Stiffness analysis of a 6-DoF serial manipulator with a counterbalance mechanism

A PROJECT REPORT

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IN

Robotics and Computer Vision

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Abstract

Keywords - counterbalance, gravity compensation, serial manipulators, stiffness modelling

Designing a counterbalance mechanism and compensating the gravity is an important and a challenging task in serial manipulators. In this report we will present a modified model for a 3-DoF gravity compensation mechanism applied on a 6-DoF serial manipulator. Then we will model the stiffness matrix for the proposed design using Virtual Joint Modelling (VJM) and Matrix Structural Analysis (MSA). Then we will compare between the two models ant try to validate the results in the best possible way.

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0.1 INTRODUCTION

When we are working with serial manipulators, as we increase the number of Degree of Freedom (DoF) and the more we add more joints and links the more we increase the mass of the robot. Having a heavy robot is a critical problem; especially from the point of view of the actuators. In more details, when we are increasing the the mass we are also increasing the gravitational torque so we need more power in the actuators in order to compensate the gravitational torque in addition to the needed torque for the task. That is why designing a counterbalance mechanism that compensate the gravitational torque in all the joints is an important problem. In this report we are proposing a new design for a 3-DoF counterbalance mechanism for a 6-DoF manipulator.

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0.2 KINEMATICS

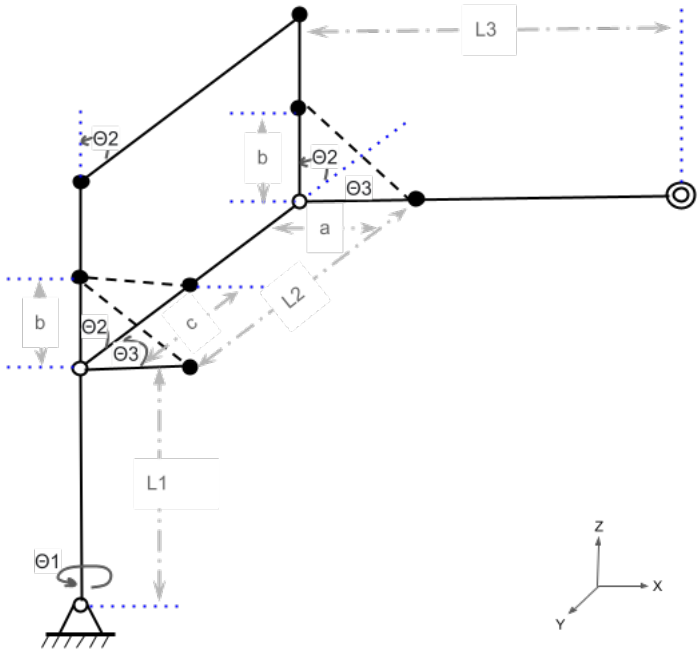


Figure 1: Kinematics scheme

As we see in the kinematics scheme, we have main chain with three revolute joints: First one is rotating around Z-axis while the second and the third joints are rotating around Y-axis. We are working only with three joints, because we considered the last three joints (wrist) as a payload. We also have two intersected parallelograms and we used the to place our three springs in a proper way trying to get a full gravity compensation in the three joints.. First we solved the forward kinematics, because we will need the rotation matrix later as we will see.

0.3 STIFFNESS OF THE LINKS

All our links are considered as a cylindrical beams, we can see below the stiffness matrix for a cylindrical beam.

$$K = \begin{bmatrix} K11 & K12 \\ K21 & K22 \end{bmatrix}$$

where:

$$K_{11} = \begin{bmatrix} \frac{E.S}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12.E.I_z}{L^3} & 0 & 0 & 0 & \frac{6.E.I_y}{L^2} \\ 0 & 0 & \frac{12.E.I_y}{L^3} & 0 & -\frac{6.E.I_y}{L^2} & 0 \\ 0 & 0 & 0 & \frac{G.J}{L} & 0 & 0 \\ 0 & 0 & -\frac{6.E.I_y}{L^2} & 0 & \frac{4.E.I_y}{L} & 0 \\ 0 & \frac{6.E.I_y}{L^2} & 0 & 0 & 0 & \frac{4.E.I_z}{L} \end{bmatrix}$$

$$K_{22} = \begin{bmatrix} \frac{E.S}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12.E.I_z}{L^3} & 0 & 0 & 0 & -\frac{6.E.I_y}{L^2} \\ 0 & 0 & \frac{12.E.I_y}{L^3} & 0 & \frac{6.E.I_y}{L^2} & 0 \\ 0 & 0 & 0 & \frac{G.J}{L} & 0 & 0 \\ 0 & 0 & \frac{6.E.I_y}{L^2} & 0 & \frac{4.E.I_y}{L} & 0 \\ 0 & -\frac{6.E.I_y}{L^2} & 0 & 0 & 0 & \frac{4.E.I_z}{L} \end{bmatrix}$$

$$K_{12} = K_{21}^T = \begin{bmatrix} -\frac{E.S}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{12.E.I_z}{L^3} & 0 & 0 & 0 & \frac{6.E.I_y}{L^2} \\ 0 & 0 & -\frac{12.E.I_y}{L^3} & 0 & -\frac{6.E.I_y}{L^2} & 0 \\ 0 & 0 & 0 & -\frac{G.J}{L} & 0 & 0 \\ 0 & 0 & \frac{6.E.I_y}{L^2} & 0 & \frac{2.E.I_y}{L} & 0 \\ 0 & -\frac{6.E.I_y}{L^2} & 0 & 0 & 0 & \frac{2.E.I_z}{L} \end{bmatrix}$$

0.4 MATRIX STRUCTURAL ANALYSIS

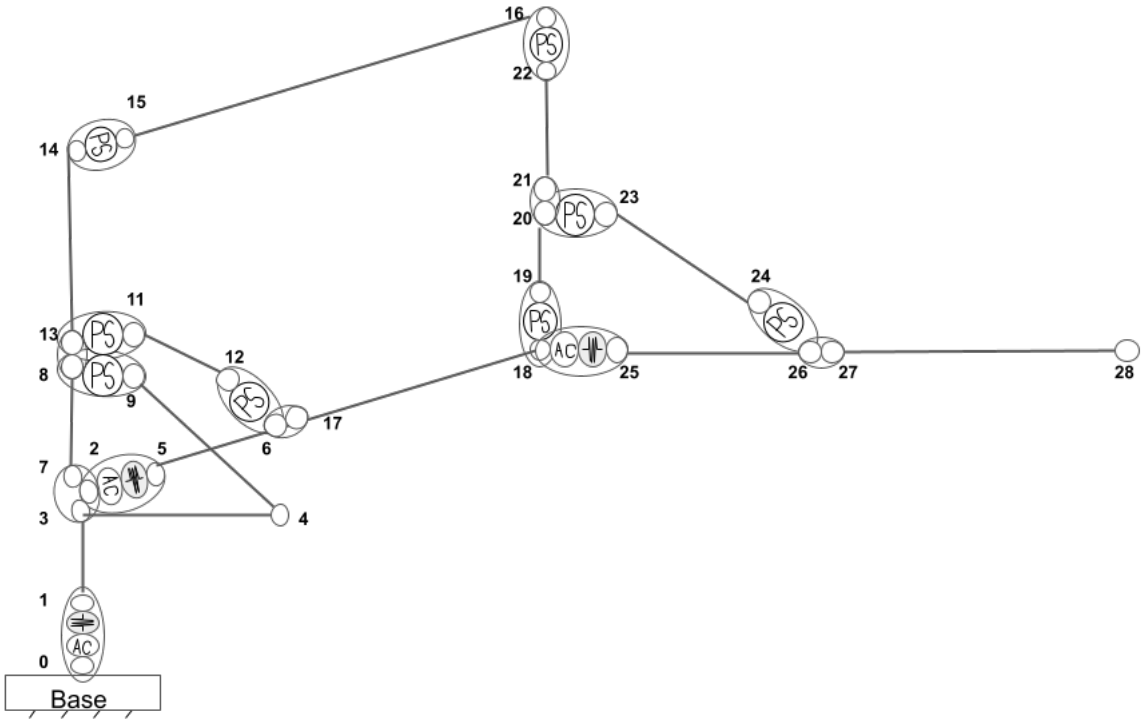


Figure 2: MSA model

First we designed the model of our manipulator with the counterbalance mechanism, and it is important to highlight the point that we modeled a worm gear between joints (2,3,5,7) because we are trying to increase the torque in this joint in order to get a full gravity compensation in it.Then we wrote the equations and we got 346 equations, we aggregated them in a proper way as we will see below, and

you can see all the matrices (A, B, C, D, E and F) in the Appendix.

$$\begin{bmatrix} -I_{168\times168} & K_{links} \\ 0_{91\times168} & A_{91\times168} \\ B_{77\times168} & 0_{77\times168} \\ C_{4\times168} & D_{4\times168} \\ E_{6\times168} & F_{6\times168} \end{bmatrix}_{346\times336} \cdot \begin{bmatrix} W_{agg} \\ \Delta t_{agr} \end{bmatrix}_{336\times1} = \begin{bmatrix} 0_{340\times1} \\ W_e \end{bmatrix}_{346\times1} \tag{1}$$

0.5 VIRTUAL JOINT MODELLING

Virtual Joint Modelling is based on Langrangian equations, and it is hard to deal with two intersected parallelograms in VJM, Although we tried but we got bad results and we could not find the best model for the counterbalance mechaninsm by using VJM in the time of the project. For the stated reasons we modeled the main chain, and as we see below we have three one-DoF actuators and three rigid links with three 6-DoF springs (one after each link).

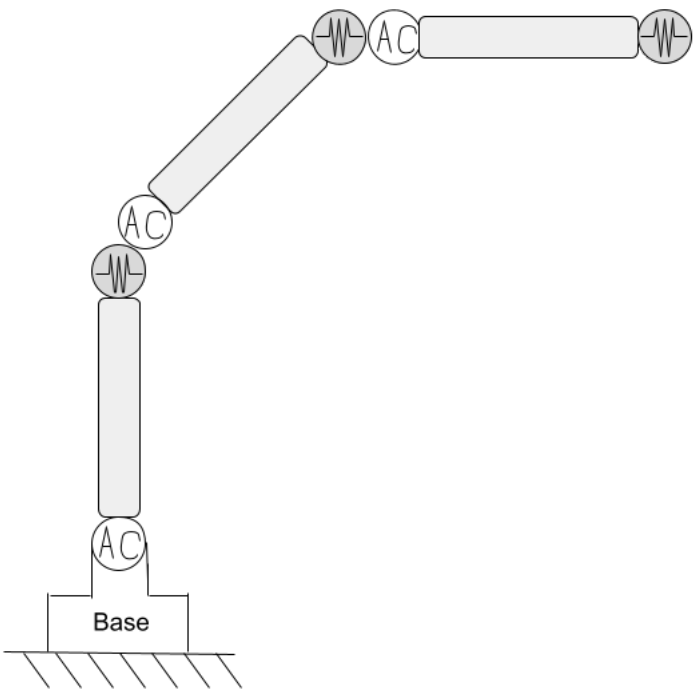


Figure 3: VJM model

This is the transformation for our VJM model:

$$T = R_z(\theta_1).T_z(l_1).T_{6DoF}(\theta_{2:7}).R_y(\theta_8).T_z(l_2).T_{6DoF}(\theta_{9:14}).R_y(\theta_{15}).T_x(l_3).T_{6DoF}(\theta_{16:21}) \tag{2}$$

and to compute the Jacobian, we need to derive the Transformation with respect to each θ, then we will aggregate the derivatives together and get the full Jacobian as following:

$$J_i = \frac{\partial T}{\partial \theta_i}; i = \{1, 2, ..., 21\} \tag{3}$$

$$J = [J_1, J_2, ..., J_{21}] \tag{4}$$

We can see below the dimension details for $K\theta$ matrix:

$$\mathbf{K}_\theta = \left[\begin{array}{c|c|c|c|c|c} \mathbf{K}_{\theta_1} & 0_{6\times6} & 0_{6\times1} & 0_{6\times6} & 0_{6\times1} & 0_{6\times6} \\ \hline 0_{6\times1} & \mathbf{K}^{22}_{6\times6} & 0_{6\times1} & 0_{6\times6} & 0_{6\times1} & 0_{6\times6} \\ \hline 0_{6\times1} & 0_{6\times6} & \mathbf{K}_{\theta_2} & 0_{6\times6} & 0_{6\times1} & 0_{6\times6} \\ \hline 0_{6\times1} & 0_{6\times6} & 0_{6\times1} & \mathbf{K}^{44}_{6\times6} & 0_{6\times1} & 0_{6\times6} \\ \hline 0_{6\times1} & 0_{6\times6} & 0_{6\times1} & 0_{6\times6} & \mathbf{K}_{\theta_3} & 0_{6\times6} \\ \hline 0_{6\times1} & 0_{6\times6} & 0_{6\times1} & 0_{6\times6} & 0_{6\times1} & \mathbf{K}^{66}_{6\times6} \end{array}\right]$$

Now we will formulate our system matrix (A), then we will invert it to get K_c, so we can compute the deflection by dividing K_c over the applied force F: $\Delta = \frac{K_c}{F}$

$$A = \begin{bmatrix} 0_{6\times6} & J\theta \\ J\theta' & -K\theta \end{bmatrix}$$

0.6 FINITE ELEMENT ANALYSIS

As we mentioned previously, we could not model our counterbalance mechanism by using —(VJM), that is why comparing (MSA) and (VJM) models together alone will not be appropriate in our case. So we decided to use Finite Element Analysis as a third model for comparison, but due the time we only managed to validate our (VJM) model by using (FEA), because modelling the counterbalancing mechanism using (FEA) was out of our time limitation for this project. In order to do the (FEA) we used Solid-Works and we used a simulator tool for (FEA), we modeled the connections between joints as a springs to represent the actuators and we put all the parameters and applied force as we did in (VSM).

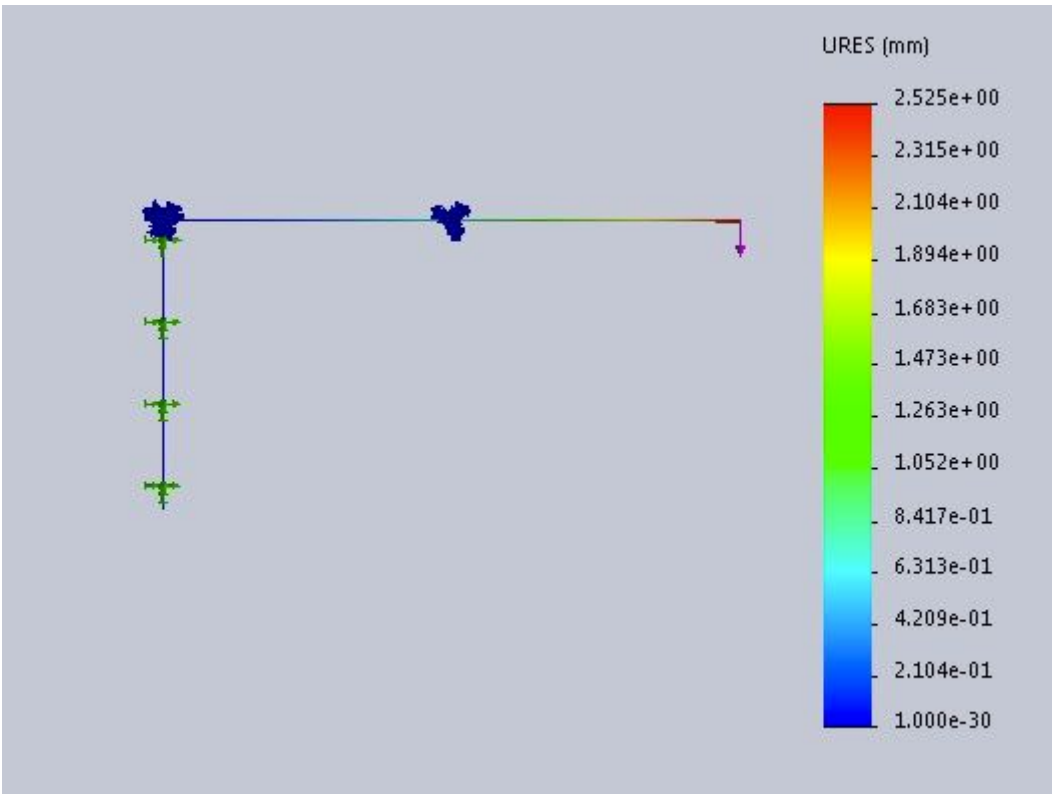
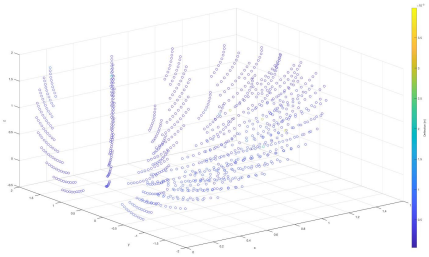
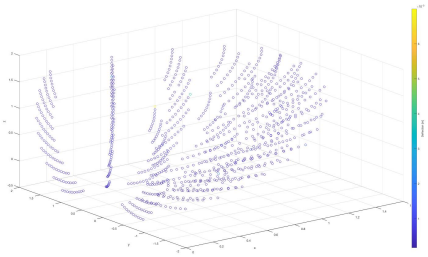


Figure 4: FEA model

0.7 Results

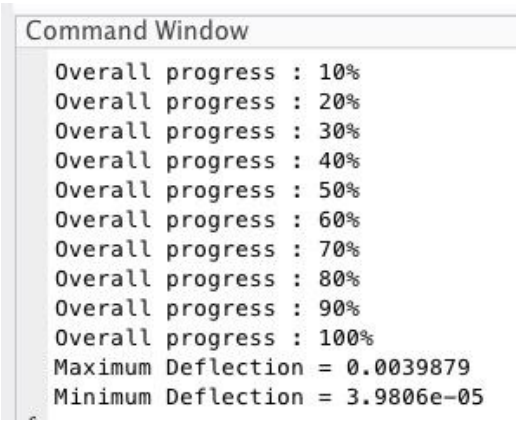


(a) MSA deflection map for a 100 N force (in all axis)



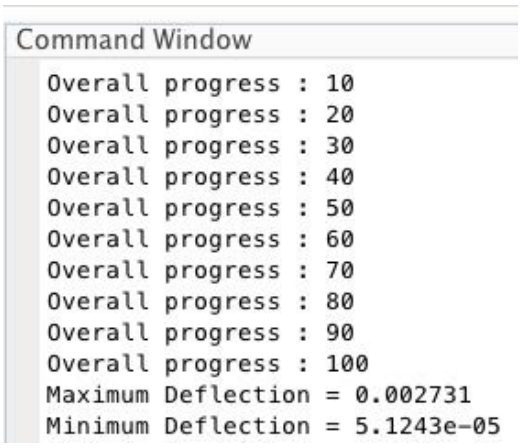
VJM deflection map for a 100 N force (in all axis)

Figure 5: Deflection Maps



```
Command Window
Overall progress : 10%
Overall progress : 20%
Overall progress : 30%
Overall progress : 40%
Overall progress : 50%
Overall progress : 60%
Overall progress : 70%
Overall progress : 80%
Overall progress : 90%
Overall progress : 100%
Maximum Deflection = 0.0039879
Minimum Deflection = 3.9806e-05
```

(a) MSA minimum and maximum deflections for a 100 N force (in all axis)



```
Command Window
Overall progress : 10
Overall progress : 20
Overall progress : 30
Overall progress : 40
Overall progress : 50
Overall progress : 60
Overall progress : 70
Overall progress : 80
Overall progress : 90
Overall progress : 100
Maximum Deflection = 0.002731
Minimum Deflection = 5.1243e-05
```

VJM minimum and maximum deflections for a 100 N force (in all axis)

Figure 6: Minimum and Maximum Deflections

	FEA	VJM	MSA
Minimum deflection [mm]	1.000 e-30	5.124 e-02	3.980 e-05
Maximum deflection [mm]	2.525 e+00	2.731 e+00	0.398 e+00

0.8 Conclusion

In the last table in the results section, we can see that (FEA) and (VJM) has almost the same maximum deflection and we can claim that is the small difference between them is due to the parameters in modelling (FEA). While we still could not validate our MSA model, even though the deflection map seems convenient and descriptive to our problem. For Future work, we want to model our robot with the counterbalance mechanism using (FEA) and validate our (MSA) modelm and we also want to find a way to model this counter balance mechanism usin (VJM) so we can compare between the three models in the two cases and validate them all.