15.7

max. Score = 1
min. Score = 0

The P.D.F of Score is

$$f(x) = \begin{cases} 4x & 0 \le x \le \frac{1}{2} \\ 4-4x & \frac{1}{2} \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

score < 0.55, student fails - - - -

a) P(student fails) = ?

$$P(\text{student fails}) = P(0 \le x \le 0.55)$$

$$= \int_{0}^{1/2}(4x)\,dx + \int_{1/2}^{0.55}(4-4x)\,dx$$

(from P.D.F)

$$= \left\{ \frac{4x^2}{2} \Big|_{0}^{1/2} \right\} + \left\{ 4x - \frac{4x^2}{2} \Big|_{1/2}^{0.55} \right\}$$

$$\Rightarrow \left\{ 2x^2 \Big|_{0}^{1/2} \right\} + \left\{ 4\left[x - \frac{x^2}{2} \right] \Big|_{1/2}^{0.55} \right\}$$

$$= \boxed{0.595} \text{ Ans}$$

(b)   what is the 50th percentile of Score Distribution?

$$F(q_{0.5}) = 0.5 \qquad F(b) = \int_0^b 4x\,dx = \frac{4x^2}{2}\Big|_0^b = 2b^2$$

$$F(q_{0.5}) = q_{0.5} \qquad 2q_{0.5}^2 = 0.5 \qquad q^2 = 0.25 \qquad q = 0.5$$

$$\boxed{q = 0.5} \quad \text{Ans}$$