

S.3

$$F(x) = 2x^2 - x^4 \quad \text{for } 0 \leq x \leq 1$$

Distribution function

a) Compute $P(1/4 \leq x \leq 3/4)$

$$P(1/4 \leq x \leq 3/4) = F(3/4) - F(1/4)$$

$\therefore F(x)$ gives us area under the curve, so
subtracting $F(3/4) - F(1/4)$ gives us area
 $1/4 (1/4 - 3/4)$ which is probability
in other sense,
so

$$\begin{aligned} P(1/4 \leq x \leq 3/4) &= F(3/4) - F(1/4) \\ &= \left[2\left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^4 \right] - \left[2\left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^4 \right] \end{aligned}$$

$$\boxed{\approx 0.6875}$$

b) P.D.F of x ?

P.D.F is derivative of $F(x)$ so

$$\begin{aligned} \text{P.D.F} &= \frac{d}{dx} (2x^2 - x^4) \\ &= \boxed{4x - 4x^3} \quad \text{for } 0 \leq x \leq 1 \end{aligned}$$