9.12

f of (X, Y) is $\begin{cases} f(x,y) = K(3x^2+8xy) & \text{for } 0 \le x \le 1 \text{ and } 0 \le y \le 2 \\ \text{and } f(x,y) = 0 & \text{otherwise} \end{cases}$

a) We need to find K. As we know total sum of probabilities have to equal to 1 e.i. in cont. case $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K(3x^2+8xy)\,dy\,dx = 1$

$\int_0^1\int_0^2 (3x^2+8xy)\,dx\,dy =$

$= \int_0^1\left(\int_0^2 (3x^2+8xy)\,dy\right)dx = \int_0^1 \left(3x^2y + 8x\frac{y^2}{2}\right)\Big|_0^2 dx =$

$= \int_0^1 (6x^2+16x)\,dx = \left(6\frac{x^3}{3} + 16\frac{x^2}{2}\right)\Big|_0^1 = \frac{6}{3} + 8 = 10$

and therefore $K = 1/10 = 0.1$

(b) $P(2X \le Y) = \int_0^1\int_{2x}^2 \frac{1}{10}(3x^2+8xy)\,dx\,dy = \frac{1}{10}\int_0^1\left(\int_{2x}^2 3x^2\,dy + \int_{2x}^2 8x\right.$

$= \frac{1}{10}\int_0^1\left(3x^2\cdot y\Big|_{2x}^2 + \overset{4}{\cancel{8}}\frac{xy^2}{\cancel{2}}\Big|_{2x}^2\right)dx =$

$= \frac{1}{10}\int_0^1 (3x^2\cdot 2 - 3x^2\cdot 2x + 16x - 16x^3)\,dx = \frac{1}{10}\int_0^1 (6x^2 - 22x^3 + 16x$

$= \frac{1}{10}\left(-\frac{22}{4} + 2 + 8\right) = \frac{1}{10}(10 - 5.5) = 0.45$