Stiffness Analysis of a 6-DoF serial Manipulator With a Counterbalance mechanism

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Under Supervision of
Prof. Alexander Klimchik
Albert Demian

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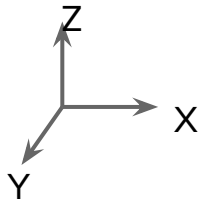
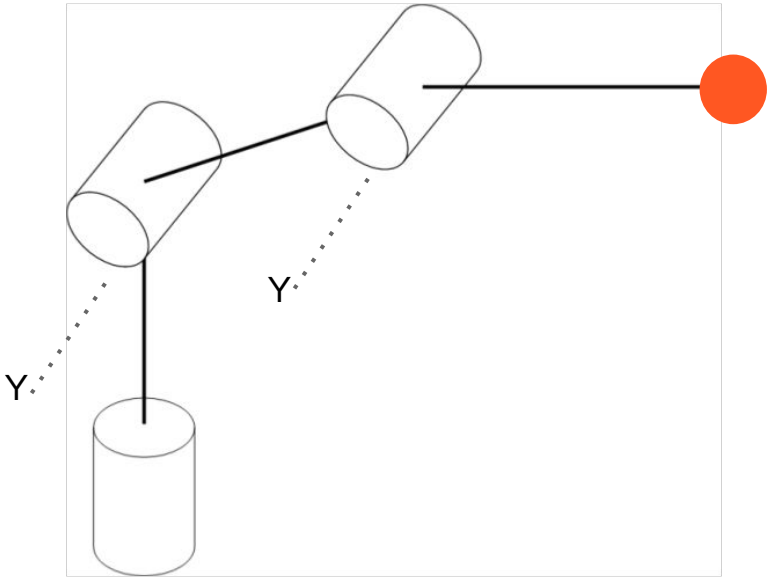
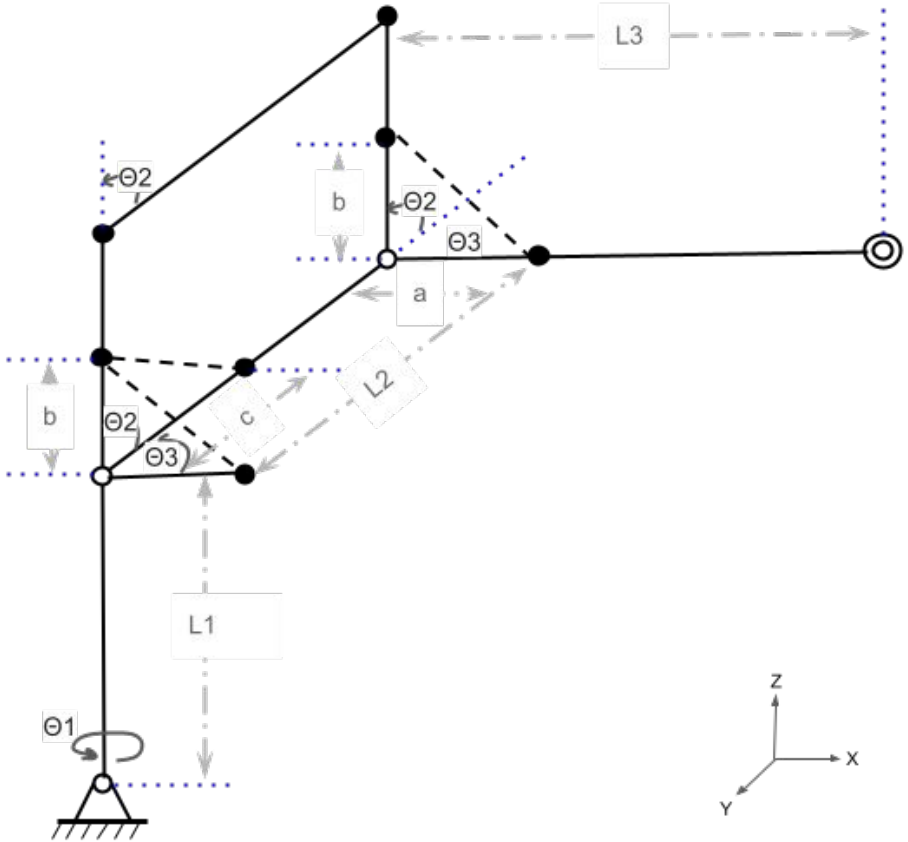
Introduction



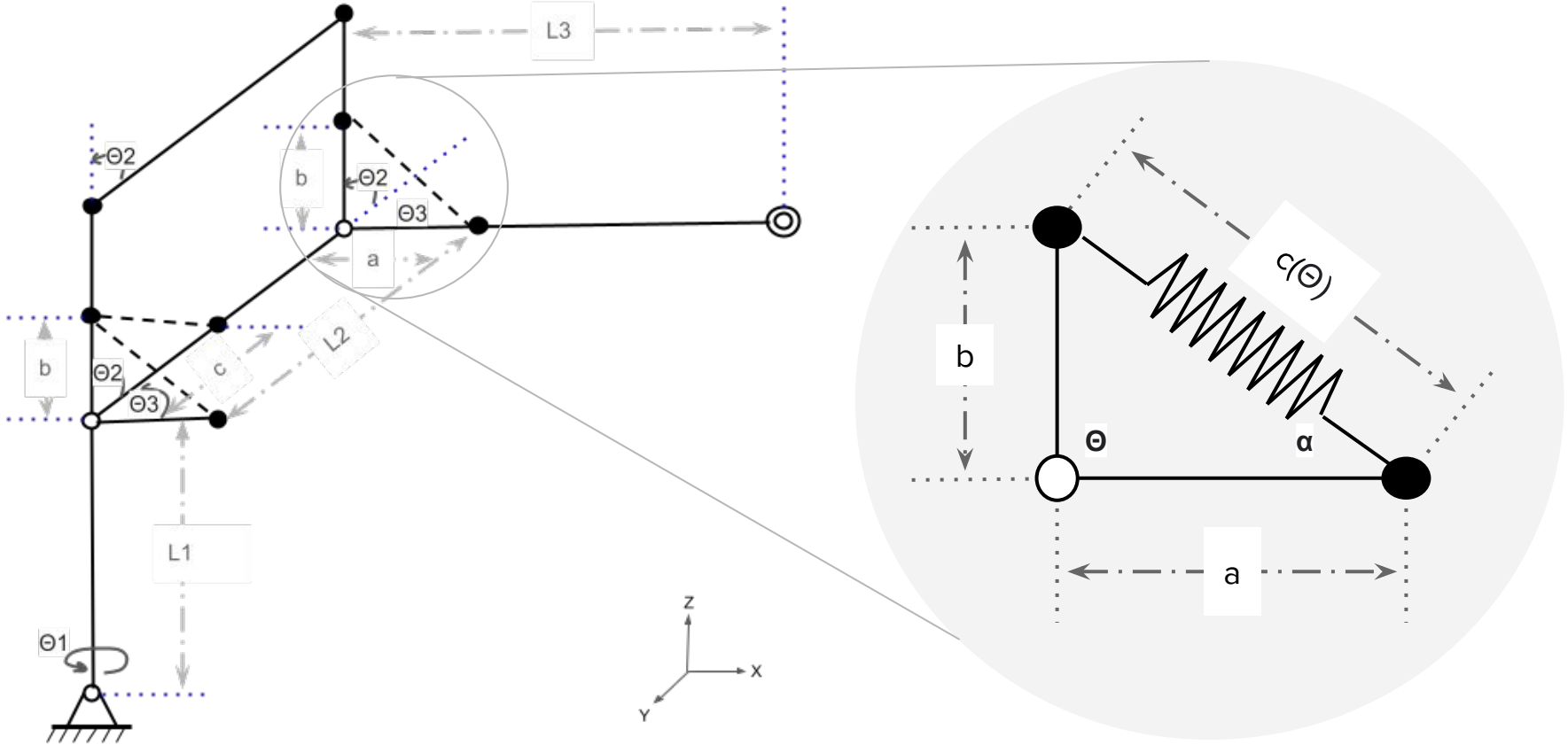


Kinematics



Kinematics

K-Springs



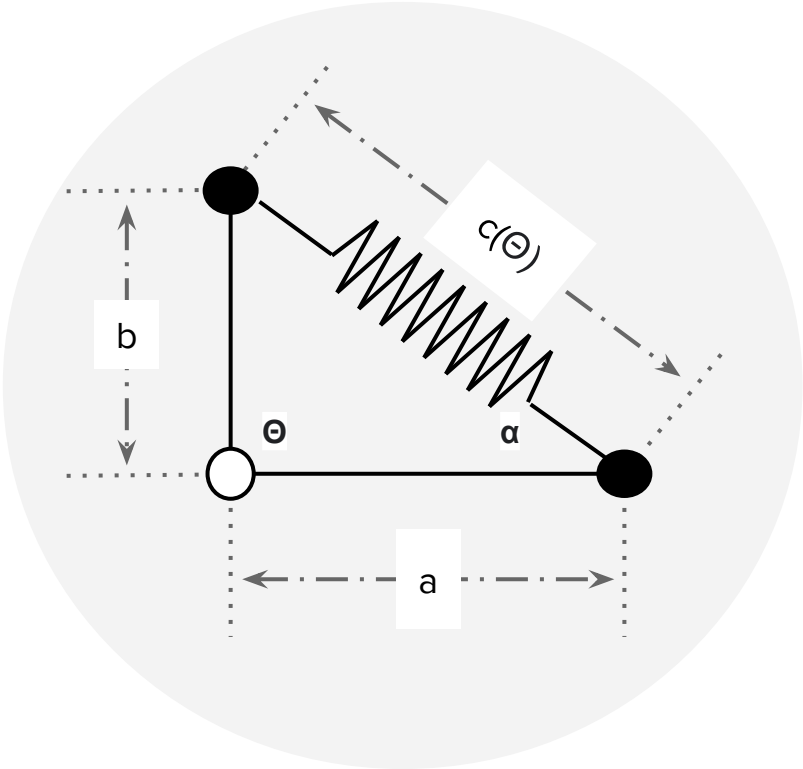
Kinematics

K-Springs

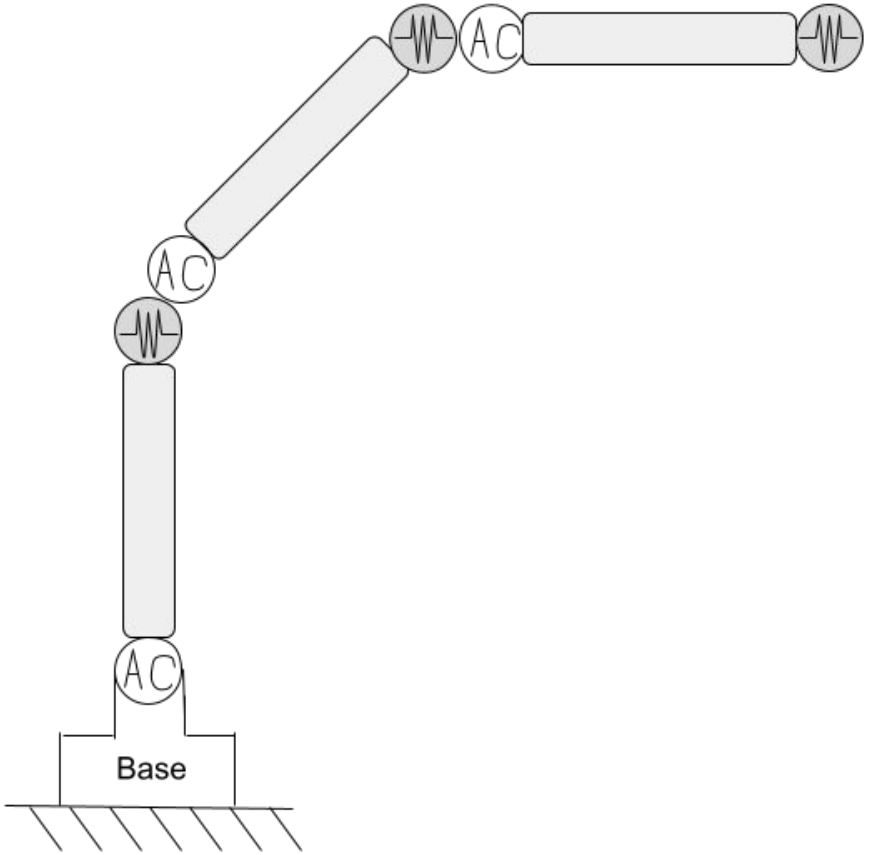
$$K_1 = \frac{m_3 * g * lc_3 + m_a * g * lc_a}{a * b}$$

$$K_2 = \frac{m_2 * g * lc_2 + m_2 * g * l_2}{c * b}$$

$$K_3 = \frac{m_3 * g * lc}{a * b}$$



Virtual Joint Modelling [VJM]



Virtual Joint Modelling [VJM]

VJM model

$$T = R_z(\theta_1).T_z(l_1).T_{6DoF}(\theta_{2:7}).R_y(\theta_8).T_z(l_2).T_{6DoF}(\theta_{9:14}).R_y(\theta_{15}).T_x(l_3).T_{6DoF}(\theta_{16:21})$$

$$J_i = \frac{\partial T}{\partial \theta_i}; i = \{1, 2, ..., 21\}$$

$$J = [J_1, J_2, ..., J_{21}]$$

Virtual Joint Modelling [VJM]

VJM model

$$\mathbf{K}_{\theta} = \left[\begin{array}{c|c|c|c|c|c} \mathbf{K}_{\theta_1} & 0_{6\times6} & 0_{6\times1} & 0_{6\times6} & 0_{6\times1} & 0_{6\times6} \\ \hline 0_{6\times1} & \mathbf{K}^{22}_{6\times6} & 0_{6\times1} & 0_{6\times6} & 0_{6\times1} & 0_{6\times6} \\ \hline 0_{6\times1} & 0_{6\times6} & \mathbf{K}_{\theta_2} & 0_{6\times6} & 0_{6\times1} & 0_{6\times6} \\ \hline 0_{6\times1} & 0_{6\times6} & 0_{6\times1} & \mathbf{K}^{44}_{6\times6} & 0_{6\times1} & 0_{6\times6} \\ \hline 0_{6\times1} & 0_{6\times6} & 0_{6\times1} & 0_{6\times6} & \mathbf{K}_{\theta_3} & 0_{6\times6} \\ \hline 0_{6\times1} & 0_{6\times6} & 0_{6\times1} & 0_{6\times6} & 0_{6\times1} & \mathbf{K}^{66}_{6\times6} \end{array}\right]$$

Virtual Joint Modelling [VJM]

VJM model

System matrix (A):

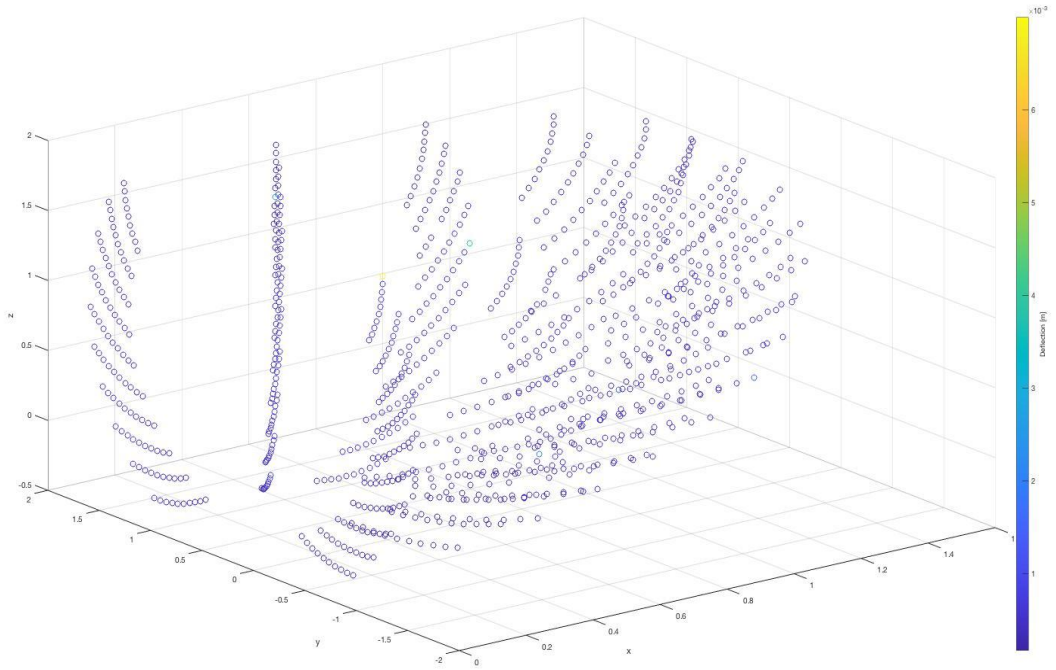
$$A = \begin{bmatrix} 0_{6\times6} & J\theta \\ J\theta' & -K\theta \end{bmatrix}$$

Kc = inv(A)

Delta = Kc/F

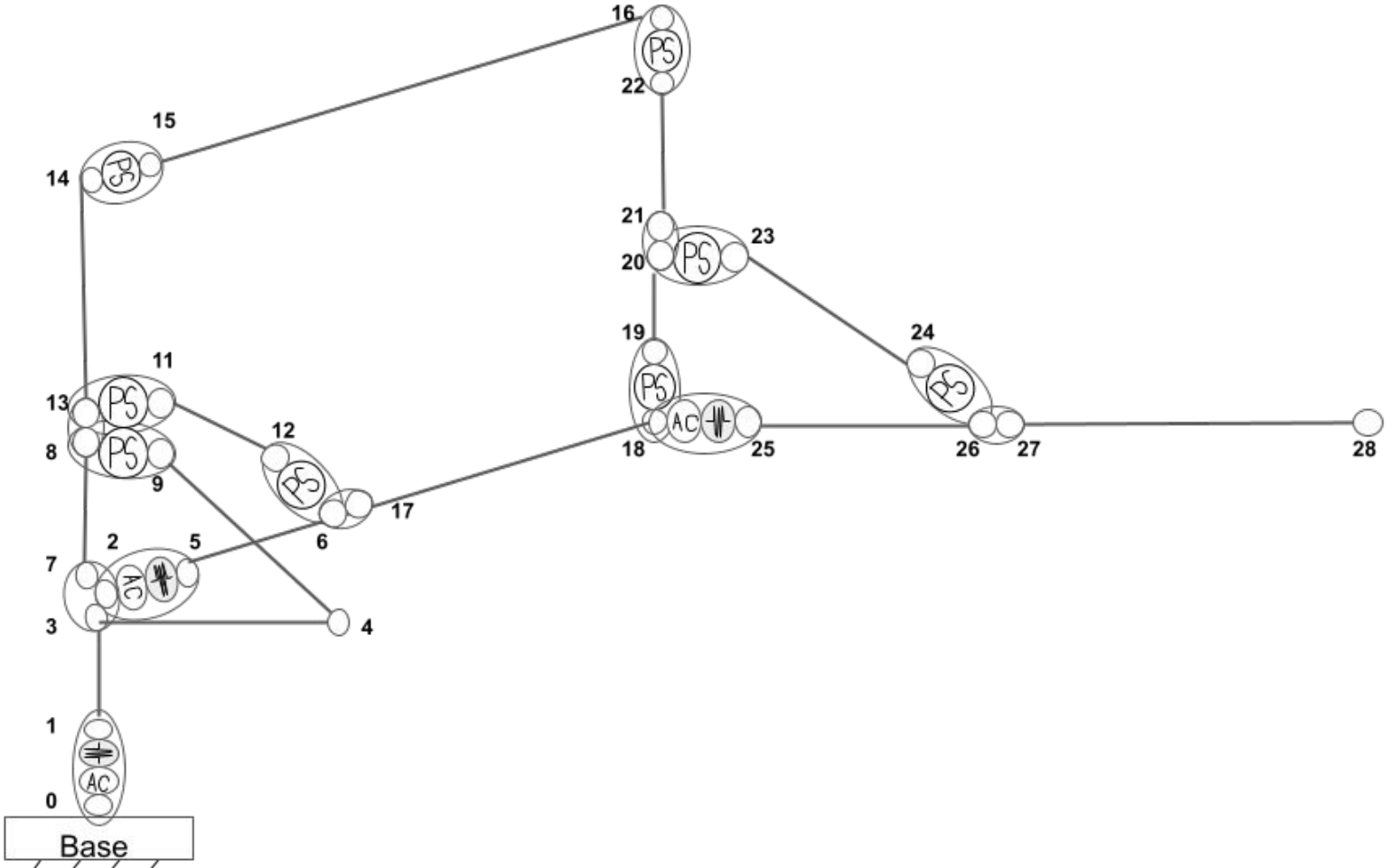
Virtual Joint Modelling [VJM]

Results




```
Command Window
  Overall progress : 10
  Overall progress : 20
  Overall progress : 30
  Overall progress : 40
  Overall progress : 50
  Overall progress : 60
  Overall progress : 70
  Overall progress : 80
  Overall progress : 90
  Overall progress : 100
  Maximum Deflection = 0.002731
  Minimum Deflection = 5.1243e-05
```

Matrix Structural Analysis [MSA]



Matrix Structural Analysis [MSA]

Aggregated model

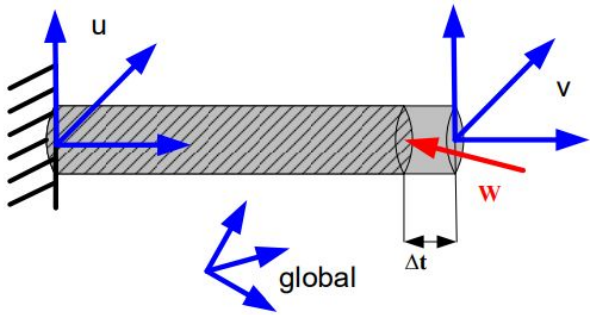
$$\begin{bmatrix} -I_{168\times168} & K_{links} \\ 0_{91\times168} & A_{91\times168} \\ B_{77\times168} & 0_{77\times168} \\ C_{4\times168} & D_{4\times168} \\ E_{6\times168} & F_{6\times168} \end{bmatrix}_{346\times336} \cdot \begin{bmatrix} W_{agg} \\ \Delta t_{agr} \end{bmatrix}_{336\times1} = \begin{bmatrix} 0_{340\times1} \\ W_e \end{bmatrix}_{346\times1}$$

Matrix Structural Analysis [MSA]

Aggregated model

$$K_{links} = \begin{bmatrix} K_{11}^{(12)} & K_{12}^{(12)} & & & & \\ K_{21}^{(12)} & K_{22}^{(12)} & & & & \\ & & K_{11}^{(34)} & K_{12}^{(34)} & & \\ & & K_{21}^{(34)} & K_{22}^{(34)} & & \\ & & & \ddots & & \\ & & & & K_{11}^{(78)} & K_{12}^{(78)} \\ & & & & K_{21}^{(78)} & K_{22}^{(78)} \\ & & & & K^{27\,28} & K^{27\,28} \\ & & & & K_{21}^{27\,28} & K_{22}^{27\,28} \end{bmatrix}$$

168 * 168



Remark

K = RRy K Ry'R'

Matrix Structural Analysis [MSA]

Aggregated model

$$
A = \begin{bmatrix}
\Lambda^r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \Lambda^r & 0 & 0 & 0 & 0 & 0 & -\Lambda^r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^r & -\Lambda^r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^r & 0 & 0 & 0 & 0 & 0 & -\Lambda^r & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \Lambda^r & 0 & 0 & 0 & 0 & 0 & -\Lambda^r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^r & 0 & -\Lambda^r & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^r & 0 & -\Lambda^r & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & -I & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & -I & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^r & -\Lambda^r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & I & 0 & 0 & 0 & 0 & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & I & 0 & 0 & 0 & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \Lambda^r & 0 & -\Lambda^r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^r & 0 & -\Lambda^r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^r & -\Lambda^r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

Matrix Structural Analysis [MSA]

Aggregated model

$$
B = \begin{bmatrix}
0&0&0&\Lambda^r&0&0&0&0&0&\Lambda^r&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&\Lambda^p&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&\Lambda^p&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
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0&0&0&0&0&0&0&\Lambda^r&\Lambda^r&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&\Lambda^p&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
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0&0&0&0&0&0&0&0&0&0&\Lambda^p&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&\Lambda^r&\Lambda^r&0&\Lambda^r&0&\Lambda^r&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&\Lambda^p&0&0&0&0&\Lambda^p&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix}
$$

Matrix Structural Analysis [MSA]

Aggregated model

$$C = \begin{bmatrix} -\Lambda^e & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^e & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Lambda^e & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^e & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda^e & 0 & 0 & 0 \end{bmatrix}$$

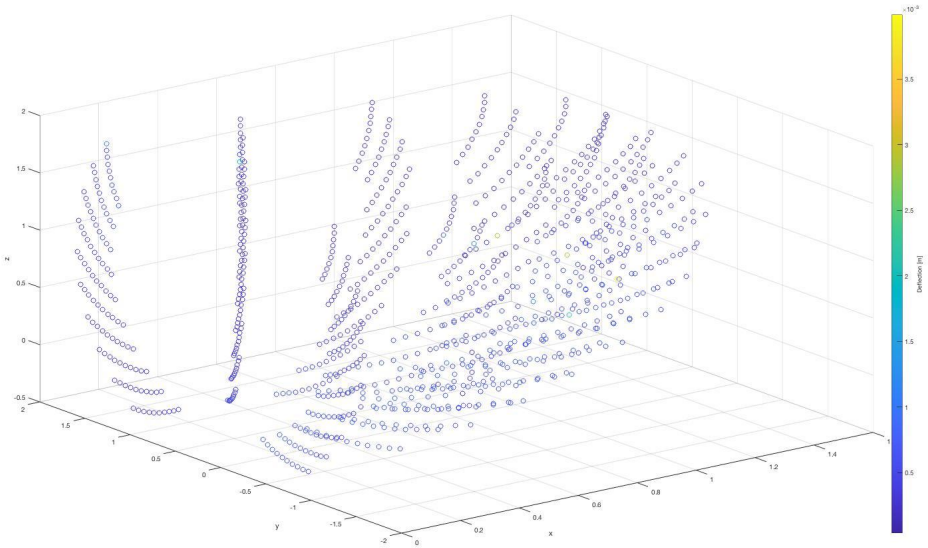
$$D = \begin{bmatrix} K_{th_1}\Lambda^e & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -K_{th_3}\Lambda^e & 0 & 0 & 0 & 0 & 0 & 0 & K_{th_1}\Lambda^e & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -K_{th_2}\Lambda^e & 0 & K_{th_2}\Lambda^e & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}$$

$$F = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Matrix Structural Analysis [MSA]

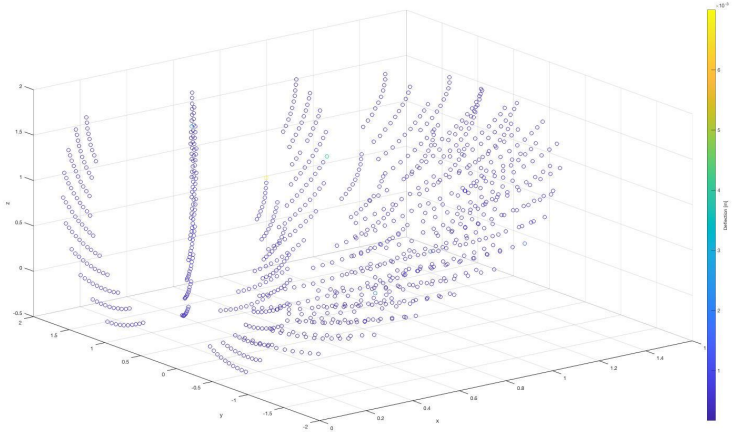
Results



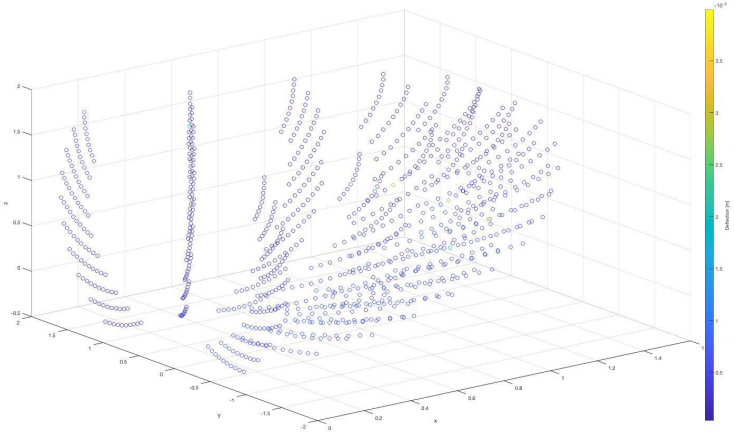
Command Window

```
Overall progress : 10%
Overall progress : 20%
Overall progress : 30%
Overall progress : 40%
Overall progress : 50%
Overall progress : 60%
Overall progress : 70%
Overall progress : 80%
Overall progress : 90%
Overall progress : 100%
Maximum Deflection = 0.0039879
Minimum Deflection = 3.9806e-05
```

VJM & MSA Comparison

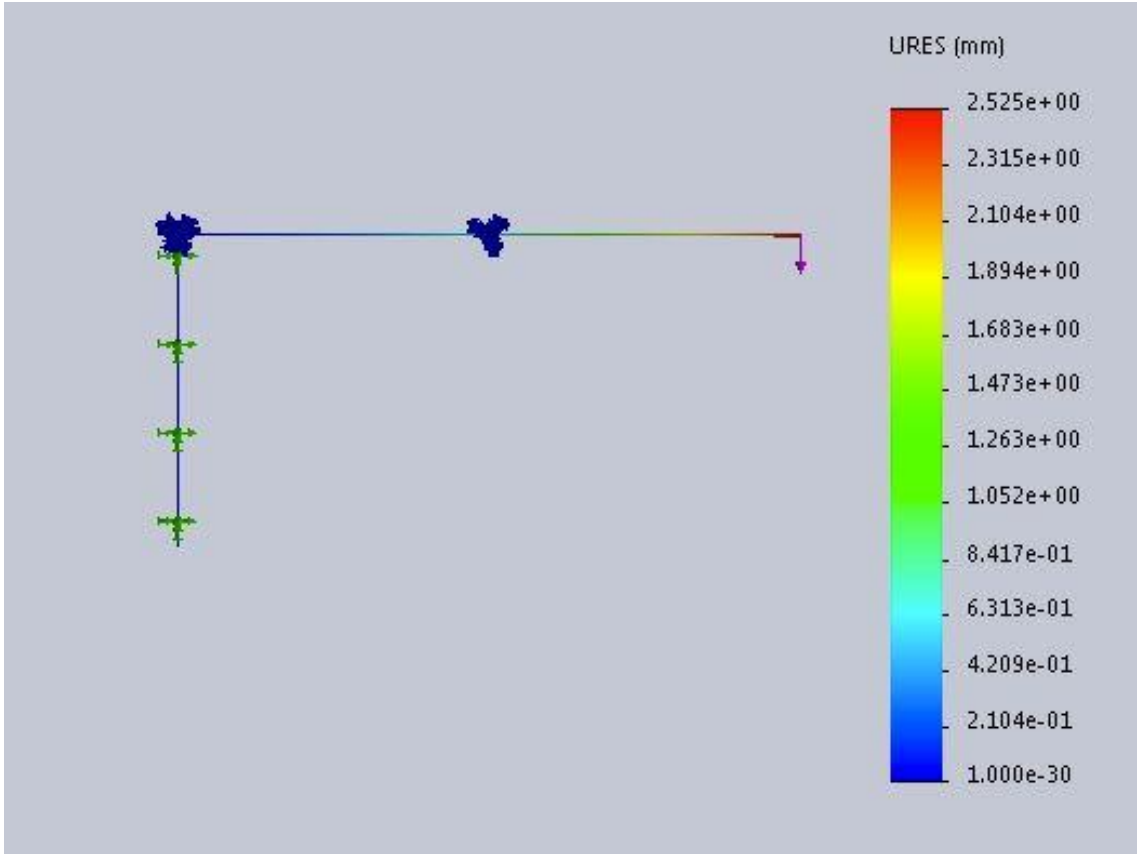


VJM



MSA

VJM & MSA Comparison



VJM & MSA Comparison

	FEA	VJM	MSA
Minimum deflection [mm]	1.000 e-30	5.124 e-02	3.980 e-05
Maximum deflection [mm]	2.525 e+00	2.731 e+00	0.398 e+00

Thank You)