1 Complexity Zoo

1.1 P

Informally: all problems that can be solved in polynomial time.

Definition 1.1.

$$\mathbf{P} = \bigcup_{k \geq 0} \text{TIME}[n^k]$$

Descriptive Complexity definitions:

Definition 1.2.

$$\mathbf{P} = \text{FO(LFP)}$$

(First Order logic extended with the Least Fixed Point operator, with successor)

Definition 1.3.

$$\mathbf{P} = \text{SO(Horn)}$$

(Second Order logic restricted with Horn)

Circuit Complexity definition:

Definition 1.4.

P = Set of problems that can be solved by a polynomial-time uniform family of boolean circuits

Notable Problems in **P**:

- 2-SAT
- 2-Colourability
- Reachability

1.2 NP

Informally: all problems that can be solved in nondeterministic polynomial time.

Definition 1.5.

$$\mathbf{NP} = \bigcup_{k \geq 0} \text{NTIME}[n^k]$$

In terms of a verifier:
Informally: The set of decision problems where a solution can be verified in polynomial time.

Descriptive Complexity Definition:

Definition 1.6.

$$\mathbf{NP} = \mathrm{SO}\exists$$

(Existential Second Order)

Notable Problems in **NP**:

- SAT
- 3-Colourability
- TSP
- Subset sum

1.3 FPT

1.4 W[1]

1.5 FPTAS

1.6 PTAS

1.7 L

1.8 NL

1.9 PSPACE

1.10 coNP

1.11 Σ_2^p

1.12 Σ_i^p

1.13 Π_2^p

1.14 Π_i^p

1.15 PH

1.16 P^{SAT}

1.17 NP^{SAT}

1.18 P/poly

1.19 P-Uniform

1.20 EXP

1.21 NC

1.22 NC_0

1.23 NC_1

1.24 NC_2

1.25 NC_i

1.26 AC_i

1.27 AC_0

1.28 AC_1

1.29 BPP

1.30 RP

1.31 co-RP

1.32 ZPP

1.33 APX

1.34 PO

1.35 PCP