به نام خدا

دانشگاه تهران
دانشکده مهندسی برق و کامپیوتر
اصول سیستم های مخابراتی

تمرین کامپیوتری چهارم

نام و نام خانوادگی	سیاوش شمس
شماره دانشجویی	۸۱۰۱۹۷۶۴۴
تاریخ ارسال گزارش	۹۹/۱۰/۲۴

فهرست گزارش سوالات

بخش ۱-

1)

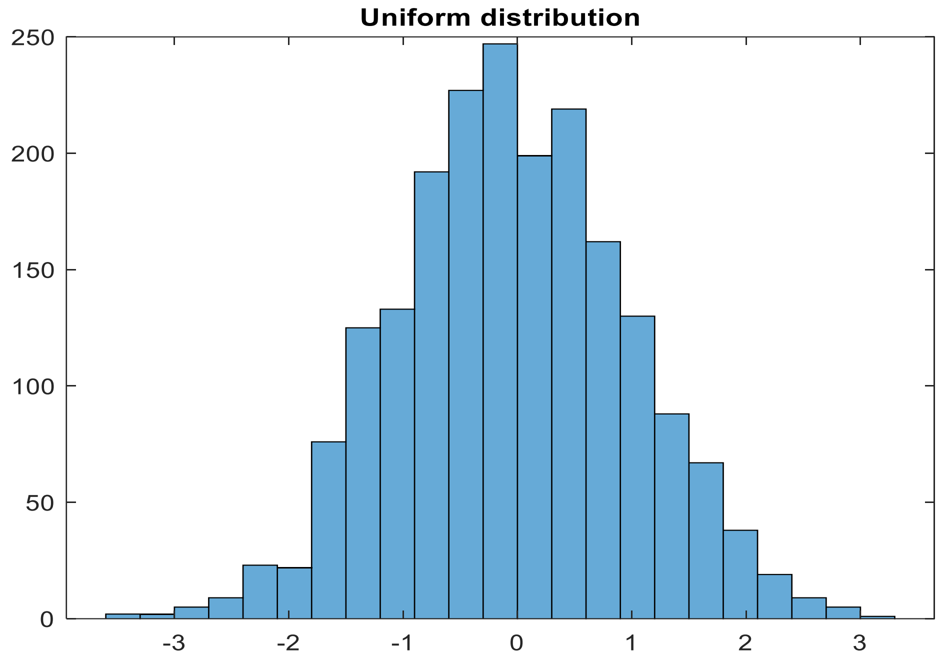

Figure 1

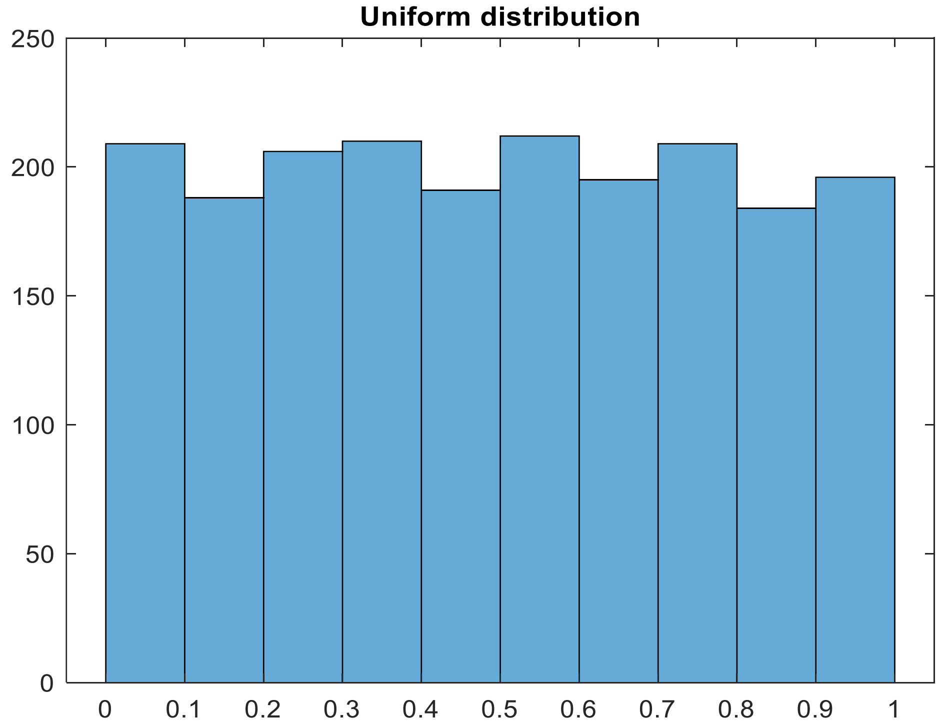

Figure 2

2)

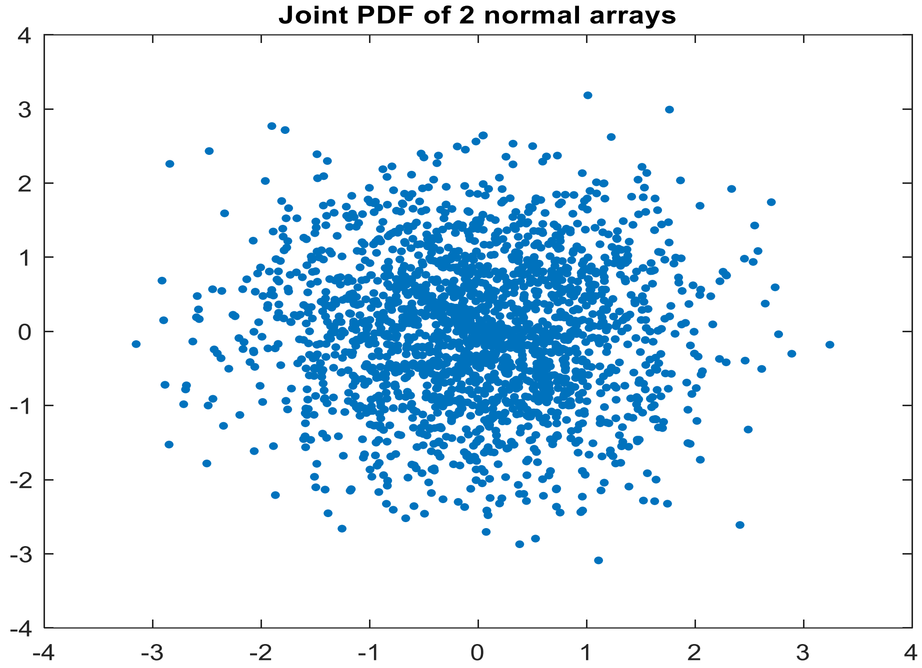

Figure3

In this case contour plot is concentric ellipses based on the variance and mean shapes of ellipses may vary

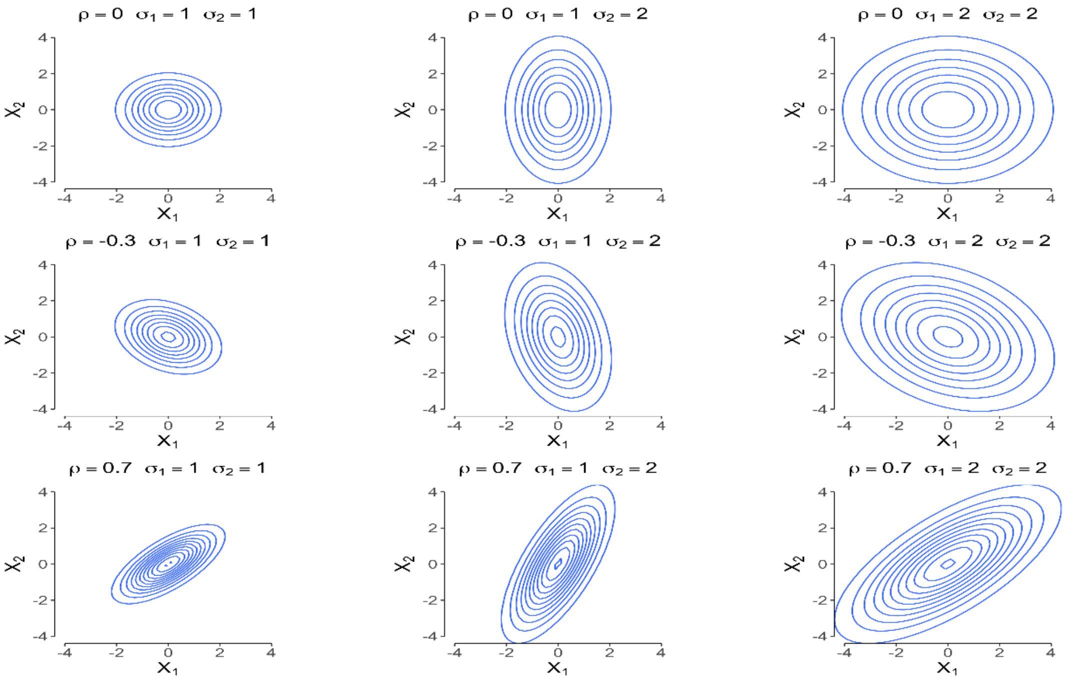

Figure4

This figure4 Is showing how contour plot of two normal distributions changes with parameters.

We know that the variance of a uniformly distributed variable is $\frac{(b-a)^2}{12}$ in this case we have

$$\frac{(2a)^2}{12} = 1 \rightarrow a = \sqrt{3}$$

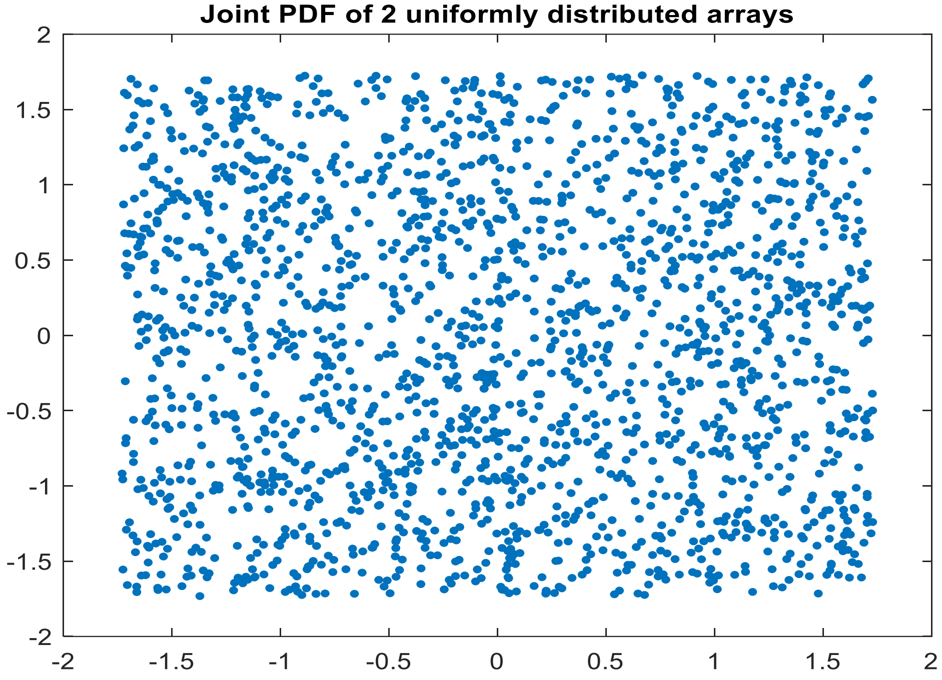

Figure5

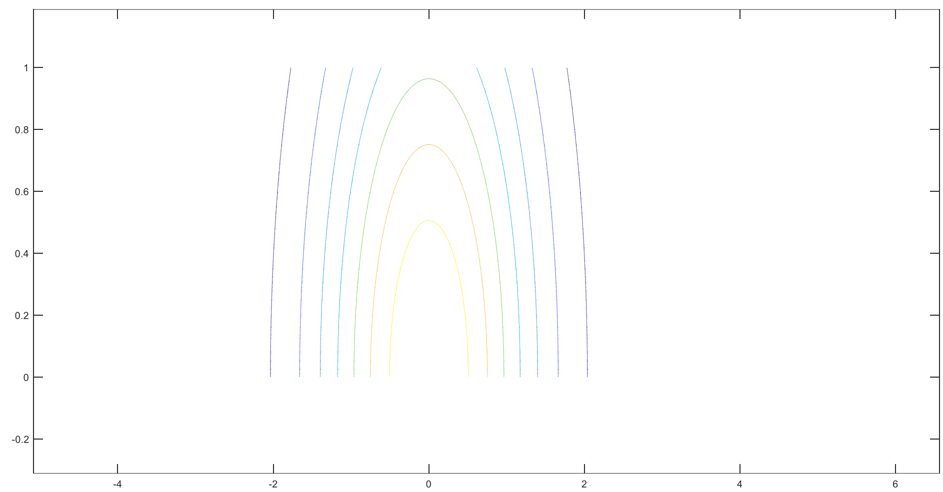

Figure6

contour plot of a normal distribution and a uniform distribution are half ellipses in the plane.

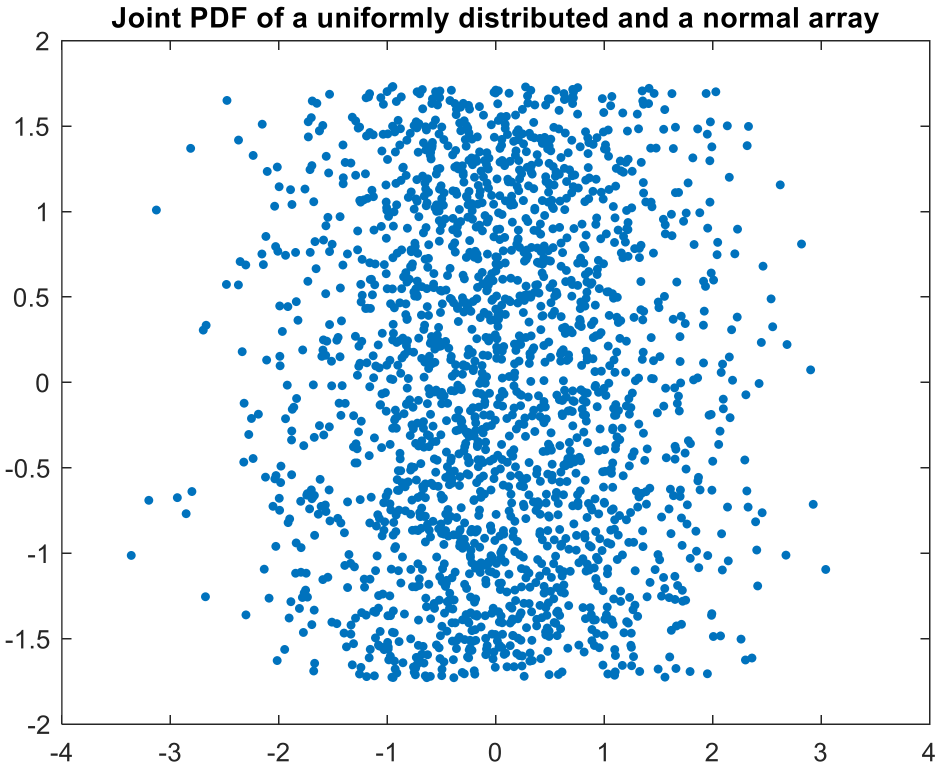

Figure7

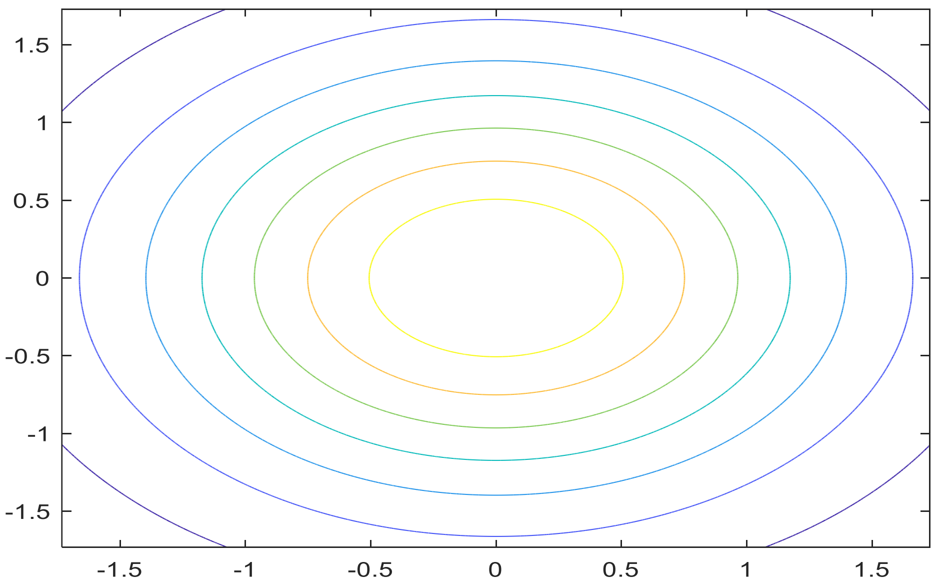

Figure8

Contour plot of 2 uniform distribution is concentric ellipses.

By observing graphs of different joint distributions, we can see that the result has the shape of each distribution mixed with the other, and theoretically joint distribution of two independent distributions is product of them, it matches the results.

3)

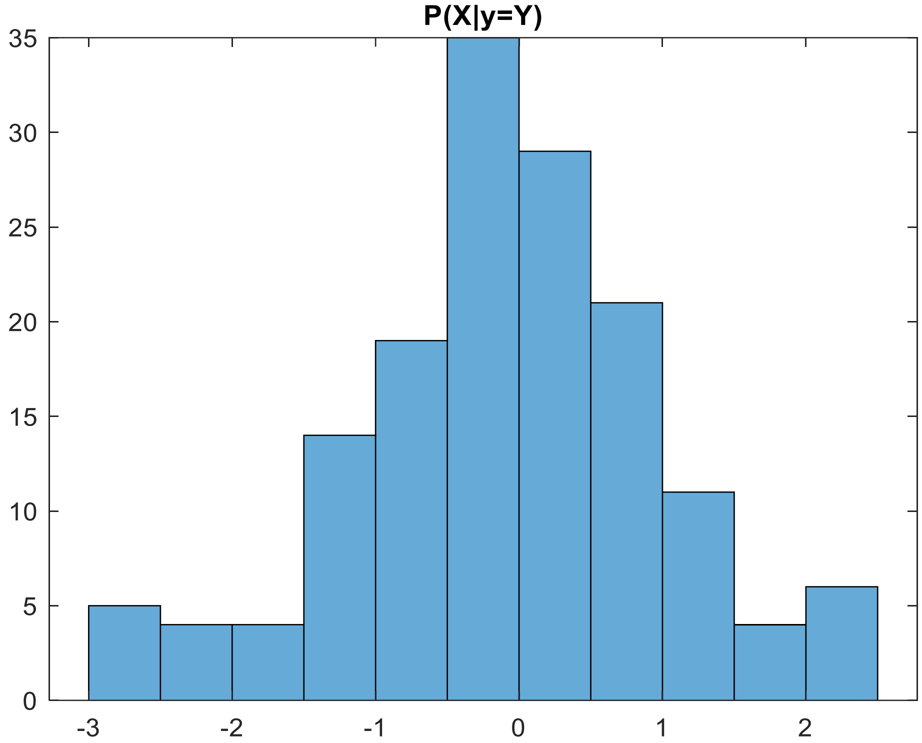

Figure9

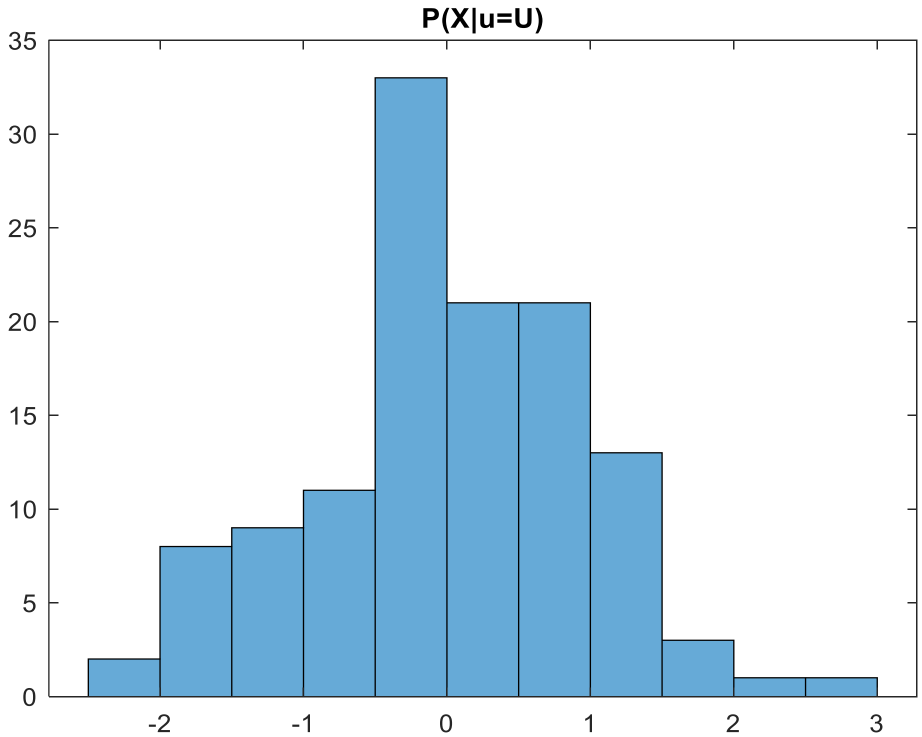

Figure10

Both results have the same distribution (normal distribution) as previous question because the random variables are independent and we know that conditional probability on 2 independent random variables doesn't change the distribution.

$$f_{y|x} = \frac{f_{xy}(x,y)}{f_x(x)} \xrightarrow{independet} \frac{f_x(x).f_y(y)}{f_x(x)}$$

But the height of the bins aren't equal to those of the previous questions, and that's because the sample size is different in these 2 questions, although the distributions remain the same.

بخش ۲-

1)

First, we use MGF to prove that linear combination of normal random variables is a random variable:

$$MXi(t) = E[e^{tXi}] = e^{\mu_i t + \frac{1}{2}\sigma^2 t^2}$$
$$\Rightarrow Ma_i x_i(t) = E[e^{t(a_i x_i)}] = e^{a_i \mu_i t_i + \frac{1}{2}\sigma^2 a_i^2 t^2}$$

Since Xi are independent with respect to each other, and Z is a linear combination of them, we know that $MY(t)$ is a product of the individual $MXi(t)$

$$MY(t) = \prod MXi(t)$$

$= e^{\sum_{i=1}^{n} a_i \mu_i t_i + \frac{1}{2}\sum_{i=1}^{n} \sigma^2 a_i^2 t^2}$

As we can see the MGF of linear combination of normal random varibles has form of the normal distribution.

Now from the MGF its clear that if we substitute $\mu = 0$ & $\sigma^2 = 1$, $a1 = \alpha$, $a2 = \sqrt{1-\alpha^2}$, Z will be normally distributed with zero mean and 1 variance.

Other solution: $E\{z\} = aE\{x\} + \sqrt{1-a^2}E\{y\} = 0$

$$E\{Z^2\} = a^2E\{x^2\} + 2a\sqrt{1-a^2}E\{xy\} + \sqrt{(1-a^2)}E\{y^2\} = a^2 + (1-a^2) = 1$$

$$E\{XZ\} = aE\{X^2\} + \sqrt{1-a^2}E\{XY\} = aE\{X^2\} = a$$

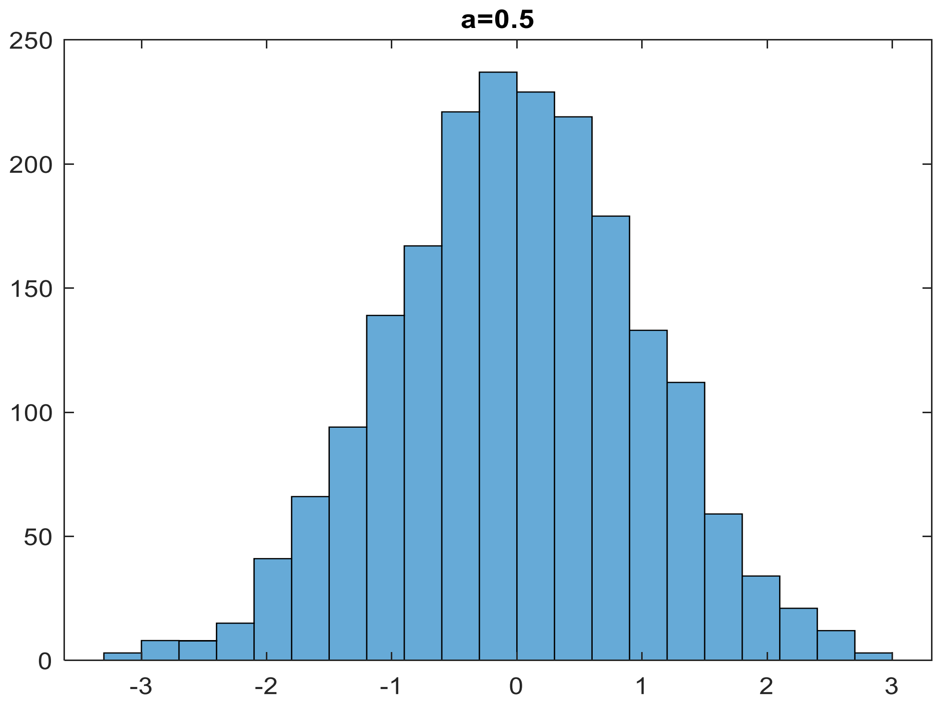

Figure11

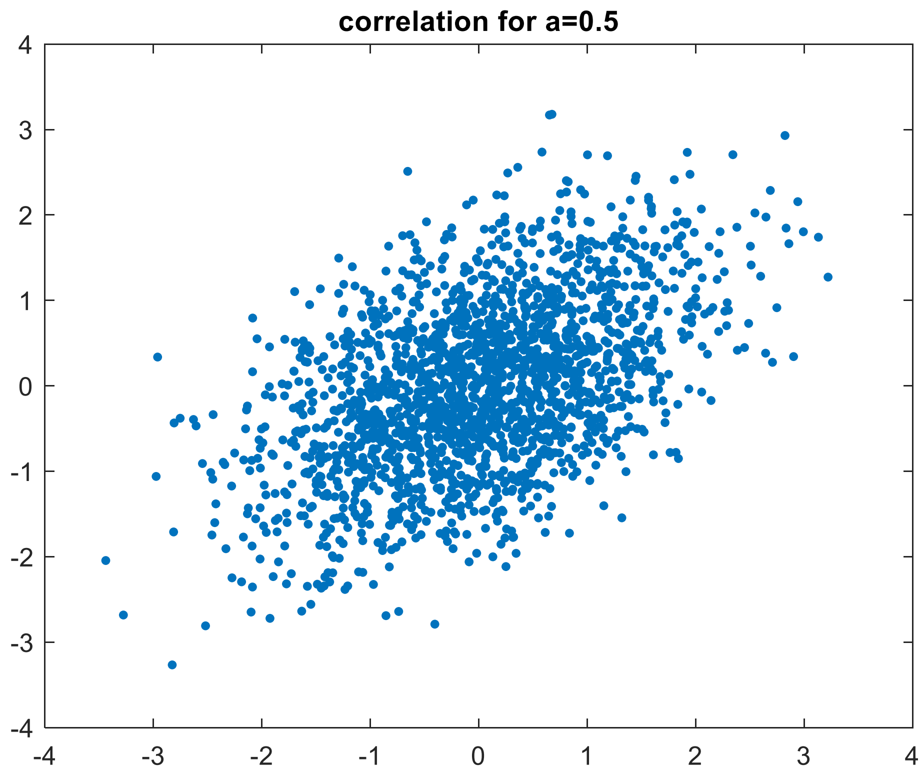

Figure12

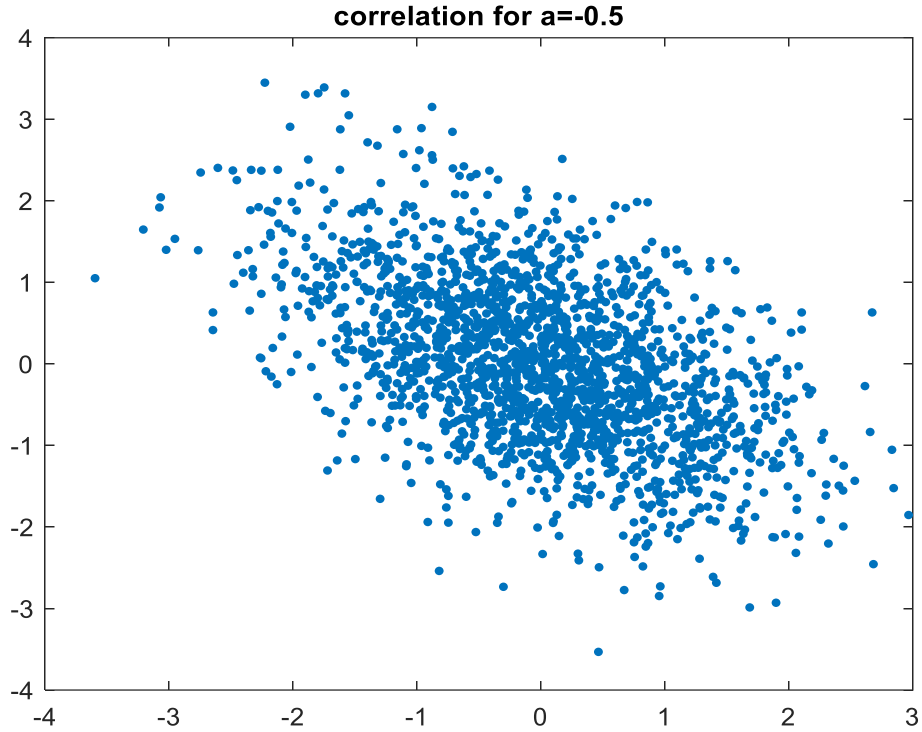

Figure13

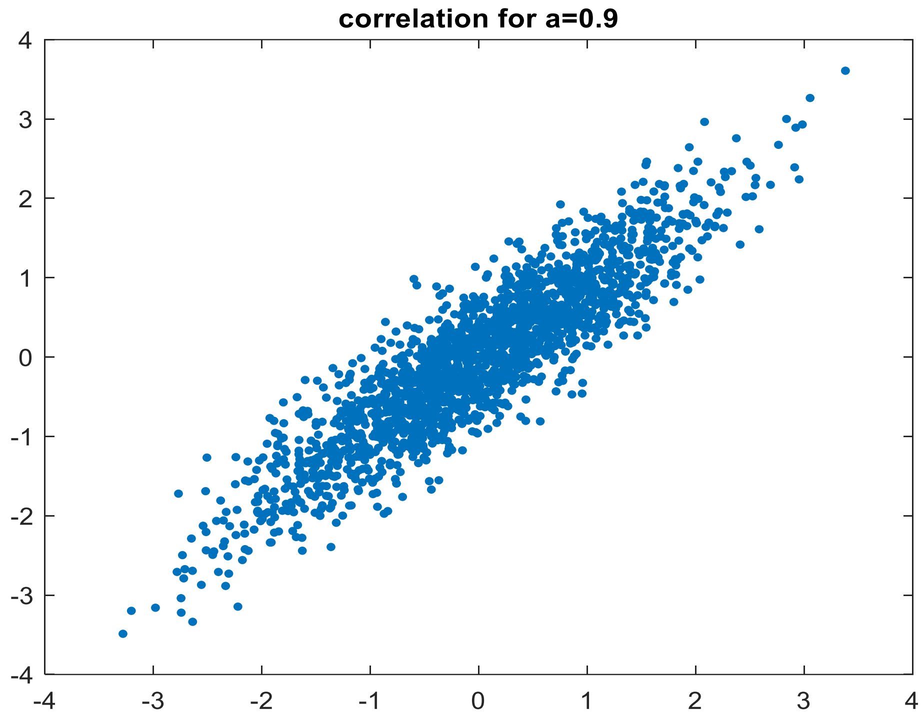

Figure14

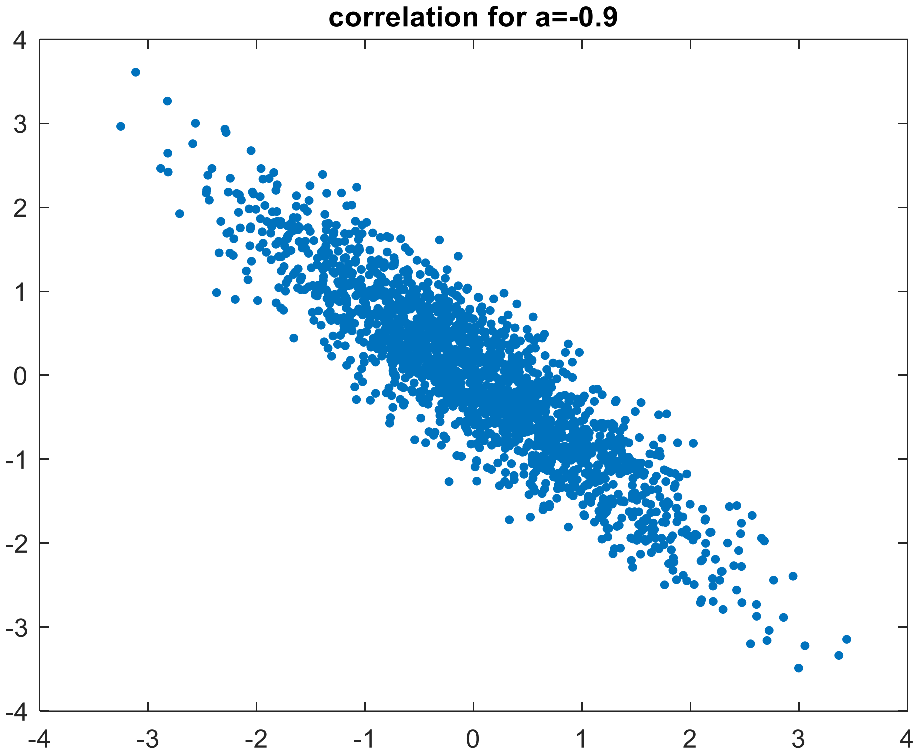

Figure15

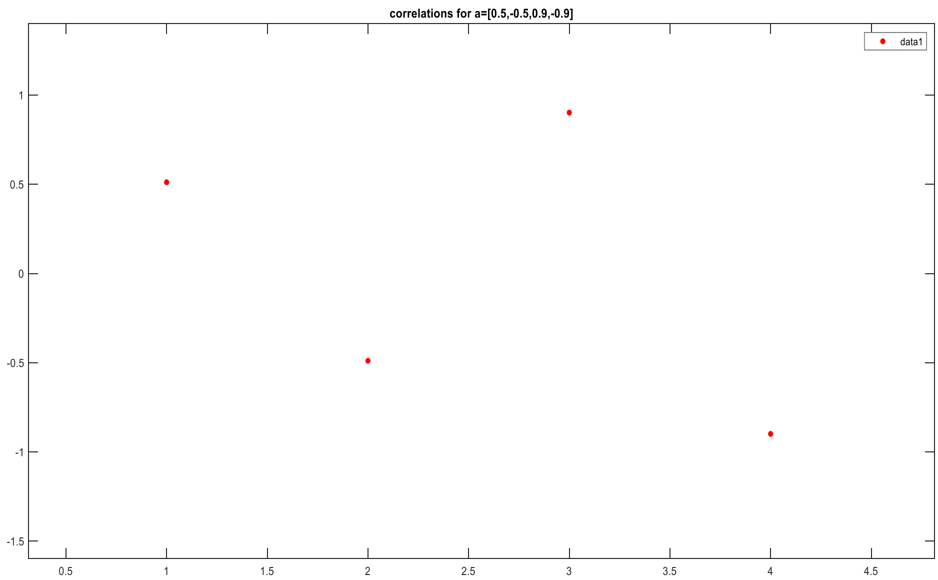

Figure16

One easy way to calculate correlation between these distributions is multiplying two vectors and then calculating mean of the result vector.

2)

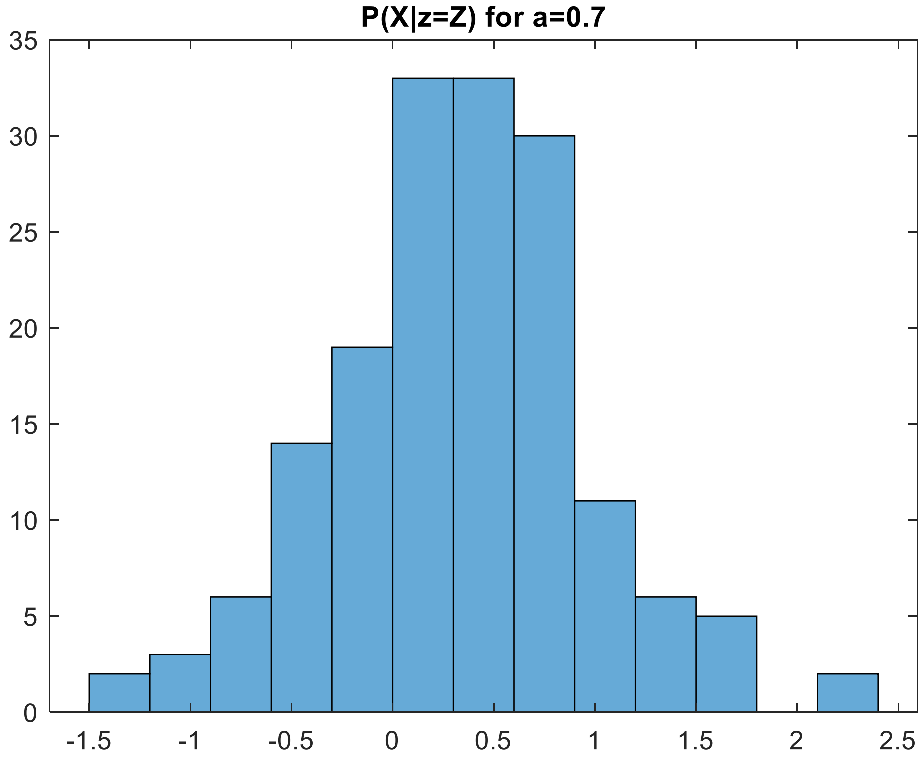

Figure17

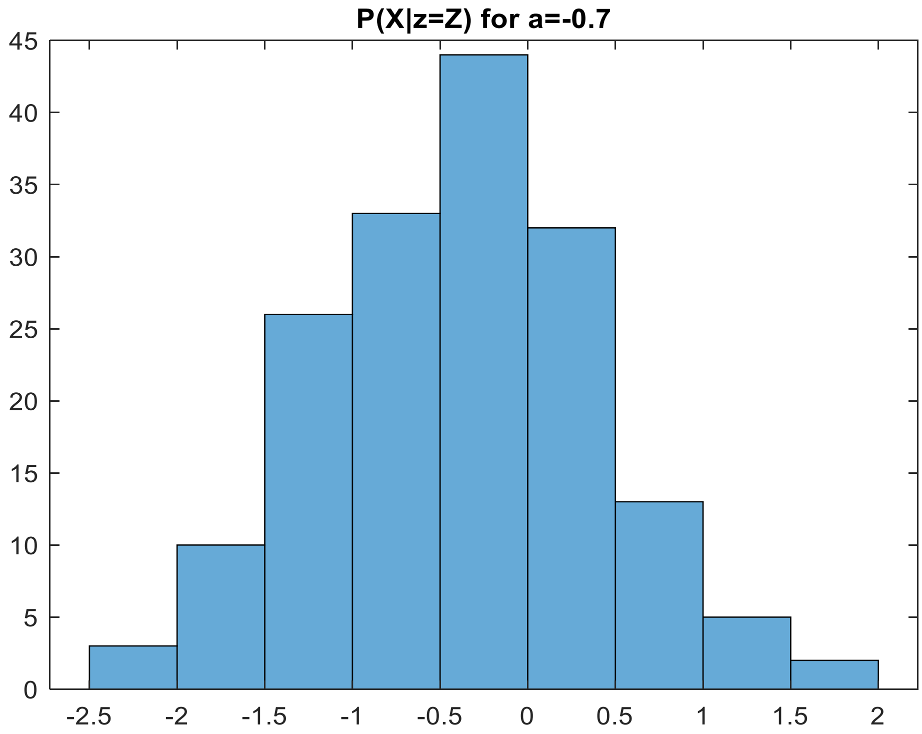

Figure18

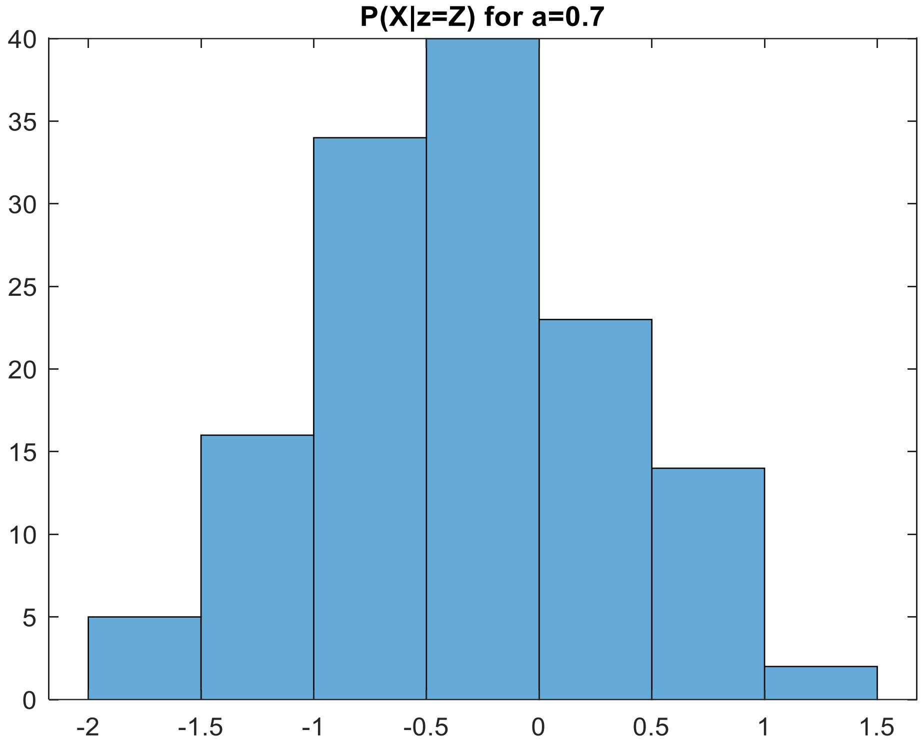

Figure19

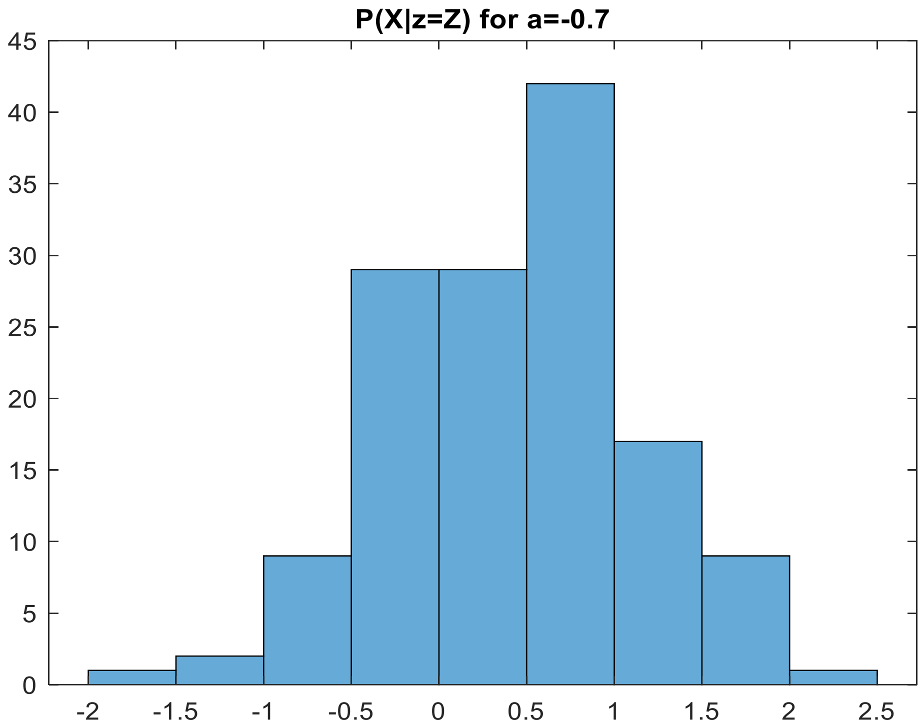

Figure20

By observing above graphs (Figure 17,18,19,20) we conclude that the result doesn't have the same distribution as variable X though it still has the form of normal distribution, the reason is: two variable Z and X aren't independent in this case and they have some correlation and in cases where correlation and interval of Z has different signs the mean will be negative and somewhere around -0.35, also the variance changes, it all means the distribution will be more drawn to one side (for example if $-0.6 < Z < -0.4$ and $\alpha = 0.7$ its more probable for X to be negative and around -0.4).

بخش ۳-

0)

Rows contain a sample of each random variable in a specific time (n) which is a random variable itself, and columns are the values of each random variable during the time, also it is deterministic. The whole matrix represents a random process.

1)

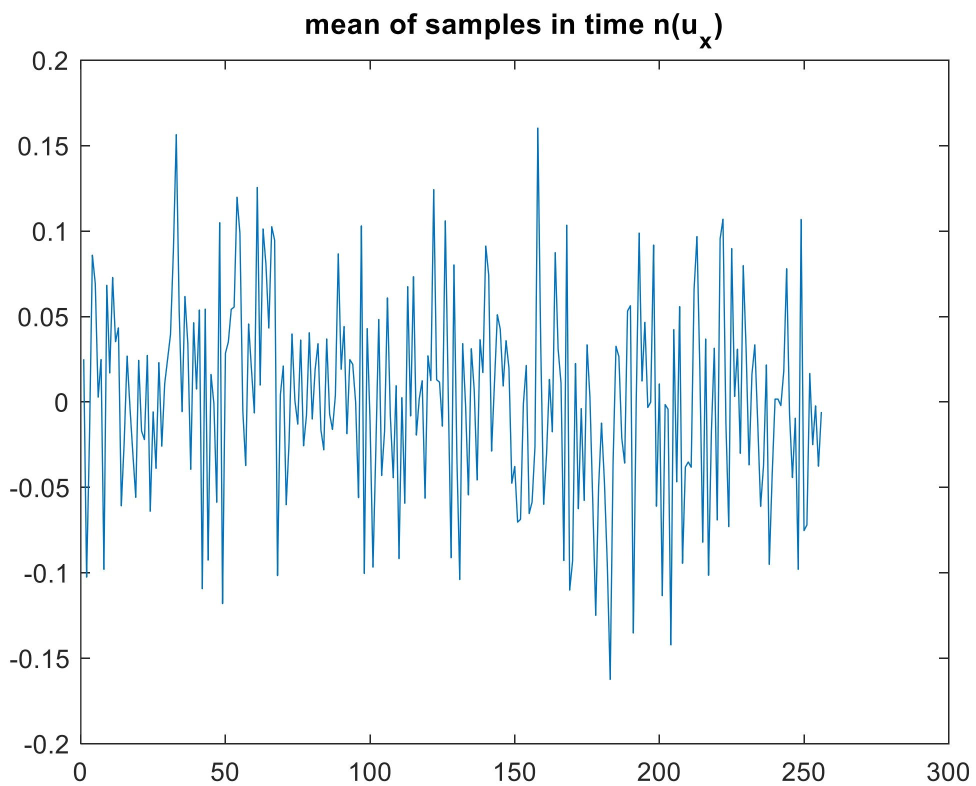

Figure21

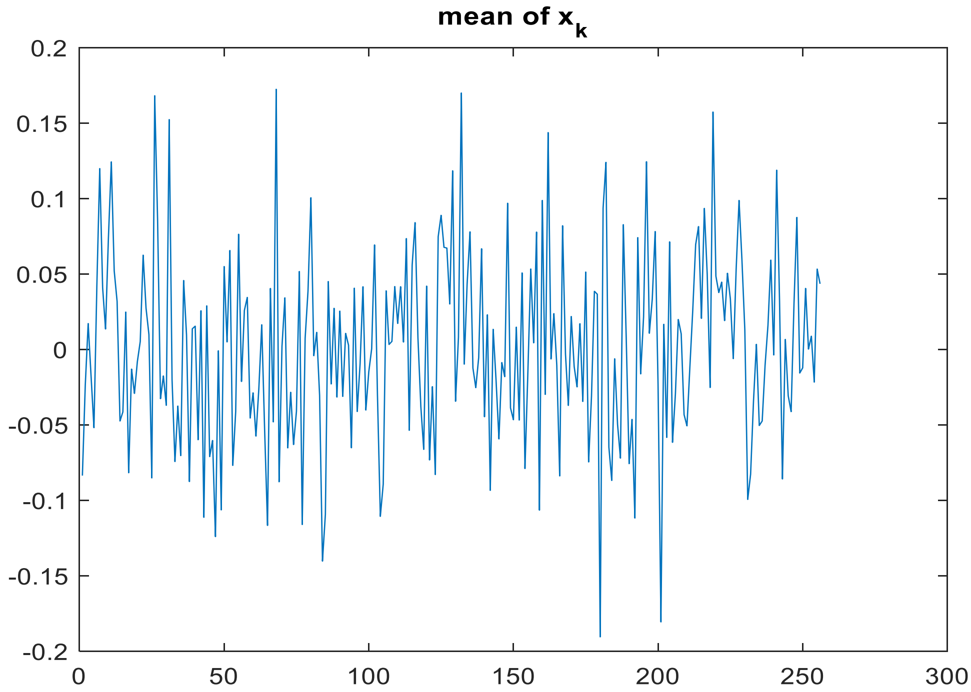

Figure22

Yes this process is **mean-ergodic** because the ensemble average equals the time average.

2)

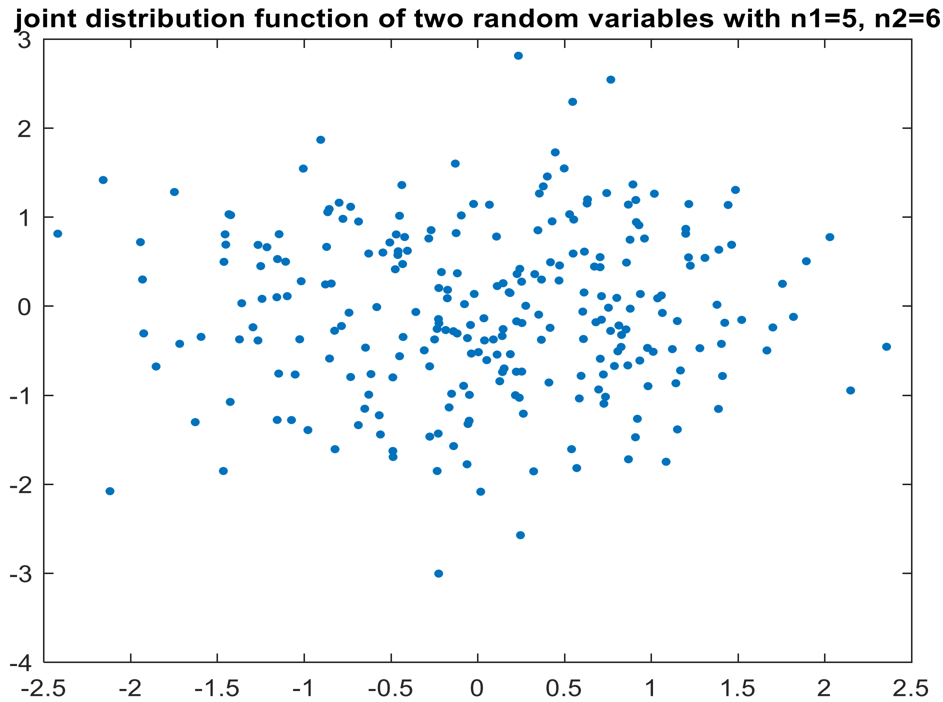

Figure23

The formula we used for calculating the correlation is $R_x(n_1, n_2) = E\{X_{n1}X_{n2}\}$

Also there is a *corr* function in *MatLab* that we can use to calculate correlation.

In Figure 21 we get $correlation = 0.009$

and according to that we conclude that these 2 random variables are not correlated.

Also from the Figure (23) we can see that the samples aren't distributed around the center/line so we conclude that they are not correlated.

بخش ۴-

1)

$$x[n] = x[0] + w[1] + w[2] + \cdots + w[n] = 0 + \sum_{i=1}^{n} w[i]$$

$$w[i] \sim N(0,1) \rightarrow x[n] \sim N(\mu_{xn}, \sigma_{xn}) \rightarrow E\{x\} = \sum_{i=1}^{n} E\{w[i]\} = 0$$

$$\rightarrow x[n] \sim (0, n) \xrightarrow{\text{LLN}} \lim_{k\rightarrow\infty} \frac{x_1[n] + \cdots + x_k[n]}{k} = \mu_{xn} = 0 \quad \blacksquare$$

2)

$$x[1] = x[0] + w[1] = w[1] \rightarrow E\{x^2[1]\} = E\{w^2[1]\} = w^2[1]$$

$$p_x[n] = E\{x^2[n]\} \ , x^2[n] = x^2[n-1] + 2x[n-1]w[n] + w^2[n]$$

$$\rightarrow \ p_x[n] = p_x[n-1] + E\{2x[n-1]w[n]\} + E\{w^2[n]\} \ \ *$$

$$R_{x,w} = \frac{cov(x[n-l], w[n])}{\sigma_x \sigma_w} \ , \qquad cov(x[n-l], w[n]) = E\{(x[n-l] - \mu_x)(w[n] - \mu_w)\}$$

$$= E\{x[n-l].w[n]\} \ , \qquad using\ previous\ part \rightarrow E\{(x[0] + \sum_{i=1}^{n-l} w[i]).(w[n])\}$$

$$= E\{x[0]w[n]\} + E\{w[1]w[n] + \cdots + E\{w[n-l]w[n]\} = w[1]E\{w[n]\} + \cdots + w[n-l]E\{w[n]\} = 0 \rightarrow \textbf{therefor x[n} - \textbf{l] and w[n] are not correlated}$$

So $p_x[n] = p_x[n-1] + 1\ if\ n \rightarrow \infty\ \ p_x[n] \rightarrow \infty$

3)

$$E\{x[n]x[n-1]\} = E\{(x[n-1]+w[n]).x[n-1]\} = E\{x^2[n-1]\} = p_x[n-1] = n-1$$

$$E\{x[n]x[n-l]\} = E\{x^2[n-l] + w[n]x[n-l] + \cdots + w[n-l+1]x[n-l]\}$$

$$= E\{x^2[n-l]\} = p_x[n-l] = n-l$$

No, its not WSS because its not stationary (correlation is variable of time)

$$\rho x(n, n-l) = \frac{n-l}{\sqrt{n(n-l)}} \quad if\ n \to \infty \quad \rho x(n, n-l) \to 1$$

4)

Figure24

By observing the graph in figure(24) we can see that in time n, the mean of random samples is zero and as the n increases the variance of the random variables increases, these results match what we got in theory in previous questions.

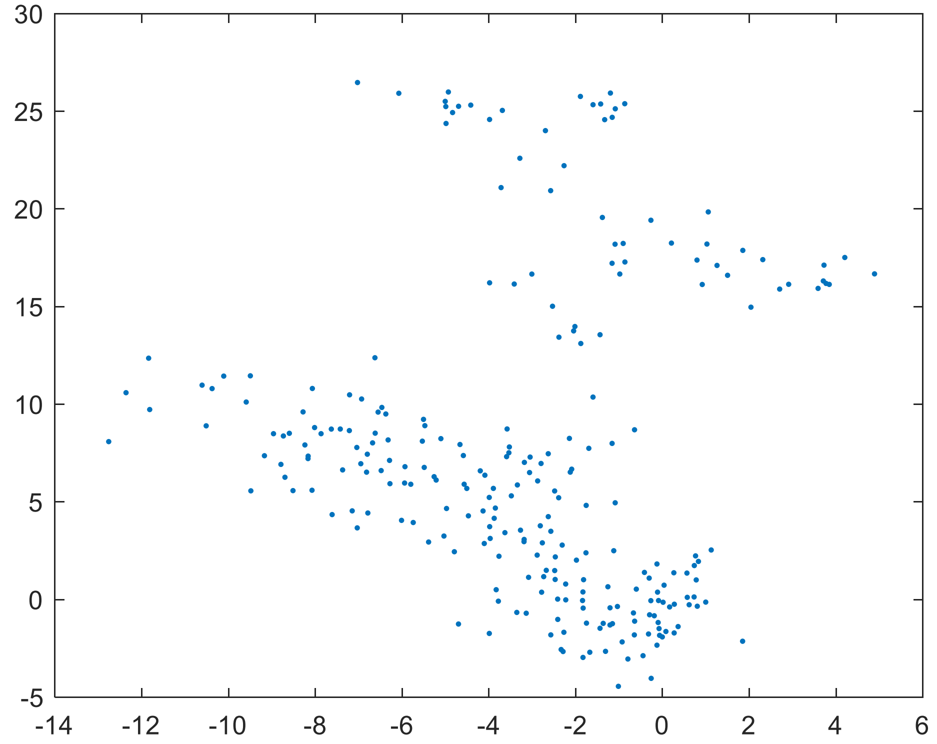

Figure25

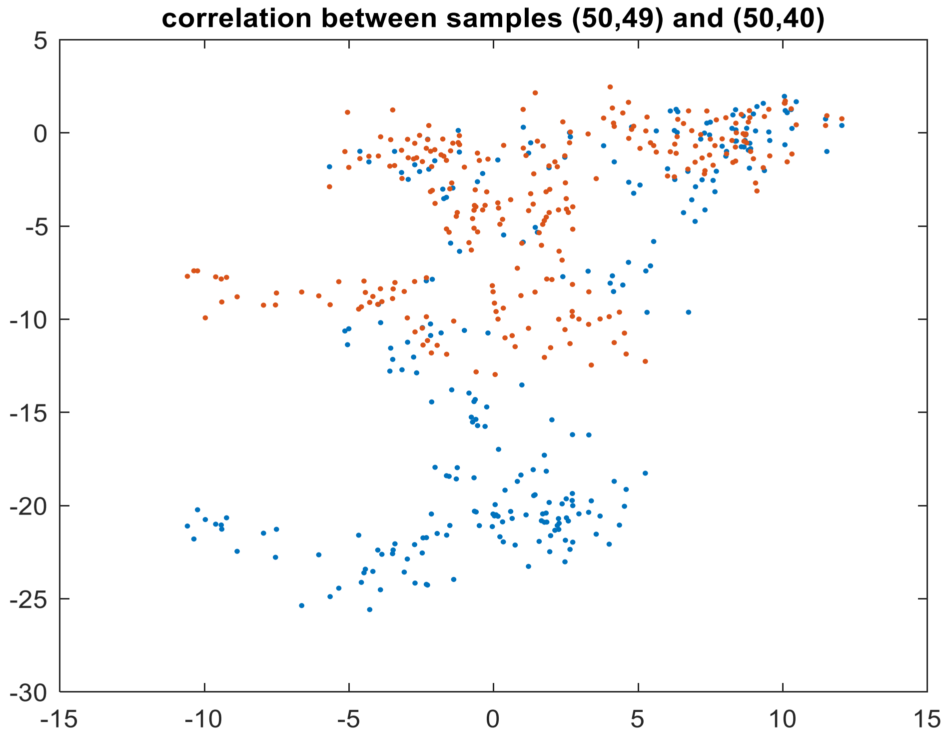

Figure26

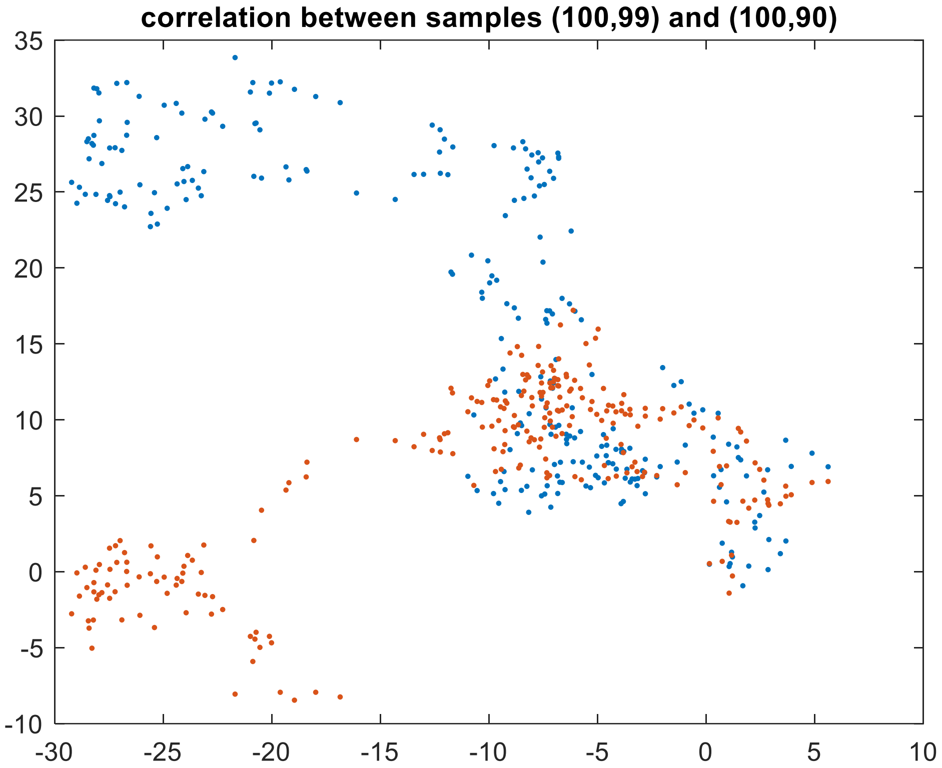
correlation between samples (100,99) and (100,90)
35
30
25
20
15
10
5
0
-5
-10
-30
-25
-20
-15
-10
-5
0
5
10

Figure27

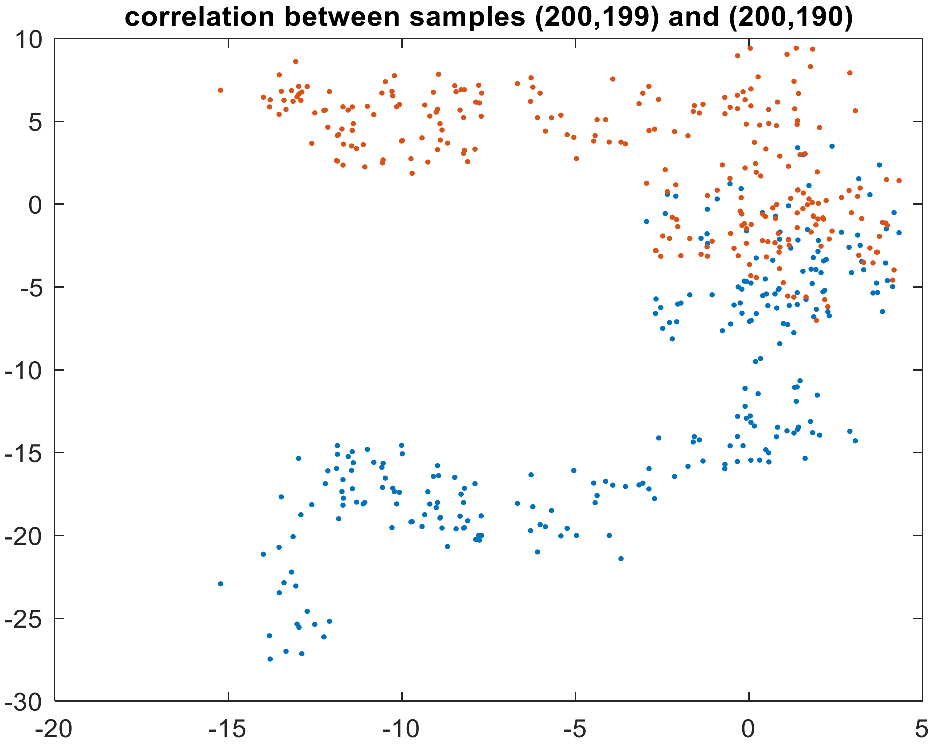

Figure28

No as we can see in above graphs figure (26,27,28) the correlation varies with time so its not WSS.

5)

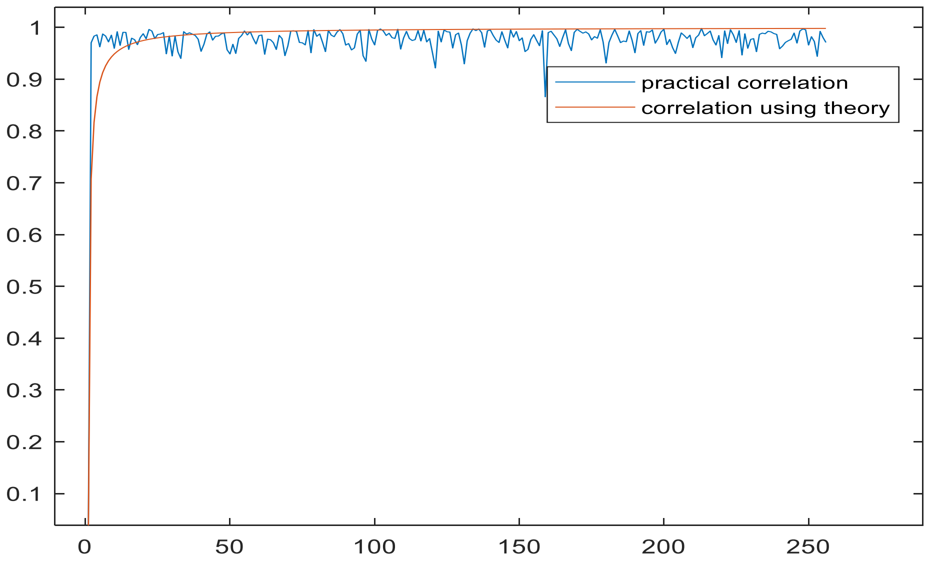

Figure29

No we cant estimate auto correlation from $\hat{r}x(n, n-1)$ because the auto-correlation varies with time (the process is not WSS).