

Towards Cohesion-Fairness Harmony: Contrastive Regularization in Individual Fair Graph Clustering

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Siamak Ghodsi¹, Seyed Amjad Seyedi², and Eirini Ntoutsi³

¹ "L3S Research Center, Leibniz University Hannover, Germany "

² "University of Kurdistan, Sanandaj, Iran "

³ "RI CODE, University of the Bundeswehr Munich, Germany "



Speaker: Siamak Ghodsi
ghodsi@l3s.de

1. Introduction

- Example
- Problem Formulation
- Fair Graph Clustering
 - Individual Fairness

2. Methodology

- NMTF
- iFairNMTF
- Interpretability

3. Experiments

- Datasets
- Metrics
- Loss Convergence
- Fairness-Cohesion

4. Conclusions

- Take-away

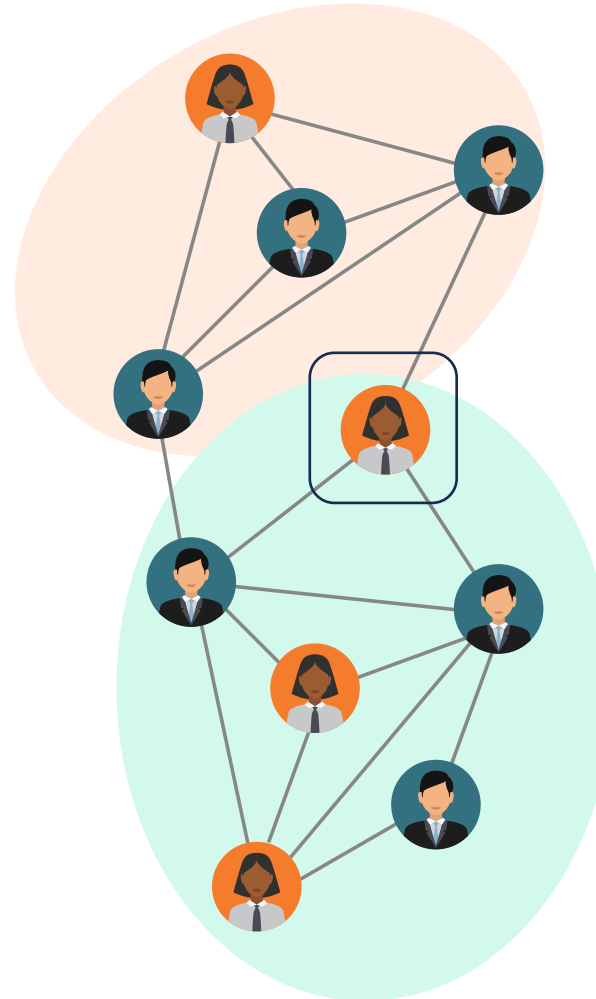
1. Introduction

2

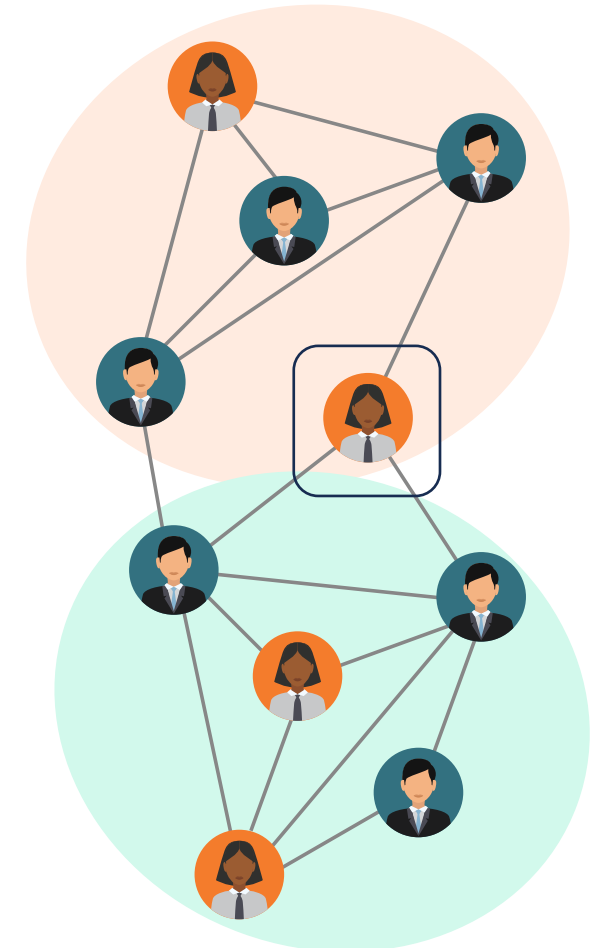
Example

- Teacher with 10 students, 4 female 6 male.
- First day of school.
- You know only the gender and friendship of students from their last year records.
- How to divide: 2 teams (clusters) for classroom assignments?

Clustering #1



Clustering #2



Problem Formulation

Inputs

- **Undirected graph** $\mathcal{G} = (V, E)$ where $V = \{v_1, v_2, \dots, v_n\}$
- **No self-loops** $E \subseteq V \times V$ is a binary set of edges
- **Adjacency matrix** $\mathbf{A} \in \mathbb{R}^{n \times n}$ encodes edge information
- **Sensitive attribute** V_S $V = \dot{\cup}_{s \in [m]} V_s$

Output

- **Non-overlapping clustering** V into $k \geq 2$ $\longrightarrow V = C_1 \dot{\cup} \dots \dot{\cup} C_k$

Fair graph clustering: Individual Fairness

Individual fairness (in i.i.d. data literature)

identically independently
distributed data

- Pair-wise node distances in the input-output space \rightarrow Lipschitz continuity condition [2, 3].

$$D(f(v_i), f(v_j)) \leq L \cdot d(v_i, v_j)$$

Individual fairness in graph clustering [4]

- Distribute representation (one-hop neighbors) of each node within clusters (max-min)

$$\delta_i = \min_{k,l \in \{1, \dots, K\}} \frac{|C_k \cap N_{v_i}|}{|C_l \cap N_{v_i}|}$$

$$Balance(C_l) = \min_{s \neq s' \in [m]} \frac{|V_s \cap C_l|}{|V_{s'} \cap C_l|}$$

1. Introduction

5

Example contd.

$$\bar{\delta} = 0.141$$

$$B = 0.33$$

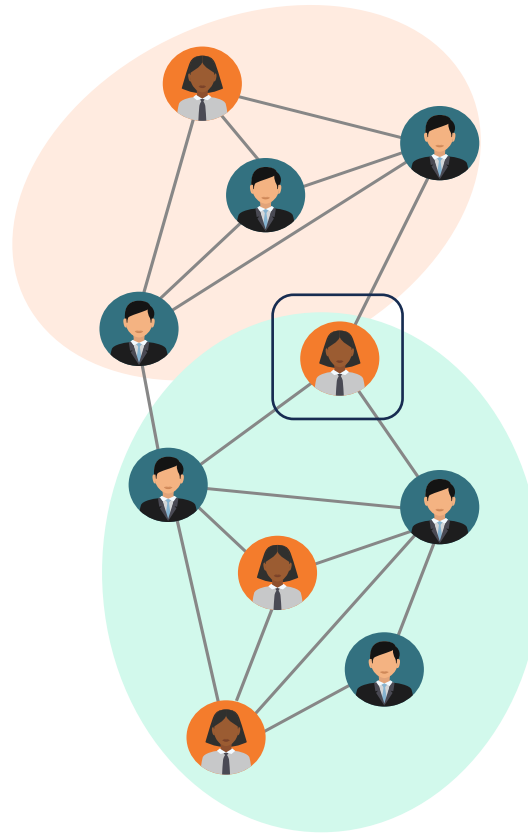
B

Group well-being ?

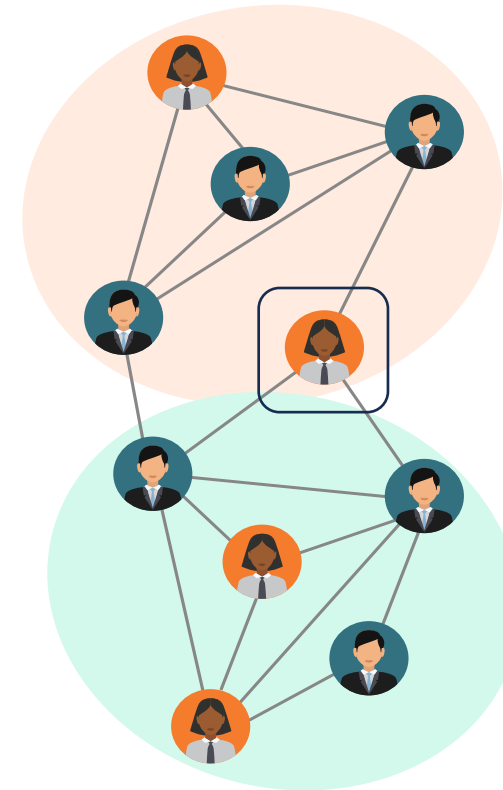
$\bar{\delta}$

Individual links ?

Clustering #1



Clustering #2



$$\bar{\delta} = 0.174 \uparrow$$

$$B = 0.66 \uparrow$$

❑ What about clustering quality? C#1 is better or C#2 ?

❑ What should be the **clustering quality vs fairness** ?

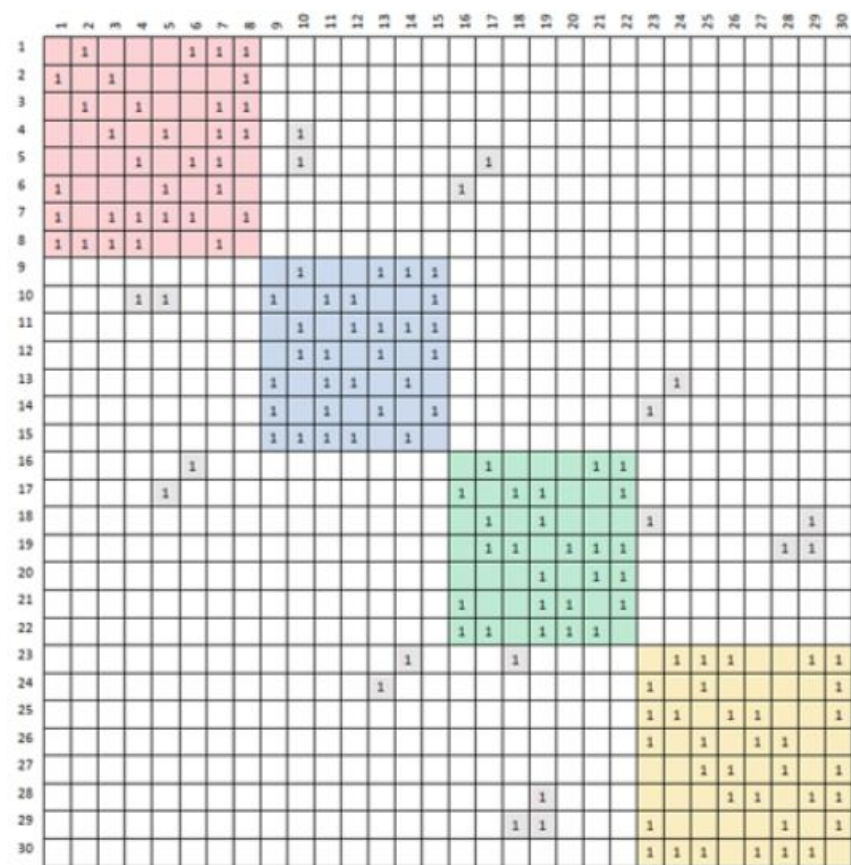
2. Methodology

6

Symmetric NMF [7]

$$\min_{H \geq 0} \|A - HH^T\|_F^2,$$

$$H \in \mathbb{R}^{n \times k}$$



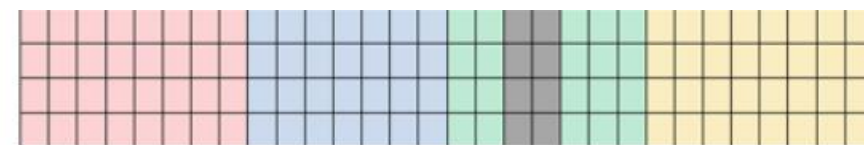
A

\approx



H

\times



H^T

$$(A_{ij} > 0)$$

$$(A_{ij} = 0)$$



$$(h_i h_j^T > 0)$$

$$(h_i h_j^T = 0)$$

Symmetric NMF

$$\min_{\mathbf{H} \geq 0} \|\mathbf{A} - \mathbf{H}\mathbf{H}^\top\|_F^2, \quad \begin{matrix} (A_{ij} > 0) \\ (A_{ij} = 0) \end{matrix} \longrightarrow \begin{matrix} (\mathbf{h}_i \mathbf{h}_j^\top > 0) \\ (\mathbf{h}_i \mathbf{h}_j^\top = 0) \end{matrix}$$

NMTF (Tri-Factorization) [8]

$$\min_{\mathbf{H}, \mathbf{W} \geq 0} \|\mathbf{A} - \mathbf{H}\mathbf{W}\mathbf{H}^\top\|_F^2, \quad A_{ij} \approx \mathbf{h}^{(i)} \mathbf{W} \mathbf{h}^{(j)\top}$$

$$\mathbf{A} \in \mathbb{R}^{n \times n}$$

$$\mathbf{H} \in \mathbb{R}^{n \times k}$$

$$\mathbf{W} \in \mathbb{R}^{k \times k}$$



Interpretability Factor

iFairNMTF

NMTF



iFair

$$\min_{\mathbf{H}, \mathbf{W} \geq 0} \|\mathbf{A} - \mathbf{H}\mathbf{W}\mathbf{H}^\top\|_F^2 + \lambda \mathcal{R}_C(\mathbf{H}),$$

$$\mathbf{C} = \mathbf{P} - \mathbf{N}$$

$$\mathcal{P}_{i,j} = \begin{cases} 1, & \text{if } g_i \neq g_j \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathcal{N}_{i,j} = \begin{cases} 1, & \text{if } g_i = g_j \\ 0, & \text{otherwise.} \end{cases}$$

$$P_{ij} = \mathcal{P}_{ij} / \sum_{r=1}^n \mathcal{P}_{ir},$$

$$N_{ij} = \mathcal{N}_{ij} / \sum_{r=1}^n \mathcal{N}_{ir},$$

iFairNMTF

$$\min_{\mathbf{H}, \mathbf{W} \geq 0} \|\mathbf{A} - \mathbf{H}\mathbf{W}\mathbf{H}^\top\|_F^2 + \lambda \mathcal{R}_C(\mathbf{H}),$$

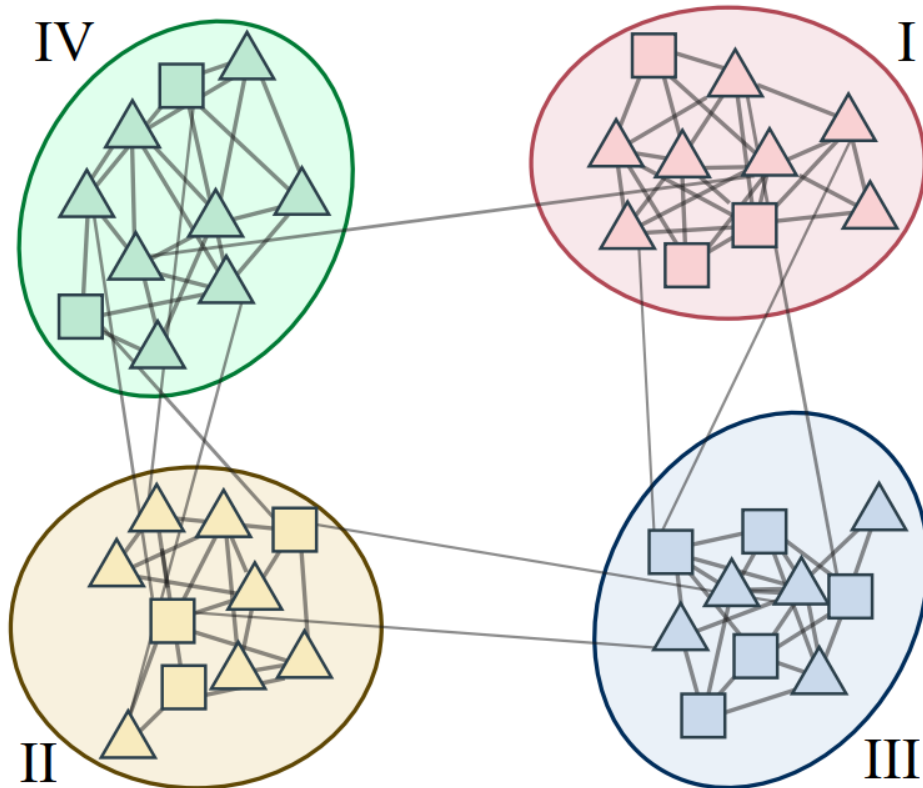
$$\min_{\mathbf{H}} \mathcal{R}_C = \sum_{i=1}^n \sum_{j=1}^n \|\mathbf{h}^{(i)} - \mathbf{h}^{(j)}\|^2 C_{ij} = \text{Tr}(\mathbf{H}^\top \mathbf{L} \mathbf{H}).$$

$$\text{iFairNMTF} \quad \longrightarrow \quad \mathcal{L} = \mathcal{L}_{\mathcal{F}} + \lambda \mathcal{R}_C$$

$$\min_{\mathbf{H}, \mathbf{W} \geq 0} \|\mathbf{A} - \mathbf{H}\mathbf{W}\mathbf{H}^\top\|_F^2 + \lambda \text{Tr}(\mathbf{H}^\top \mathbf{L} \mathbf{H})$$

iFairNMTF (Interpretability)

$$\min_{\mathbf{H}, \mathbf{W} \geq 0} \|\mathbf{A} - \mathbf{H} \mathbf{W} \mathbf{H}^\top\|_F^2 + \lambda \text{Tr}(\mathbf{H}^\top \mathbf{L} \mathbf{H})$$



	I	II	III	IV
I	2.9	0	4E-12	7E-13
II	0	1.10	5E-08	5E-04
III	4E-12	5E-08	2.74	0
IV	7E-13	5E-04	0	1.91

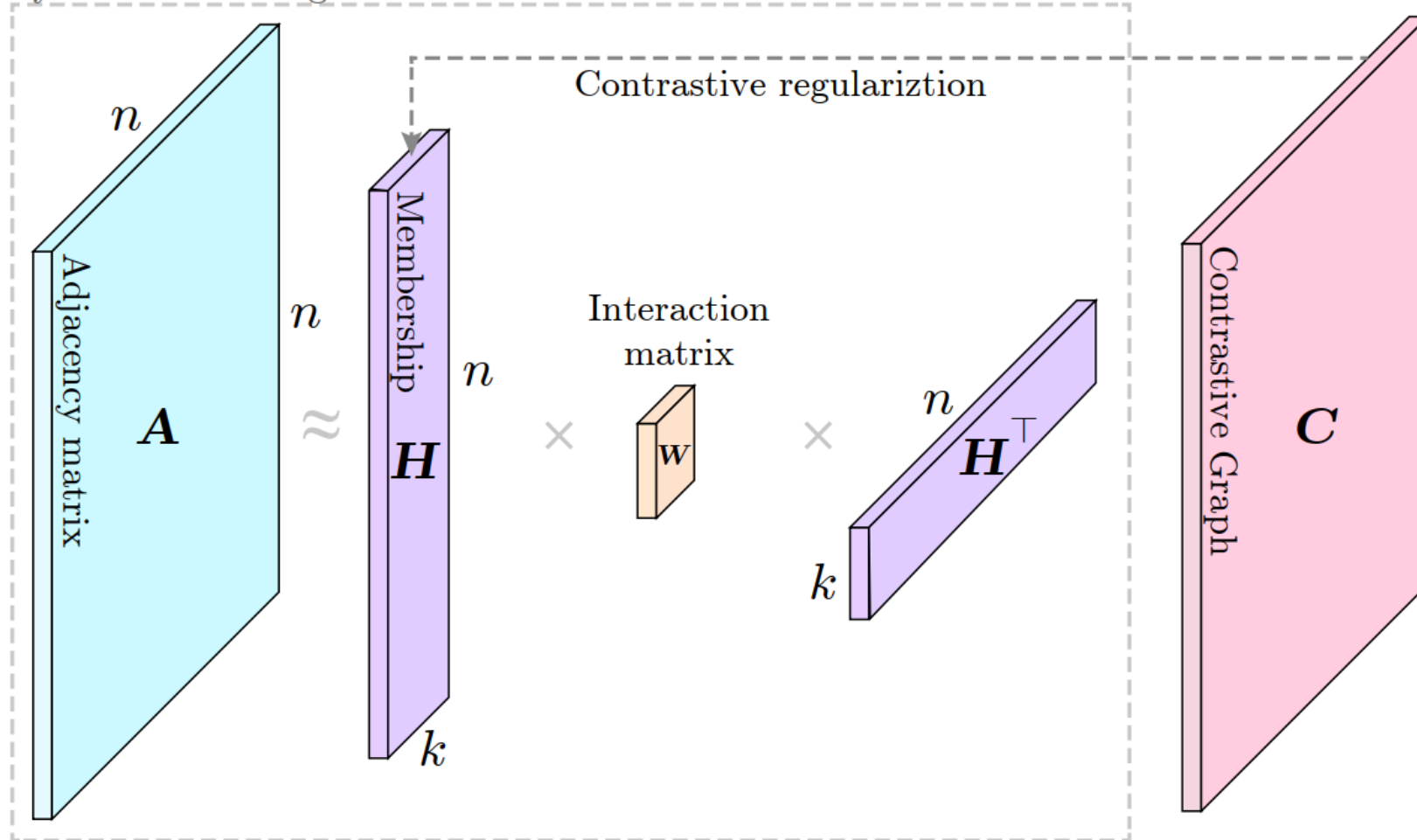
\mathbf{W}

Fig. 4: Interpretability of \mathbf{W} factor for a 40-node graph divided to 4 clusters. Shapes indicate groups.

iFairNMTF

$$\min_{\mathbf{H}, \mathbf{W} \geq 0} \|\mathbf{A} - \mathbf{H}\mathbf{W}\mathbf{H}^\top\|_F^2 + \lambda \text{Tr}(\mathbf{H}^\top \mathbf{L} \mathbf{H}),$$

Symmetric Nonnegative Matrix Tri-Factorization



Datasets

Network	$ V $		$ E $		Sensitive Attribute	$ g $	$ c $	ρ
	raw	clean	raw	clean				
SBM	2,000	-	267,430	-	attribute	5	5	0.1338
	5,000	-	978,959	-	attribute	5	5	0.0783
	10,000	-	2,603,190	-	attribute	5	5	0.0521
Friendship	134	127	406	396	gender	2	-	0.04949
Facebook	156	155	1,437	1,437	gender	2	-	0.1204
DrugNet	293	193	284	273	ethnicity	3	-	0.01473
NBA	403	403	8,285	8,285	nationality	2	-	0.10228
LastFM	7,624	5,576	27,806	19,587	country	6	-	0.00126

$$\rho = \frac{2|E|}{|V|(|V| - 1)}, \quad \rho = \frac{2m}{n(n - 1)}$$

Metrics

Cohesion (Modularity) [10]

$$Q = \frac{1}{|E|} \sum_{i,j} \left(A_{ij} - \frac{\deg(i)\deg(j)}{|E|} \right) \delta(c_i, c_j)$$

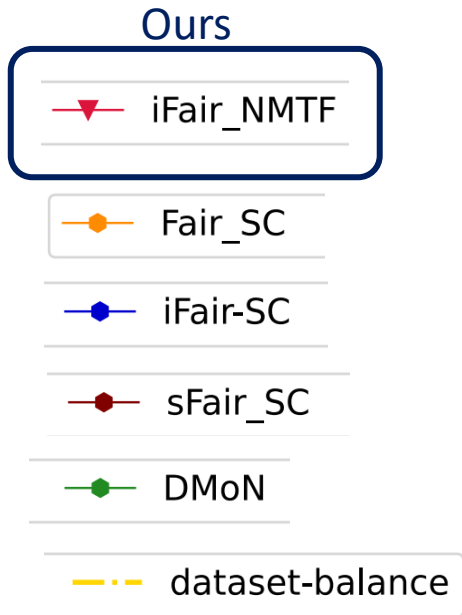
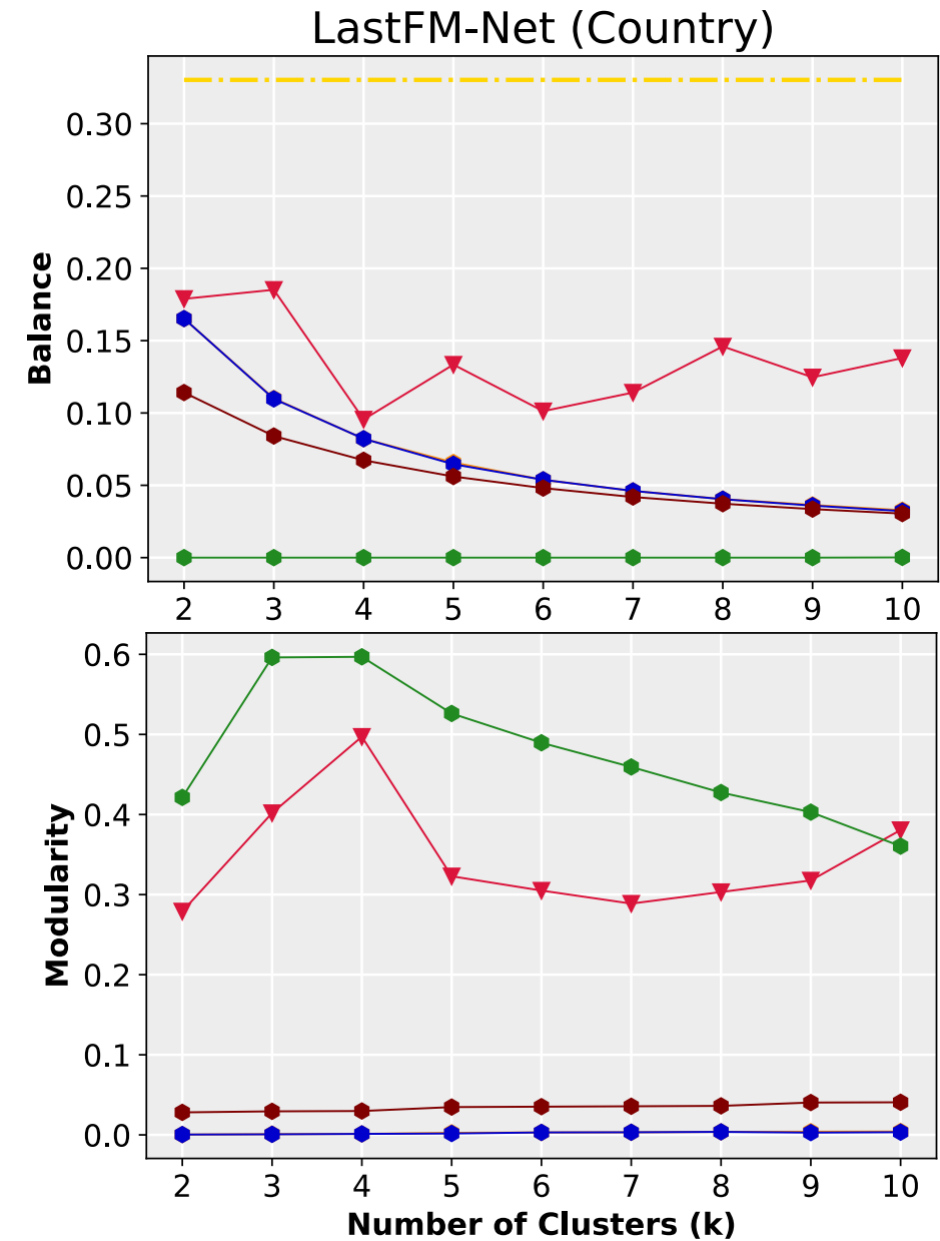
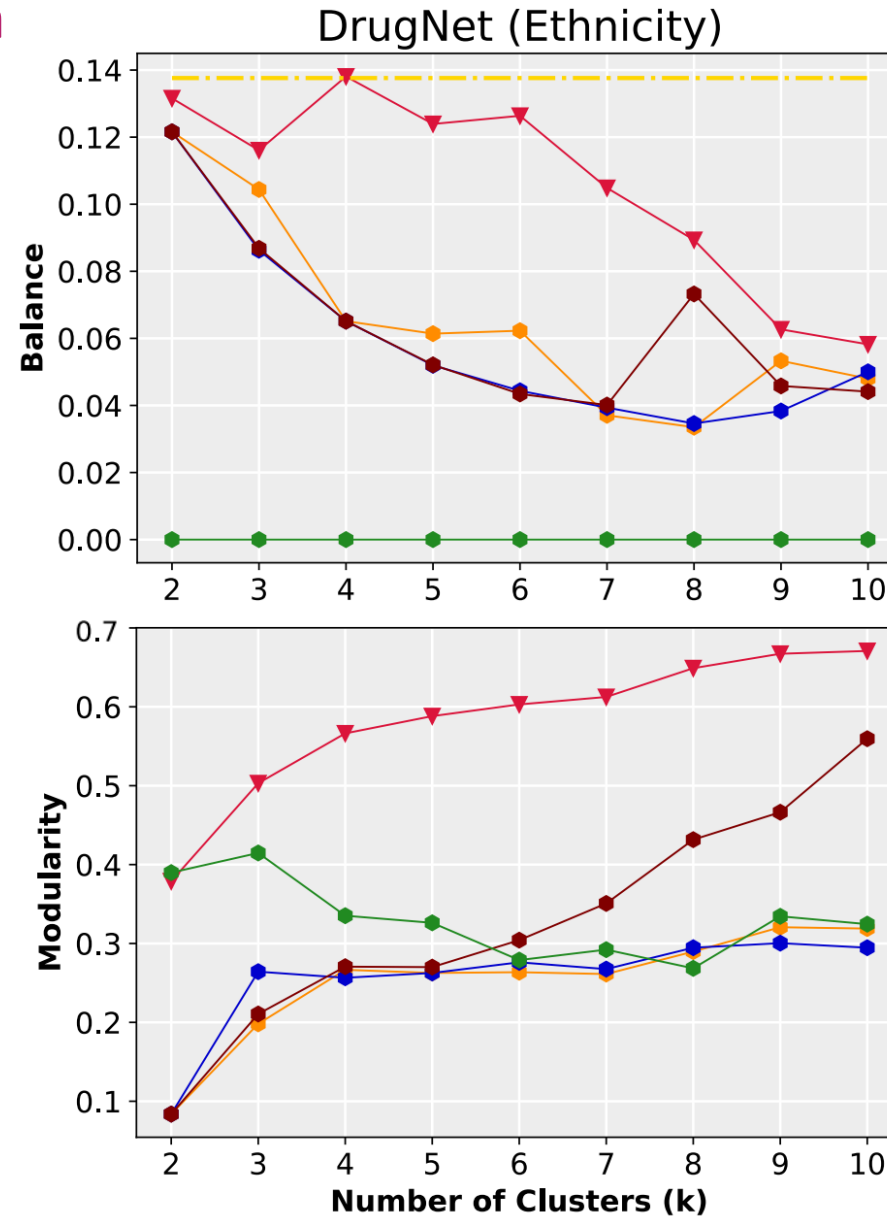
Balance score [1, 9]

$$\text{Balance}(C_l) = \min_{s \neq s' \in [m]} \frac{|V_s \cap C_l|}{|V_{s'} \cap C_l|}$$

$$B = \frac{1}{k} \sum_{l=1}^k \text{Balance}(C_l),$$

3. Experiments

Fairness-Cohesion

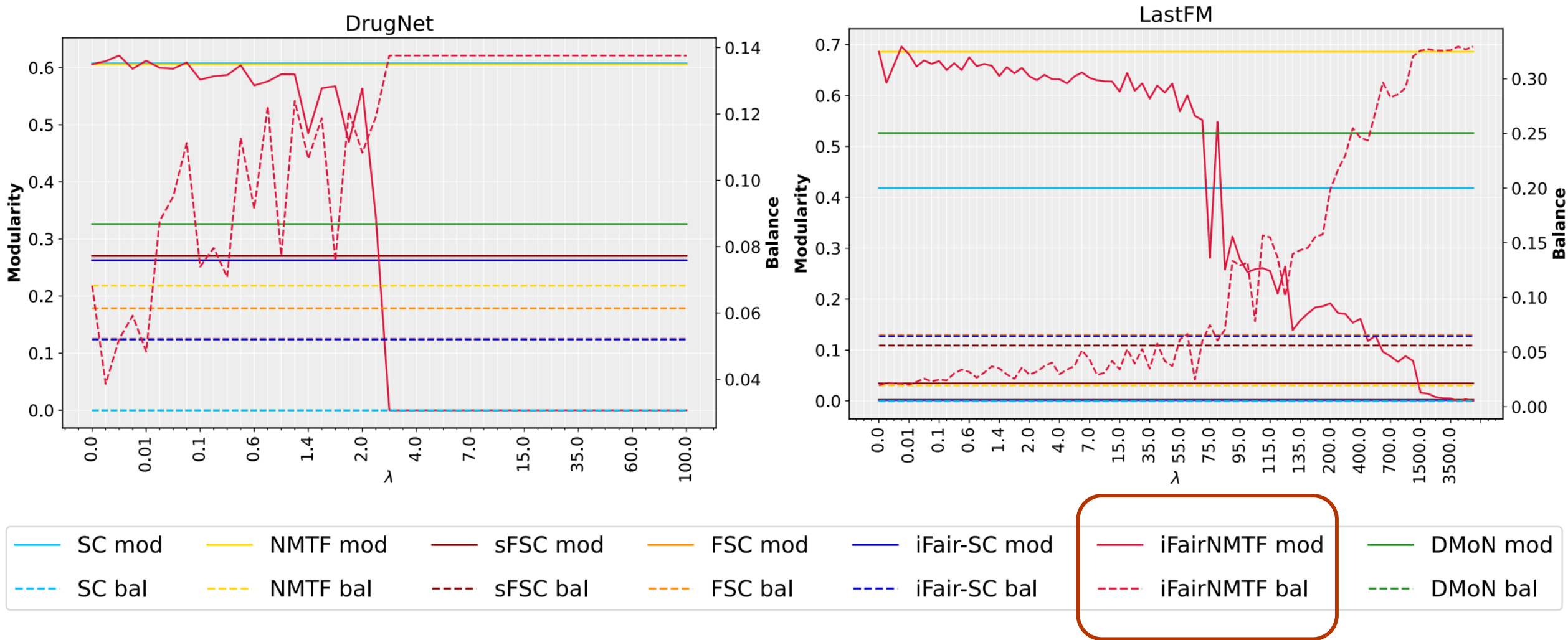


Fairness-Cohesion

Fixed number of clusters $k=5$. Best value of λ for our model according to next slide. Best B is **bold-underlined** and best Acc/Q with **boldfaced gray**

Network	FairSC		sFairSC		iFairSC		DMoN		iFairNMTF	
	B	Q	B	Q	B	Q	B	Q	B	Q
Diaries	<u>0.708</u>	0.612	<u>0.809</u>	0.684	0.699	0.647	0.263	0.145	0.648	0.640
Facebook	0.327	0.449	<u>0.602</u>	0.500	0.330	0.448	0.268	0.048	<u>0.514</u>	0.509
Friendship	0.391	0.483	<u>0.485</u>	0.627	0.374	0.392	0.183	0.140	<u>0.631</u>	0.669
DrugNet	0.052	0.263	0.052	0.270	<u>0.061</u>	0.263	0.000	0.326	<u>0.124</u>	0.588
NBA	0.083	0.000	<u>0.323</u>	0.113	0.072	0.000	0.036	0.057	<u>0.286</u>	0.150
LastFM	0.065	0.003	0.056	0.035	<u>0.066</u>	0.002	0.000	0.526	<u>0.069</u>	0.600
	B	Acc	B	Acc	B	Acc	B	Acc	B	Acc
SBM-2K	<u>0.575</u>	0.588	—	—	0	0.799	—	—	<u>0.953</u>	0.958
SBM-5K	<u>0.995</u>	0.998	—	—	0	0.799	—	—	<u>0.941</u>	0.962
SBM-10K	<u>0.999</u>	0.999	—	—	0	0.600	—	—	<u>1</u>	1

Fairness-Cohesion (Parameter Selection)



Choice of λ is problem/dataset specific

Take-Away

- Previous works include fairness in **rigid (hard-constrained)** graph-clustering frameworks, but our model proposes a **flexible (adjustable)** degree of **trade-off** between individual **fairness** and **cohesion** (clustering objective).
- A **novel contrastive regularization** (individual fairness constraint): takes not just Lipschitz condition into account but also group membership of nodes.
- **Individual-fair** by definition but also **group-fair by design**.
- The **first work** to incorporate fairness in an **NMF** framework.
- It enables users/policy-makers to enforce required degree of fairness in compromise to accuracy.

Future Outlook

- Multi-objective techniques to effectively balance fairness and cohesion objectives.
- Extend to group fairness notions and fusion ideas.
- Investigating individual-level measures for further evaluation.

- [1] Kleindessner, Matthäus, et al., "Guarantees for spectral clustering with fairness constraints", In: International Conference on Machine Learning. PMLR, 2019.
- [2] Dwork, Cynthia, Hardt, Moritz, Pitassi, Toni, Reingold, O., Zemel, Richard.S, "Fairness through awareness", In: Proceedings of the 3rd ITCS Conference. pp. 214–226 (2012)
- [3] Zemel, Richard.S., Wu, Yu, Swersky, Kevin, Pitassi, Toni, Dwork, Cynthia, "Learning fair representations ", In: International Conference on Machine Learning. PMLR, 2013.
- [4] Gupta, Shubham, and Ambedkar Dukkipati, "Consistency of Constrained Spectral Clustering under Graph Induced Fair Planted Partitions.", In: Advances in Neural Information Processing Systems. 2022.
- [5] Wang, Yu-Xiong, and Yu-Jin Zhang, "Nonnegative matrix factorization: A comprehensive review", In: IEEE Transactions on knowledge and data engineering 25.6 (2012): 1336-1353.
- [6] Cai, Deng, et al, "Graph regularized nonnegative matrix factorization for data representation", In: IEEE transactions on pattern analysis and machine intelligence 33.8 (2010): 1548-1560.
- [7] Ding, Chris, Xiaofeng He, and Horst D. Simon, "On the equivalence of nonnegative matrix factorization and spectral clustering", In: Proceedings of the 2005 SIAM international conference on data mining. Society for Industrial and Applied Mathematics, 2005.
- [8] Pei, Y., Chakraborty, N., Sycara, K.P., "Nonnegative matrix tri-factorization with graph regularization for community detection in social networks", In: IJCAI. pp. 2083–2089. AAAI Press (2015).
- [9] Dong, Yushun, et al., "Fairness in Graph Mining: A Survey", In: arXiv preprint arXiv:2204.09888 (2022).
- [10] Chakraborty, Tanmoy, et al., "Metrics for community analysis: A survey", IN: ACM Computing Surveys (CSUR) 50.4 (2017): 1-37.
- [11] Flavio Chierichetti, Ravi Kumar, Silvio Lattanzi, and Sergei Vassilvitskii, " Fair clustering through fairlets ", In: Advances in Neural Information Processing Systems. (2017).



Siamak Ghodsi

ghodsi@l3s.de



@GhodsiSiamak



@siamak-ghodsi



S. Amjad Seyedi

amjadseyedi@uok.ac.ir



Elrini Ntoutsli

elrini.ntoutsli@unibw.de

For code and Supplemental material, refer to **iFairNMTF** at:

<https://github.com/SiamakGhodsi/iFairNMTF>

Thank you for your attention

Introducing NMF

The diagram illustrates the Non-negative Matrix Factorization (NMF) process. It shows a large orange rectangle labeled X on the left, followed by an approximation symbol \approx . To the right of the symbol is a tall orange rectangle labeled U , followed by a multiplication symbol \times , and then a teal rectangle labeled V . Below the rectangles, the dimensions are specified: $X \in \mathbb{R}^{m \times n}$, $U \in \mathbb{R}^{m \times d}$, and $V \in \mathbb{R}^{d \times n}$. A note below the multiplication symbol states $d \leq \min(m, n)$.

$$X \approx U \times V$$

$d \leq \min(m, n)$

$X \in \mathbb{R}^{m \times n}$ $U \in \mathbb{R}^{m \times d}$ $V \in \mathbb{R}^{d \times n}$

Feature Learning

Dataset

Feature Factor
Basis Matrix

Representation
Reconstruct Weight

Face Recognition

Dataset

Basis Images
Eigen Faces

Image Reconstruct
Coefficient

Clustering

Dataset

Centroids

Cluster Indicator

Loss Convergence

$$\mathcal{L} = \mathcal{L}_{\mathcal{F}} + \lambda \mathcal{R}_{\mathcal{C}}$$

$$\min_{\mathbf{H}, \mathbf{W} \geq 0} \|\mathbf{A} - \mathbf{H}\mathbf{W}\mathbf{H}^{\top}\|_F^2 \\ + \lambda \text{Tr}(\mathbf{H}^{\top} \mathbf{L} \mathbf{H}),$$

