A Comprehensive Framework for Control of Complex Networks

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Introduction

The study of Complex Networks is a new branch of science that has recently attracted the attention of researchers from various fields. Brain networks, power networks, and social networks are examples of complex networks. Controlling complex networks is of great importance in science and engineering. Network control means steering the network states to the desired value. Despite the recent development of control theory, we still lack a framework for controlling complex networks. In this project, we provide a comprehensive framework for optimal network control. Moreover, we present examples of the implementation of this framework on real-world networks.

Methods

In complex networks, we usually do not have the ability to access all network nodes. Therefore, we will only have access to a limited number of nodes to control the network. Consequently, determining controllable nodes in order to achieve the controllability of the entire network is of particular importance. Full controllability of the network will be possible with different sets of nodes, so it is necessary to determine the most optimal set of nodes to minimize the control energy (cost). To determine the minimum number of controllable nodes, we examine a method based on the maximum matching algorithm and then mention its limitations, and then we examine a method based on the maximum geometric multiplicity [1] for this purpose. According to the aforementioned method, the minimum number of nodes required to control a complex network with dynamics $\dot{x}=Ax+Bu$ will be equal to the maximum geometric multiplicity of matrix A. Also, by performing elementary column operations on the $\lambda I-A$ matrix, the location of dependent columns will be determined, and the selected combination of nodes corresponding to these columns can be candidates to make the network controllable. Therefore, it is necessary to choose the most optimal combination possible to minimize the control energy. For this purpose, we solve an optimization problem by the Projected Gradient Method (Projected Gradient Method) [2]. Since this method has the problem of converging to the local minimum, we develop this method and propose a method based on Monte Carlo scenarios [2] (PGME). Finally, the importance of nodes will be determined in order to minimize the control energy. Then, according to the type of problem (discrete or continuous time, control horizon, and system dynamics), the appropriate method to find the optimal input is specified, and we solve the modeled problem. However, in reality, we usually do not have enough knowledge about the dynamics of complex natural networks, so it is necessary to provide a data-driven method to control such networks [3]. For this purpose, we provide expressions based on approximate data and closed forms of control inputs. In this method, data is obtained by applying non-optimal inputs (usually random) and collecting the system response.

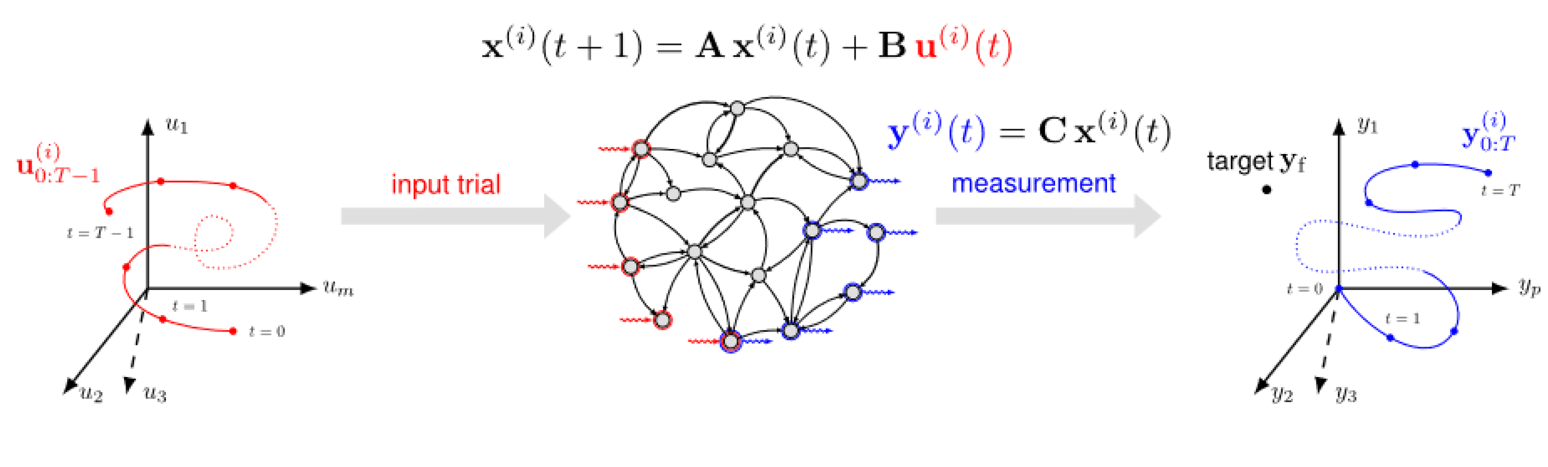

Figure 1. An illustrative diagram showing the basics of controlling a network

Results

Examples of implementations of our proposed framework for control of various complex networks

1. Controlling opinions dynamic in a social network in Fig. 2
2. Desynchronization of the activity of different regions of the brain in order to improve the treatment process of epilepsy and stop convulsive attacks in Fig. 3 [4].
3. Phase and frequency control of generators in a power network after fault in Fig. 4 and 5

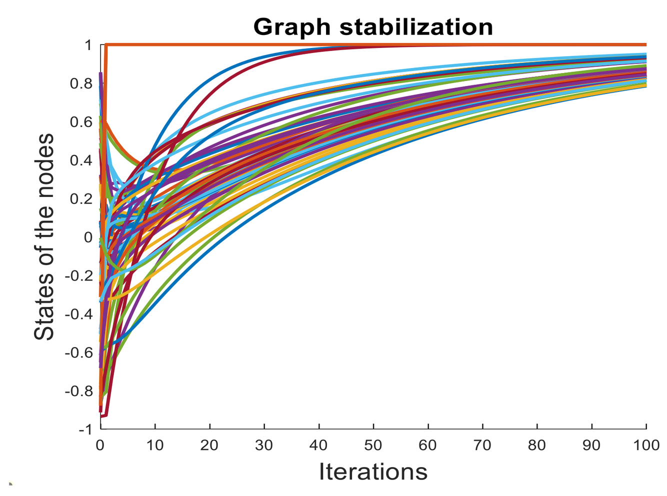

Figure 2. Opinions of people in a social network (-1 indicates the most disagreement and +1 indicates the most agreement.)

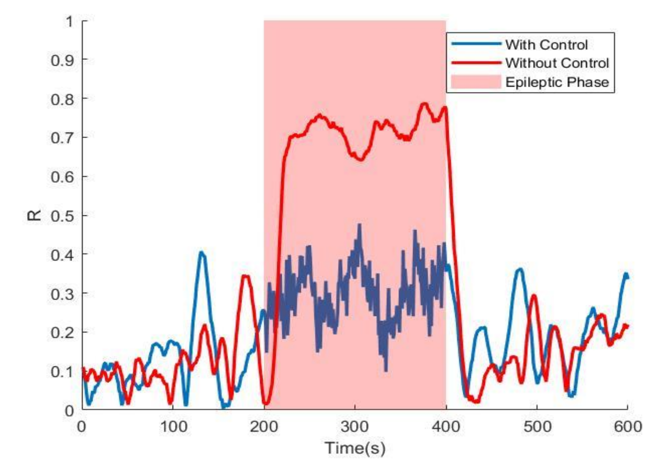

Figure 3: Brain synchronization measure (red color indicates the uncontrolled response, blue indicates the controlled response, and the colored area indicates the seizure phase.)

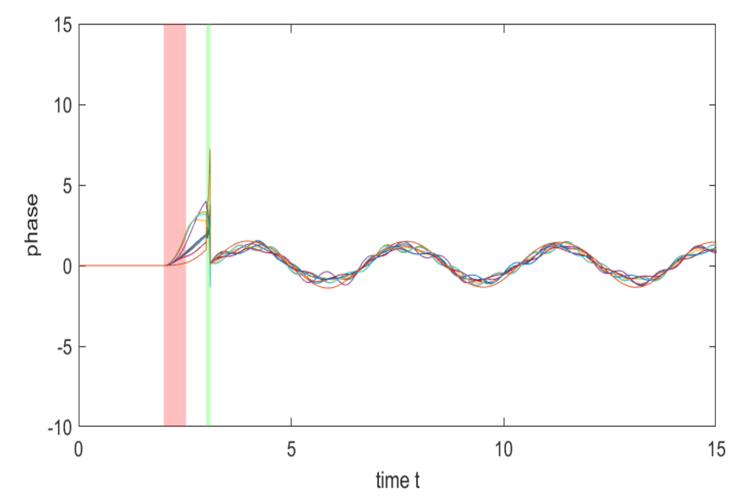

Figure 4: The phase of the generators over time (the error occurred in the red area for 0.5 seconds and was resolved, and a fault was entered in the control horizon for 0.1 seconds in the green area)

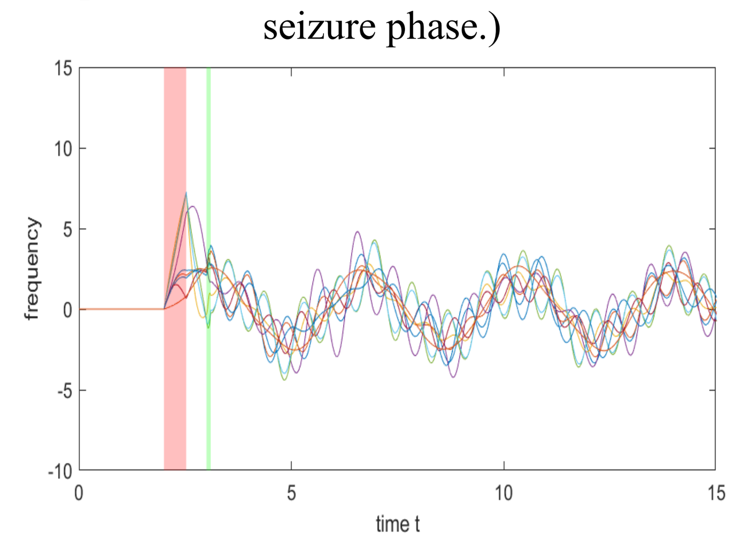

Figure 5: The frequency of generators over time (the error occurred in the red area for 0.5 seconds and was resolved, and an error was entered in the control horizon for 0.1 seconds in the green area)

Conclusion

While the previous researches focused on only one element of complex network control, and each had the mentioned limitations, this project addresses various aspects of complex network control. Among the innovations and achievements of this project is the optimal selection of driver nodes, the Implementation of numerical methods of optimization, examining a wide range of types of networks, and proposing a potential method for treating neurological diseases such as epilepsy.

References

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