Propositional logic

A proposition is simply a statement.

A proposition is a statement that can be either true or false.

$\wedge \rightarrow$ meet (and)

$\vee \rightarrow$ join (or)

$\sim \rightarrow$ negation (not)

$\Rightarrow$ implies

$\Leftrightarrow$ iff

Conjunction: If p and q are arbitrary propositions, then the conjunction of p and q is

$p \wedge q$

and will be true iff both p and q are true

Truth table:

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

Disjunction : If p and q are arbitary propositions, Then the disjunction of p and q $p \vee q$ and will be true iff either p or q is true or both p and q are true

Truth table :

p	q	$p \vee q$
F	F	F
F	T	T
T	F	T
T	T	T

Definition : If p and q are arbitrary proposition Then the conditional of p and q is written

$$p \Rightarrow q$$

and will be true iff either p is false or q is ~~written~~ ~~$p \Rightarrow q$~~ True.

Negation : If p is a proposition than negation of p written $\sim p$ and will be true if p is false.

Truth table :

p	$\sim p$
F	T
T	F

p	q	$p \Rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

Definition: If P and q are arbitrary proposition then the biconditional of P and q written $P \Leftrightarrow$ and will be true iff both p and q are true or false.

P	q	$P \Leftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

Question: For thanee propositions P, q and p prove the distributive law.

Soln: For three propositions P, q and p the distribution law is

$$(P \wedge q) \vee p = (P \vee p) \wedge (q \vee p)$$

You can prove this by theoritically or by truth table.

Tautologies:

A tautology is a formula which is always true for every value of its propositional variables.

Example: Prove $[(P \to Q) \wedge P] \to Q$ is a tautology.

The truth table is as follows:

P	Q	$P \to Q$	$(P \to Q) \wedge P$	$[(P \to Q) \wedge P] \to Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

As we can see every value of $[(P \to Q) \wedge P] \to Q$ is True, it is a tautology.

Contradictions

A contradiction is a formulae which is always false for every value of its propositional variables.

Example: Prove $(P \vee Q) \wedge [(\sim P) \wedge (\sim Q)]$ is a contradiction.

Proof: Do yourself.