Supplementary Material for Towards Cohesion-Fairness Harmony: Contrastive Regularization in Individual Fair Graph Clustering

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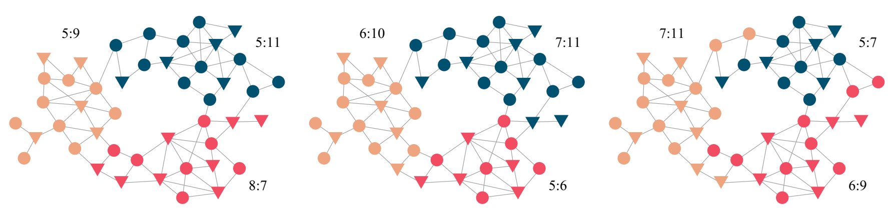
S1 The iFairNMTF complementary details

An illustrative example is shown in Figure S1, comparing iFairNMTF's clustering results with three different degrees of enforced fairness regularization on a toy graph with 45 nodes divided into three clusters and two groups. Cluster and group divisions are described in the caption. In Figure S1(a), the model with no fairness penalty prioritizes the clustering objective and obtains a lower Balance (potentially biased clusters). The model in Figure S1(b), enforces an intermediate fairness regularization and well equilibrates the objectives Modularity (indicating clustering cohesion) and Balance (indicating fairness). However, in Figure S1(c), a large penalty is enforced and the model prioritizes fairness leading to well-distributed but less modular clusters.



(a) min-Balance $= 0.455$ Modularity $= 0.605$ (b) min-Balanace $= 0.600$ Modularity $= 0.591$ (c) min-Balance $= 0.637$ Modularity $= 0.547$

Fig. S1: Comparison of Modularity (Eq. (S1)) and minimum Balance (min-Balance(C) $=$ $\min\{\text{Balance}(C_1), \text{Balance}(C_2), \text{Balance}(C_3)\}$ where Balance is computed according to Eq. (S3)) for three different partitionings of a graph, constituting 45 nodes with 27 ○ and 18 ▽ representing sensitive groups. Colors identify clusters. The effect of fairness penalty: a) No penalty: the model prioritizes clustering performance, potentially leading to biased clusters. b) An intermediate penalty: equilibrates clustering and fairness objectives, producing more equitable representations. c) A large penalty: prioritizes fairness leading to well-distributed clusters but potentially with lower performance.

S2 Experimental Evaluation

S2.1 Datasets: Full Details

In this section, the complementary details of the datasets of Table S1 are provided. Table S1 summarizes the statistics of the datasets. It illustrates the raw and cleaned numbers of nodes and edges with $|V|$, and $|E|$ respectively. The cardinality of $|g|$ indicates the number of sensitive groups and $|c|$ indicates the ground-true number of classes (of the SBM networks).

Table S1: Dataset statistics. $|V|$, $|E|$, $|g|$ and $|c|$ indicate the number of nodes, edges, groups, and ground-truth classes respectively.

Network	$\|V\|$		$\|E\|$		**Sensitive Attribute**	$\|g\|$	$\|c\|$
	raw	clean	raw	clean			
	2,000	-	267,430	-	attribute	5	5
SBM	5,000	-	978,959	-	attribute	5	5
	10,000	-	2,603,190	-	attribute	5	5
Friendship	134	127	406	396	gender	2	-
Facebook	156	155	1,437	1,437	gender	2	-
DrugNet	293	193	284	273	ethnicity	3	-
NBA	403	403	8,285	8,285	nationality	2	-
LastFM	7,624	5,576	27,806	19,587	country	6	-

Synthetic (SBM): The Synthetic networks used in the paper experiments are generated according to an extension of the popular Stochastic Block Model (SBM) model [3]. The SBM is a random graph model that has been widely used to study the performance of clustering algorithms. According to [4], this extension of the SBM can generate two or more meaningful ground-truth partitionings such that only one of these ground-truths are fair (distributed w.r.t. the sensitive attribute). In the traditional SBM, there is a ground-truth clustering of the vertex set V = [n] into k clusters, and in a random graph generated from the model, two vertices i and j are connected with a probability that only depends on which clusters i and j belong to.

In the extended SBM-generator that we use, we assume that V = [n] comprises m groups $V = V_1 \dot{\cup} \ldots \dot{\cup} V_m$ and is partitioned into k ground-truth non-overlapping clusters $V = C_1 \dot{\cup} \ldots \dot{\cup} C_k$ such that $|V_s \cap C_l|/|C_l| = \eta_s$, $s \in [m]$, and $l \in [k]$ for some $\eta_1, \ldots, \eta_m \in (0, 1)$ with $\sum_{s=1}^{m} \eta_s = 1$. Hence, in every cluster each group is represented with the same fraction as in the whole data set V and this ground-truth clustering is fair. The edges that connect the set of vertices V on the graph are defined between every arbitrary pair of vertices i and j with a certain probability $\Pr(i, j)$ that depends only on whether i and j are in the same cluster (or not) and on whether i and j are in the same group (or not). More specifically:

$$\Pr(i,j) = \begin{cases} a, \ i \text{ and } j \text{ in same cluster and in same group,} \\ b, \ i \text{ and } j \text{ not in same cluster, but in same group,} \\ c, \ i \text{ and } j \text{ in same cluster, but not in same group,} \\ d, \ i \text{ and } j \text{ not in same cluster, not in same group,} \end{cases}$$

and assume that $a > b > c > d$. In our experiments, similar to [4, 8] we choose the probabilities proportional to the number of nodes of the network such that $a = 10p$, $b = 7p$, $c = 4p$, and $d = 1p$ where $p = \left(\frac{log(n)}{n}\right)^{2/3}$.

Real: The three high school friendship networks; *Facebook, Friendship*, and *Contact-Diaries* datasets [5] are collected from a French high school based on three different strategies but from the same statistical population. Vertices correspond to students and are split into two groups of males and females. The Contact diaries network is constructed based on students' face-to-face contacts measured through their contact diaries. The Friendship network constitutes self-reported surveys of students' friendship connections and Facebook is collected from their online social network profiles.

The *DrugNet* [9], which is a network encoding acquaintanceship between drug users in Hartford, CT, can be either used with ethnicity as a sensitive attribute constituting three ethnic groups of African Americans, Latinos, and others or gender (i.e. male and female). In our experiments, we have used the ethnicity feature because it has three imbalanced groups and better challenges algorithms.

The *LastFMNet* [7] contains mutual follower relations among users of Last.fm, a recommendation-based online radio and music community in Asia. LastFMNet was collected from public API in 2020 and used to study the distribution of vertex features on graphs.

NBA is an extension of a Kaggle[4] dataset containing relationships between around 400 NBA basketball players [2]. It has demographics including nationality, age, salary, and a number of statistical performance indicators of players in the 2016-2017 season. The nationality of players is used as a sensitive attribute categorizing them to US and non-US players.

S2.2 Metrics

In the paper, accuracy is used for measuring clustering assignment quality on synthetic networks. For real-world networks, since the ground truth cluster structures are unknown, Newman's modularity measure [1, 6] is adopted which basically analyzes the homogeneity of clusters by calculating the difference in the proportion of internal links in each cluster for a given partitioning compared to

[4] https://www.kaggle.com/datasets/noahgift/social-power-nba

the expected proportion of edges in a null graph with the same degree distribution as of the original graph:

$$Q = \frac{1}{|E|} \sum_{i,j} \left(A_{ij} - \frac{deg(i)deg(j)}{|E|} \right) \delta(c_i, c_j), \tag{S1}$$

where $|E|$ is the total number of edges in the graph, $deg(i)$ and $deg(j)$ are degrees of each arbitrary pair of i and j nodes, c_i and c_j are cluster assignments of nodes i and j, respectively and $\delta(c_i, c_j)$ is the Kronecker delta function, indicating 1 if its arguments are equal, and 0 otherwise. In other terms, $\delta(c_i, c_j)$ equals 1 only if nodes i and j belong to the same cluster (community). Modularity can range between $[-1, 1]$. We measure the fairness of clustering in terms of the average balance (B) over k clusters such that:

$$B = \frac{1}{k} \sum_{l=1}^{k} Balance(C_l), \tag{S2}$$

Where Balance(C_l) calculates the minimum group proportion of C_l according to Equation (S3):

$$Balance(C_l) = \min_{s \neq s' \in [m]} \frac{|V_s \cap C_l|}{|V_{s'} \cap C_l|}, \tag{S3}$$

The average balance is a common measure that was also previously used by [4, 8]. The minimum balance of each cluster can range between $[0, 1]$, thus their average also ranges between $[0, 1]$. For both modularity and average balance, higher values indicate better results.

S2.3 Convergence Analysis

Figure S2, illustrates how the proposed model's loss converges w.r.t. the number of iterations. In particular, the objective value (i.e. the loss) of the model is calculated according to Equation (S4) in each iteration of the optimization. As demonstrated in the plots, for all datasets, the loss values converge immediately in the initial 50 iterations. Note that, loss values in Figure S2 are shown in scientific notation with a power of $1e2$.

$$\min_{\boldsymbol{H}, \boldsymbol{W} \geq 0} \|\boldsymbol{A} - \boldsymbol{H}\boldsymbol{W}\boldsymbol{H}^\top\|_F^2 + \lambda \mathrm{Tr}(\boldsymbol{H}^\top \boldsymbol{L}\boldsymbol{H}), \tag{S4}$$

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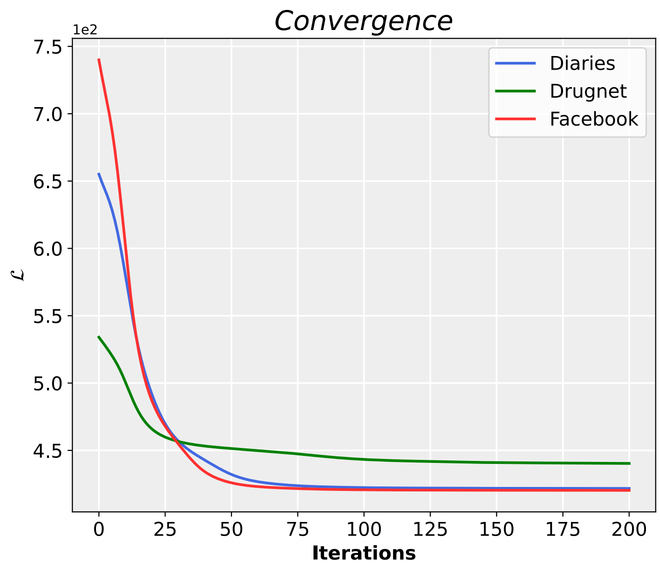

Fig. S2: iFairNMTF loss convergence as a function of the number of iterations on three datasets Contact Diaries, Drugnet, Facebook for a parameter value of $\lambda = 1$ and $k = 5$ clusters.

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