Outerspace

Q1 $f_Y(y) = \begin{cases} \frac{3}{8}y^2 & y \in (0,2) \\ 0 & \text{otherwise} \end{cases}$

$0 < a < 2$

$$F_Y(a) = \int_{-\infty}^{0} f_Y(y) + \int_{0}^{a} f_Y(y)$$

$$= 0 + \int_0^a \frac{3}{8}y^2$$

$$= \frac{3}{8}\frac{y^3}{3}\Big|_0^a = \frac{a^3}{8}.$$

$a > 2$

$$F_Y(a) = \int_{-\infty}^{0} f_Y(y) + \int_{0}^{2} f_Y(y) + \int_{2}^{\infty} f_Y(y)$$

$$= \frac{3}{8}\int y^2$$

$$= \frac{8}{8}y^3/3\Big|_0^2 = \frac{y^3}{8}\Big|_0^2 = 1$$

$$F_Y(a) = \begin{cases} \frac{y^3}{8} + 1 \end{cases}$$

$F_Y(a) = P(Y \le a) \Rightarrow F_X(Y \le a)$

where $Y = g(X)$

$F_Y(a) = \frac{a^2}{8} + 1$

$F_X(Y \le a) = \int f_X \, dx$ $\quad f_X = \begin{cases} \frac{1}{b-a} & a \le x \le b \end{cases}$

$0 < a \le 1$

$= \int_{-\infty}^{0} f_X \, dx + \int_{0}^{a} f_X \, dx + \int_{a}^{\infty} f_X \, dx$

$= \frac{x-a}{b-a} \Big|_0^a = \frac{x}{1} \quad (a=0, b=1)$

$f_X(Y)$

$0 < Y < 1$

$= \int_{-\infty}^{0} f_X \, dx + \int_{0}^{1} f_X \, dx + \int_{1}^{\infty} f_X \, dx$

$= 1$

$F_X(Y \le a) = x + 1.$

$$F_Y(a) = P(Y \le a) = F_X(Y \le a)$$

$$\frac{y^3}{8} + 1 = x + 1$$

$$\frac{y^3}{8} = x$$

$$y^3 = 8x$$

$$\boxed{y = 2x^{1/3}}$$