method and results

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Method

Adam

Adam is A Method for Stochastic Optimization proposed in 2015 from Diederik P Kingma that only need the first-order gradient. The Stochastic Gradient descent (SGD) is often used when the objective function is typically non-convex (as in our case). The "Ada" is derived from "adaptive", meaning this method change the learning rate over time according to gradients before. The detailed proof and explanation can be found in Diederik's paper. Here we just extracted the fake code part from original paper to clarify.

Algorithm:

1. Reqiured: α: Stepsize
   $\beta_1, \beta_2 \in [0, 1)$: Exponential decay rates for the moment estimates
   $f(\boldsymbol{\theta})$: The objective function with parameter vetctor $\boldsymbol{\theta}$
   ϵ controls the converge
2. Required: $\boldsymbol{\theta}_0$: Initial guess of parameters
   $\mathbf{m}_0 \leftarrow \mathbf{0}$: Initialize the 1st moment vector as $\mathbf{0}$
   $\mathbf{v}_0 \leftarrow \mathbf{0}$: Initialize the 1st moment vector as $\mathbf{0}$
   $t \leftarrow 0$:Initialize time step =0
   while $\boldsymbol{\theta}_t - \boldsymbol{\theta}_{t-1} > \epsilon$ not converge, do
   $t \leftarrow t + 1$
   $\boldsymbol{g}_t \leftarrow \nabla_\theta f_t(\boldsymbol{\theta}_{t-1})$: Get gradients w.r.t objective function at timestep t
   $\boldsymbol{m}_t \leftarrow \beta_1 \cdot \boldsymbol{m}_{t-1} + (1 - \beta_1) \cdot \boldsymbol{g}_t$:Update biased first moment estimate
   $\boldsymbol{v}_t \leftarrow \beta_2 \cdot \boldsymbol{v}_{t-1} + (1 - \beta_2) \cdot \boldsymbol{g}_t^2$:Update biased second moment estimate
   $\hat{\boldsymbol{m}}_t \leftarrow \boldsymbol{m}_t/(1 - \beta_1^t)$: Compute bias-corrected first moment estimate
   $\hat{\boldsymbol{v}}_t \leftarrow \boldsymbol{v}_t/(1 - \beta_2^t)$: Compute bias-corrected second raw moment estimate
   $\boldsymbol{\theta}_t = \boldsymbol{\theta}_{t-1} - \alpha \cdot \hat{\boldsymbol{m}}_t/(\sqrt{\hat{\boldsymbol{v}}_t + \epsilon}$: Update parameters
   End while Return $\boldsymbol{\theta}_t$ Result parameters
3. Defult setting:$\beta_1 = 0.9 \quad \beta_2 = 0.999 \quad \alpha = 0.001 \quad \epsilon = 10^{-8}$

Notes: g_t^t indicate the element-wise t power like $(g_t)^t$. Similarly, β_1^t and β_2^t also means the β_1 and β_2 to the power of t. In our case, we set the maximun time step $t = 10000$ to decrease the computation.

Loss function:

$$f = \sum_{i=1}^{n} (y_i - \frac{a}{1 + exp(-b(t - c))})^2$$

Gradient for parametrs a,b,c:

$$\nabla f(t,a) = \sum_{i=1}^{n} (\frac{2a}{(1+e^{(-bt+bc)})^2} - \frac{2y}{1+e^{(-bt+bc)}})$$

$$\nabla f(t,b) = -\sum_{i=1}^{n} (\frac{2a^2 e^{(-bt+bc)}}{(1+e^{(-bt+bc)})^3} + \frac{2ae^{(-bt+bc)}(c-t)y}{(1+e^{(-bt+bc)})^2})$$

$$\nabla f(t,c) = -\sum_{i=1}^{n} (\frac{2a^2 be^{(-bt+bc)}}{(1+e^{(-bt+bc)})^3} + \frac{2abe^{(-bt+bc)}}{(1+e^{(-bt+bc)})^2})$$

The initail guess of a,b,c in each country: a_0=two times the cumulative case in 24 May, b_0=0.3, c_0=40. For some special countries for example China and South Korea, the intial guess are adjusted for many times and the iteration also increases.

Results

Task 1.1

After applying the Adam algorithm in 116 countries, we get the estimated a,b,c values for each country in Table 1. The maximum a value is 138340 from Italy.The b value ranges from 0.085(Singapore) to 3.857(Trinidad and Tobago). The c value changes from 70 (China, Taiwan) to 4(Uzbekistan).

country_region	a_value	b_value	c_value
Afghanistan	342	0.202	37
Albania	269	0.173	17
Algeria	723	0.258	30
Andorra	345	0.344	22
Argentina	970	0.315	23
Armenia	514	0.288	23
Australia	4072	0.293	58
Austria	10760	0.275	28
Azerbaijan	365	0.184	30
Bahrain	795	0.118	29
Bangladesh	99	0.244	18
Belarus	102	0.276	19
Belgium	8530	0.254	49
Bolivia	81	0.192	16
Bosnia and Herzegovina	352	0.292	19
Brazil	4507	0.380	27
Brunei	98	0.381	7
Bulgaria	459	0.253	16
Burkina Faso	252	0.363	14
Cambodia	168	0.317	56
Canada	5462	0.338	58
Chile	1862	0.318	21
Netherlands	11170	0.239	26
New Zealand	505	0.420	27
Nigeria	102	0.407	25
North Macedonia	309	0.325	27
Norway	5557	0.175	26
Oman	361	0.125	40
Pakistan	1774	0.326	26
Panama	715	0.321	14
Paraguay	74	0.195	19
Peru	678	0.322	16
Philippines	1091	0.240	54
Poland	1821	0.283	20
Portugal	4741	0.335	22
Qatar	889	0.175	19
Romania	1783	0.256	29
Russia	979	0.291	53
Rwanda	107	0.356	11
San Marino	230	0.191	19
Saudi Arabia	1551	0.288	23
Senegal	357	0.217	27
Serbia	627	0.286	18
Singapore	1262	0.085	67
Slovakia	254	0.332	13
Slovenia	805	0.200	16
South Africa	1303	0.343	20
Spain	79759	0.257	52
Sri Lanka	105	0.459	51
Sweden	4381	0.171	52
Switzerland	19766	0.261	28
Taiwan*	576	0.097	70
Thailand	1634	0.306	62
Trinidad and Tobago	53	3.857	6
Tunisia	419	0.242	24
Turkey	3770	0.537	13
Ukraine	212	0.395	21
United Arab Emirates	652	0.114	62
United Kingdom	16258	0.279	53
Uruguay	184	0.548	6
US	106991	0.389	29
Uzbekistan	50	0.729	4
Venezuela	95	0.426	5
Vietnam	418	0.102	69

country_region	a_value	b_value	c_value
China	78732	0.223	18
Colombia	777	0.335	18
Congo (Kinshasa)	115	0.360	14
Costa Rica	375	0.268	18
Cote d'Ivoire	342	0.857	15
Croatia	958	0.310	29
Cuba	122	0.363	13
Cyprus	272	0.234	15
Denmark	3258	0.170	24
Dominican Republic	640	0.498	23
Ecuador	2180	0.449	23
Egypt	806	0.193	39
Estonia	569	0.235	22
Finland	1570	0.216	55
France	39932	0.148	64
Georgia	151	0.140	29
Germany	65957	0.259	57
Ghana	300	0.332	15
Greece	1499	0.182	27
Guatemala	23	0.589	6
Honduras	32	0.549	8
Hungary	393	0.266	20
Iceland	1311	0.213	25
India	1060	0.253	54
Indonesia	1389	0.266	22
Iran	49441	0.131	33
Iraq	642	0.143	30
Ireland	2673	0.309	24
Israel	4055	0.304	33
Italy	138340	0.183	53
Jamaica	20	0.331	5
Japan	2195	0.094	60
Jordan	326	0.302	21
Kazakhstan	69	0.529	5
Kenya	237	0.320	18
Korea, South	8801	0.284	40
Kuwait	564	0.088	36
Kyrgyzstan	279	0.546	9
Latvia	411	0.270	22
Lebanon	829	0.169	35
Liechtenstein	55	0.500	15
Lithuania	432	0.451	25
Luxembourg	2213	0.354	24
Malaysia	3231	0.222	59
Malta	242	0.248	17
Martinique	135	0.251	18
Mauritius	115	0.492	7
Mexico	748	0.317	25
Moldova	273	0.285	16
Monaco	60	0.272	25
Montenegro	124	0.507	8
Morocco	357	0.291	22

Table 1. Estimated a,b,c values in each country

Untill 24 May, It is estimaed that there are 27 countries that pass the midpoint. They are : Belarus, Brunei, Cambodia, China, Denmark,Estonia, Guatemala, Honduras , Iran, Jamaica, Japan, Kazakhstan, Korea South, Liechtenstein, Norway, Pakistan, Peru, Qatar, San Marino, Slovakia, Slovenia, Sri Lanka, Sweden,Trinidad and Tobago, Uruguay,Uzbekistan, Venezuela.

If we define the cumulative cases at 24 May surpass the 80% of a value in corresponding country is "appraoching the end". Then there are 15 countries: Brunei, China, Guatemala, Honduras, Jamaica, Kazakhstan,Korea South, Liechtenstein, San Marino, Slovakia, Sri Lanka, Trinidad and Tobago, Uruguay, Uzbekistan, Venezuela.

Task 1.2

We select three kinds of countries to do the visualization: 1) The early stages of COVID-19 outbreak, no deliberate intervention implemented. Representatives: Afghanistan and Vietnam. 2) Outbreak stage, the government intervention hasn't come into effect. Representatives: UK and US. 3)After the outbreak and the govrnment interventions have been effective. Representatives: China and South Korea. The a,b,c values of above 6 example countries are as follow:

country_region	a_value	b_value	c_value
Afghanistan	342	0.202	37
China	78732	0.223	18
Korea, South	8801	0.284	40
United Kingdom	16258	0.279	53
US	106991	0.389	29
Vietnam	418	0.102	69

Table 2. Estimated a,b,c values in 6 countries

The data from 25 May to 5 April (11 days) is used as test data to examine the predictivity of fitted model. The MSEs of training data(data before 24 May) and test data are as follow. Because the original data itself is relatively large, so the calculated MSE seems to be large.

Country	Train_error
Afghanistan	2.080206e+01
China	4.077602e+06
Korea_South	4.471121e+04
United_Kingdom	9.472004e+03
US	1.871744e+05
Vietnam	5.664849e+01

Table 3. MSE of train data

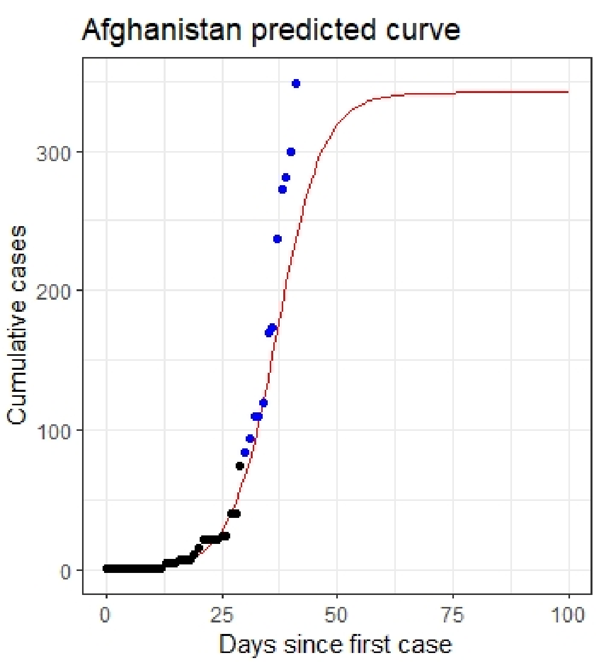
Country	test_error
Afghanistan	3.200053e+03
China	1.211702e+07
Korea_South	9.565317e+05
United_Kingdom	2.690240e+08
US	1.428445e+10
Vietnam	8.978671e+01

Table 4. MSE of test data

But if we visualize the model fitted value(red line) and observed values(train data is black and test data is blue). In the following plot, the fitted logistic curve fits the train data well, but deviations from test data in those two countries are different. The Afghanistan and Vietnam are both at the initial outbreak, so a dramatic increase of cases can be expected.

The maximum cases(a=342) is expected to be reached around the 60th day in Afghanistan. The deviation of test data before around 1 April is smaller than that after 1 April. But the data in April 5, apparently exceeds the estimaed a value, which denote the bias of our fitted model since we built the model only based on the data before 24 May.

For Vietnam, the The maximum cases(a=418) is expected to be reached around the 120th day. The fitteness of both train and test data is good.



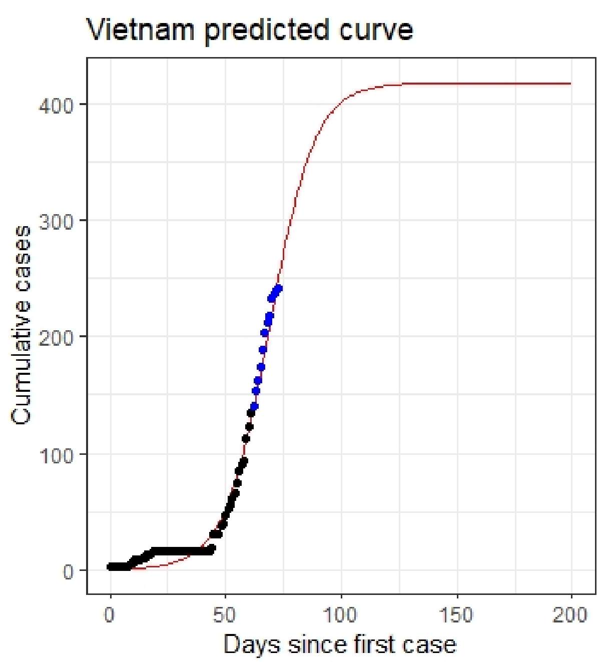


Figure 1. Afghanistan and Vietnam fitted and predicted values

In second kind country is as follow. The estimated a values are 16258 and 106991 for UK and US respectively.And the estimated stable stage when a is reach is 70th day and 50th day for UK and US respectively. For both of them, the red line fits black train data very well. But the increase of cases after 25 May is soaring, which is far away from the fitted line. To some extend, the Figure 2 denotes the lack of predictivity beause the lack of data when we built the model.

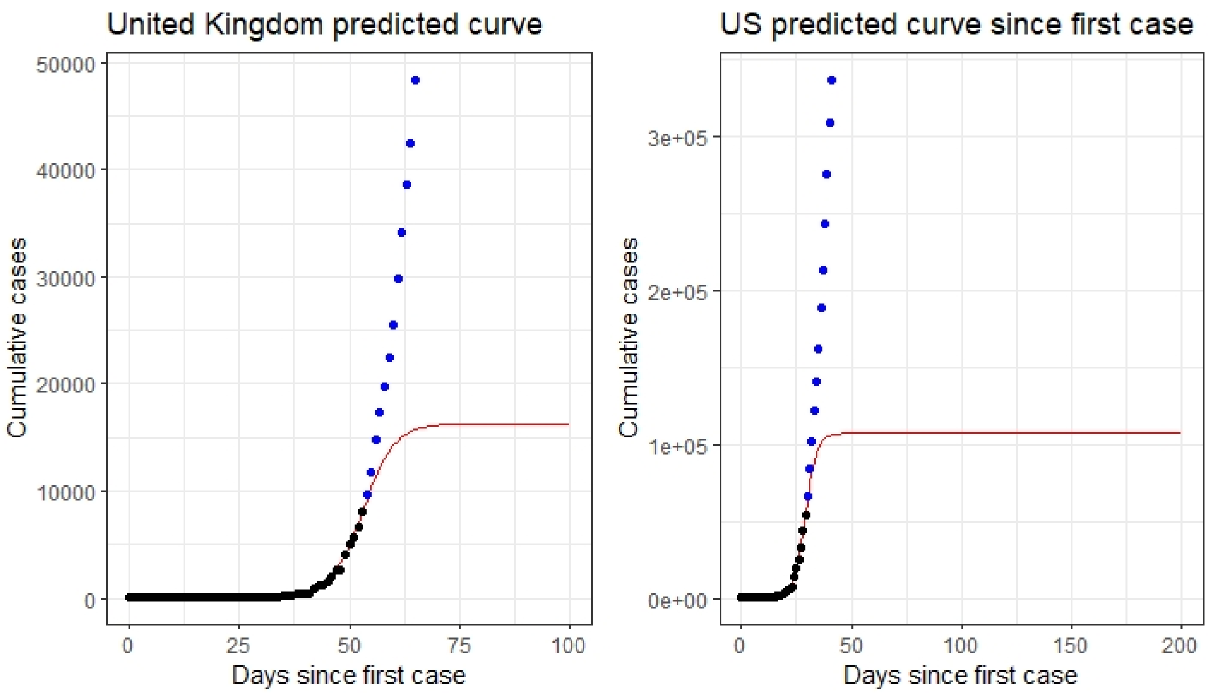


Figure 2. UK and US fitted and predicted values

In third kind country, who breakout reported at early Jan, their growths are very similar to each other. Old problem of fitted model re-appears that it estimates both of them already reached the end of spreading. But in fact both of them is undering increase cases after May 25. But the increase of cases is much slighter than UK and US. And the increase in China after 25 May is more flat given 1) it may already enters the stable part, which means the increase slows and 2) the interventions China takes may paly an important role.

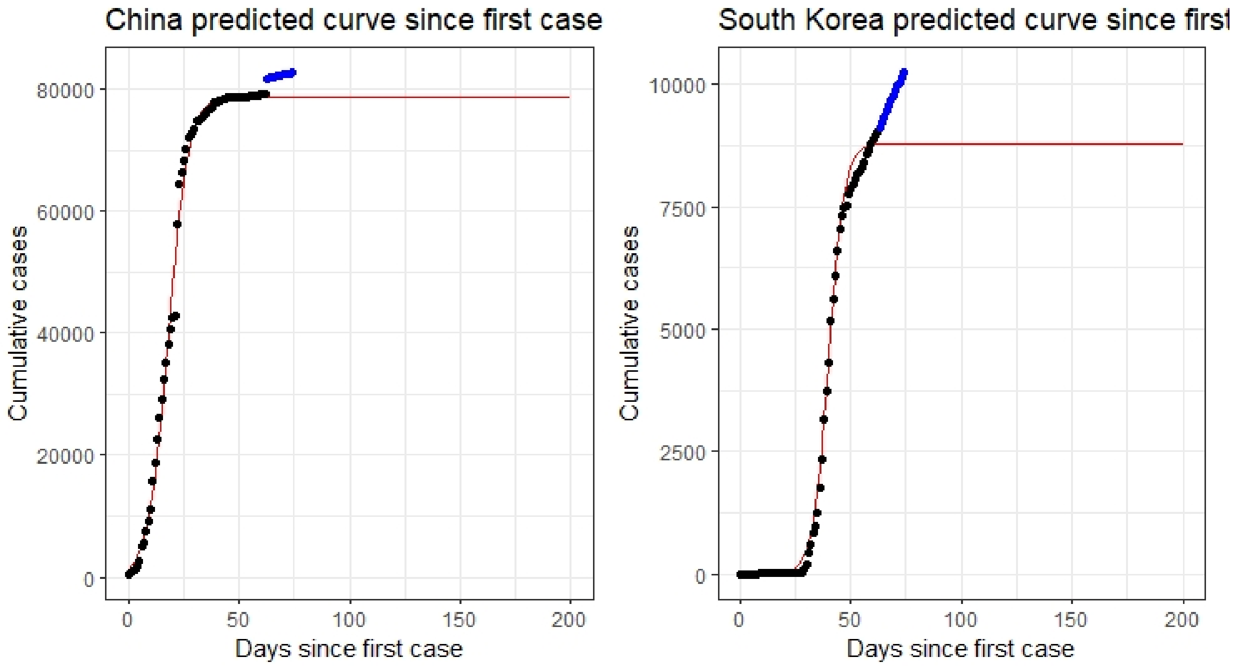


Figure 3. China and South Korea fitted and predicted values