For all pairwise disjoint ~~sets~~ partitions $\{B_1 \ldots B_k\}$,

The random Probability measure G on these partitions is said to be Dirichlet Process if

$$G(B_1), \ldots G(B_k) \sim Dir\left(\alpha G_0(B_1), \ldots \alpha G_0(B_k)\right)$$

If $B_1 = P_0(A)$
$B_2 = 1 - P_0(A)$

then, $$P(A) \sim Dir\left(\underset{a}{\alpha P_0(A)}, \underset{b}{\alpha(1-P_0(A))}\right)$$

Or we can also say

$$P(A) \sim Beta\left(\underset{a}{\alpha P_0(A)}, \underset{b}{\alpha(1-P_0(A))}\right)$$

We know that for a Dirichlet the mean is

$$E[\theta_j] = \frac{\alpha_j}{A}$$

$$Var[\theta_j] = \frac{\alpha_j}{A(A+1)} - \frac{\alpha_j^2}{A^2(A+1)}$$

So, $$E[\theta_j \times \theta_k] = \frac{\Gamma(A)}{\Gamma(A+2)} \times \frac{\Gamma(\alpha_j+1)}{\Gamma(\alpha_j)} \times \frac{\Gamma(\alpha_k+1)}{\Gamma(\alpha_k)} = \frac{\alpha_j \alpha_k}{(A+1)(A)}$$

$$Covar(\theta_j, \theta_k) = E[\theta_j \theta_k] - E(\theta_j) E[\theta_k]$$

$$= \frac{\alpha_j \alpha_k}{A(A+1)} - \frac{\alpha_j \alpha_k}{A^2} = \frac{-\alpha_j \alpha_k}{A^2(A+1)}$$

$$= -\frac{\cancel{\alpha^2} P_0(A)(1-P_0(A))}{\cancel{\alpha^2}(\alpha+1)}$$

So, $$\boxed{\text{Covariance}(B_1, B_2) = -\frac{P_0(B_1)\, P_0(B_2)}{(\alpha+1)}}$$

Since Covariance is negative between two disjoint sets, Correlation is also negative.

~~The Negative Correlation~~

The Intuition of Correlation is that the degree of correlation declines with separating distance becoming more and more negative. In this way, the neighboring points of a space should be positively correlated.

However, In a Dirichlet Process the samples present in two adjacent sets induces negative correlation on them, without taking any effect of the distance between those two sets.