Towards Cohesion-Fairness Harmony: Contrastive Regularization in Individual Fair Graph Clustering

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Example

- Teacher with 10 students, 4 female 6 male.
- First day of school.
- You know only the gender and friendship of students from their last year records.
- How to divide: 2 teams (clusters) for classroom assignents?

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- First day of school.
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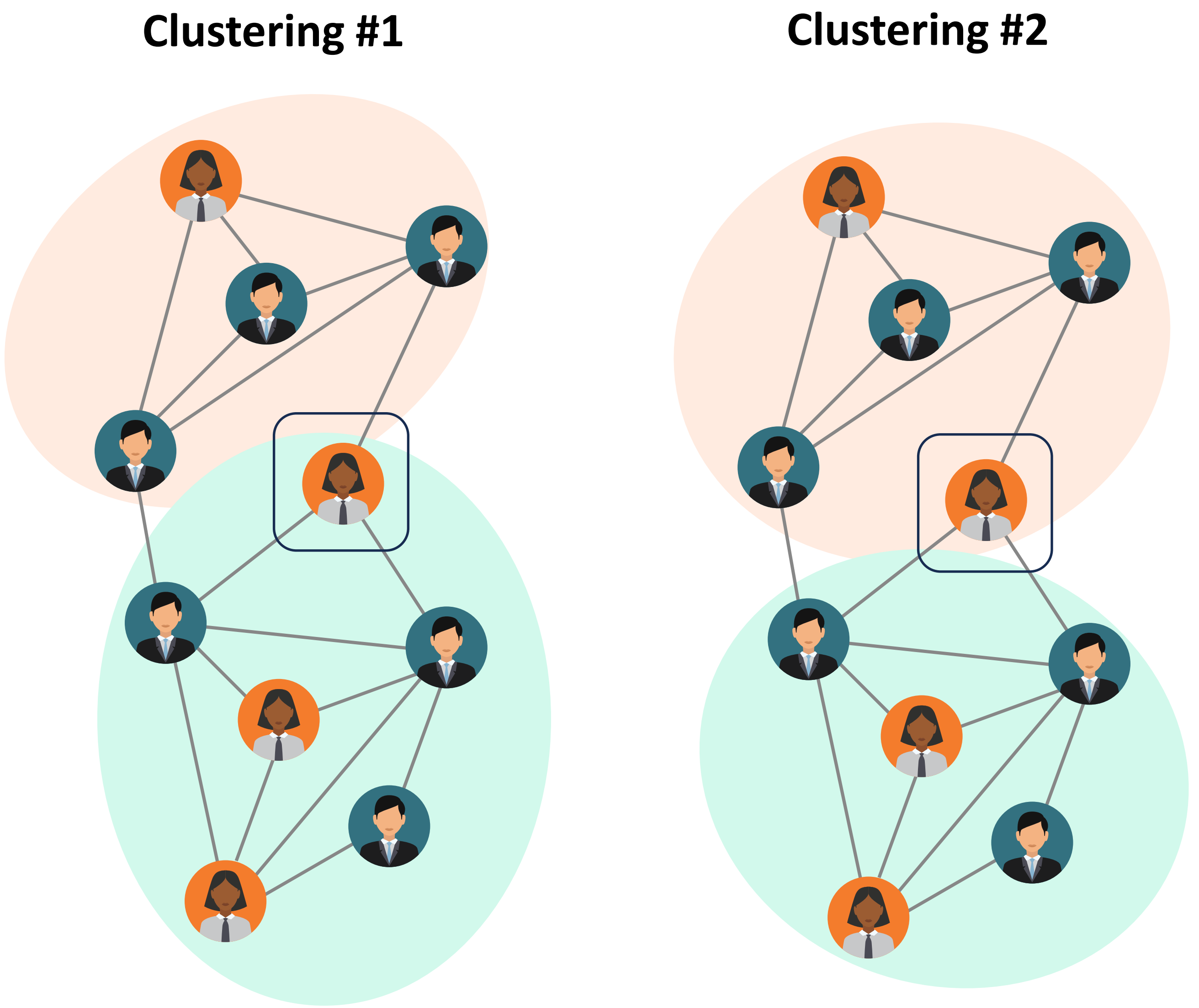

Problem Formulation

Inputs

- **Undirected graph** $\mathcal{G} = (V, E)$ where $V = \{v_1, v_2, \ldots, v_n\}$
- **No self-loops** $E \subseteq V \times V$ is a binary set of edges
- **Adjacency matrix** $\boldsymbol{A} \in \mathbb{R}^{n \times n}$ encodes edge information
- **Sensitive attribute V_s** $V = \dot{\bigcup}_{s \in [m]} V_s$

Output

- **Non-overlapping clustering** V into $k \geq 2$ $\longrightarrow$ $V = C_1 \dot{\cup} \ldots \dot{\cup} C_k$

Fair graph clustering: Individual Fairness

Individual fairness (in i.i.d. data literature)

- Pair-wise node distances in the input-output space → Lipschitz continuity condition [2, 3].

$$D(f(v_i), f(v_j)) \leq L \cdot d(v_i, v_j)$$

Fair graph clustering: Individual Fairness

identically independently distributed data

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$$D(f(v_i), f(v_j)) \leq L \cdot d(v_i, v_j)$$

Individual fairness in graph clustering [4]

- Distribute representation (one-hop neighbors) of each node within clusters (max-min)

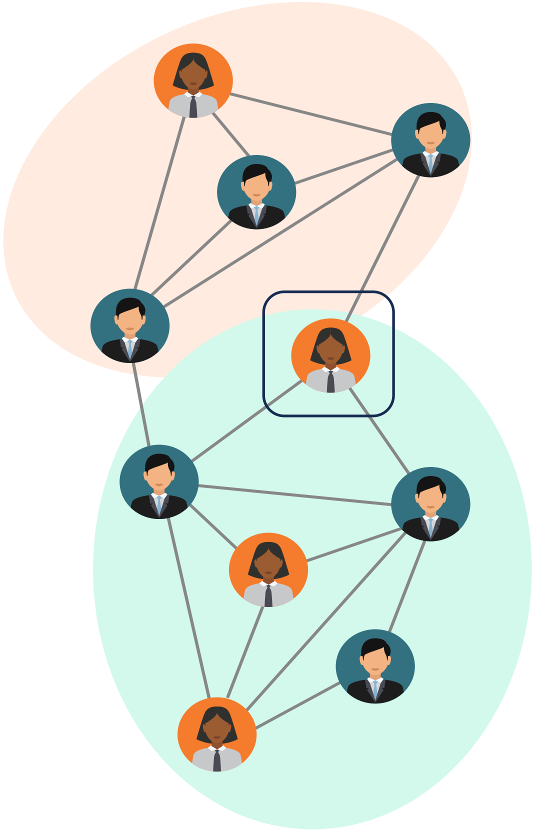
$$\delta_i = min_{k,l \in \{1,\ldots,K\}} \frac{|C_k : \cap N_{v_i}|}{|C_l : \cap N_{v_i}|} \qquad Balance(C_l) = \min_{s \neq s' \in [m]} \frac{|V_s \cap C_l|}{|V_{s'} \cap C_l|}$$

Example contd.

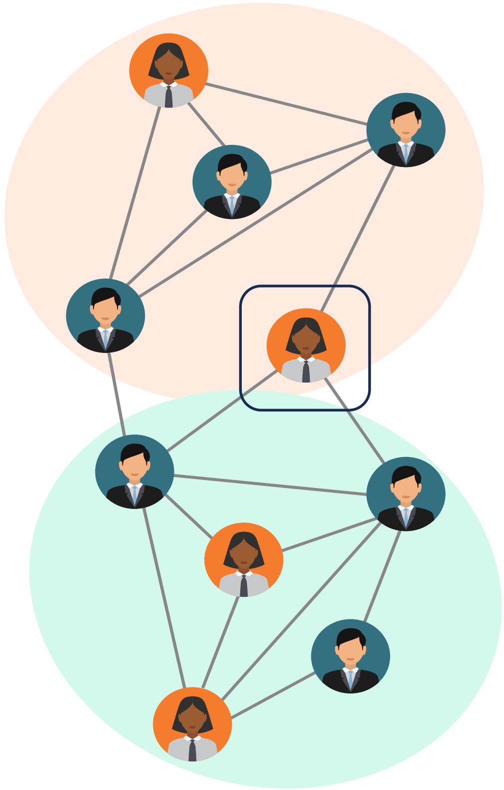
$$\bar{\delta} = 0.141$$

$$B = 0.33$$

Clustering #1



Clustering #2



$$\bar{\delta} = 0.174 \uparrow$$

$$B = 0.66 \uparrow$$

Example contd.

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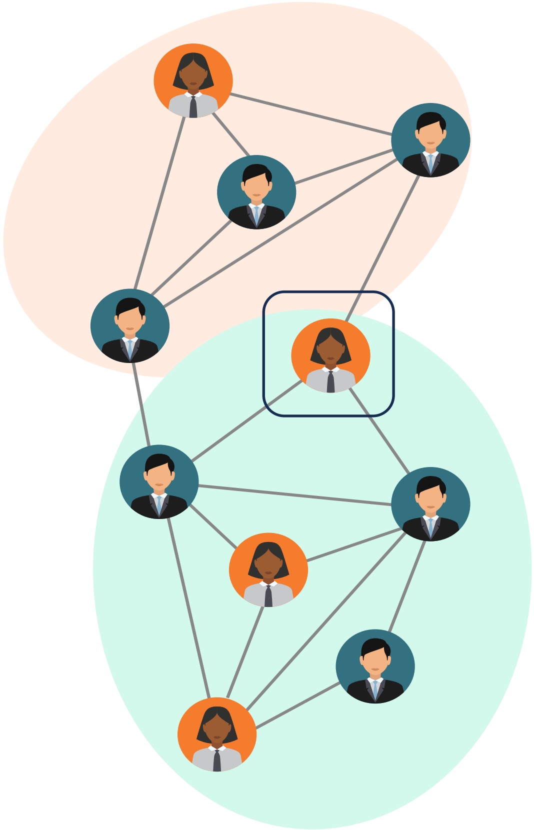
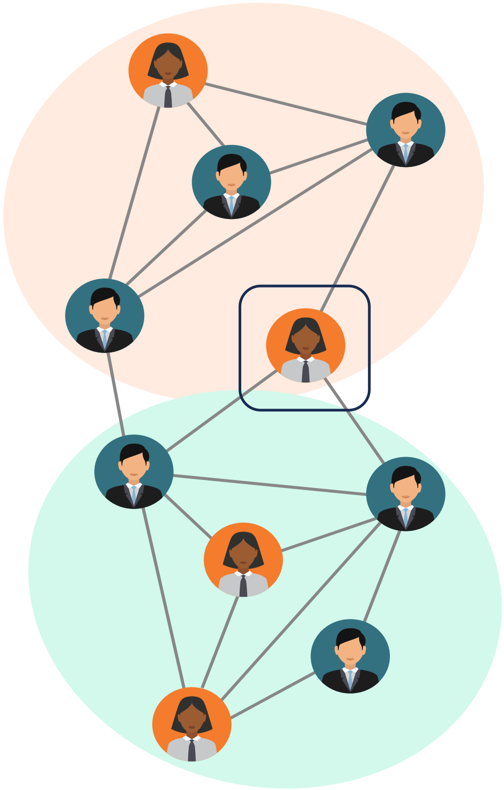
Clustering #1



Clustering #2



$\bar{\delta} = 0.174 \uparrow$

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$\boldsymbol{B}$ Group well-being ?

$\boldsymbol{\bar{\delta}}$ Individual links ?

Example contd.

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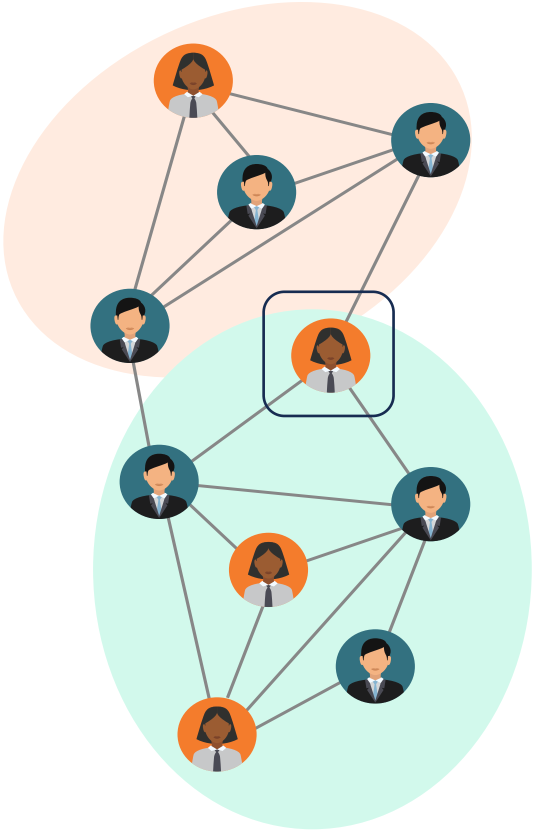
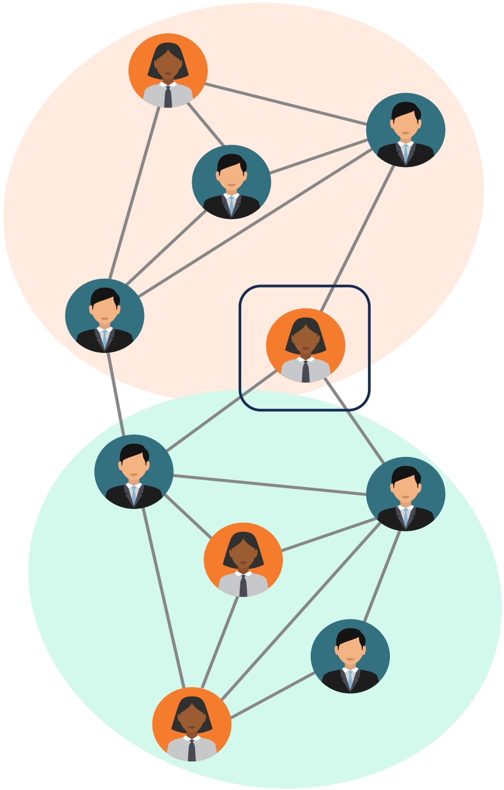
Clustering #1



Clustering #2



$\bar{\delta} = 0.174$ ↑

$B = 0.66$ ↑

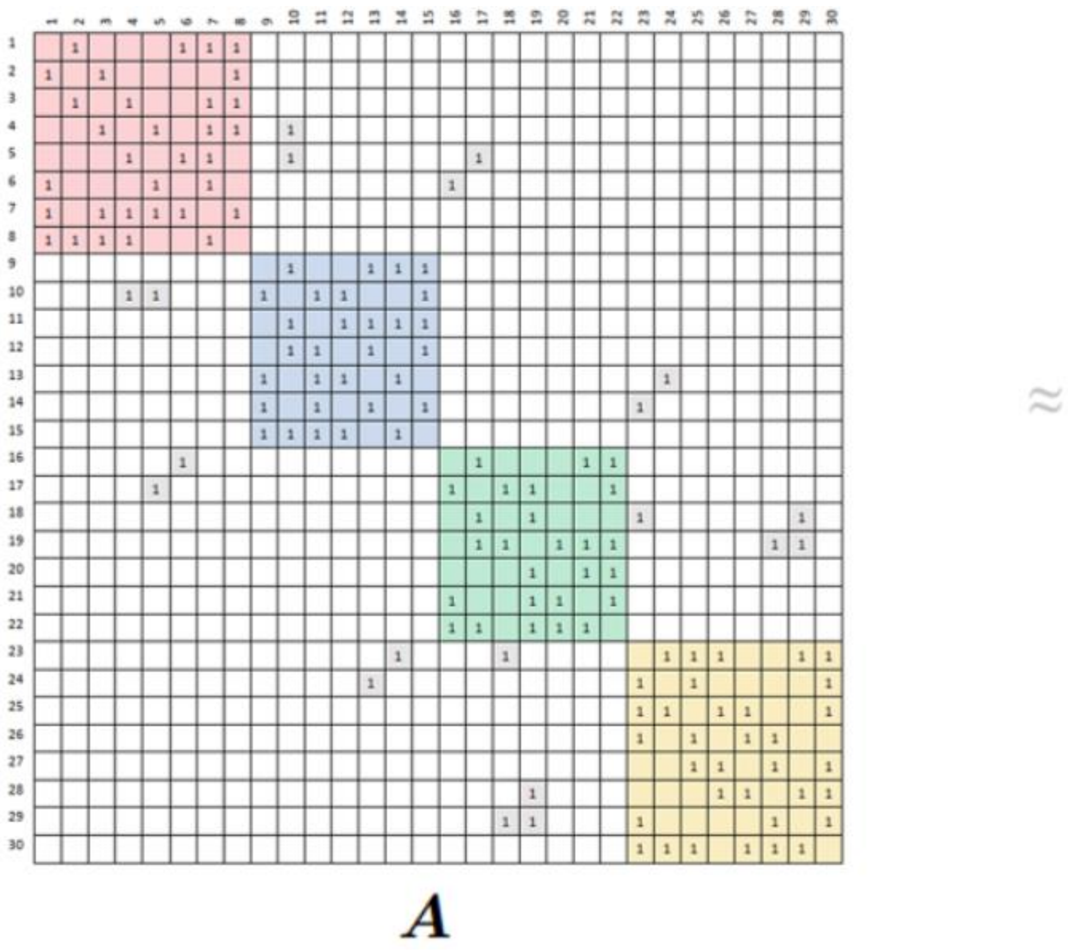
$\boldsymbol{B}$ — Group well-being ?

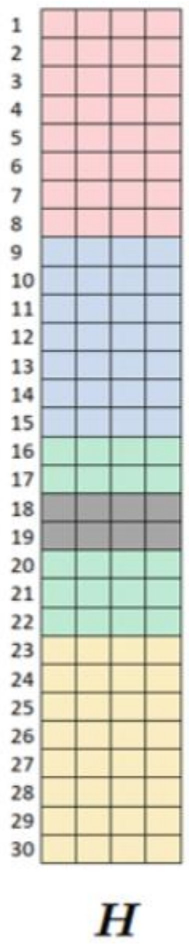
$\boldsymbol{\bar{\delta}}$ — Individual links ?

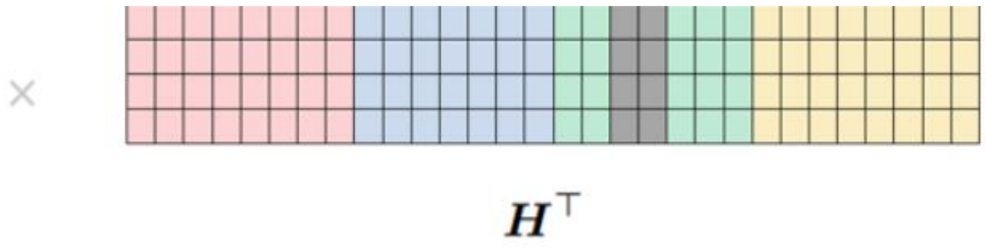
- What about clustering quality? C#1 is better or C#2 ?
- What should be the **clustering quality vs fairness** ?

Symmetric NMF [7]

$$\min_{H \geq 0} \|A - HH^\top\|_F^2, \qquad H \in \mathbb{R}^{n \times k}$$







$$(A_{ij} > 0) \rightarrow (h_i h_j^\top > 0)$$

$$(A_{ij} = 0) \rightarrow (h_i h_j^\top = 0)$$

Symmetric NMF

$$\min_{\boldsymbol{H}\geq 0} \|\boldsymbol{A} - \boldsymbol{H}\boldsymbol{H}^\top\|_F^2,$$

$$(A_{ij} > 0) \longrightarrow (\boldsymbol{h}_i\boldsymbol{h}_j^\top > 0)$$

$$(A_{ij} = 0) \qquad (\boldsymbol{h}_i\boldsymbol{h}_j^\top = 0)$$

NMTF (Tri-Factorization) [8]

$$\min_{\boldsymbol{H},\boldsymbol{W}\geq 0} \|\boldsymbol{A} - \boldsymbol{H}\boldsymbol{W}\boldsymbol{H}^\top\|_F^2,$$

$$A_{ij} \approx \boldsymbol{h}^{(i)}\boldsymbol{W}\boldsymbol{h}^{(j)^\top}$$

$$\boldsymbol{A} \in \mathbb{R}^{n\times n}$$

$$\boldsymbol{H} \in \mathbb{R}^{n\times k}$$

$$\boldsymbol{W} \in \mathbb{R}^{k\times k} \longrightarrow$$ **Interpretability Factor**

Symmetric NMF

$$\min_{\boldsymbol{H} \geq 0} \|\boldsymbol{A} - \boldsymbol{H}\boldsymbol{H}^\top\|_F^2,$$

$$(A_{ij} > 0) \longrightarrow (\boldsymbol{h}_i \boldsymbol{h}_j^\top > 0)$$

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NMTF (Tri-Factorization) [8]

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$$\boldsymbol{H} \in \mathbb{R}^{n \times k}$$

$$\boxed{\boldsymbol{W} \in \mathbb{R}^{k \times k}} \longrightarrow \text{Interpretability Factor}$$

iFairNMTF

NMTF → iFair

$$\min_{\boldsymbol{H},\boldsymbol{W}\geq 0} \|\boldsymbol{A} - \boldsymbol{H}\boldsymbol{W}\boldsymbol{H}^\top\|_F^2 + \lambda \mathcal{R}_{\boldsymbol{C}}(\boldsymbol{H}),$$

iFairNMTF

NMTF → iFair

$$\min_{\boldsymbol{H},\boldsymbol{W}\geq 0} \|\boldsymbol{A} - \boldsymbol{H}\boldsymbol{W}\boldsymbol{H}^\top\|_F^2 + \lambda\mathcal{R}_{\boldsymbol{C}}(\boldsymbol{H}),$$

$$\boldsymbol{C} = \boldsymbol{P} - \boldsymbol{N}$$

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$$\boldsymbol{C} = \boldsymbol{P} - \boldsymbol{N}$$

$$\mathcal{P}_{i,j} = \begin{cases} 1, & \text{if } g_i \neq g_j \\ 0, & \text{otherwise.} \end{cases} \qquad \mathcal{N}_{i,j} = \begin{cases} 1, & \text{if } g_i = g_j \\ 0, & \text{otherwise.} \end{cases}$$

$$P_{ij} = \mathcal{P}_{ij} / \sum_{r=1}^{n} \mathcal{P}_{ir}, \qquad N_{ij} = \mathcal{N}_{ij} / \sum_{r=1}^{n} \mathcal{N}_{ir},$$

iFairNMTF

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$$\min_{\boldsymbol{H}} \mathcal{R}_{\boldsymbol{C}} = \sum_{i=1}^{n}\sum_{j=1}^{n} \|\boldsymbol{h}^{(i)} - \boldsymbol{h}^{(j)}\|^2 C_{ij} = \mathrm{Tr}(\boldsymbol{H}^\top \boldsymbol{L}\boldsymbol{H}).$$

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iFairNMTF $\longrightarrow$ $\mathcal{L} = \mathcal{L}_{\mathcal{F}} + \lambda \mathcal{R}_{\boldsymbol{C}}$

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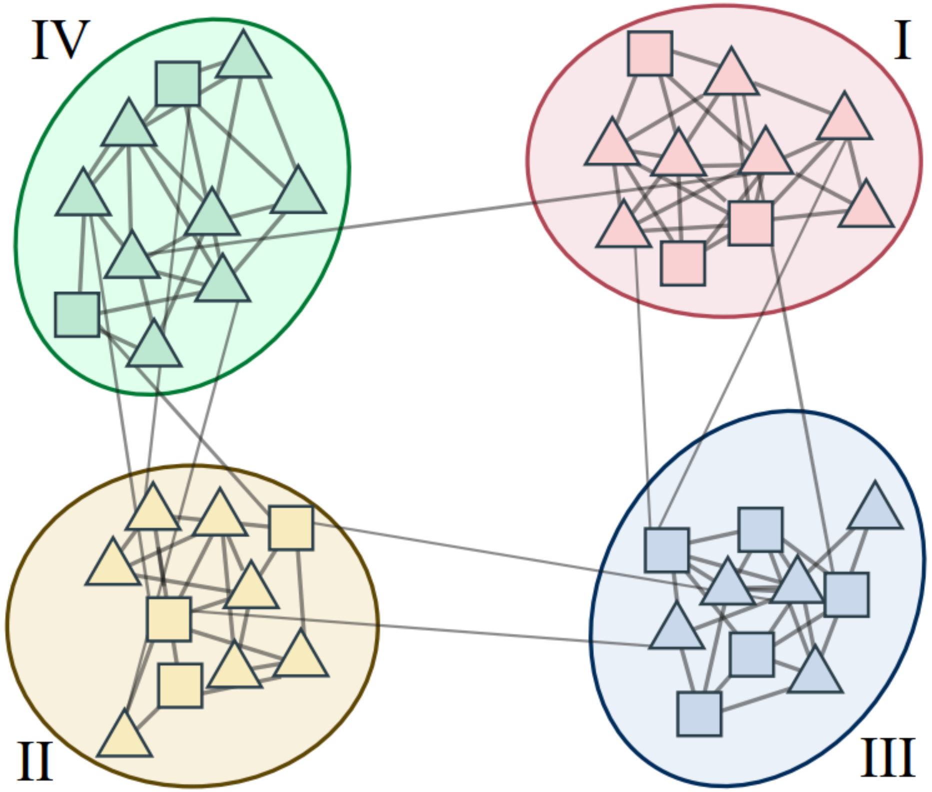
iFairNMTF $\longrightarrow$ $\mathcal{L} = \mathcal{L}_{\mathcal{F}} + \lambda \mathcal{R}_{\boldsymbol{C}}$

$$\boxed{\min_{\boldsymbol{H},\boldsymbol{W}\geq 0} \|\boldsymbol{A} - \boldsymbol{H}\boldsymbol{W}\boldsymbol{H}^\top\|_F^2 + \lambda \mathrm{Tr}(\boldsymbol{H}^\top \boldsymbol{L}\boldsymbol{H})}$$

iFairNMTF (Interpretability)

$$\min_{\boldsymbol{H},\boldsymbol{W}\geq 0} \|\boldsymbol{A} - \boldsymbol{H}\boxed{\boldsymbol{W}}\boldsymbol{H}^\top\|_F^2 + \lambda \mathrm{Tr}(\boldsymbol{H}^\top \boldsymbol{L}\boldsymbol{H})$$



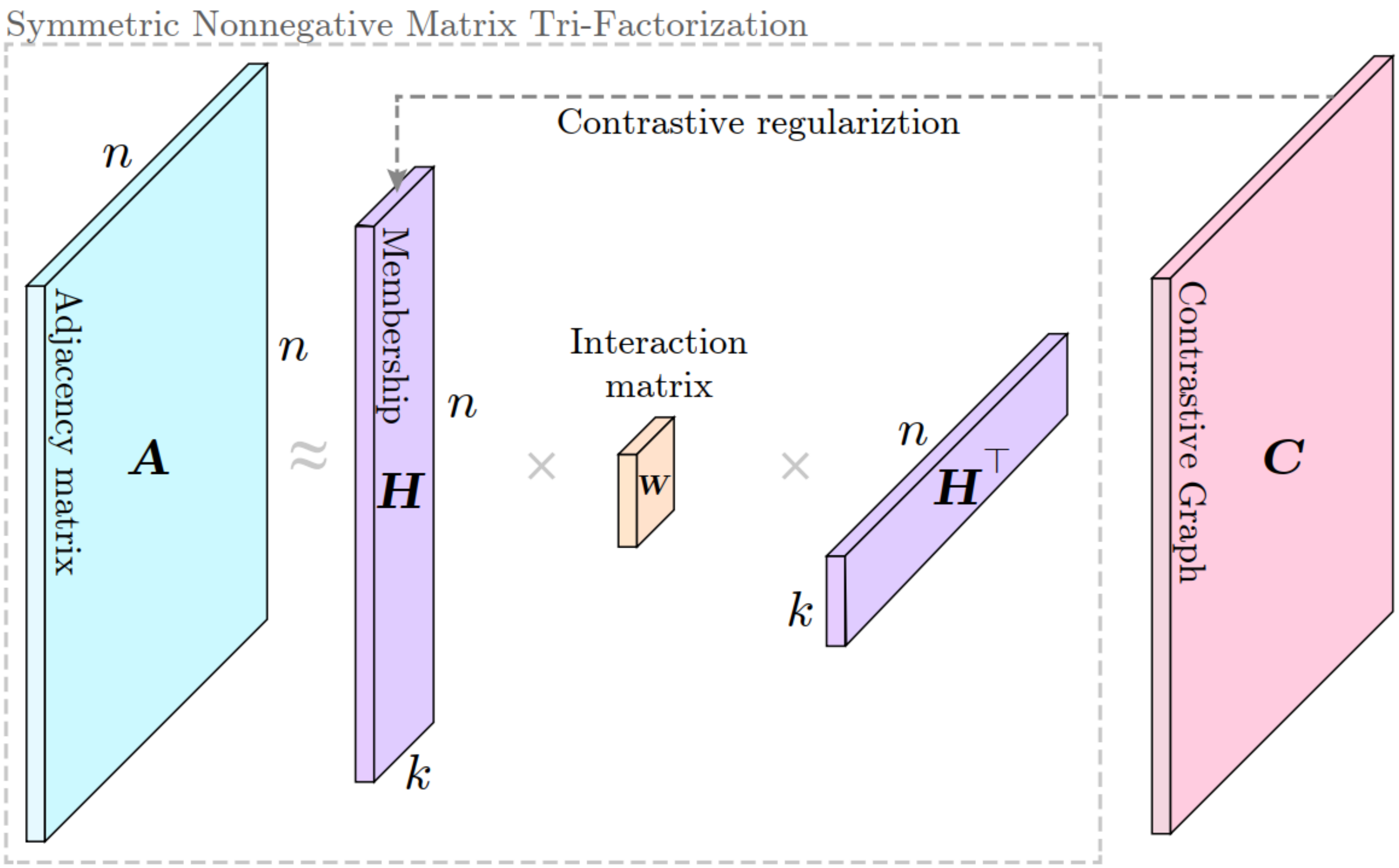
	I	II	III	IV
I	2.9	0	4E-12	7E-13
II	0	1.10	5E-08	5E-04
III	4E-12	5E-08	2.74	0
IV	7E-13	5E-04	0	1.91

$\boldsymbol{W}$

Fig. 4: Interpretability of $\boldsymbol{W}$ factor for a 40-node graph divided to 4 clusters. Shapes indicate groups.

iFairNMTF

$$\min_{\boldsymbol{H},\boldsymbol{W}\geq 0} \|\boldsymbol{A} - \boldsymbol{H}\boldsymbol{W}\boldsymbol{H}^\top\|_F^2 + \lambda \mathrm{Tr}(\boldsymbol{H}^\top \boldsymbol{L}\boldsymbol{H}),$$



Datasets

Network	$\lvert V \rvert$		$\lvert E \rvert$		Sensitive Attribute	$\lvert g \rvert$	$\lvert c \rvert$	ρ
	raw	clean	raw	clean				
SBM	2,000	-	267,430	-	attribute	5	5	0.1338
	5,000	-	978,959	-	attribute	5	5	0.0783
	10,000	-	2,603,190	-	attribute	5	5	0.0521
Friendship	134	127	406	396	gender	2	-	0.04949
Facebook	156	155	1,437	1,437	gender	2	-	0.1204
DrugNet	293	193	284	273	ethnicity	3	-	0.01473
NBA	403	403	8,285	8,285	nationality	2	-	0.10228
LastFM	7,624	5,576	27,806	19,587	country	6	-	0.00126

$$\rho = \frac{2|E|}{|V|(|V|-1)}, \qquad \rho = \frac{2m}{n(n-1)}$$

Metrics

Cohesion (Modularity) [10]

$$Q = \frac{1}{|E|} \sum_{i,j} \left(A_{ij} - \frac{deg(i)deg(j)}{|E|} \right) \delta(c_i, c_j)$$

Metrics

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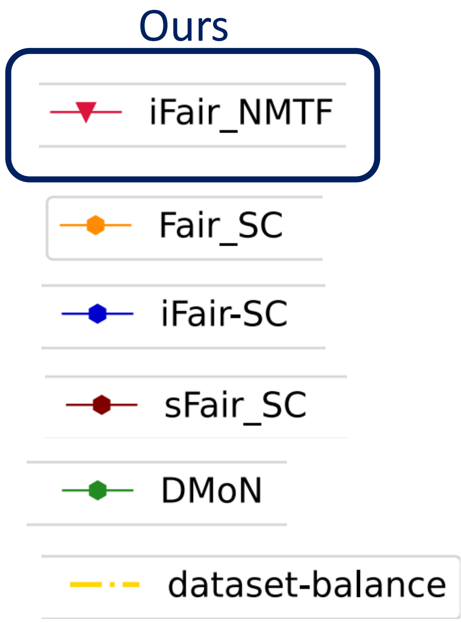
$$Q = \frac{1}{|E|}\sum_{i,j}\left(A_{ij} - \frac{deg(i)deg(j)}{|E|}\right)\delta(c_i, c_j)$$

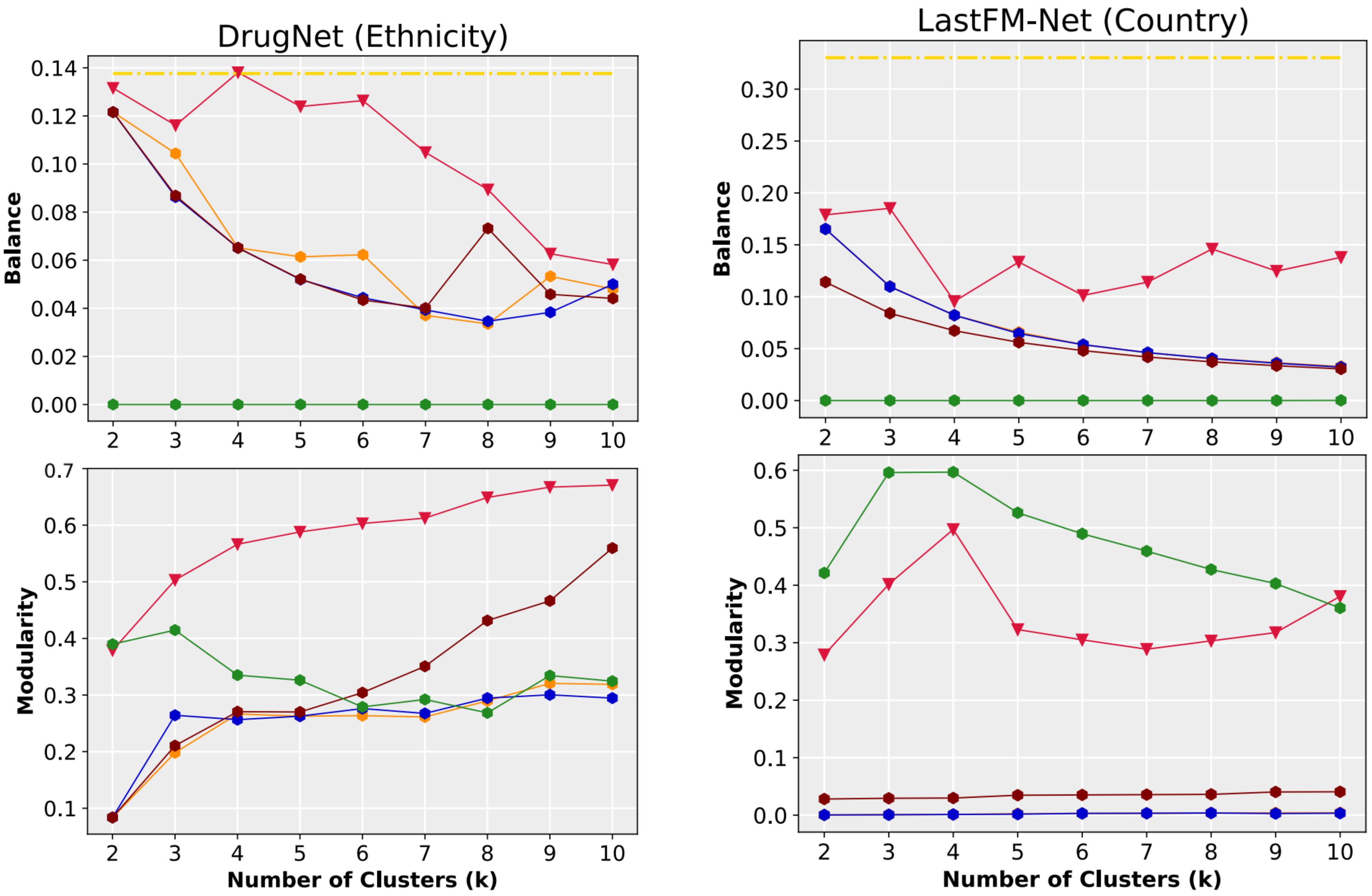
Balance score [1, 9]

$$Balance(C_l) = \min_{s \neq s' \in [m]} \frac{|V_s \cap C_l|}{|V_{s'} \cap C_l|}$$

$$B = \frac{1}{k}\sum_{l=1}^{k} Balance(C_l),$$

Fairness-Cohesion



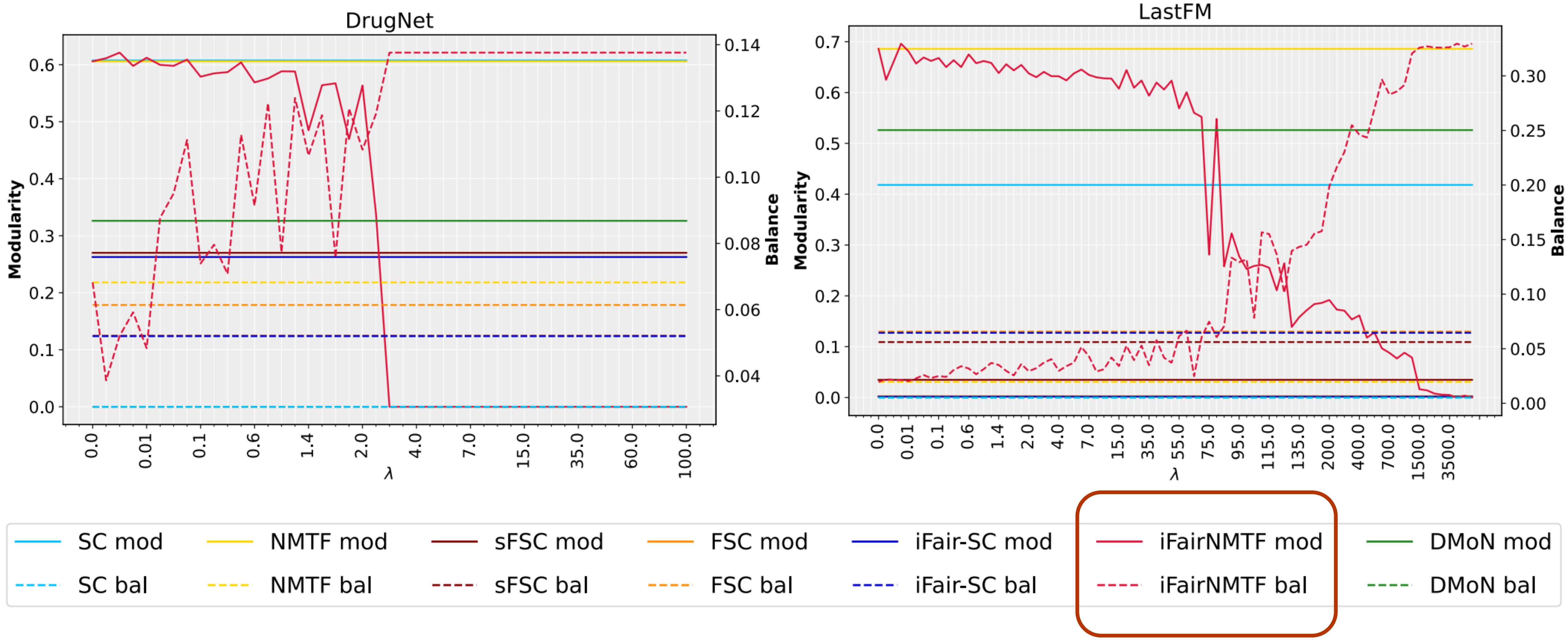


Fairness-Cohesion

Fixed number of clusters k=5. Best value of λ for our model according to next slide. Best B is **bold-underlined** and best Acc/Q with **boldfaced gray**

Network	FairSC		sFairSC		iFairSC		DMoN		iFairNMTF	
	B	Q	B	Q	B	Q	B	Q	B	Q
Diaries	0.708	0.612	**0.809**	**0.684**	0.699	0.647	0.263	0.145	0.648	0.640
Facebook	0.327	0.449	**0.602**	0.500	0.330	0.448	0.268	0.048	0.514	**0.509**
Friendship	0.391	0.483	0.485	0.627	0.374	0.392	0.183	0.140	**0.631**	**0.669**
DrugNet	0.052	0.263	0.052	0.270	0.061	0.263	0.000	0.326	**0.124**	**0.588**
NBA	0.083	0.000	**0.323**	0.113	0.072	0.000	0.036	0.057	0.286	**0.150**
LastFM	0.065	0.003	0.056	0.035	0.066	0.002	0.000	0.526	**0.069**	**0.600**
	B	Acc	B	Acc	B	Acc	B	Acc	B	Acc
SBM-2K	0.575	0.588	–	–	0	0.799	–	–	**0.953**	**0.958**
SBM-5K	**0.995**	**0.998**	–	–	0	0.799	–	–	0.941	0.962
SBM-10K	0.999	0.999	–	–	0	0.600	–	–	1	**1**

Fairness-Cohesion (Parameter Selection)



Choice of λ is problem/dataset specific

Take-Away

- Previous works include fairness in **rigid (hard-constrained)** graph-clustering frameworks, but our model proposes a **flexible (adjustable)** degree of **trade-off** between individual **fairness and cohesion** (clustering objective).
- A **novel contrastive regularization** (individual fairness constraint): takes not just Lipschitz condition into account but also group membership of nodes.
- **Individual-fair** by definition but also **group-fair by design**.
- The **first work** to incorporate fairness in an **NMF** framework.
- It enables users/policy-makers to enforce required degree of fairness in compromise to accuracy.

Future Outlook

- Multi-objective techniques to effectively balance fairness and cohesion objectives.
- Extend to group fairness notions and fusion ideas.
- Investigating individual-level measures for further evaluation.

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For code and Supplemental material, refer to **iFairNMTF** at:

https://github.com/SiamakGhodsi/iFairNMTF

Thank you for your attention

2. Methodology

Introducing NMF

$$X \approx U \times V$$

$d \leq min(m, n)$

$\boldsymbol{X} \in \mathbb{R}^{m \times n}$ $\boldsymbol{U} \in \mathbb{R}^{m \times d}$ $\boldsymbol{V} \in \mathbb{R}^{d \times n}$

Feature Learning	Dataset	Feature Factor Basis Matrix	Representation Reconstruct Weight
Face Recognition	Dataset	Basis Images Eigen Faces	Image Reconstruct Coefficient
Clustering	Dataset	Centroids	Cluster Indicator

Loss Convergence

$$\mathcal{L} = \mathcal{L}_{\mathfrak{F}} + \lambda \mathfrak{R}_{\boldsymbol{C}}$$

$$\min_{\boldsymbol{H},\boldsymbol{W}\geq 0} \|\boldsymbol{A} - \boldsymbol{H}\boldsymbol{W}\boldsymbol{H}^\top\|_F^2$$

$$+ \lambda \mathrm{Tr}(\boldsymbol{H}^\top \boldsymbol{L}\boldsymbol{H}),$$