Registration No: - | 2 | 2 | 0 | 1 | 2 | 0 | 2 | 4 | 5 | 5 |

Total Number of Pages: 02 B.Tech / 22CM4BS01T

4th Semester Regular Examination: 2023-24
DISCRETE STRUCTURE
BRANCH: CSE, CST, IT
Time: 3 Hours
Max Marks: 100
Q Code: Q062

Answer Question No.1 (Part-1) which is compulsory, any EIGHT from Part-II and any TWO from Part-III.
The figures in the right hand margin indicate marks.

Part-I

Q No.		CO	Level		
Q1				**Short Answer Type Questions (Answer All-10)**	**(02x10)**
	a)	1	1	Define a contradiction and give an example of it.	2
	b)	1	1	Define a countable set. Is the set of rational numbers countable?	2
	c)	1	1	Write the truth table of the compound statement $p \vee q$.	2
	d)	1	2	How many integers between 1 and 500 are divisible by 3 or 5 ?	2
	e)	2	2	Define the generating function of a numeric function. Hence find the generating function corresponding to the sequence 1, 1, 1,...., 1,....	2
	f)	4	1	Give an example of a group containing exactly two elements. Justify your answer.	2
	g)	2	2	Let $A = \{1,2,3,4\}$ and R be a relation on A such that $R = \{(a,b): a \in A, b \in B \ and \ a < b\}$. Find R^2.	2
	h)	2	1	Define a lattice and give an example of it.	2
	i)	3	1	Define a circuit. How is it different from path ?	2
	j)	3	1	What is a complete binary tree?	2

Part-II

Q No.		CO	Level		
Q2				**Focused-Short Answer Type Questions- (Answer Any Eight out of Twelve)**	**(06x08)**
	a)	1	2	Show that statements $[\neg(p \vee q)] \vee [(\neg p) \wedge q]$ and $\neg p$ are logically equivalent.	6
	b)	2	1	Define relation matrix of a relation. Let $A = \{1,2,3,4\}$ and $R = \{(1,3),(3,2),(3,4),(1,2),(4,1)\}$ be a relation defined on A. Find the relation matrix of the relation R.	6
	c)	4	2	Show that intersection of two sub-groups of a group is a sub-group. What will happen to their union.	6
	d)	2	2	Show that a poset $(P, \prec)$ has exactly one greatest element if such an element exists.	6

Q No.		CO	Level	Question	Marks
	e)	2	3	In how many ways can 7 students arrange themselves in a row, if 2 particular students always take the corner seats.	6
	f)	2	3	If R is an equivalence relation on a set of positive integers defined as $x\,R\,y$ if and only if $x \equiv y \pmod 4$. Find the equivalence class of 2.	6
	g)	1	3	Prove the statement 'if $3n+1$ is even, then n is odd ' using the method of proof by contradiction.	6
	h)	4	2	Show that the set of fourth roots of unity is a group under multiplication.	6
	i)	1	2	Show that the statement $(\sim P \wedge (P \vee Q)) \rightarrow Q$ is a tautology.	6
	j)	2	3	Show that the lattice $L = \{1,2,3,5,30\}$ is non-distributive under the partial order relation.	6
	k)	3	3	What is a pendent vertex ? Show that the total number of pendent vertices in a full binary tree with n vertices is $(n+1)/2$.	6
	l)	3	2	Define the degree of a vertex of a graph. Show that the sum of the degrees of the vertices of a graph is twice of its number of edges.	6

Part-III

Q No.		CO	Level		
				Long Answer Type Questions (Answer Any Two out of Four)	**(02x16)**
Q3	a)	1	3	UsingMathematical induction ,prove that $5^{2n}-2^{2n}$ is divisible by 7.	8
	b)	1	2	Define principal cunjunctive normal form of a given formula. Write the PNCF of $P \wedge (P \vee Q)$.	8
Q4	a)	2	3	Show that $C_o + C_2 + \ldots\ldots = C_1 + C_3 + \ldots\ldots = 2^{n-1}$.	8
	b)	2	3	Define a modular Lattice. Show that for the modular lattice $(L, \prec)$, $x \wedge z = y \wedge z$ and $x \vee z = y \vee z \Rightarrow x = y$ for all $x \prec y,\ x, y, z \in L$.	8
Q5	a)	2	2	Find the solution of the recurrence relation $a_r = -2a_{r-1} + 3a_{r-2} + 2^r$.	8
	b)	4	2	Show that a non-empty subset H of a group G is a sub-group of G if and only if for all $a, b \in H$, $ab^{-1} \in H$.	8
Q6	a)	3	2	What is a minimal spanning tree? Write Prim's algorithm for finding the minimal spanning tree.	8
	b)	3	3	Let G be a connected plannar simple graph with e edges and v vertices. Let r be the number of regions in a planar representation of G. Then prove that $r = e - v + 2$.	8