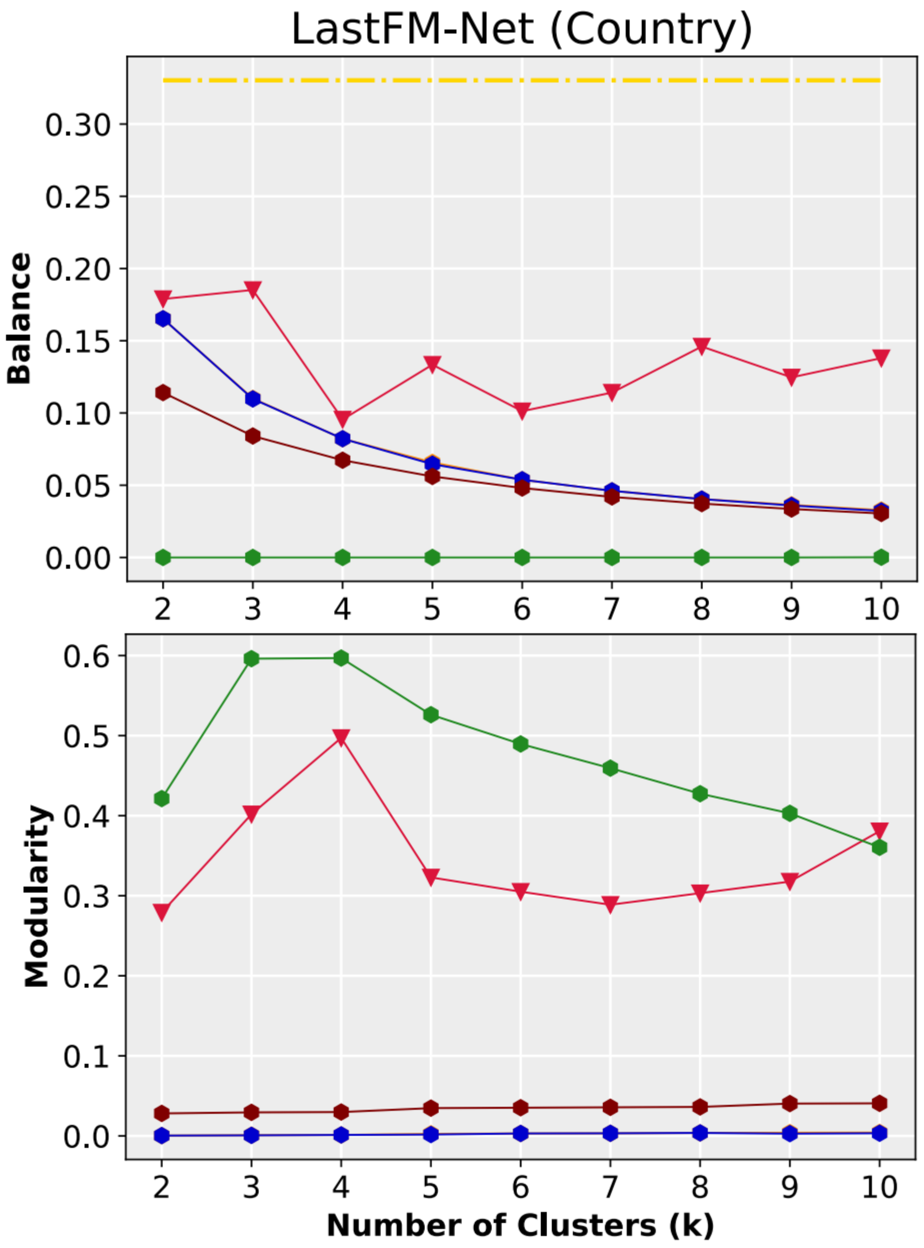
Fixed number of clusters k=5. Best value of λ for our model according to slide 19. Best B is **bold-underlined** and best Acc/Q with **boldfaced gray**

Network	FairSC		sFairSC		iFairSC		DMoN		iFairNMTF	
	B	Q	B	Q	B	Q	B	Q	B	Q
Diaries	0.708	0.612	**0.809**	**0.684**	0.699	0.647	0.263	0.145	0.648	0.640
Facebook	0.327	0.449	**0.602**	0.500	0.330	0.448	0.268	0.048	0.514	**0.509**
Friendship	0.391	0.483	0.485	0.627	0.374	0.392	0.183	0.140	**0.631**	**0.669**
DrugNet	0.052	0.263	0.052	0.270	0.061	0.263	0.000	0.326	**0.124**	**0.588**
NBA	0.083	0.000	**0.323**	0.113	0.072	0.000	0.036	0.057	0.286	**0.150**
LastFM	0.065	0.003	0.056	0.035	0.066	0.002	0.000	0.526	**0.069**	**0.600**
	B	Acc	B	Acc	B	Acc	B	Acc	B	Acc
SBM-2K	0.575	0.588	–	–	0	0.799	–	–	**0.953**	**0.958**
SBM-5K	**0.995**	**0.998**	–	–	0	0.799	–	–	0.941	0.962
SBM-10K	0.999	0.999	–	–	0	0.600	–	–	**1**	**1**

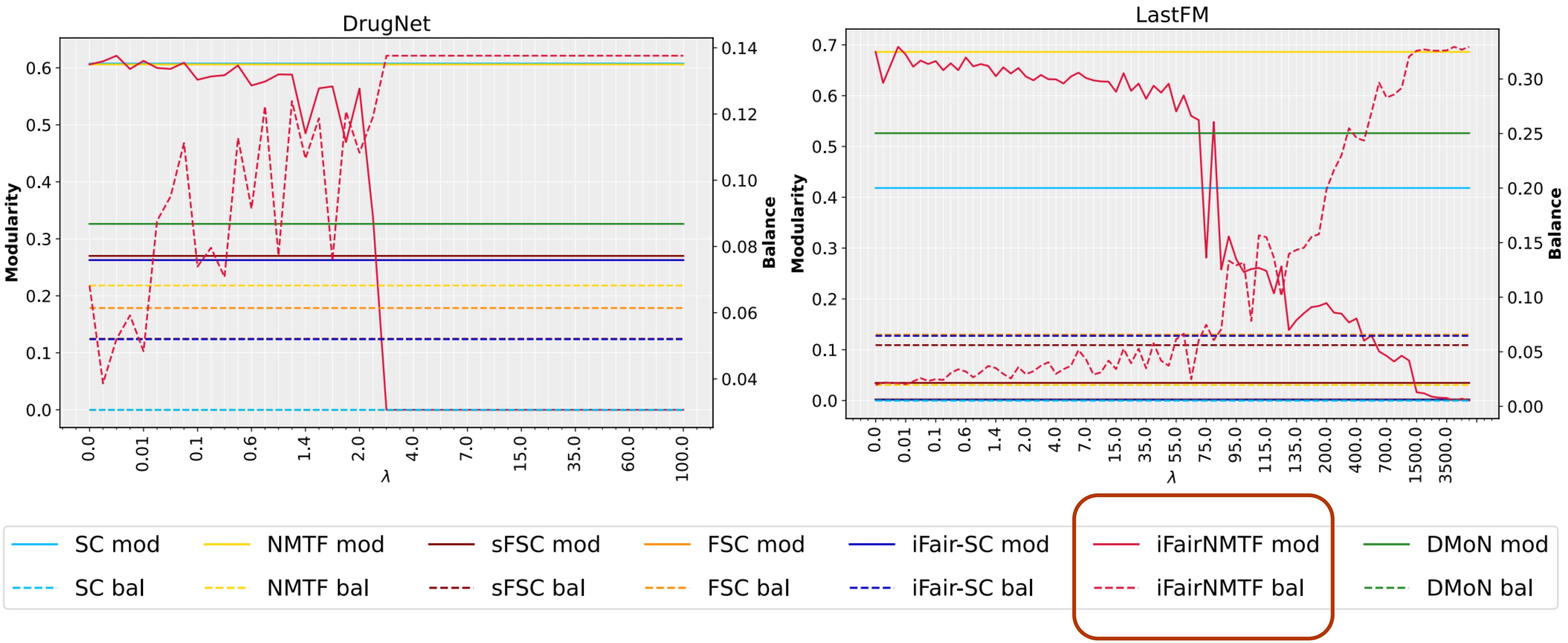
Fairness-Cohesion







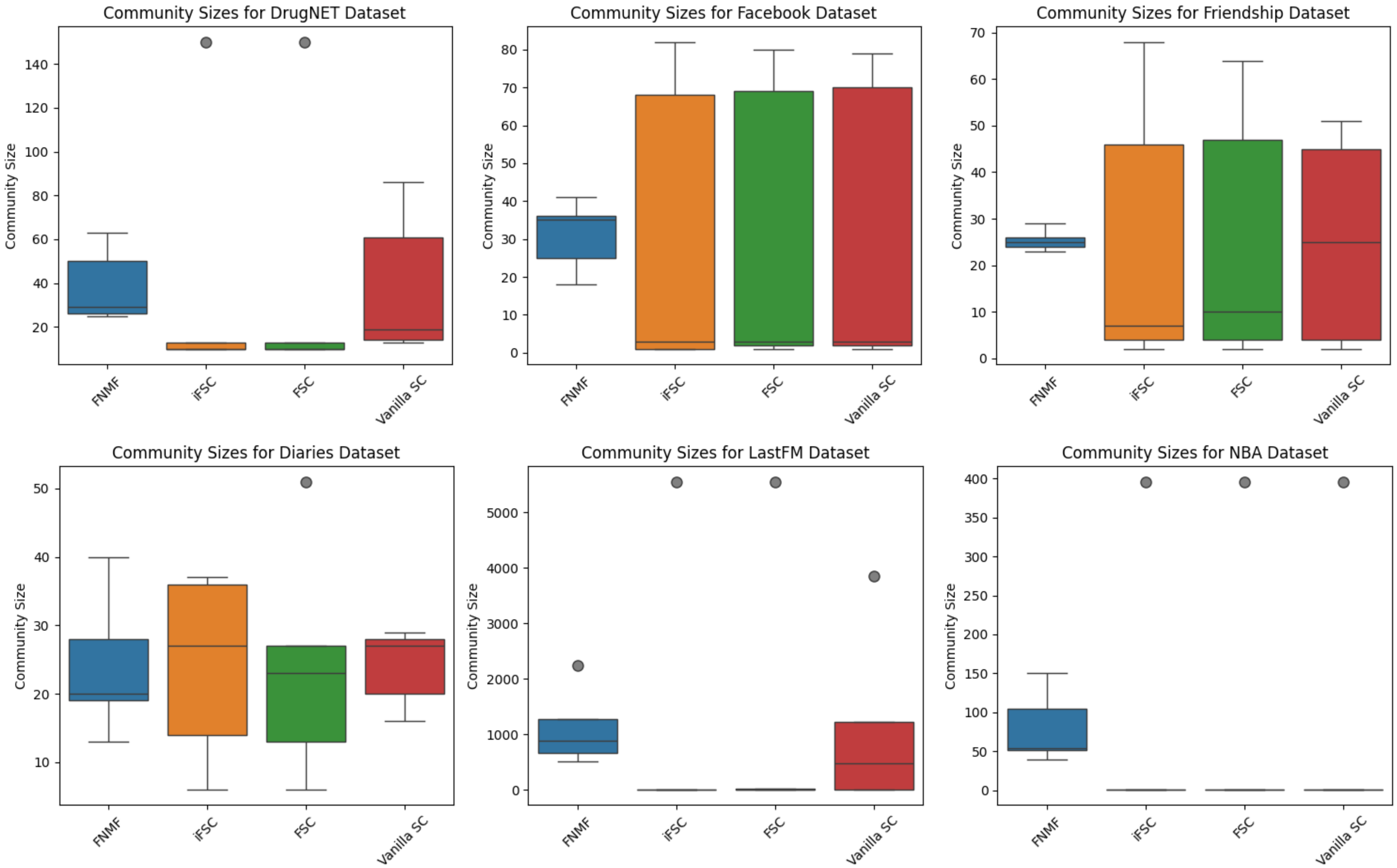
Fairness-Cohesion (Parameter Selection)



Choice of λ is problem/dataset specific

3. Experiments

Cluster-sizes



- **Flexible (adjustable)** degree of **trade-off** between individual **fairness and cohesion** (clustering objective) compared to existing **hard-constrained** graph-clustering frameworks.
- **Contrastive regularization**: takes Lipschitz condition (individual fairness) and also group membership of nodes into account.
- **Individual-fair** by definition but also **group-fair by design**.
- The **first work** to incorporate fairness in **NMF** framework.
- It enables users/policy-makers to enforce required degree of fairness in compromise to accuracy.

Future Outlook

- Multi-objective techniques to effectively balance fairness and cohesion objectives.
- Extend to group fairness notions and fusion ideas.
- Investigating further network characteristic evaluations to uncover clustering/fairness correlations.

5. References

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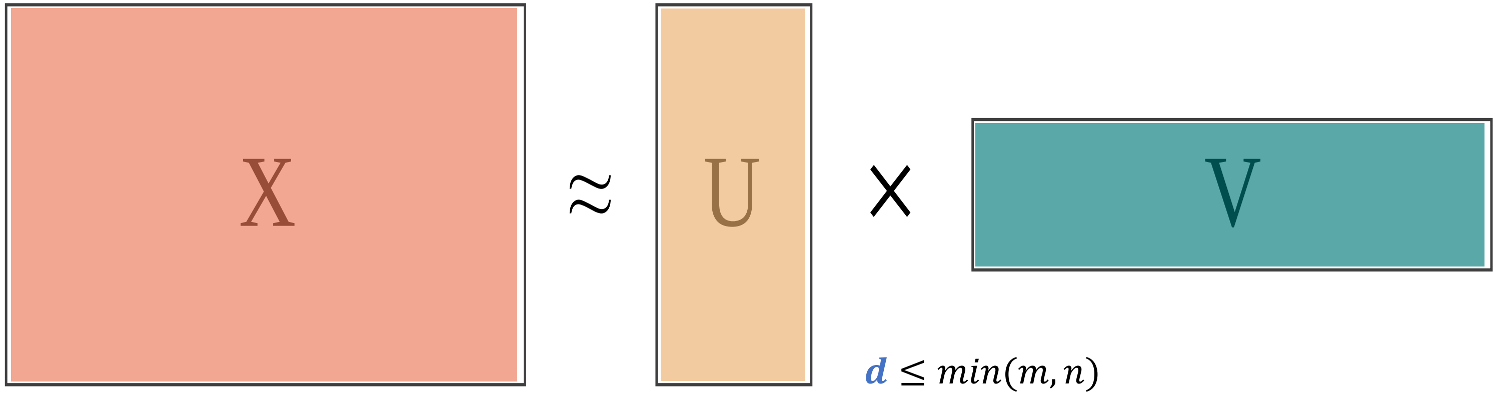
For code and Supplemental material,

https://github.com/SiamakGhodsi/iFairNMTF

Scan me!

Thank you for your attention

Introducing NMF



$d \leq min(m, n)$

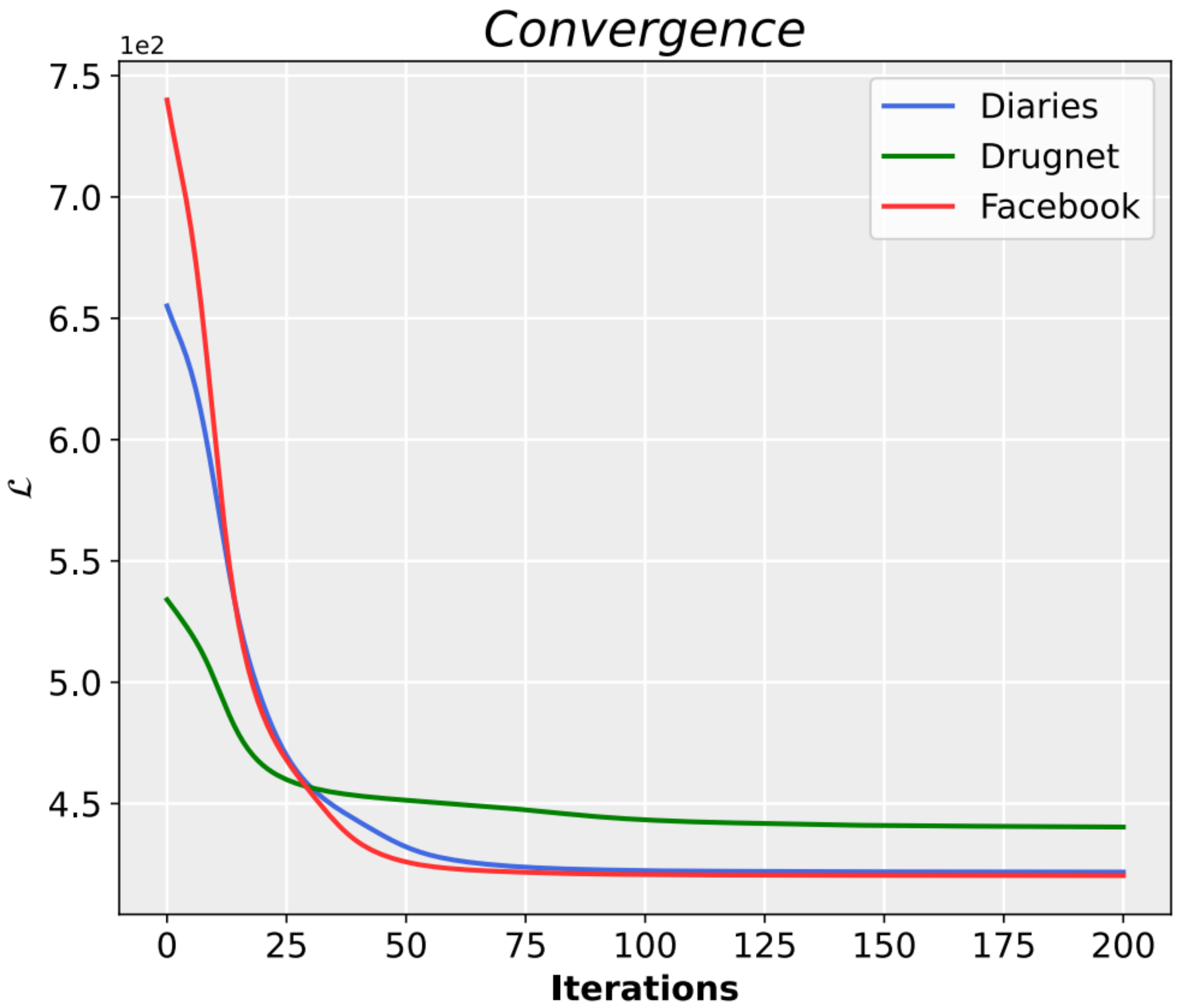
$\boldsymbol{X} \in \mathbb{R}^{m \times n}$ $\boldsymbol{U} \in \mathbb{R}^{m \times d}$ $\boldsymbol{V} \in \mathbb{R}^{d \times n}$

Feature Learning	Dataset	Feature Factor Basis Matrix	Representation Reconstruct Weight
Face Recognition	Dataset	Basis Images Eigen Faces	Image Reconstruct Coefficient
Clustering	Dataset	Centroids	Cluster Indicator

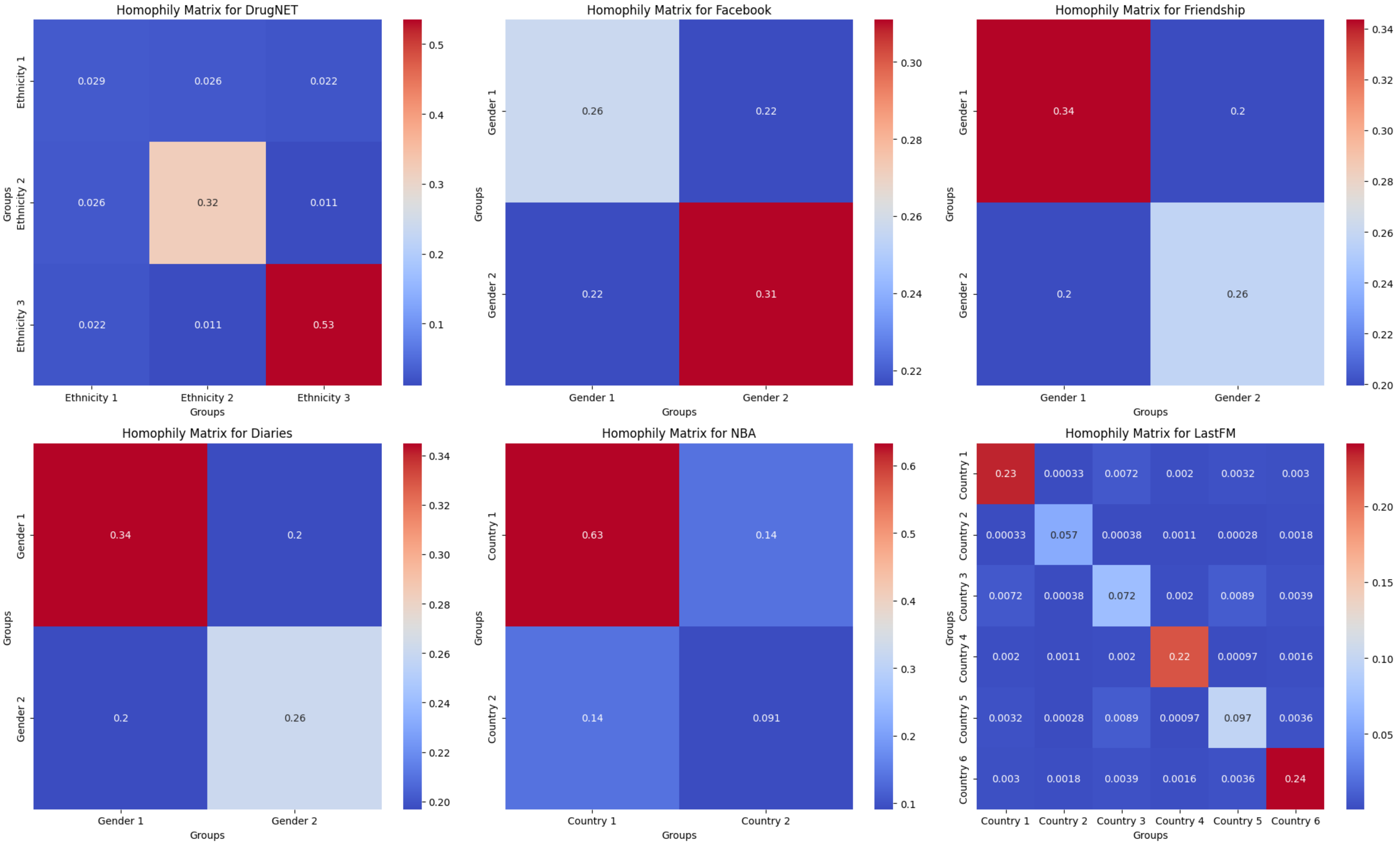
Loss Convergence

$$\mathcal{L} = \mathcal{L}_{\mathfrak{F}} + \lambda \mathfrak{R}_{\boldsymbol{C}}$$

$$\min_{\boldsymbol{H},\boldsymbol{W}\geq 0} \|\boldsymbol{A} - \boldsymbol{H}\boldsymbol{W}\boldsymbol{H}^\top\|_F^2 \quad + \lambda \mathrm{Tr}(\boldsymbol{H}^\top \boldsymbol{L}\boldsymbol{H}),$$



Datasets: Homophily And Heterophily



Datasets: Network Factors

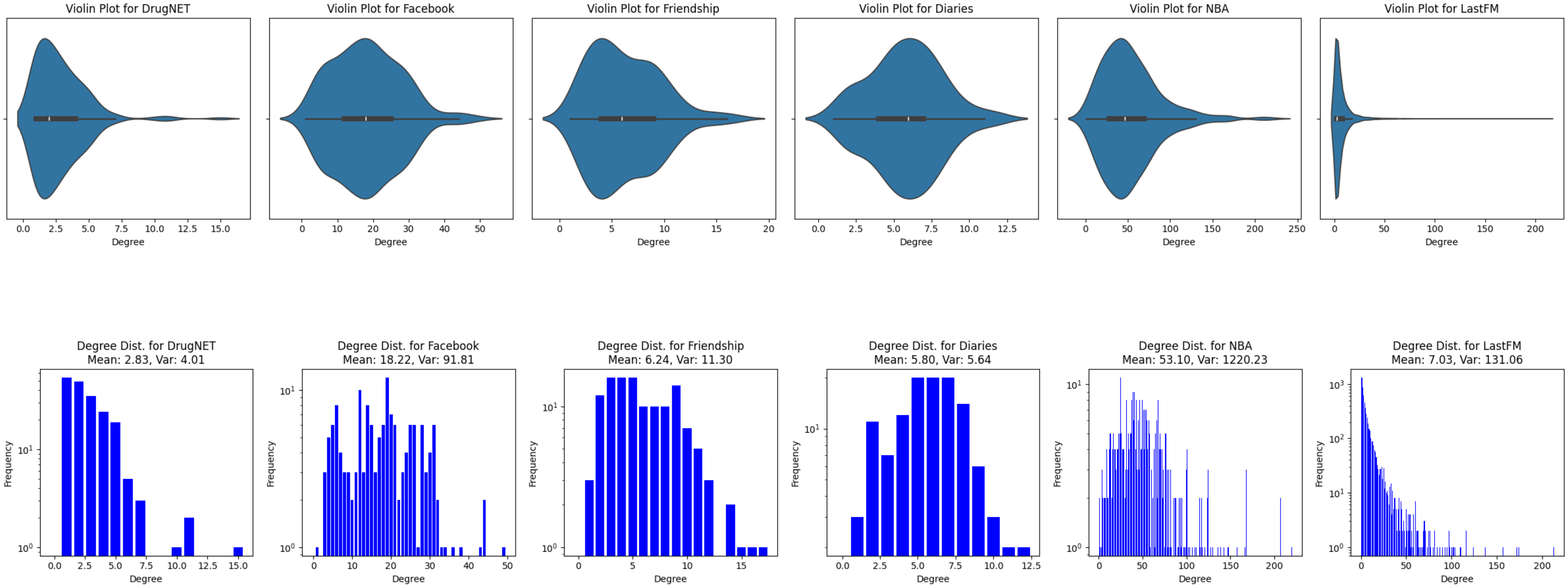
- Degree mean
- Clustering coefficient
- Assortativity coefficient

- Heterophily index
- Homophily index
- Degree centrality
- Betweenness centrality

- Closeness centrality
- Edge density
- Graph sparsity

	Degree mean	Clustering coefficient	Assortativity coefficient	Homophily	Heterophily	Degree centrality	Betweenness centrality	Closeness centrality	Edge density	Graph sparsity
DrugNET	2.83	0.14	0.79	0.88	0.12	0.01	0.03	0.15	0.01	0.99
Facebok	18.22	0.62	0.13	0.57	0.43	0.12	0.01	0.41	0.12	0.88
Friendship	6.24	0.54	0.20	0.60	0.40	0.05	0.02	0.25	0.05	0.95
Diaries	5.80	0.45	0.21	0.61	0.39	0.05	0.04	0.19	0.05	0.95
NBA	53.10	0.34	0.22	0.72	0.28	0.13	0.00	0.52	0.13	0.87
LastFM	7.03	0.21	0.90	0.92	0.08	0.00	0.00	0.19	0.00	1.00

Datasets: Node degree distributions



Correlation of individual fairness to network factors