Ex 7.7

Let X be a r.v

$$P.D.F = f(x) = \begin{cases} 4x - 4x^3 & \text{for } 0 \leq x \leq 1 \\ \text{for } x < 0, & 0 \\ \text{for } x > 1, & 0 \end{cases}$$

a) $\mathbb{E}(2X+3) = ?$

first, let's compute $E(X)$

$$E(X) = \int_{-\infty}^{+\infty} (x f x) \cdot dx$$

$$E(X) = \int_0^1 (x)(4x - 4x^3)\, dx$$ (since function has only values in [0, 1], else 0)

$$E(X) = \int_0^1 (4x^2 - 4x^4)\, dx$$

$$= 4 \int_0^1 (x^2 - x^4)\, dx$$

$$= 4 \left\{ \frac{x^3}{3} - \frac{x^5}{5} \Big|_0^1 \right\}$$

$$\Rightarrow 4 \left\{ \left(\frac{1}{3} - \frac{1}{5}\right) - (0 - 0) \right\}$$

$$= 4 \left\{ \frac{5-3}{15} \right\} = \boxed{8/15}$$

$$E(2X+3) = 2 \cdot E(X) + 3$$
$$= 2\left(\frac{8}{15}\right) + 3$$
$$= \boxed{61/15}$$

(b) $Var(2X+3) = ?$

first let us compute $Var(X)$

$$Var(X) = E(X^2) - (E[X])^2 \quad \text{—— (A)}$$

$$E(X^2) = \int_{-\infty}^{+\infty} (x^2) \cdot f(x)\, dx$$
$$= \int_0^1 (x^2) \cdot (4x - 4x^3)\, dx$$
$$= \int_0^1 (4x^3 - 4x^5)\, dx$$
$$\Rightarrow 4 \int_0^1 (x^3 - x^5)\, dx$$
$$\Rightarrow 4 \left\{ \frac{x^4}{4} - \frac{x^6}{6} \Big|_0^1 \right\}$$
$$= 4 \left\{ \left(\frac{1}{4} - \frac{1}{6}\right) - \left(\frac{0}{\cdot} - 0\right) \right\} = 1/3$$

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$$Var(X) = E(X^2) - (E[X])^2$$
$$= \left(\frac{1}{3}\right) - \left(\frac{8}{15}\right)^2$$
$$= \frac{11}{225}$$

so

$$Var(2X+3) = 4Var(X)$$
$$= 4\left(\frac{11}{225}\right)$$
$$\boxed{= \frac{44}{225}}$$ Ans

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