Dynamic of Nonlinear Robotic Systems Final Project.

Siba Issa

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Figure 1: Mitsubishi robot RV-1A

Abstract

This report of the final project in Dynamics of Non Linear Robotics course for 1^{st} year master students at Innopolis University. I have worked with mitsubishi robot RV-1A with a ZXZ wrist configuration. In this report we will state our work in the kinematic and dynamic part, passing through the trajectory planning and you will find in the end of this report the results of our code while all the implementations and code are available on GitHub.

1 Introduction

In this introduction we will talk about the mechanical structure of Mitsubishi RV-1A arm. As it is clear in (Figure 1) that this robot has 6 Degree of freedom, and it is an Anthropomorphic arm since it has a 3 revolute joints in the arm part.The wrist part has a three revolute joints too.

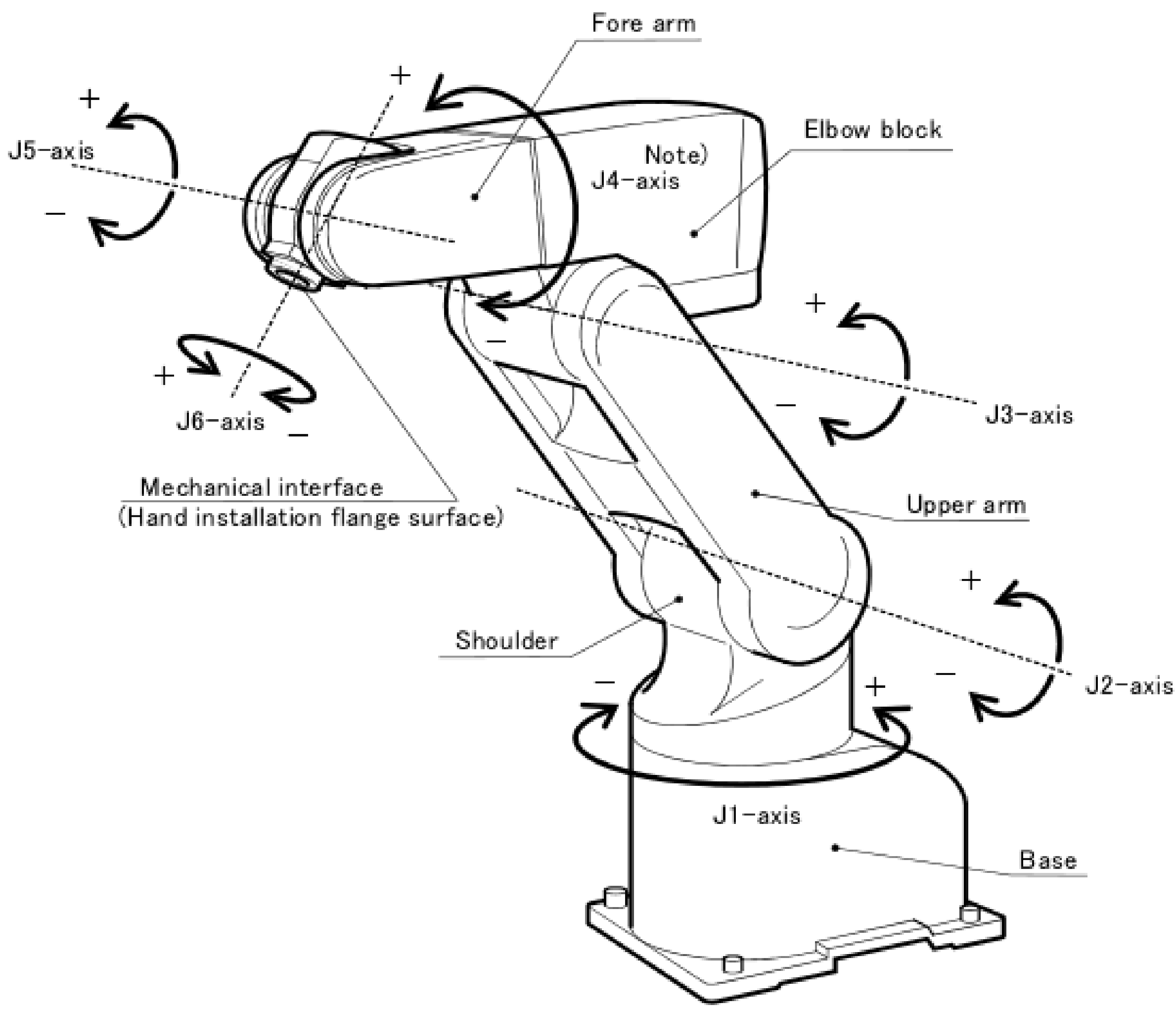


Figure 2: Mitsubishi RV-1A mechanical structure

2 Kinematic Analysis

The first step to get in our kinematics analysis is to draw our kinematic scheme and to place the Cartesian coordinate frames of the robot. And as we see below in (Figure 2) that the wrist configuration of our model is ZXZ.

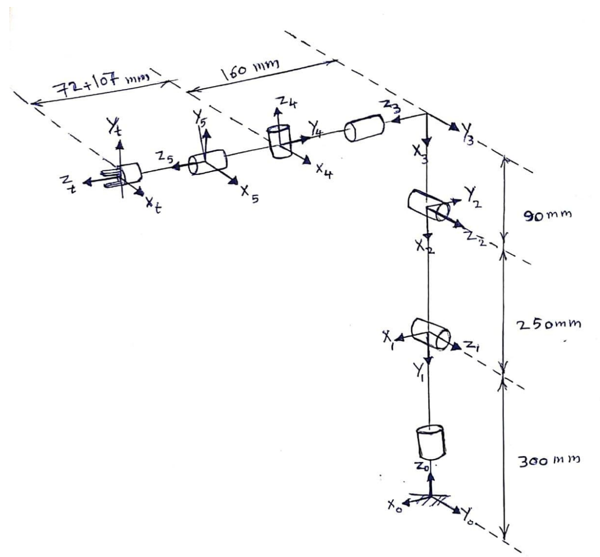


Figure 3: Mitsubishi RV-1A mechanical structure

2.1 Forward Kinematics

In order to solve the forward kinematics problem; Denavit Hartenberg is used. so sfter placing Cartesian coordinate frames and specifying our wrist configuration,the DH parameters of the robot were found and are shown in (Table 1).

i	θ	d	α	a
0-1	θ_1	0.3	$\frac{-\pi}{2}$	0
1-2	θ_2	0	0	-0.25
2-3	θ_3	0	$\frac{\pi}{2}$	-0.09
3-4	θ_4	0.16	$\frac{-\pi}{2}$	0
4-5	θ_5	0	$\frac{\pi}{2}$	0
5-6	θ_6	0.179	0	0

Table 1: DH- Parameters

Then for each joint of the robot, populate anew T matrix with the following values:

$$T_i^{i-1} = \begin{bmatrix} cos(\theta_i) & -sin(\theta_i) \cdot cos(\alpha_i) & sin(\theta_i) \cdot sin(\alpha_i) & r \cdot cos(theta_i) \\ sin(\theta_i) & cos(\theta_i) \cdot cos(\alpha_i) & -cos(\theta_i) \cdot sin(\alpha_i) & r \cdot sin(theta_i) \\ 0 & sin(\alpha_i) & cos(\alpha_i) & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then we multiply all of the matrices together starting with the first joint all the way up to the end effector. The final T matrix will contain the position and the orientation of the end effector.

$$T_6^0 = T_1^0 \cdot T_2^1 \cdot T_3^2 \cdot T_4^3 \cdot T_5^4 \cdot T_6^5$$

3 Jacobian Analysis

The Jacobian matrix provides the transition between the velocity of the joint variables and the linear and angular velocities of the end effector. The dimension of this matrix is 6x6. In order to compute the Jacobian; first of all we have to calculate these transformations:

$$T_1^0 = R_z(q_1) \cdot T_z(0.3) \tag{1}$$

$$T_2^0 = T_1^0 \cdot R_y(q_2) \cdot T_z(0.25) \tag{2}$$

$$T_3^0 = T_2^0 \cdot R_y(q_3) \cdot T_z(0.09) \cdot T_x(0.160) \tag{3}$$

$$T_4^0 = T_3^0 \cdot R_x(q_4) \tag{4}$$

$$T_5^0 = T_4^0 \cdot R_y(q_5) \tag{5}$$

$$T_6^0 = T_5^0 \cdot R_x(q_6) \cdot T_x(0.072 + 0.107) \tag{6}$$

The Jacobian for revolute joints is calculated using the following equation:

$$J_i = \begin{bmatrix} Z_{i-1} \times (O_n - Oi - 1) \\ Z_{i-1} \end{bmatrix}$$

where O_i is the positional vector from T_i transformation matrix, and Z_i is the rotation matrix from T_i transformation cross product the column vector corresponds to the axis of rotation. The Jacobian will be written in the following format:

$$J = \begin{bmatrix} J_1 & J_2 & J_3 & J_4 & J_5 & J_6 \end{bmatrix}$$

3.1 Inverse Kinematics

As we know that $q = f^{-1}(x)$, where $X = [P_x, P_y, P_z, \varphi, \theta, \Psi]$ and $q = [q_1, q_2, ..., q_n]$ and we can express the final Transformation (from the first joint to the end effector) as following:

$$T_n^0 = \begin{bmatrix} s_x & n_x & a_x & p_x \\ s_y & n_y & a_y & p_y \\ s_z & n_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

according to the inverse Kinematic model:

$$T_6^0 = T_1^0(q_1)T_2^1(q_2)....T_6^5 \tag{8}$$

$$T_1^0(q_1)T_6^0 = T_2^1(q_2)....T_5^4 T_6^5 \tag{9}$$

$$T_6^1(q_1) = T_6^1(q_2...q_6) - > q_1 \tag{10}$$

when three revolute joints whose axes intersect at a point (as it happens in joints 4, 5, 6) there wont be a change in direction of the end effector while the position will be the same, so:

$$P_6^0 = P_5^0 = P_4^0 \tag{11}$$

and:

$$\begin{bmatrix} P_{x4} \\ P_{y4} \\ P_{z4} \\ 1 \end{bmatrix} = T_1^0(q_1)T_2^1(q_2)T_3^2(q_3)T_4^3(q_4) \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{12}$$

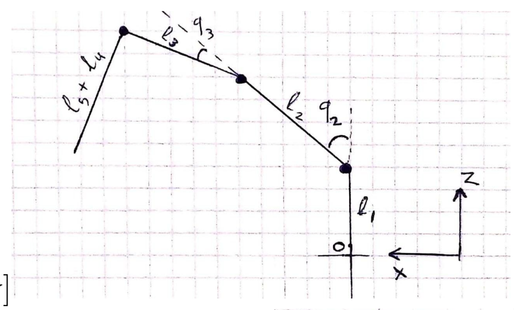
4 Dynamic Analysis

In this section, the dynamic model of Mitsubishi RV-1A robot is discussed. Dynamics is the field of science that studies the motions of objects under a cause and effect relationship with the forces applied to them. Lagrange-Euler equations were used to obtain the dynamic model. This method is an energy-based approach. lagrangian of the mechanical system can be defined as a function of the generalized coordinates. The general dynamic equation is obtained by:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial V}{\partial q_j} = P_j \tag{13}$$

Where, T is the total kinetic energy, V is the total potential energy of the system.P_j is the generalized force, t is time, q_j is the generalized coordinates and $\dot{q}_j$ is the generalized velocity.

First of all, we need to determine the center of mass - which in our case in the end of each link - and write the equations of each link projected on each axes.

[Side view]


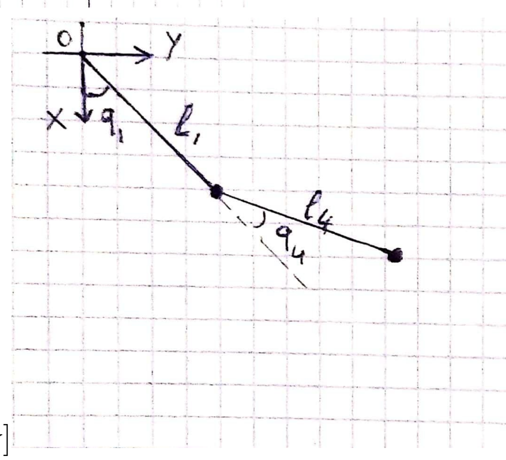
[Top view]


Figure 4: Mitsubishi robot RV-1A Top and Side views

$x_1 = l1.c1$
$x_2 = l1.c1 + l2.c2$
$x_3 = l1.c1 + l2.c2 + l3.c(q_2 + q_3)$
$x_4 = l1.c1 + l2.c2 + l3.c(q_2 + q_3) + l4.c(q_1 + q_4)$
$x_5 = l1.c1 + l2.c2 + l3.c(q_2 + q_3) + l4.c(q_1 + q_4)$
$x_6 = l1.c1 + l2.c2 + l3.c(q_2 + q_3) + l4.c(q_1 + q_4) + l6.c(q_1 + q_4 + q_6)$

$y_1 = l1.s1$
$y_2 = l1.s1$
$y_3 = l1.s1$
$y_4 = l1.s1 + l4.s(q_1 + q_4)$
$y_5 = l1.s1 + l4.s(q_1 + q_4) + l5.c5$
$y_6 = l1.s1 + l4.s(q_1 + q_4) + l5.c5 + l6.s(q_1 + q_4 + q_6)$

$z_1 = 0$
$z_2 = l2.s2$
$z_3 = l2.s2 + l3.c(q_2 + q_3)$
$z_4 = l2.s2 + l3.c(q_2 + q_3)$
$z_5 = l2.s2 + l3.c(q_2 + q_3) + l5.s5$
$z_6 = l2.s2 + l3.c(q_2 + q_3) + l5.s5$

Now we have to compute translation and rotation jacobian matrices in order to compute $M_(q)$.
let us compute the translation jacobians:

$$J_v^i = \begin{bmatrix} \frac{\partial x_i}{q_1} & ... & \frac{\partial x_i}{q_6} \\ \frac{\partial y_i}{q_1} & ... & \frac{\partial y_i}{q_6} \\ \frac{\partial z_i}{q_1} & ... & \frac{\partial z_i}{q_6} \end{bmatrix} \tag{14}$$

$$J_v^1 = \begin{bmatrix} -l1.s1 & 0 & 0 & 0 & 0 & 0 \\ l1.c1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$J_v^2 = \begin{bmatrix} -l1.s1 & -l2.s2 & 0 & 0 & 0 & 0 \\ l1.c1 & 0 & 0 & 0 & 0 & 0 \\ 0 & l2.c2 & 0 & 0 & 0 & 0] \end{bmatrix}$$
$$J_v^3 = \begin{bmatrix} -l1.s1 & -l3.s23 - l2.s2 & -l3.s23 & 0 & 0 & 0 \\ l1.c1 & 0 & 0 & 0 & 0 & 0 \\ 0 & l2.c2 - l3.s23 & -l3.s23 & 0 & 0 & 0 \end{bmatrix}$$
$$J_v^4 = \begin{bmatrix} -l4.s14 - l1.s1 & -l3.s23 - l2.s2 & -l3.s23 & -l4.s14 & 0 & 0 \\ l4.c14 + l1.c1 & 0 & 0 & l_4.c14 & 0 & 0 \\ 0 & l2.c2 - l3.s23 & -l3.s23 & 0 & 0 & 0 \end{bmatrix}$$
$$J_v^5 = \begin{bmatrix} -l4.s14 - l1.s1 & -l3.s23 - l2.s2 & -l3.s23 & -l4.s14 & 0 & 0 \\ l4.c14 + l1.c1 & 0 & 0 & l4.c14 & -l5.s5 & 0 \\ 0 & l2.c2 - l3.s23 & -l3.s23 & 0 & l5.c5 & 0 \end{bmatrix}$$
$$J_v^6 = \left[\begin{matrix} -l4.s14 - l1.s1 - l6.s146, -l3.s23 - l2.s2 & -l3.s23 & -l4.s14 - l6.s146 & 0 \\ l4.c14 + l1.c1 + l6.c146 & 0 & 0 & l4.c14 + l6.c146 \\ -l4.s14 - l1.s1 - l6.s146 & -l3.s23 - l2.s2 & -l3.s23 & -l4.s14 - l6.s146 \end{matrix}\right.$$
and now we will move to the rotation jacobian:

$$J_w^1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \; J_w^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \; J_w^3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$J_w^4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \; J_w^5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \; J_w^6 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

now we can compute $M(q)$ matrix, by using the following formula:

$$M(q) = \sum_{i=1}^{6} m_i J_v^{iT} J_v^i + J_w^{i\,T} R_i I R_i^T J_w^i \tag{15}$$

where m_i is the mass of the i^{th} joint, and I is the inertia matrix in which we will consider it as a unit matrix for simplicity (but we can change it whenever we want regarding our model)

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{16}$$

so, our $M(q)$ matrix will be in the following form:

$$M(q) = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} & m_{15} & m_{16} \\ m_{21} & m_{22} & m_{23} & m_{24} & m_{25} & m_{26} \\ m_{31} & m_{32} & m_{33} & m_{34} & m_{35} & m_{36} \\ m_{41} & m_{42} & m_{43} & m_{44} & m_{44} & m_{46} \\ m_{51} & m_{52} & m_{53} & m_{54} & m_{55} & m_{56} \\ m_{61} & m_{62} & m_{63} & m_{64} & m_{65} & m_{66} \end{bmatrix} \tag{17}$$

Then we calculate $C(q, \dot{q})$matrix, by using the following formula:

$$C_{ij} = \sum_{k=1}^{6} c_{ijk} \dot{q}_k \tag{18}$$

and we will also get a matrix as following:

$$C(q, \dot{q}) = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{44} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{bmatrix} \tag{19}$$

and the $g(q)$ matrix is:

$$g = \sum_{k=1}^{6} (J_{wi}^k)^T g_0 \tag{20}$$

finally we can derive the dynamic model by using Euler-Lagrange approach by substituting in the next formula:

$$M(q) + C(q, \dot{q})\dot{q} + g(q) = \tau \tag{21}$$

5 Implementation

our task is to move the end effector from the red point (start point) in (Figure 5) to the green point (end point) through yellow point (middle point).

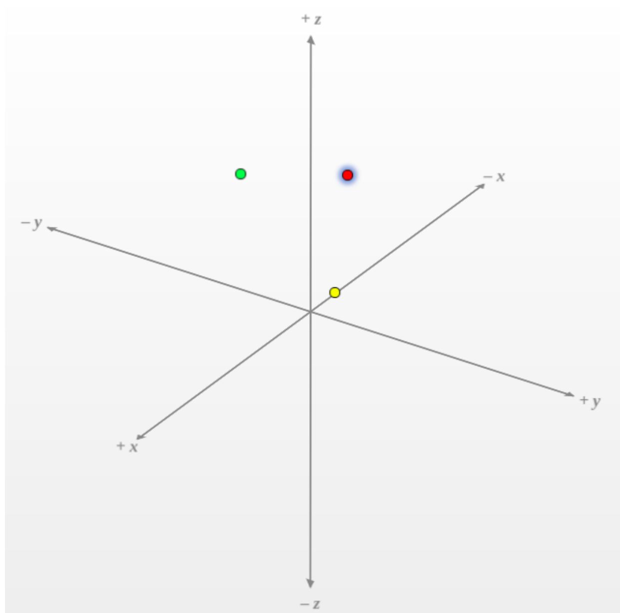


Figure 5: Task positions in Cartesian space

while we can see below the points in the Cartesian space (their rotations and positions).

$$T_{start} = \begin{bmatrix} 1.0000 & 0 & 0 & -0.3400 \\ 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 1.0000 & 0.6390 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \tag{22}$$

$$T_{mid} = \begin{bmatrix} -0.9967 & 0.0231 & -0.0773 & 0.0753 \\ -0.0511 & 0.5609 & 0.8263 & 0.1964 \\ 0.0625 & 0.8276 & -0.5579 & 0.2085 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \tag{23}$$

$$T_{end} = \begin{bmatrix} -0.4178 & 0.8903 & -0.1811 & -0.1337 \\ -0.5045 & -0.3931 & -0.7687 & -0.5156 \\ -0.7556 & -0.2297 & 0.6134 & 0.5834 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \tag{24}$$

After defining my task, we have to move from Cartesian space to joint space by using Inverse Kinematics:

$$q_{start} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} q_{mid} = \begin{bmatrix} -5.7853 \\ 3.7521 \\ 2.4453 \\ 1.2217 \\ 2.1942 \\ 1.7433 \end{bmatrix} q_{end} = \begin{bmatrix} 7.5921 \\ 0.5532 \\ 11.1868 \\ 3.4214 \\ 0.0872 \\ 5.7279 \end{bmatrix} \tag{25}$$

Now as we have our points in the joint space we can start planning our trajectory, butt first we need to specify the maximum velocity and acceleration for each joint. The maximum velocities have token from the Specifications Manual and converted to [radian] while the maximum acceleration values have chosen randomly:

$$v_{max} = \begin{bmatrix} 3.1459 \\ 1.5708 \\ 2.35619 \\ 3.14159 \\ 3.14159 \\ 3.66519 \end{bmatrix} a_{max} = \begin{bmatrix} 12 \\ 8 \\ 10 \\ 4 \\ 4 \\ 2 \end{bmatrix} \tag{26}$$

The trajectory planning has done by using polynomial method (5^{th} degree). polynomial's variables have determined by solving this equation $Ax = B$, where A and B are as following:

$$A = \begin{bmatrix} t_0^5 & t_0^4 & t_0^3 & t_0^2 & t_0 & 1 \\ t_f^5 & t_f^4 & t_f^3 & t_f^2 & t_f & 1 \\ 5t_0^4 & 4t_0^3 & 3t_0^2 & 2t_0 & 1 & 0 \\ 5t_f^4 & 4t_f^3 & 3t_f^2 & 2t_f & 1 & 0 \\ 20t_0^3 & 12t_0^2 & 6t_0 & 2 & 0 & 0 \\ 20t_f^3 & 12t_f^2 & 6t_f & 2 & 0 & 0 \end{bmatrix} a_{max} = \begin{bmatrix} q_{i0} \\ q_{if} \\ \dot{q_{i0}} \\ \dot{q_{if}} \\ \ddot{q_{i0}} \\ \ddot{q_{if}} \end{bmatrix} \tag{27}$$

Figure 5 shows trajectory planning results.

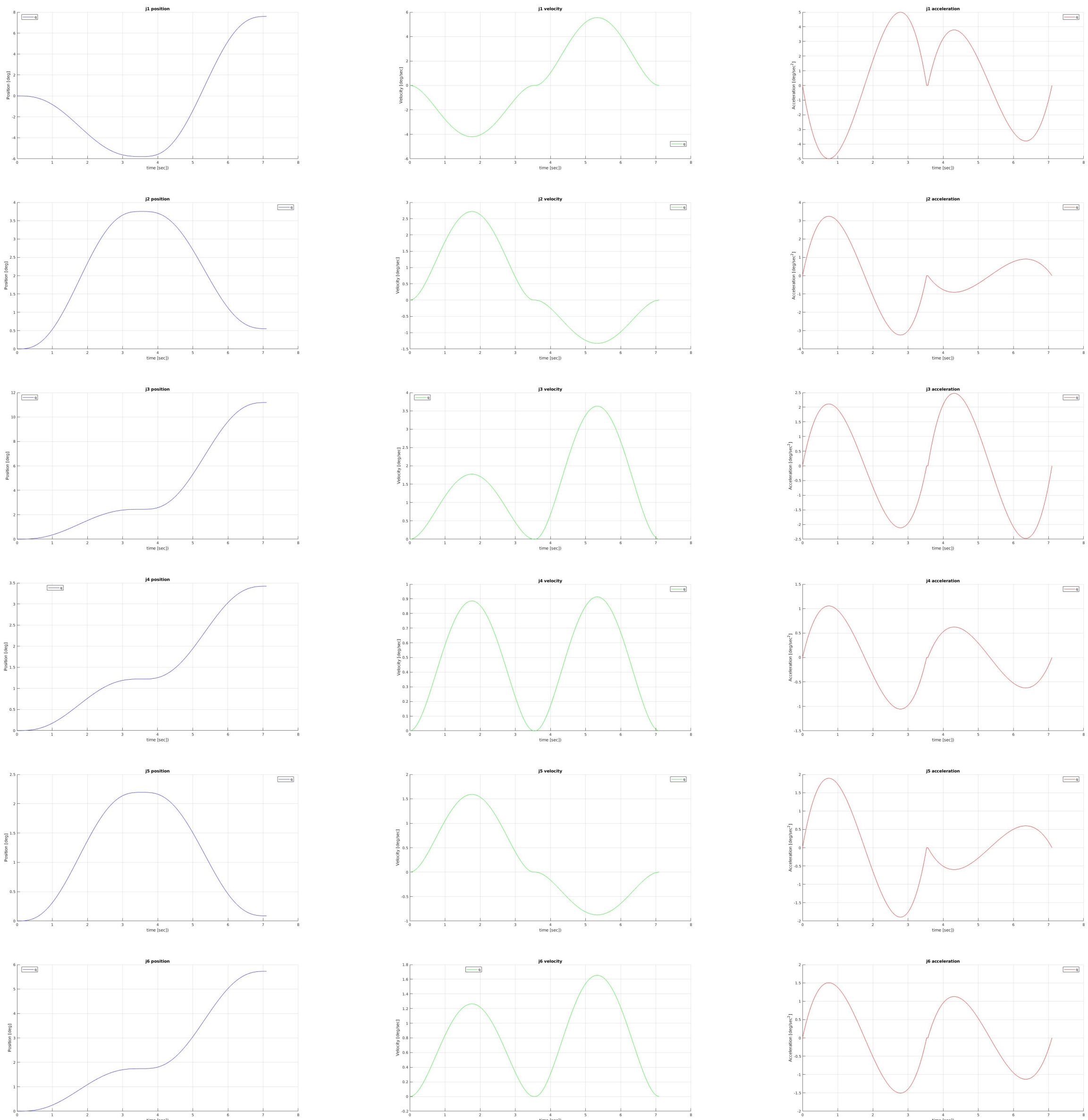


Figure 6: Trajectory Function outputs: the first column on the left is joint position $[rad]$, the second column is velocity $[rad/sec]$ and the third column is acceleration $[rad/sec^2]$. And each row is related to each joint (the first is for the first joint ..etc)