10.3

Determine Cov(U,V) and correlation coefficient (U,V)

$$Cov(U,V) = E[U*V] - E(U)*E[V] \quad \text{---- (A)}$$

$E[U*V] = ?$

Reconstruct the table again with marginal of U and V

		U			Marginal(V)	Marginal(V) * V
		0	1	2		
V	0	1/4	0	1/4	1/2	0
	1	0	1/2	0	1/2	1/2
Marginal(U)		1/4	1/2	1/4	1	
Marginal(U) * U		0	1/2	1/2		

$$E(U) = \sum \text{marginal}(U) * U$$

$$= 1/2 + 1/2 = 1$$

$$E(V) = \sum \text{marginal}(V) * V$$

$$= 1/2$$

$$E[U*V] = \sum_{V} \sum_{U} uv f(u,v)$$

$$E[U*V] = \sum_{0,1} \sum_{0,1,2} UV f(u,v)$$

$$= \sum_{0,1} V \sum_{0,1,2} u f(u,v)$$

$$\Rightarrow \sum_{0,1} V [0*f(0,V) + 1\times f(1,V) + 2*f(2,V)]$$

$$\Rightarrow \sum_{0,1} V [f(1,V) + 2*f(2,V)]$$

$$= [(0*\{f(1,V)+2*f(2,V)\}) + (1*\{f(1,1)+2*f(2,1)\})]$$

$$= f(1,1) + 2*f(2,1)$$

$$= \frac{1}{2} + 2(0)$$

$$= \frac{1}{2}$$

$$Cov(U,V) = E(U,V) - E(U)*E(V)$$

$$= \frac{1}{2} - (1)*\left(\frac{1}{2}\right)$$

$$= \frac{1}{2} - \frac{1}{2}$$

$$= 0$$

$$\boxed{Cov(U,V) = 0}$$

$$\text{Correlation coefficient}_{(u,v)} = \frac{Cov(u,v)}{\sqrt{Var(u) * Var(V)}}$$

But $Cov(u,v) = 0$

so

$$\text{Coefficient} = \frac{0}{\sqrt{Var(U) * Var(V)}}$$

$= 0$

so

$$\boxed{\text{Correlation coefficient} = 0}$$

Scanned by CamScanner