

Static & Flutter Analysis of Airfoil and Wing

Course Project Report

Aeroelasticity

MTech. Aerospace Engineering

by

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This report presents a Computational Aeroelasticity study focused on the static divergence and flutter behavior of a wing. Aeroelasticity deals with the interaction of aerodynamic, elastic, and inertial forces, where divergence represents a static instability, and flutter is a dynamic aeroelastic instability leading to destructive oscillations.

Methodology

The study utilized classical flutter theory and torsion–bending equations to predict aeroelastic instabilities. A MATLAB-based computational framework was developed, implementing strip theory and modal analysis to compute divergence speed, flutter speed, and associated damping characteristics. Results The MATLAB implementation yielded the following results: - Flutter Speed ≈ 0.05 m/s (numerical artifact, practically much higher). - Divergence Speed ≈ 65.25 m/s. The Frequency vs Airspeed and Damping vs Airspeed plots highlight the onset of instabilities. Divergence speed marks the point of monotonic instability, while flutter speed corresponds to oscillatory instability with negative damping

Matlab Code:

```
clc;
clear;

%% ===== USER INPUT =====

model = 3; % 1=quasi-static, 2=1+aero-damp, 3=quasi-steady unsteady
U = 0:0.05:300; % airspeed vector
m = 200; % plunge mass
Sa = 225; % plunge-twist coupling (structural)
Ia = 502.4; % torsional inertia
rho = 1.225; % air density
c = 0.2; % chord
b = c/2; % semi-chord
a = 0.46; % elastic-axis offset (in chord units)
Kh = 3750; % plunge stiffness
Ka = 1983395.7; % torsional stiffness
lambda = [-30,0,30];
% structural M,K
M_struct = [m, Sa; Sa, Ia];
```

```
K_struct = [Kh, 0; 0, Ka];
```

```
% non-circulatory added-mass (used in models 3 & 4)
```

```
M_add_nc = pi*rho*b^2 * [1, -b*a; -b*a, b^2*(a^2 + 1/8)];
```

```
% storage
```

```
num_modes = 4;
```

```
roots_all = zeros(num_modes, length(U));
```

```
%% === LOOP OVER AIRSPEEDS ===
```

```
for i = 1:length(U)
```

```
Ui = U(i)^2;
```

```
% circulatory aero-stiffness & damping at this Ui
```

```
K_circ = [0,          2*pi*rho*Ui^2*b;  
          0,          -2*pi*rho*Ui^2*b^2*(a+0.5)];
```

```
C_circ = [2*pi*rho*Ui*b,    2*pi*rho*Ui*b^2*(0.5 - a);  
          -2*pi*rho*Ui*b^2*(a+0.5), -2*pi*rho*Ui*b^3*(a+0.5)*(0.5 - a)];
```

```
C_add_nc = pi*rho*b^2 * [0,    Ui;  
                        0, Ui*b*(0.5-a)];
```

```
% pick M, C, K based on model
```

```
switch model
```

```
case 1 % quasi-static, no aero damping
```

```
    M_curr = M_struct;  
    C_curr = zeros(2);  
    K_curr = K_struct + K_circ;
```

```
case 2 % quasi-static + aero damping
```

```
    M_curr = M_struct;  
    C_curr = C_circ;  
    K_curr = K_struct + K_circ;
```

```
case 3 % quasi-steady unsteady (C(k)=1)
```

```
    M_curr = M_struct + M_add_nc;  
    C_curr = C_circ + C_add_nc;  
    K_curr = K_struct + K_circ;
```

```
otherwise
```

```
    error('Model must be 1, 2, 3');
```

```
end
```

```

% eigen-analysis

lambda = flutter_analysis(M_curr, K_curr, C_curr);

if i==1
    roots_all(:,i) = lambda;
else
    roots_all(:,i) = sort_roots(roots_all(:,i-1), lambda);
end
end

%% ===== POST-PROCESSING =====

damp = real(roots_all);
freq = imag(roots_all);

flutter_speed = NaN;
divergence_speed = NaN;
tol_im = 1e-3;

% divergence: real>0 & |imag| small
for i = 1:length(U)
    lam = roots_all(:,i);
    if any(real(lam)>0 & abs(imag(lam))<tol_im)
        divergence_speed = U(i);
        break
    end
end

% flutter: real>0 & |imag| large
for i = 1:length(U)
    lam = roots_all(:,i);
    if any(real(lam)>1e-8 & abs(imag(lam))>=tol_im)

```

```

        flutter_speed = U(i);
        break
    end
end

fprintf('Divergence speed: %.2f m/s\n', divergence_speed);
fprintf('Flutter speed: %.2f m/s\n', flutter_speed);

%% ===== PLOTTING =====

figure('Position',[100,100,900,700])
subplot(2,1,1), hold on
for m_idx=1:num_modes
    plot(U, freq(m_idx,:), 'LineWidth',1.5);
end
xline(flutter_speed, 'Label','Flutter','Color','r','LineWidth',1)
xline(divergence_speed,'Label','Divergence','Color','b','LineWidth',1)
title('Frequency vs Airspeed'), xlabel('U (m/s)'), ylabel('Frequency'), grid on, box on

subplot(2,1,2), hold on
for m_idx=1:num_modes
    plot(U, damp(m_idx,:), 'LineWidth',1.5);
end
xline(flutter_speed, 'Label','Flutter','Color','r','LineWidth',1)
xline(divergence_speed,'Label','Divergence','Color','b','LineWidth',1)
title('Damping vs Airspeed'), xlabel('U (m/s)'), ylabel('Damping'), grid on, box on

%% === SUPPORT FUNCTIONS ===

function lam = flutter_analysis(M,K,C)
    n = size(M,1);

```

```

A = [zeros(n), eye(n);
     -M\K,   -M\C];
[~,eig_val]=(eig(A));
lam=diag(eig_val);
end

function sorted = sort_roots(prev, curr)

n = length(prev);
sorted = zeros(n,1);
used = false(n,1);
for i = 1:n
    [~,j] = min(abs(prev(i) - curr(~used)));
    avail = find(~used);
    sorted(i) = curr(avail(j));
    used(avail(j)) = true;
end
end

```

```

function Ck = Compute_Ck(k)

% Theodorsen function  $H_1^2/(H_1^2+i H_0^2)$ 
if k==0
    Ck = 1;
else
    H1 = besselh(1,2,k);
    H0 = besselh(0,2,k);
    Ck = H1/(H1 + 1i*H0);
end
end

```

