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**Iterative and projection methods for solving Linear Systems**

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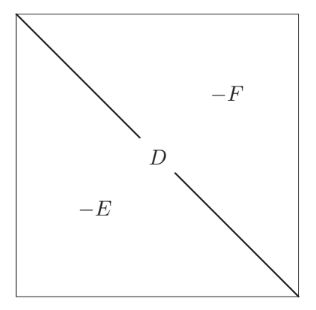
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Systems of linear algebraic equations are often found in various areas of applied mathematics because many physical phenomena are modeled by systems of PDE. Moreover, systems of non-linear equations can often be approximated by linear systems. Thereby methods for their solving play an important role in many applied fields, including linear programming, engineering and econometrics. Let us consider some methods for solving large sparse linear systems.

The first iterative methods used for solving large linear systems were based on relaxation of the coordinates. Beginning with a given approximate solution, these methods modify the components of the approximation, one or a few at a time and in a certain order, until convergence is reached. Each of these modifications, called relaxation steps, is aimed at annihilating one or a few components of the residual vector. Now, these techniques are rarely used separately.

Suppose given an real coefficient matrix and a right-hand side real vector The problem considered is: find belonging to such that Begin with the decomposition in which D is the diagonal of A, −E its strict lower part, and −F its strict upper part. Due to the nature of the iterative method, it is necessary to present an initial approximation 

There are a lot of iteration methods, but the classical one is the Jacobi iteration. It determines the i-th component of the next approximation to annihilate the i-th component of the residual vector. So the iterative formula of Jacobi iteration method: Similarly, the Gauss-Seidel iteration corrects the i-th component of the current approximate solution, in the order i = 1, 2,... ,n, again to annihilate the i-th component of the residual. However, this time the approximate solution is updated immediately after the new component is determined. Formula of Gauss-Seidel iteration method: There is only a slight difference between the Jacobi and Gauss–Seidel Iterations. Gauss-Seidel immediately updates the component to be corrected at the current step, and uses the updated approximate solution to compute the residual vector needed to correct the next component. However, the Jacobi iteration uses the same previous approximation for this purpose.

Another popular iteration method in application is the Block Relaxation Schemes. Block relaxation schemes are generalizations of the “point” relaxation schemes described above. They update a whole set of components at each time, typically a subvector of the solution vector, instead of only one component in the Jacobi iteration method.

Most of the existing practical iterative techniques for solving large linear systems of equations utilize a projection process in one way or another. A projection process represents a canonical way for extracting an approximation to the solution of a linear system from a subspace.

In projection techniques we use a subspace of to find an approximate solution to the above problem. If K⊆ is search subspace and if m = dim K then we can describe subspace K⊆ using m independent orthogonality conditions: the residual vector is orthogonal to m of linearly independent vectors. These m defines another subspace L⊆, dim L = m which is called the subspace of constraints or left subspace. Projection methods can be orthogonal or oblique. In orthogonal techniques, subspace L is the same as K , but in oblique techniques subspace L is different from K .

Let , ..., be column-vectors forming the basis of K⊆ , w1 , ...,wm are column-vectors forming the basis of L⊆ and V=[ ,..., ], W=[ ,...,] are n×m matrices. If is the approximate solution then the orthogonal conditions lead us to the following linear system for the vector y : . If m×m matrix is nonsingular then approximate solution can be found The approximate solution is defined only if is a nonsingular matrix, but this condition can be false even if matrix A is nonsingular.

Thus, properties of orthogonal and oblique projection techniques are a framework for solving linear systems. For instance, methods using the concept of Krylov subspaces such as FOM and GMRES are based on projection techniques.

**List of terms**

FOM (Full Orthogonalization Method) — метод полной ортогонализации

GMRES (Generalized Minimal RESidual method) — обобщенный метод минимальных невязок

Iterative method — итерационный метод

PDE (Partial Derivative Equations) - уравнения в частных производных

approximate — приближенный

arbitrary — произвольный

block relaxation scheme — блочная схема релаксации

convergence — сходимость

nonsingular — невырожденный

oblique — косой

operator — оператор

orthogonal — ортогональный

particular — частный

projector — проектор

proposition — утверждение

residual — невязка

residual — невязка

respectively — соответственно

span — линейная оболочка

sparse — разреженный

subspace — подпространство

technique — метод

to assume — предполагать

to obtain — получать

**List of useful phrases**

an arbitrary square matrix — произвольная квадратная матрица

approximate solution — приближенное решение

can be represented as … — может быть представлено как …

exact solution — точное решение

let us denote — обозначим

linearly independent vectors — линейно независимые векторы

oblique projection process onto K orthogonally to L — косая проекция на K ортогонально L

oblique projection — косая проекция

orthogonal conditions — условия ортогональности

pair of subspaces — пара подпространств

positive definite matrix — положительно определенная матрица

projection techniques — проекционные методы

residual vector — вектор невязки

special case — частный случай

starting approximate vector — начальный вектор приближения

subspace of constraints — подпространство ограничений

to be invariant under operator A — быть инвариантным для оператора A solution obtained from this method — решение, полученное этим методом convergence properties — свойства сходимости

to form the basis — составлять базис

to minimize the vector a over the subspace B — минимизировать вектор a на подпространстве B

**Bibliography**

1. Saad, Y. Iterative Methods for Sparse Linear Systems. Second Edition, 2003, 105 - 147.