Quantum Information Project

\$ Becoming Rich with Quantum \$

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Contents

Problem Statement

Portfolio-optimization: choosing from a set of N possible stocks, the subset that optimizes our financial objectives:

- Maximize returns
- Minimize risk

Optimization problem:
Find z such as it minimizes

$$C(\mathbf{z}) = \lambda \mathbf{z}^T \Sigma \mathbf{z} - (1-\lambda)\mathbf{u}\cdot\mathbf{z}$$

$$\mathbf{z} \in \{-1, 0, +1\}^N$$

Figure: Positions vector for a 8 stock portfolio.

$\lambda \in 0, 1$: risk parameter
$\mathbf{u}$: expected returns
Σ: covariance matrix
$\mathbf{z}$: positions vector

Introduction to QAOA

QAOA: Quantum Approximate Optimization Algorithm:

1. Choose ansatz parametrized by θ : $|\psi(\theta)\rangle$
2. Energy (cost): $E(\theta) = \langle\psi(\theta)|H_c|\psi(\theta)\rangle$
3. Find $\theta^* = argmin\ E(\theta)$
4. Ground state $|\psi^*\rangle \approx |\psi(\theta^*)\rangle$

GOAL: Find Hamiltonian

$$H_c|z\rangle = C(z)|z\rangle$$

Adiabatic Theorem: A physical system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough and if there is a gap between the eigenvalue and the rest of the Hamiltonian's spectrum.

1. Encode problem as a simple Hamiltonian.
2. Prepare quantum system in the ground state.
3. Adiabatically evolve the simple Hamiltonian into the problem Hamiltonian.

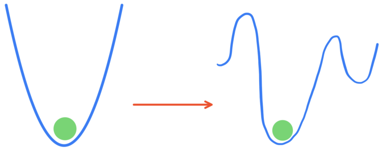


Figure: Time evolution of the ground state under adiabatic evolution of the Hamiltonian.

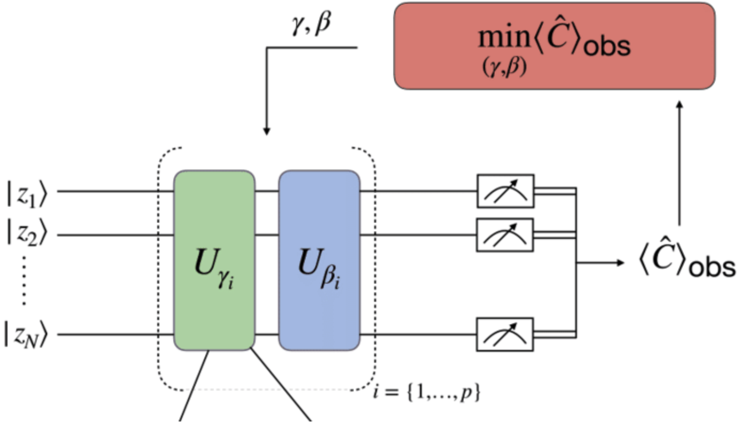


Figure: Schematic of QAOA.
ref: Qualifying quantum approaches for hard industrial optimization problems. A case study in the field of smart-charging of electric vehicles

Mathematical Implementation

Time-evolution operator: $|\psi(t)\rangle = U(t)|\psi(0)\rangle = e^{-iHt/\hbar}|\psi(0)\rangle$

How to implement such operator in a quantum circuit?

Idea: **Matrix Exponentiation**: $e^M = \sum_{k=0}^{\infty} \frac{1}{k!} M^k$

$$e^{-i\theta X} = R_X(2\theta) = \begin{pmatrix} \cos(\theta) & -i\sin(\theta) \\ -i\sin(\theta) & \cos(\theta) \end{pmatrix}$$

$$e^{-i\theta Y} = R_Y(2\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$$

$$e^{-i\theta Z} = R_Z(2\theta) = \begin{pmatrix} e^{-i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix}$$

Over to the Quantum Picture

Initial variable $z_i \in \{-1, 0, +1\}$

Spin encoding : $z_i = \frac{s_i^+ - s_i^-}{2}, \; s_i \in \{-1, +1\}$

Which can then be plugged in the cost function

$$C_{RR}(\mathbf{z}) = \lambda \sum_{ij} \sigma_{ij} z_i z_j - \sum_i (1-\lambda)\mu_i z_i \;\rightarrow$$

$$\rightarrow C_{RR}(\mathbf{s}) = \lambda \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\sigma_{ij}}{4} \left(s_i^+ s_j^+ - s_i^+ s_j^- - s_i^- s_j^+ + s_i^- s_j^- \right) - (1-\lambda) \sum_{i=1}^{N} \frac{\mu_i}{2} \left(s_i^+ - s_i^- \right)$$

For unconstrained QAOA, $B = \sum_{i=1}^{n} \sigma_x^i$ (or) $H_M = \sum_{i=1}^{n} X_i$

Introducing soft constraints with penalty function

$$P_{INV}(\mathbf{z}) = A\left(\sum_{i=1}^{N} z_i - D\right)^2$$

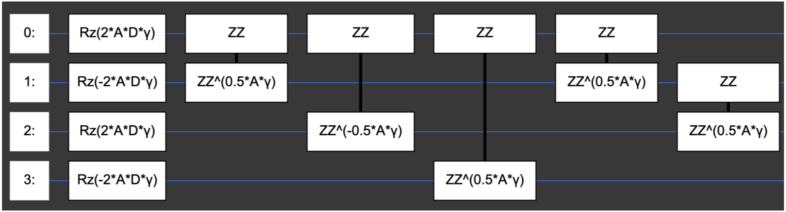
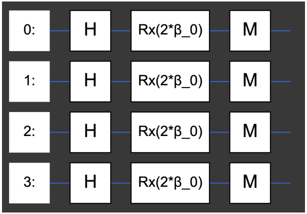
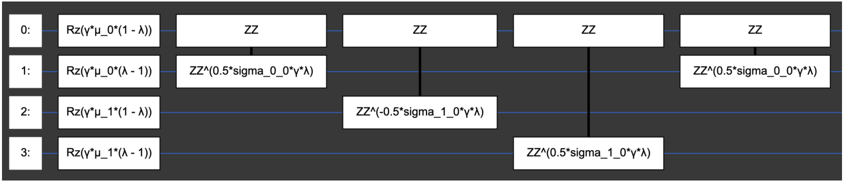


Figure: Constraints



(a)



(b)

Figure: From left to right, (a)initialisation of states - Hadamard, Mixer and Measurments and (b)risk-returns

Initialisation for hard:

$$|\psi_0\rangle = (|01\rangle)^{\otimes D} \otimes \left(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle\right)^{\otimes(N-D)}$$

Constrained Optimization Problems $\rightarrow$ Quantum Alternating Operator Ansatz

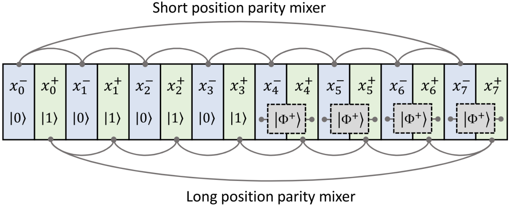
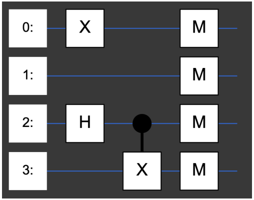


Fig. 1: Initial state for the hard constraint form of the investment constraint. The two parity mixers are depicted together with the Bell state entanglement for an example where $N = 8$, $D = 4$, $K = 5$.

Figure: Ref: arXiv:1911.05296

Blocks of the circuit(hard)



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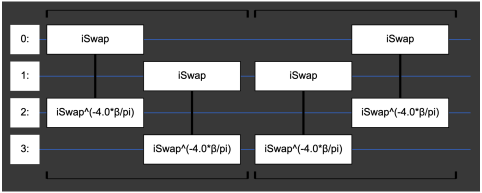


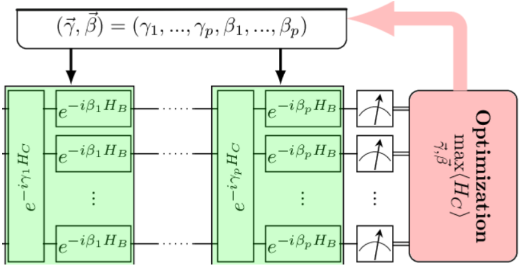
(b)

Figure: From left to right, (a)initialisation of states(hard) and Measurments and (b)Mixer for hard

Circuit parameters: β,γ

$$\beta = [0,\ 2\pi(\frac{1}{N}),\ 2\pi(\frac{2}{N}),...,\ 2\pi(\frac{N-1}{N})]$$

$$\gamma = [0,\ 2\pi(\frac{1}{N}),\ 2\pi(\frac{2}{N}),...,\ 2\pi(\frac{N-1}{N})]$$



Ran it for 4 stock portfolio case: z = [1,0,1,0]

Parameters

- p (circuit depth)
- Iterations
- Number of samples
- Fraction of interest

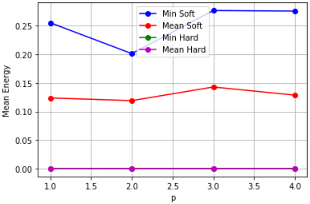
Standard Case

- p = 2
- Iterations = 10
- Number of samples = 10
- Fraction of interest = 0.2

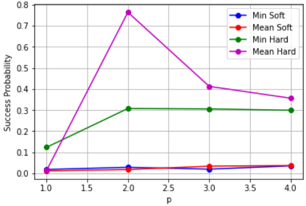
Approaches

- Minimum (Min)
- Mean (Mean)

Result: Finding the optimal p



(a)



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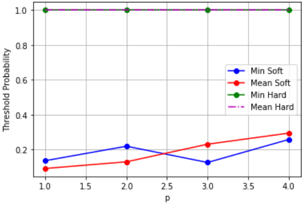
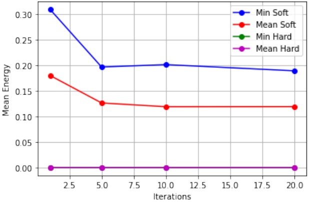


(c)

Figure: Values of p-depth = [1, 2, 3, 4]. Optimal value = 2.

Finding the optimal iterations



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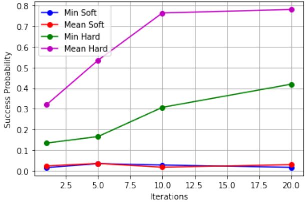

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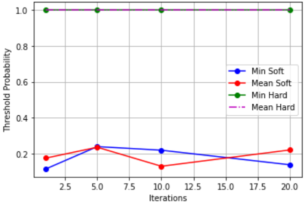
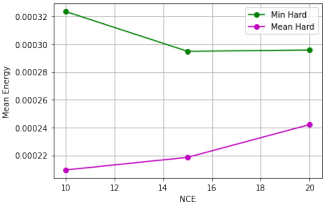


(c)

Figure: Values of iterations = [1,5,10,20]. Optimal value = 10.

Finding the optimal number of samples



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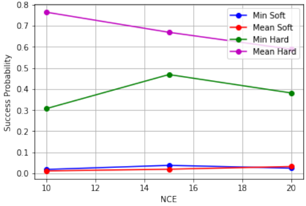

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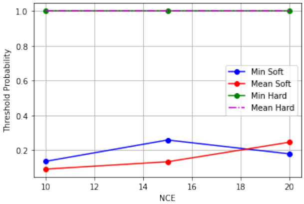

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Figure: Values of number of samples = [10,15,20]. Optimal value = 10.

Finding the optimal fraction of interest

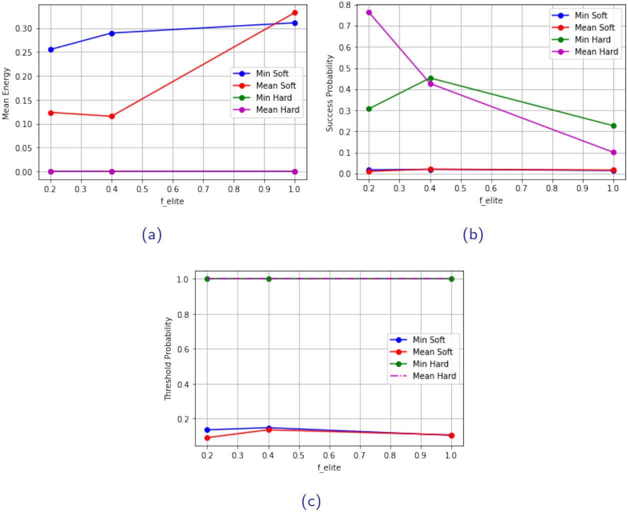
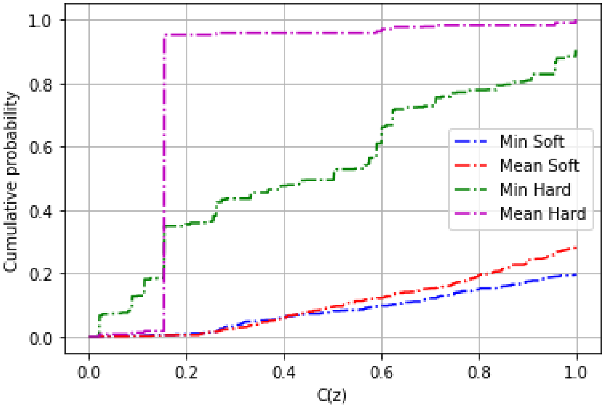


Figure: Values of fraction of interest = [0.2,0.4,1]. Optimal value = 0.2.



(a)

Figure: 8 real-world stocks such as Amazon, Apple, Google, Intel, Meta, Netflix, Starbucks and Tesla were taken. Optimal values found from previous graphs were taken, and all 4 different algorithms were analysed.

λ=0.1

1 1 1 -1

0 -1 1 1

λ=0.9

1 -1 1 1

0 0 1 0

Conclusion and Further Vision

- We were able to implement our own QAOA circuit with soft and hard constraints.
- We found the optimal configuration of parameters for the 4-stock portfolio case.
- We used our implementation on 8 real-world stocks to obtain the best investments for $\lambda = 0.1$ (high return) and $\lambda = 0.9$ (low risk).
- We aren't rich yet :(

References

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END OF PRESENTATION
NO QUESTIONS THANK YOU!