Ex 3.16

D = "You have the disease"

T = "Test says so"

$P(D) = 1\% = 0.01$ $\qquad$ $P(T|D) = 0.98$

$P(T^c|D^c) = 0.95$

a) $P(D|T) = ?$

$$P(D|T) = \frac{P(T|D) \cdot P(D)}{P(T)} \quad \text{— (A)}$$

(Bayes Theorem)

whereas

$$P(T) = P(T \cap D) + P(T \cap D^c)$$

positive result

True positive

false positive

so

$$P(T) = [P(T|D) \cdot P(D)] + [P(T|D^c) \cdot P(D^c)]$$

$$P(T) = [0.98 \times 0.01] + [(1 - P(T^c|D^c)) \cdot 0.99]$$

$$P(T) = [0.98 \times 0.01] + [0.05 \times 0.99]$$

$$= 0.0098 + 0.0495$$

$$= 0.0593$$

Putting values in eq^n (A)

$$P(D|T) = \frac{P(D \cap T)}{P(T)} = \frac{0.0098}{0.0593} = \boxed{0.1652}$$

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b) Ambiguous case: It is written that "independent repitition of the test which is not obvious because both tests are related to each other i.e (A Person would only have the second test "Z" if the result of first is Positive, so they are related and clearly there is dependency)

So to solve this we take two cases. Case 1 where they are related and case2 where they are independent. so

Case 1

Here the $P(D|T)$ would be used as our base Probability and

$Z$ = test 2 says Positive

$P(D) = 0.1653$ $\quad P(Z^c|D^c) = 0.95$

$P(Z|D) = 0.98$

$P(D|Z) = ?$

- Again, just using the Bayes Theorem

$$P(D|Z) = \frac{P(D \cap Z)}{P(Z)} \quad \text{— (A)}$$

where

$$P(Z) = P(D \cap Z) + P(D^c \cap Z)$$
$$= [P(D) \cdot P(Z|D)] + [P(D^c) \cdot P(Z|D^c)]$$

- $P(D) = 0.1653$

$P(Z|D) = 0.98$

$P(D^c) = 1 - P(D) = 1 - 0.1653 = 0.8347$

$P(Z|D^c) = 1 - P(Z^c|D^c)$
$= 1 - 0.95 = 0.05$

putting values

- $P(Z) = (0.98 \times 0.1653) + (0.05 \times 0.8347)$

$P(Z) = 0.1619 + 0.0417 = 0.2036$

so

$$P(D|Z) = \frac{0.1619}{0.2036} = 0.795 \approx 79.5\%$$

so

- $\boxed{P(D|Z) \approx 0.795}$

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Case 2 : independence

It is said that this test is indepent of the previous one..

This case is only logical if we are told in advance that you have the disease. Then all of the tests becomes irrelevant and in that case

$$P(Z \cap T) = P(Z) . P(T) \quad — (B)$$

So right now, that's the formula we have to use...

~~P(D|~~

$$P(D \mid Z \cap T) = ?$$

where $Z$ and $T$ are independent

$$P(D \mid Z \cap T) = \frac{P(D \cap Z \cap T)}{P(Z \cap T)} \quad — \text{(C)}$$

$$P(D \cap Z \cap T) = P(Z \mid D) * P(T \mid D) * P(D)$$

(conditional Independence)

$$P(D \cap Z \cap T) = 0.98 \times 0.98 * 0.01$$

$$\approx 0.0960$$

And

$$P(z \cap T) = [P(z|D) * P(T|D) * P(D)] +$$

$$[P(z|D^c) * P(T|D^c) * P(D^c)]$$

(law of total probability)

$$P(z \cap T) = [0.98 \times 0.98 \times 0.01] + [(1 - P(z^c|D^c)) * 1 - P(T^c|D^c) * P(D^c)]$$

$$= [0.98 \times 0.98 \times 0.01] + [0.05 \times 0.05 \times 0.99]$$

$$\approx 0.012079$$

$$P(z \cap T) = 0.012079$$

$$P(D|z \cap T) = 0.00960 / 0.012079 \approx \boxed{0.7951}$$

So

$P(D|z \cap T)$ is approx equal to

$P(D|z)$ which is $\boxed{0.7951}$