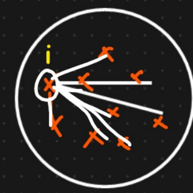
Silhouette Clustering

(1)

$\boxed{a(i)}$



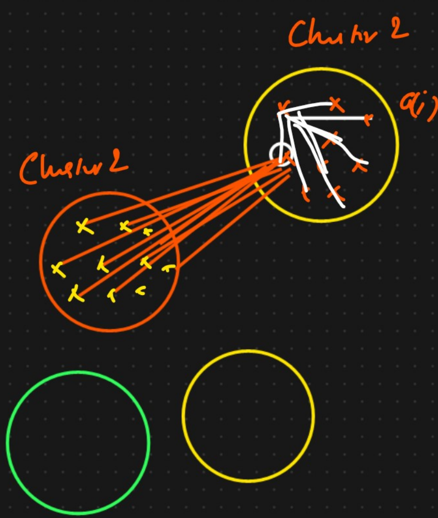
For data point $i \in C_I$ (data point i in the cluster C_I), let

$$a(i) = \frac{1}{|C_I| - 1} \boxed{\sum_{j \in C_I, i \neq j} d(i,j)}$$

be the mean distance between i and all other data points in the same cluster, where $|C_I|$ is the number of points belonging to cluster i, and $\mathrm{d(i,j)}$ is the distance between data points i and j in the cluster C_I (we divide by $|C_I| - 1$ because we do not include the distance $\mathrm{d(i,i)}$ in the sum). We can interpret $\mathrm{a(i)}$ as a measure of how well i is assigned to its cluster (the smaller the value, the better the assignment).

(2)

b(i)



We then define the mean dissimilarity of point i to some cluster C_J as the mean of the distance from i to all points in C_J (where $C_J \neq C_I$).

For each data point $i \in C_I$, we now define

$$b(i) = \min_{J \neq I} \frac{1}{|C_J|} \boxed{\sum_{j \in C_J} d(i,j)}$$

to be the *smallest* (hence the $\min$ operator in the formula) mean distance of i to all points in any other cluster (i.e., in any cluster of which i is not a member). The cluster with this smallest mean dissimilarity is said to be the "neighboring cluster" of i because it is the next best fit cluster for point i.

③ Silhouette Score

small ⇐ $\boxed{a_i/b_i}$

$\boxed{a_i > b_i}$ ⇒ Good cluster

$\boxed{a_i < b_i}$ ⇒ bad cluster

We now define a *silhouette* (value) of one data point i

$$\boxed{s(i) = \frac{b(i) - a(i)}{\max\{a(i), b(i)\}}, \text{ if } |C_I| > 1}$$ ⇒ -1 to 1

and

$$s(i) = 0, \text{ if } |C_I| = 1$$

Which can be also written as:

$$s(i) = \begin{cases} \boxed{1 - a(i)/b(i),} & \text{if } a(i) < b(i) \\ 0, & \text{if } a(i) = b(i) \\ \boxed{b(i)/a(i) - 1,} & \text{if } a(i) > b(i) \end{cases}$$

From the above definition it is clear that

$$\{-1 \le s(i) \le 1\}$$