Problem statement

In this report, the field amplitudes of the forward and backward propagating wave within a structure of N pairs of dielectric layers is determined using coupled wave theory and the exact matrix approach. For layer 1 and 2, the permittivity and thickness are ε_1 and L_1 and ε_2 and L_2 respectively. Outside this N pairs of dielectric layers, permittivity everywhere is ε_2. The permeability is μ_o throughout the structure.

Input parameters

The input parameters used for the numerical simulation are provided in the following table,

Refractive index of Layer 1, n1	1.55
Refractive index of Layer 2, n2	1.5
Free space optical wavelength, λ	1 μm
Length of layer 1, L1	0.1612μm
Length of layer 2, L2	0.1666μm
Number of unit cells,N	100
Number of dielectric layers	200
Length of an unit cell, Λ	0.32795μm
Length of the entire index perturbation, N$\times\Lambda$	32.7956 μm
Relative Electric permittivity, outside the perturbation ε_{ro}	2.25
Relative Electric permittivity in L1, ε_{r1}	2.4025
Relative Electric permittivity in L2, ε_{r2}	2.25

Figure 1: Simulation parameters

Part 1: Matrix theory

In this section, the electric field amplitude for the forward and backward propagating waves throughout the N layer structure is calculated using the matrix theory. In addition, the reflection and transmission coefficient for optical power are determined.

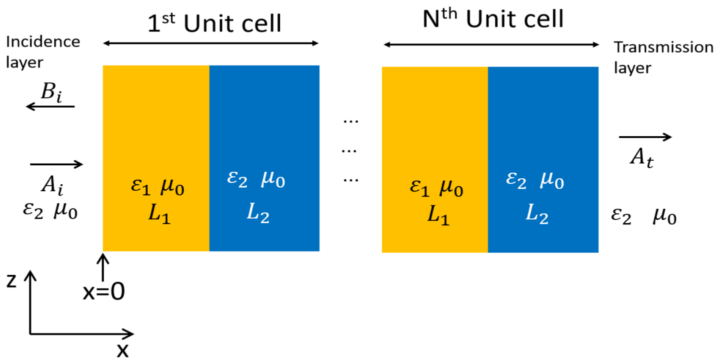


Figure 2: Bragg stack

The transverse electric field equation is given by,

$$\vec{E_n} = e^{j(\omega t-\beta z)}\{A_n e^{-jk_{nx}(x-x_n)} + B_n e^{jk_{nx}(x-x_n)}\}\vec{a_y}$$

Where, A_n and B_n are coefficients of the incident and reflected field amplitudes respectively. The wave vector magnitude along the z-direction in both layers is $\beta = 0$. The propagation vector magnitude along the x direction in the two layers is given by

$$k_{ix} = \sqrt{\omega^2\varepsilon_2\mu_o - \beta^2} = \sqrt{\omega^2 n_2^2\varepsilon_o\mu_o} = \frac{\omega n_2}{c} = \frac{2\pi n_2}{\lambda}$$
$$k_{1x} = \sqrt{\omega^2\varepsilon_1\mu_o - \beta^2} = \sqrt{\omega^2 n_1^2\varepsilon_o\mu_o} = \frac{\omega n_1}{c} = \frac{2\pi n_1}{\lambda}$$
$$k_{2x} = \sqrt{\omega^2\varepsilon_2\mu_o - \beta^2} = \sqrt{\omega^2 n_2^2\varepsilon_o\mu_o} = \frac{\omega n_2}{c} = \frac{2\pi n_2}{\lambda}$$
$$k_{tx} = \sqrt{\omega^2\varepsilon_2\mu_o - \beta^2} = \sqrt{\omega^2 n_2^2\varepsilon_o\mu_o} = \frac{\omega n_2}{c} = \frac{2\pi n_2}{\lambda}$$

From the theory of 1D bragg structures, it is known that,

$$D_n^{TE} P_n^{TE} \begin{bmatrix} A_n \\ B_n \end{bmatrix} = D_{n+1}^{TE} \begin{bmatrix} A_{n+1} \\ B_{n+1} \end{bmatrix}$$

$$D_{n+1}^{TE} P_{n+1}^{TE} \begin{bmatrix} A_{n+1} \\ B_{n+1} \end{bmatrix} = D_{n+2}^{TE} \begin{bmatrix} A_{n+2} \\ B_{n+2} \end{bmatrix}$$

Where,

$$D_n^{TE} = \begin{bmatrix} 1 & 1 \\ \frac{k_{nx}}{\mu_o} & -\frac{k_{nx}}{\mu_o} \end{bmatrix} \qquad D_{n+1}^{TE} = \begin{bmatrix} 1 & 1 \\ \frac{k_{n+1,x}}{\mu_o} & -\frac{k_{n+1,x}}{\mu_o} \end{bmatrix}$$

$$P_n^{TE} = \begin{bmatrix} e^{-jk_{nx}L_n} & 0 \\ 0 & e^{jk_{nx}L_n} \end{bmatrix} \qquad P_{n+1}^{TE} = \begin{bmatrix} e^{-jk_{n+1,x}L_{n+1}} & 0 \\ 0 & e^{jk_{n+1,x}L_{n+1}} \end{bmatrix}$$

The following matrices are defined

$$A = \begin{bmatrix} 1 & 1 \\ k_{2,x} & -k_{2,x} \end{bmatrix}^{-1}$$

$$F = \begin{bmatrix} e^{-jk_{2,x}L_2} & 0 \\ 0 & e^{jk_{2,x}L_2} \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 1 \\ k_{t,x} & -k_{t,x} \end{bmatrix}^{-1}$$

$$B = \begin{bmatrix} 1 & 1 \\ k_{1,x} & -k_{1,x} \end{bmatrix}$$

$$Y = \begin{bmatrix} 1 & 1 \\ k_{2,x} & -k_{2,x} \end{bmatrix}$$

$$C = \begin{bmatrix} e^{-jk_{1,x}L_1} & 0 \\ 0 & e^{jk_{1,x}L_1} \end{bmatrix}$$

$$Z = \begin{bmatrix} e^{-jk_{2,x}L_2} & 0 \\ 0 & e^{jk_{2,x}L_2} \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 1 \\ k_{1,x} & -k_{1,x} \end{bmatrix}^{-1}$$

$$R = \begin{bmatrix} 1 & 1 \\ k_{1,x} & -k_{1,x} \end{bmatrix}^{-1}$$

$$E = \begin{bmatrix} 1 & 1 \\ k_{2,x} & -k_{2,x} \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & 1 \\ k_{i,x} & -k_{i,x} \end{bmatrix}$$

Thus, for the problem at hand, we have the following relation between the electric field amplitudes in each layer,

$$\begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ k_{1,x} & -k_{1,x} \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ k_{i,x} & -k_{i,x} \end{bmatrix} \begin{bmatrix} A_i \\ B_i \end{bmatrix} = RS \begin{bmatrix} A_i \\ B_i \end{bmatrix}$$

$$\begin{bmatrix} A_2 \\ B_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ k_{2,x} & -k_{2,x} \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ k_{1,x} & -k_{1,x} \end{bmatrix} \begin{bmatrix} e^{-jk_{1,x}L_1} & 0 \\ 0 & e^{jk_{1,x}L_1} \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = ABC \begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = (ABC)(RS) \begin{bmatrix} A_i \\ B_i \end{bmatrix}$$

$$\begin{bmatrix} A_3 \\ B_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ k_{1,x} & -k_{1,x} \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ k_{2,x} & -k_{2,x} \end{bmatrix} \begin{bmatrix} e^{-jk_{2,x}L_2} & 0 \\ 0 & e^{jk_{2,x}L_2} \end{bmatrix} \begin{bmatrix} A_2 \\ B_2 \end{bmatrix} = DEF \begin{bmatrix} A_2 \\ B_2 \end{bmatrix} = (DEF)(ABC)(RS) \begin{bmatrix} A_i \\ B_i \end{bmatrix}$$

$$\begin{bmatrix} A_4 \\ B_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ k_{2,x} & -k_{2,x} \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ k_{1,x} & -k_{1,x} \end{bmatrix} \begin{bmatrix} e^{-jk_{1,x}L_1} & 0 \\ 0 & e^{jk_{1,x}L_1} \end{bmatrix} \begin{bmatrix} A_3 \\ B_3 \end{bmatrix} = ABC \begin{bmatrix} A_3 \\ B_3 \end{bmatrix} = (ABC)(DEF)(ABC)(RS) \begin{bmatrix} A_i \\ B_i \end{bmatrix}$$

Similarly,

$$\begin{bmatrix} A_5 \\ B_5 \end{bmatrix} = (DEF)(ABC)(DEF)(ABC)(RS) \begin{bmatrix} A_i \\ B_i \end{bmatrix}$$

$$\begin{bmatrix} A_6 \\ B_6 \end{bmatrix} = (ABC)(DEF)(ABC)(DEF)(ABC)(RS) \begin{bmatrix} A_i \\ B_i \end{bmatrix}$$

$$\vdots$$

$$\begin{bmatrix} A_{200} \\ B_{200} \end{bmatrix} = (ABC)(DEF).......(ABC)(DEF)(ABC)(DEF)(ABC)(RS) \begin{bmatrix} A_i \\ B_i \end{bmatrix}$$

$$\begin{bmatrix} A_t \\ B_t \end{bmatrix} = XYZ \begin{bmatrix} A_{200} \\ B_{200} \end{bmatrix} = (XYZ)(ABC)(DEF).......(ABC)(DEF)(ABC)(DEF)(ABC)(RS) \begin{bmatrix} A_i \\ B_i \end{bmatrix}$$

Thus, we have the following relationship between the coefficients in the transmission and the incident layer,

$$\begin{bmatrix} A_t \\ B_t \end{bmatrix} = T \begin{bmatrix} A_i \\ B_i \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} A_i \\ B_i \end{bmatrix}$$

Where,

T=(XYZ)(ABC)(DEF).......(ABC)(DEF)(ABC)(DEF)(ABC)(RS)

To obtain the transfer matrix numerically in matlab, a for loop can be constructed to repeatedly perform matrix multiplication starting from the left of the equation. The matrices A, B, C, D, E, F, X, Y, Z, R and S defined as previously explained. The code for determining the transfer matrix is as follows,

```
T=X*Y*Z;
for i =1:199
    if(mod(i,2))
        T=T*D*E*F;
    else
        T=T*A*B*C;
    end
end
T=T*R*S;
```

Since the matrix multiplication is carried from the left side, the transfer matrix is initialized with the value of 'X*Y*Z'. As we proceed with the multiplication, if the index of the cell is even, the 'T' matrix is multiplied with the matrices 'D*E*F' and if the index is even the 'T' matrix is multiplied with the matrices 'A*B*C'. Once the loop is over, we will have the relation between 'A_t' and 'B_t' and 'A_1' and 'B_1'. To complete the transfer matrix multiplication, the 'T' matrix is finally multiplied with the matrices 'R*S'.

For the problem at hand, we have the following values,

$k_{1x} = 9.7389 * 10^6\ m^{-1}$

$k_{2x} = 9.4247 * 10^6\ m^{-1}$

$k_{ix} = 9.4247 * 10^6\ m^{-1}$

$k_{tx} = 9.4247 * 10^6\ m^{-1}$

$L_1 = 0.1612\ \mu$m

$L_2 = 0.1666\ \mu$m

Once, the values are substituted into the matrices A, B, C, D, E, F, X, Y, Z, R and S and the transfer matrix calculation loop is carried out , the transfer matrix is,

$$T = \begin{bmatrix} 12.45060 & -12.4129 \\ -12.4129 & 12.45060 \end{bmatrix}$$

Thus, we get the relation between the electric field amplitudes in the incidence and the reflected layers as follows,

$$\begin{bmatrix} A_t \\ B_t \end{bmatrix} = \begin{bmatrix} 12.45060 & -12.4129 \\ -12.4129 & 12.45060 \end{bmatrix} \begin{bmatrix} A_i \\ B_i \end{bmatrix}$$

$$\implies \begin{bmatrix} A_i \\ B_i \end{bmatrix} = \begin{bmatrix} 13.29447 & 13.254260 \\ 13.254260 & 13.29447 \end{bmatrix} \begin{bmatrix} A_t \\ B_t \end{bmatrix}$$

We know that in the transmission layer,

$$\begin{bmatrix} A_t \\ B_t \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\implies \begin{bmatrix} A_i \\ B_i \end{bmatrix} = \begin{bmatrix} T_{11}^{-1} \\ T_{21}^{-1} \end{bmatrix} = \begin{bmatrix} 13.294479 \\ 13.25426 \end{bmatrix}$$

In the problem statement, it is given that the incident field has an amplitude of 1 V/m. Hence, the matrix values are multiplied by a constant value, $1/T_{11}^{-1}$ to get an unity amplitude. This gives us the following matrix values,

$$\implies \begin{bmatrix} A_i \\ B_i \end{bmatrix} = \begin{bmatrix} 1 \\ 0.996974 \end{bmatrix}$$

$$\implies \begin{bmatrix} A_t \\ B_t \end{bmatrix} = \begin{bmatrix} 0.0752191 \\ 0 \end{bmatrix}$$

Now that the value of A_i and B_i is determined, the previously explained matrix relationship between the field amplitudes in adjacent layers is used to calculate the field amplitude in all the dielectric layers in the structure.

Thus, the graph showing the variation,

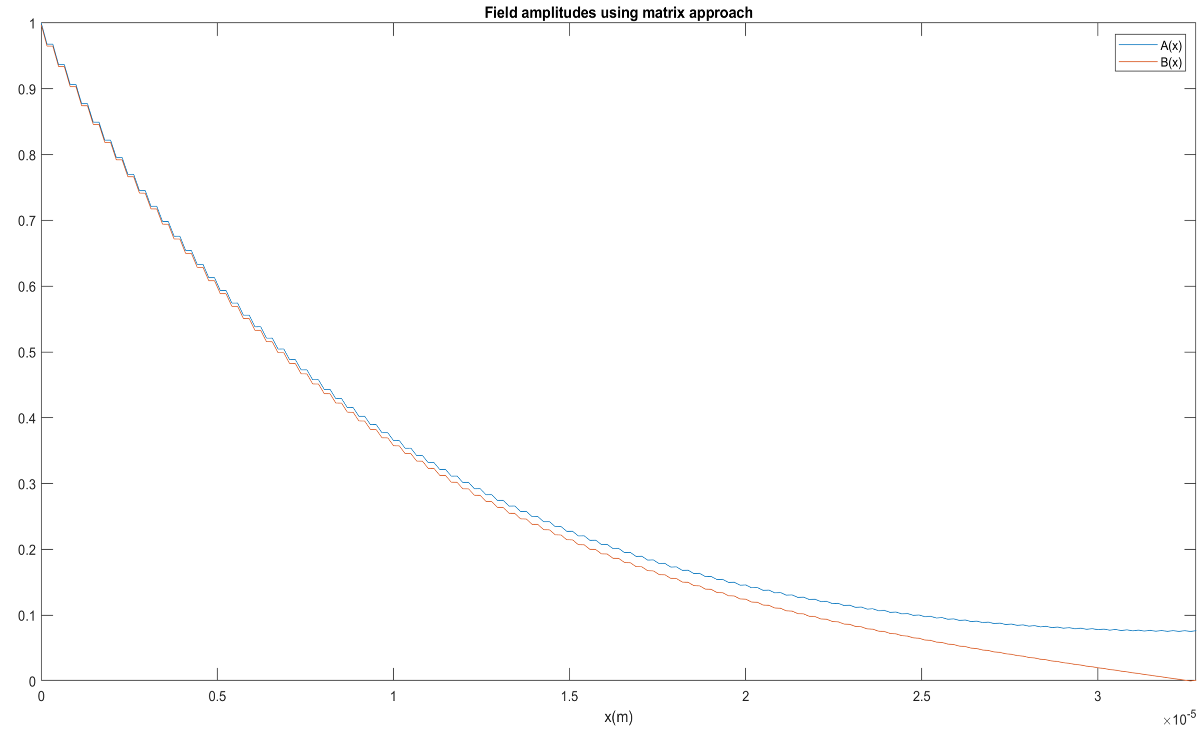


Figure 3: Field amplitude variation inside the dielectric layers using matrix approach

As expected, the amplitudes A(x) and B(x) have an exponential decay within the index perturbation. B(x) drops to 0 while A(x) remains non-zero.

The reflection co-efficient is,

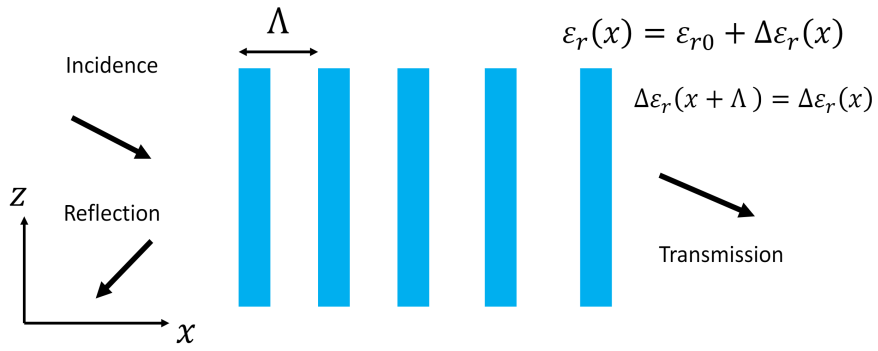
$$R = \left|\frac{B_i}{A_i}\right|^2 = \left|\frac{1}{0.996974}\right|^2 = 0.993958591679348$$

The transmission co-efficient is,

$$T = \left|\frac{\mu_i k_{tx}}{\mu_t k_{ix}}\right|^2 \left|\frac{A_t}{A_i}\right|^2 = \left|\frac{0.0752191}{1}\right|^2 = 0.00565$$

Part 2 : Coupled Wave Theory

In this section, the electric field amplitude for the forward and backward propagating waves throughout the N layer structure is calculated using the coupled wave approach. In addition, the reflection and transmission coefficient for optical power are determined. To apply the coupled wave theory, we have to determine how the relative permitivity varies within and outside the index perturbation



In coupled wave approach, we have two possible solutions depending on phase match conditions. First, the phase mismatch has to determined.

$$\Delta k = 2k_x - \frac{2\pi}{\Lambda}$$

For the problem at hand, the parameters are ,

$k_x = \frac{2\pi}{\lambda}n_o = \frac{2\pi}{10^{-6}} \times 1.5 = 9.4247 \times 10^6 m^{-1}$

$\Lambda = L_1 + L_2 = \frac{\lambda}{4n_1} + \frac{\lambda}{4n_2} = \frac{\lambda}{4}\left[\frac{n_1+n_2}{n_1 n_2}\right] = 0.32795\mu m$

$$\implies \Delta k = 2(9.4247 \times 10^6) - \frac{2\pi}{0.32795 \times 10^-6} = -309009.1m^{-1}$$

Since we have a phase mismatch($\Delta k \neq 0$), the solutions for the amplitude of forward and backward propagating waves in the layered structure are given by,

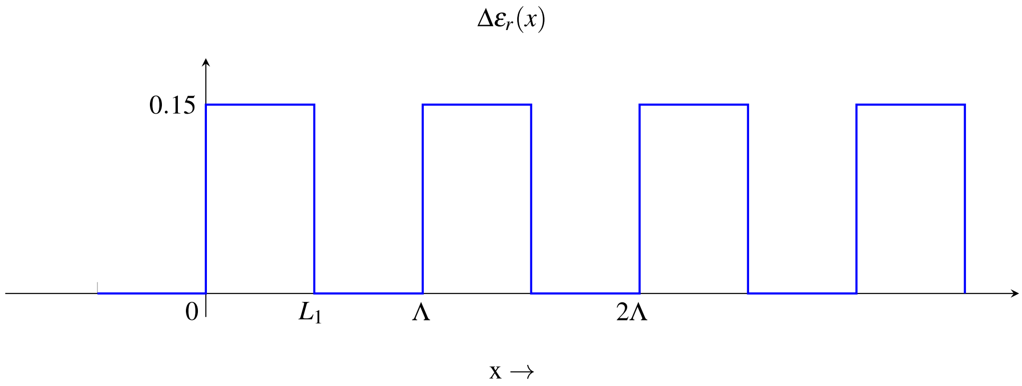
$$A(x) = exp(\frac{j\Delta kx}{2})\frac{scosh[s(L-x)] + j\frac{\Delta k}{2}sinh[s(L-x)]}{scosh[sL] + j\frac{\Delta k}{2}sinh[sL]}A(0)$$

$$B(x) = exp(\frac{-j\Delta kx}{2})\frac{-j\kappa^* sinh[s(L-x)]}{scosh[sL] + j\frac{\Delta k}{2}sinh[sL]}A(0)$$

with

$$s^2 = |\kappa|^2 - \left(\frac{\Delta k}{2}\right)^2$$

It is given that outside the index perturbation, the refractive index is $n_o = 1.5$, hence $\varepsilon_{ro} = 2.25$. In dielectric layers with refractive index n=1.55 within the index perturbation, $\varepsilon_r = 2.4025$. In dielectric layers with refractive index n=1.5 within the index perturbation, $\varepsilon_r = 2.25$. Thus, in the equation $\varepsilon_r(x) = \varepsilon_{ro} + \Delta\varepsilon_r(x)$. Thus the variation of index, $\Delta\varepsilon_r(x)$, is as shown below,



It can be observed that the $\Delta\varepsilon_r(x)$ is a square wave, with a constant value of 0.1525 from x = 0 to x = L_1 and 0 from x = L_1 to $x = \Lambda$. Thus, the value of $\Delta\varepsilon_r(x)$ within the dielectric layers of the N layer structure is given by

$$\Delta\varepsilon_r(x) = \begin{cases} 0.1525, & \text{if } n = 1.55 \\ 0, & \text{if } n = 1.5 \end{cases}$$

The coupling constant κ is given by,

$$\kappa = \frac{\omega^2}{2k_x c^2}\widetilde{\Delta\varepsilon}_{r,-1}$$

and

$$\widetilde{\Delta\varepsilon}_{r,m} = \frac{1}{\Lambda}\int_{-\frac{\Lambda}{2}}^{\frac{\Lambda}{2}} \Delta\varepsilon_r(x)e^{-jm\frac{2\pi x}{\Lambda}}dx = \frac{1}{\Lambda}\int_0^{\Lambda} \Delta\varepsilon_r(x)exp^{-jm\frac{2\pi x}{\Lambda}}dx$$

For m=-1,

$$\widetilde{\Delta\varepsilon}_{r,-1} = \frac{1}{\Lambda}\left[\int_0^{L_1} \Delta\varepsilon_r(x)e^{+j\frac{2\pi x}{\Lambda}}dx + \int_{L_1}^{\Lambda} \Delta\varepsilon_r(x)e^{+j\frac{2\pi x}{\Lambda}}dx\right]$$

$$= \frac{1}{\Lambda}\left[\int_0^{L_1} \Delta\varepsilon_r(x)e^{+j\frac{2\pi x}{\Lambda}}dx\right]$$

$$= \frac{0.1525}{\Lambda}\left[\int_0^{L_1} e^{+j\frac{2\pi x}{\Lambda}}dx\right]$$

$$= \frac{0.1525}{j2\pi}\left[e^{+j\frac{2\pi}{\Lambda}L_1} - 1\right]$$

$$= \text{ 0.001249 + 0.04851i}$$

With the value for $\widetilde{\Delta\varepsilon}_{r,-1}$ determined, coupling constant κ can be calculated.

$$\kappa = \frac{\omega^2}{2k_x c^2}\widetilde{\Delta\varepsilon}_{r,-1} = \frac{(\frac{2\pi}{\lambda})^2}{2(\frac{2\pi}{\lambda}n_o)}\widetilde{\Delta\varepsilon}_{r,-1} = \frac{2\pi}{2n_o\lambda}\widetilde{\Delta\varepsilon}_{r,-1}$$

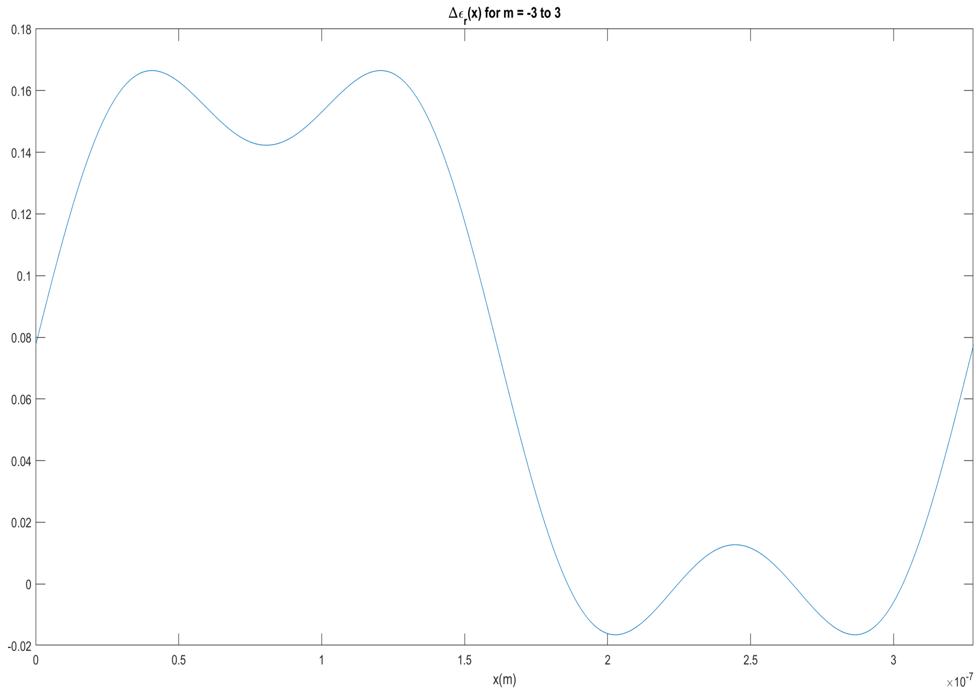
Plugging in the values for n_o, λ and $\widetilde{\Delta\varepsilon}_{r,-1}$, the value of κ is determined as,

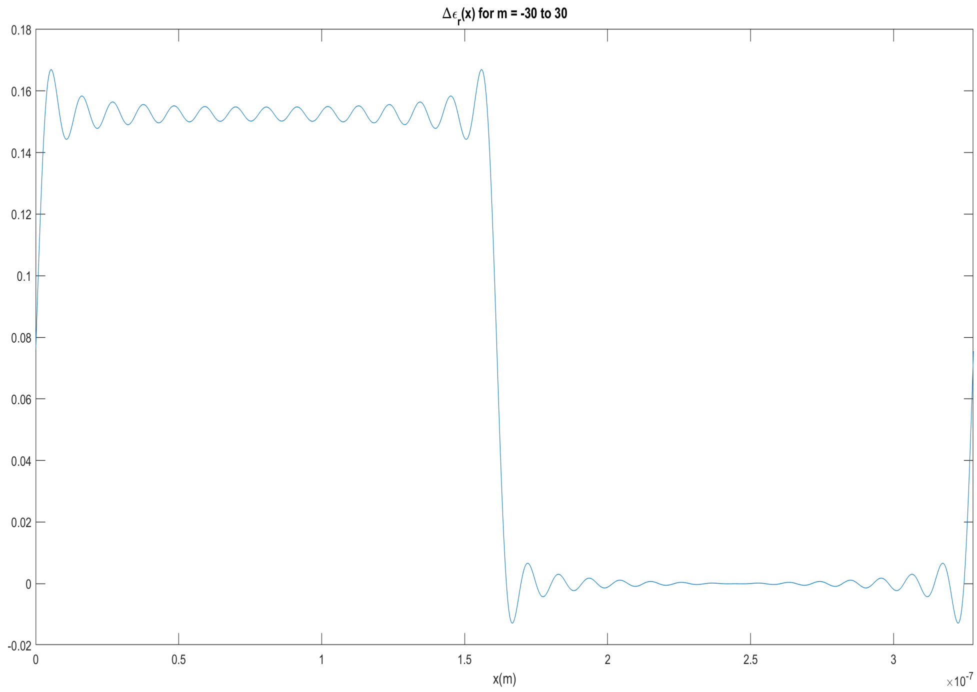
$$\kappa = 2616.836 + 101599.26i$$

The accuracy of the value of the co-efficient can be analyzed by finding the coefficients of the Fourier series given below and constructing the square wave and comparing it with the expected square wave for the value of $\Delta\varepsilon_r(x)$

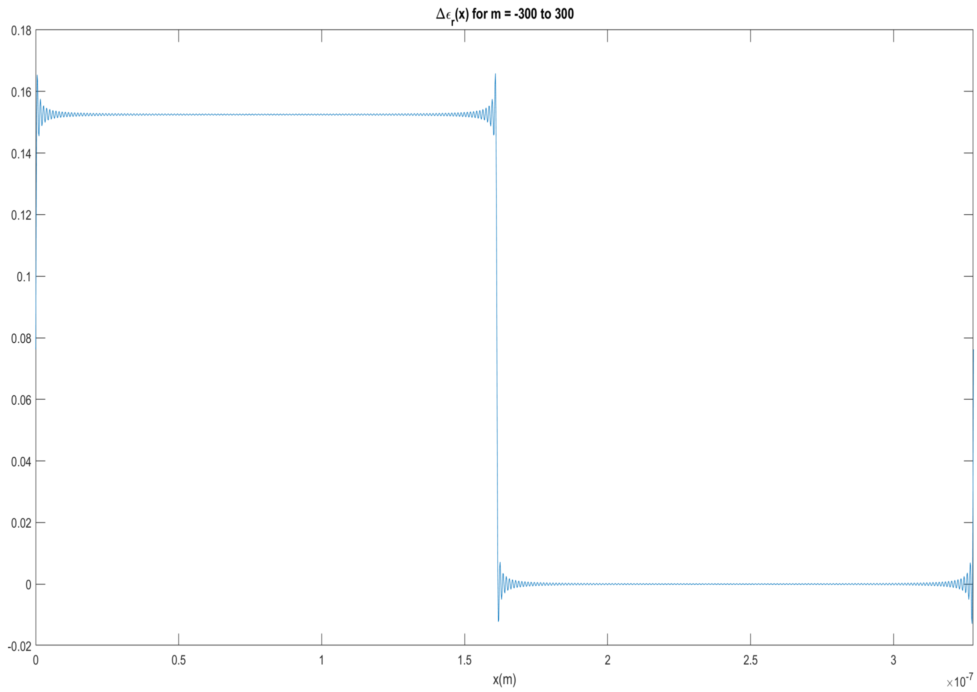
$$\Delta\varepsilon_r(x) = \sum_{m=-\infty}^{+\infty}\widetilde{\Delta\varepsilon}_{r,m}e^{jm\frac{2\pi x}{\Lambda}}$$

$\widetilde{\Delta\varepsilon}_{r,m}$ for different values of 'm' is calculated and plugged into the Fourier series to construct the square wave for $\Delta\varepsilon_r(x)$ in the range from x=0 to x=Λ. The square output is shown for different values of 'm'

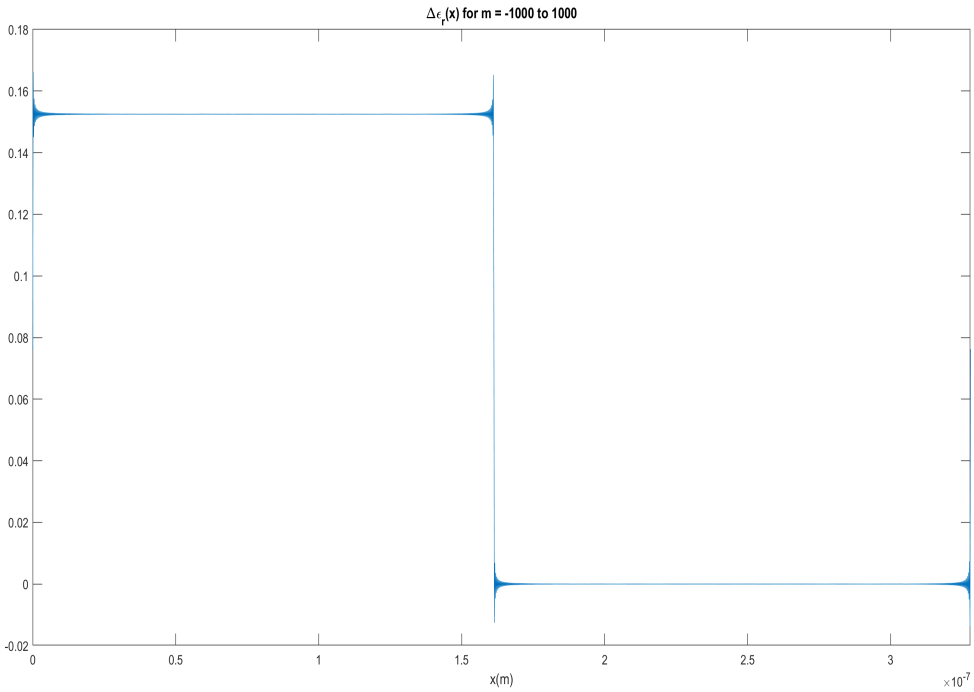


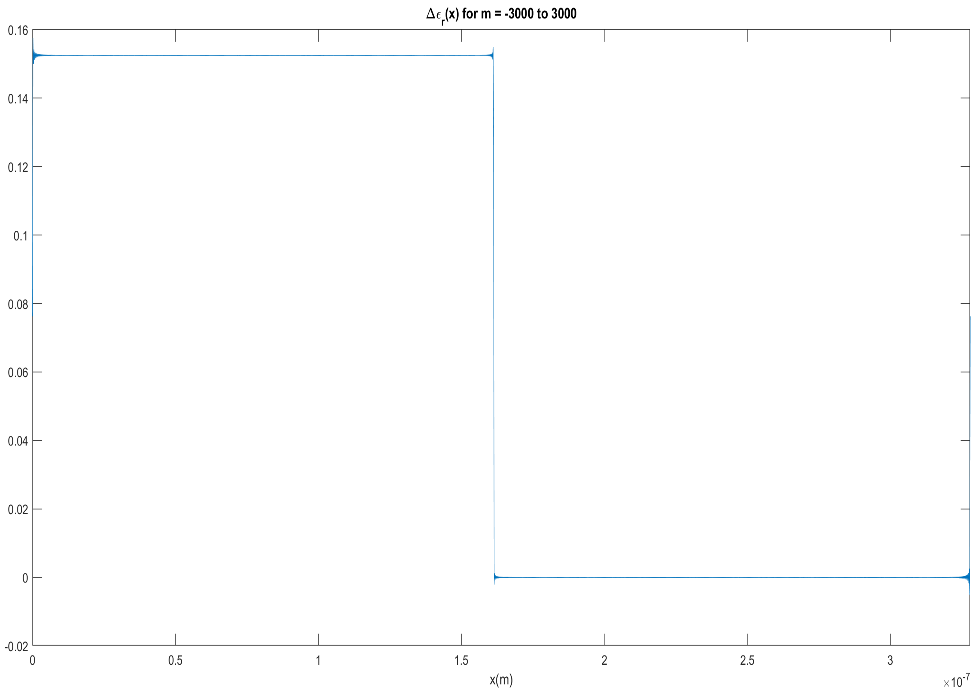


$\Delta\epsilon_r(x)$ for m = -30 to 30
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
0
0.5
1
1.5
2
2.5
3
x(m)
$\times 10^{-7}$



$\Delta\epsilon_r(x)$ for m = -300 to 300
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
0
0.5
1
1.5
2
2.5
3
x(m)
$\times 10^{-7}$





It can be observed that as we increase the number of coefficients, the square wave becomes more and more accurate. The square is exactly as expected and this validates the correctness of the Fourier co-efficient $\Delta\varepsilon_{r,-1}$ used for the calculation of κ. Thus, with the value calculated for κ and Δk , the value of 's' can be determined,

$$s^2 = |\kappa|^2 - \left(\frac{\Delta k}{2}\right)^2$$

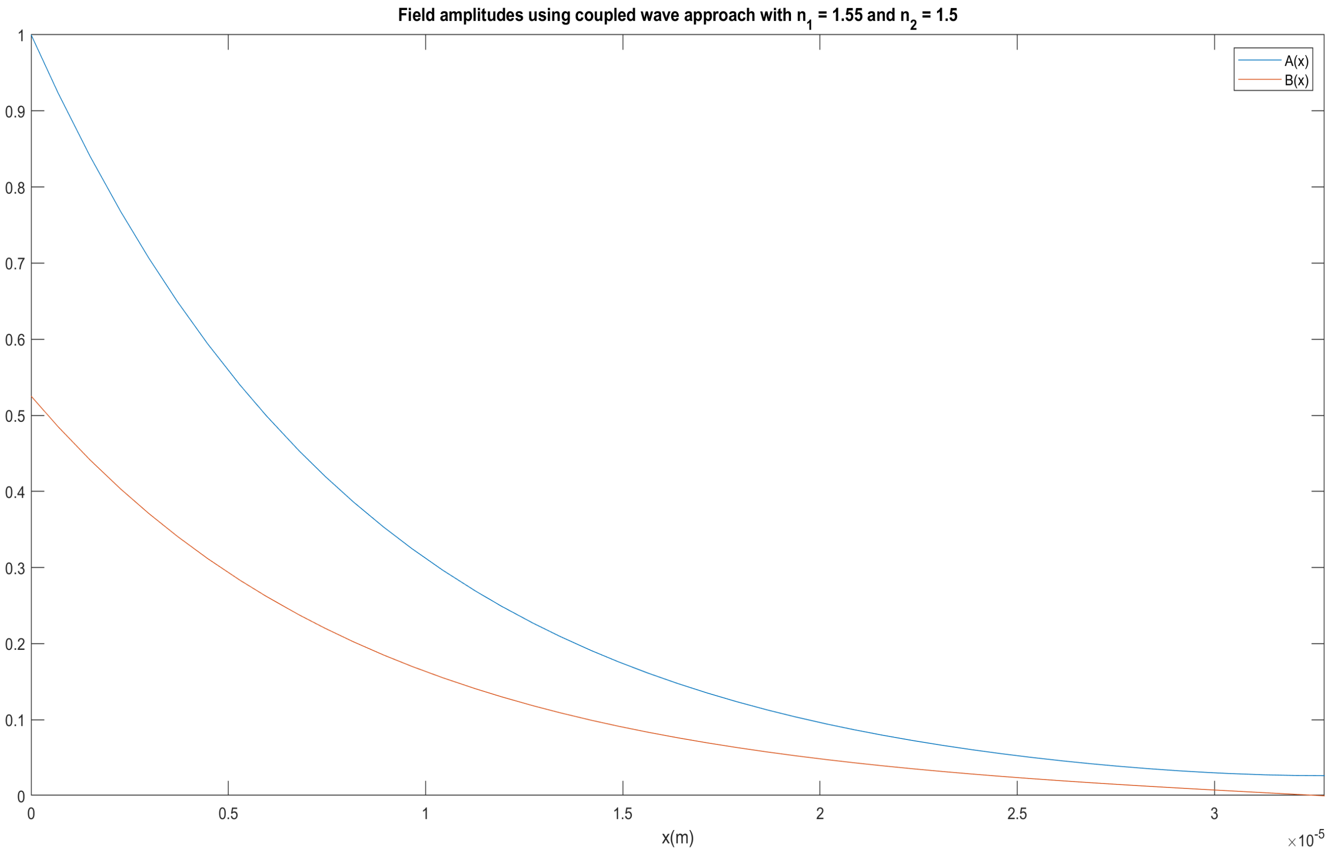
$$\implies |s| = 1.1637 \times 10^5$$

With the values for s , κ and Δk, the field amplitudes of the forward and backward propagating waves can be determined

$$A(x) = exp(\frac{j\Delta kx}{2})\frac{scosh[s(L-x)] + j\frac{\Delta k}{2}sinh[s(L-x)]}{scosh[s(L)] + j\frac{\Delta k}{2}sinh[s(L)]}A(0)$$

$$B(x) = exp(\frac{-j\Delta kx}{2})\frac{-j\kappa^* sinh[s(L-x)]}{scosh[s(L)] + j\frac{\Delta k}{2}sinh[s(L)]}A(0)$$

Using the above equations, the determined field amplitude variation in the N dielectric layer structure is ,



It can observed that the field amplitudes decrease exponentially inside the structure. The amplitude of the forward propagating wave has a decreasing amplitude but does not drop to zero whereas the amplitude of the backward propagating wave drops to zero at the transmission layer as expected. The reflection and transmission co-efficient are,

The reflection co-efficient is,

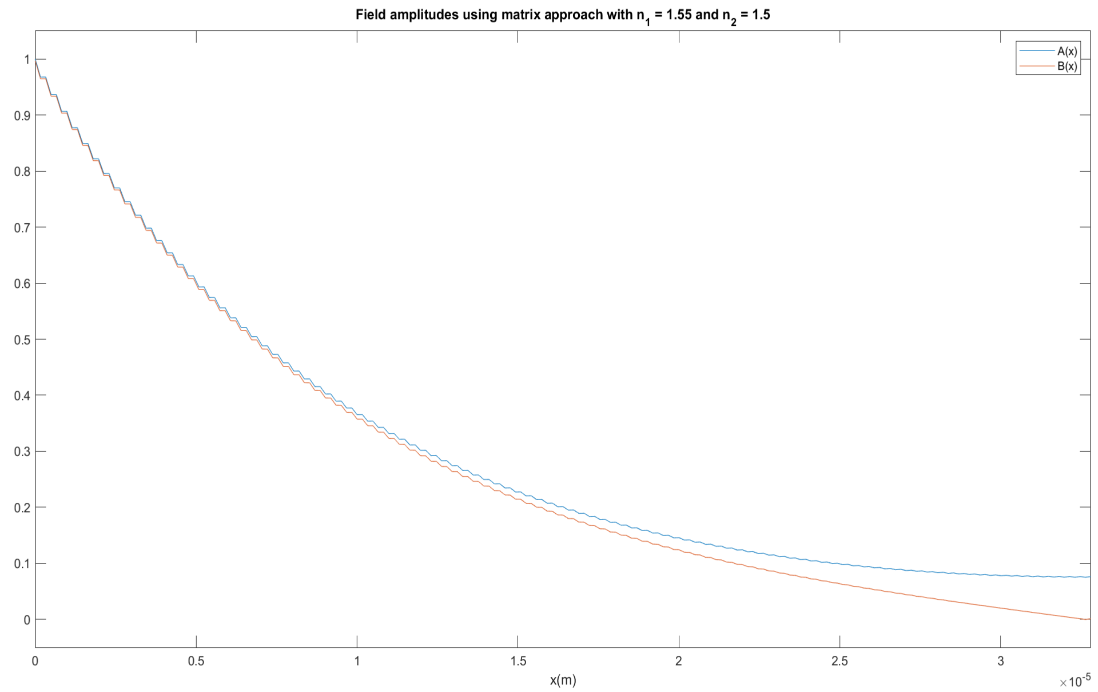
$$R = \left|\frac{B(0)}{A(0)}\right|^2 = \left|\frac{sinh(sL)}{scosh(sL) + j\frac{\Delta k}{2}sinh(sL)}\right|^2 = 0.27588$$

The transmission co-efficient is,

$$T = \left|\frac{A(L)}{A(0)}\right|^2 = \left|\frac{s}{scosh(sL) + j\frac{\Delta k}{2}sinh(sL)}\right|^2 = 0.0007$$

Part 3 : Comparison of Matrix approach and Coupled wave theory

In this section, a comparison between the matrix approach and coupled wave approach is carried out by determining the field amplitudes for the forward and backward propagating wave for different values of refractive index 'n_1'. The comparison is started with n_1 = 1.55 and the analysis is carried out to see how the two methods compare with each other as the value of n_1 is decreased



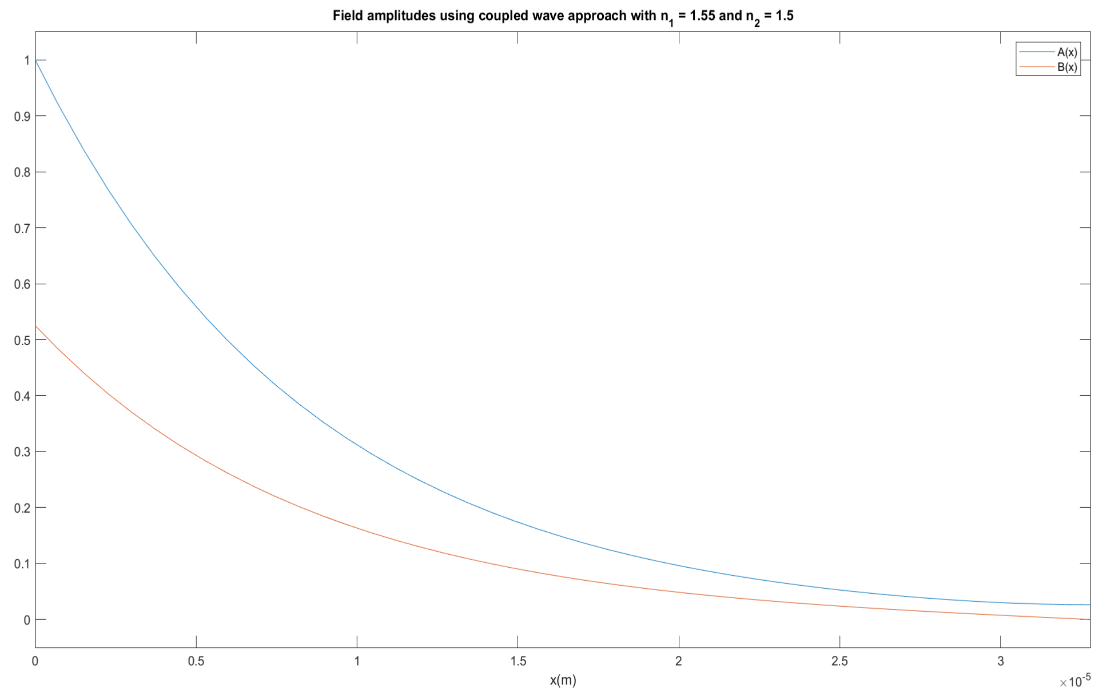
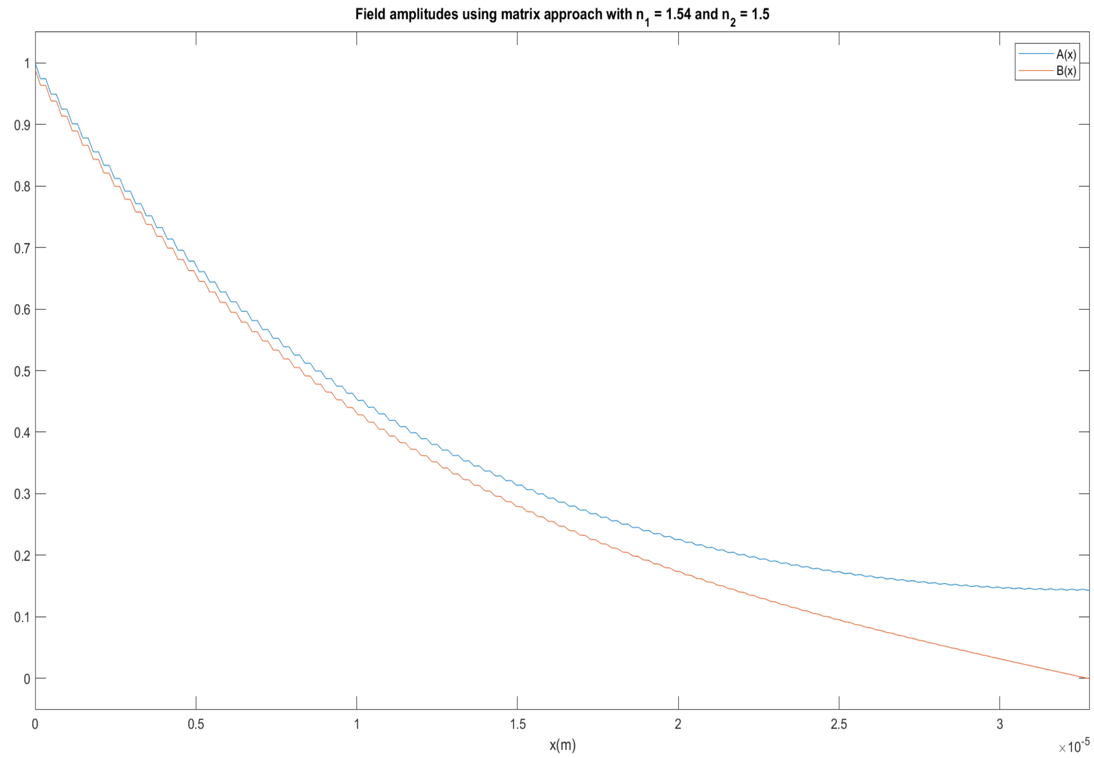


Figure 4: Comparison with $n_1 = 1.55$



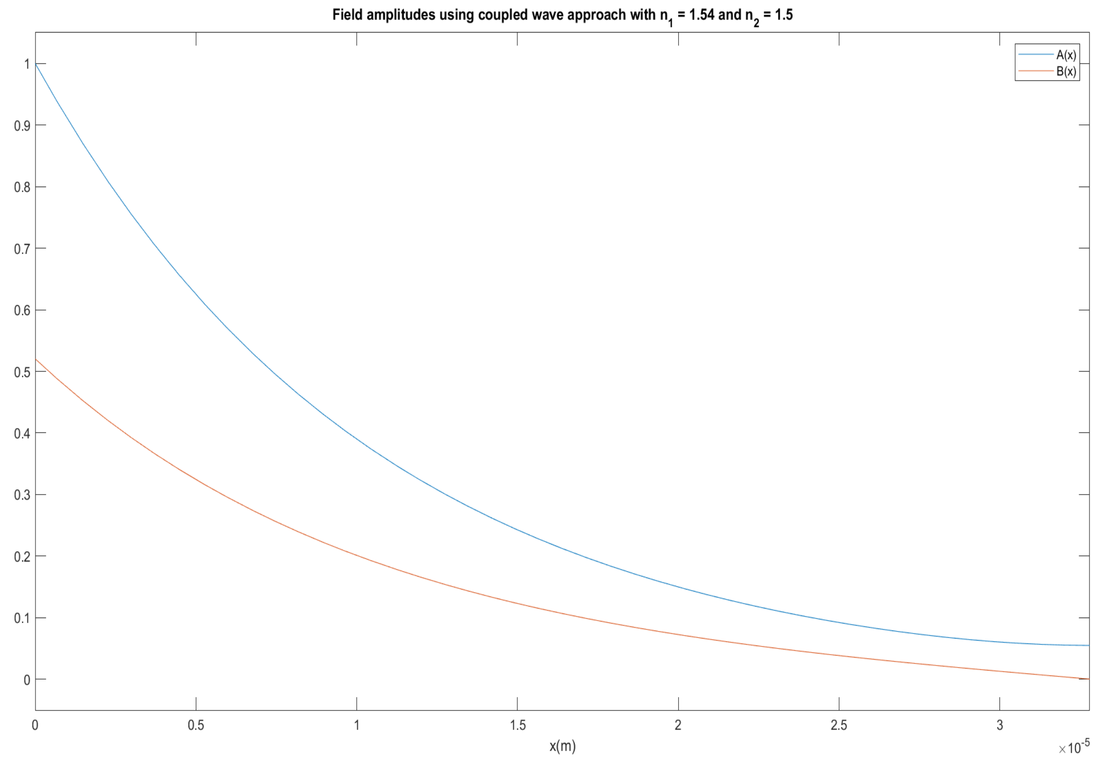


Figure 5: Comparison with $n_1 = 1.54$

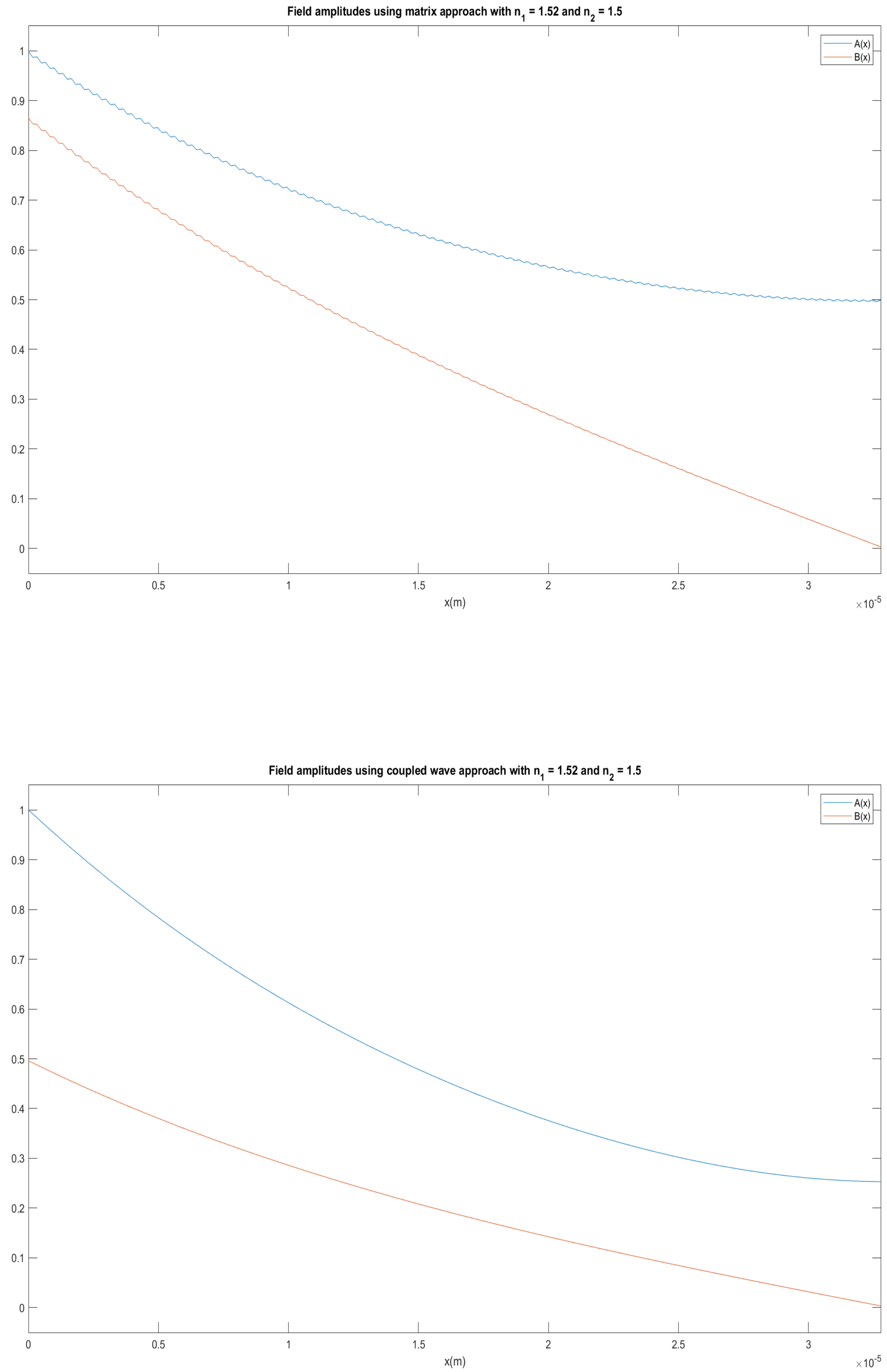
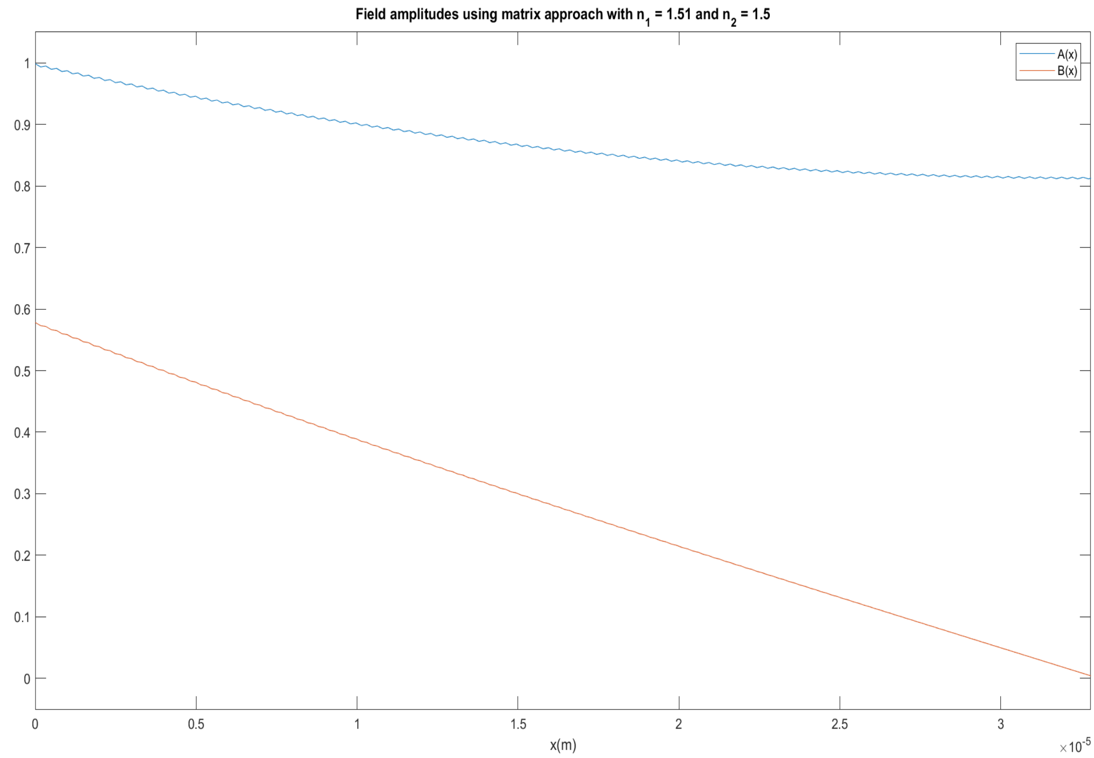


Figure 6: Comparison with $n_1 = 1.52$



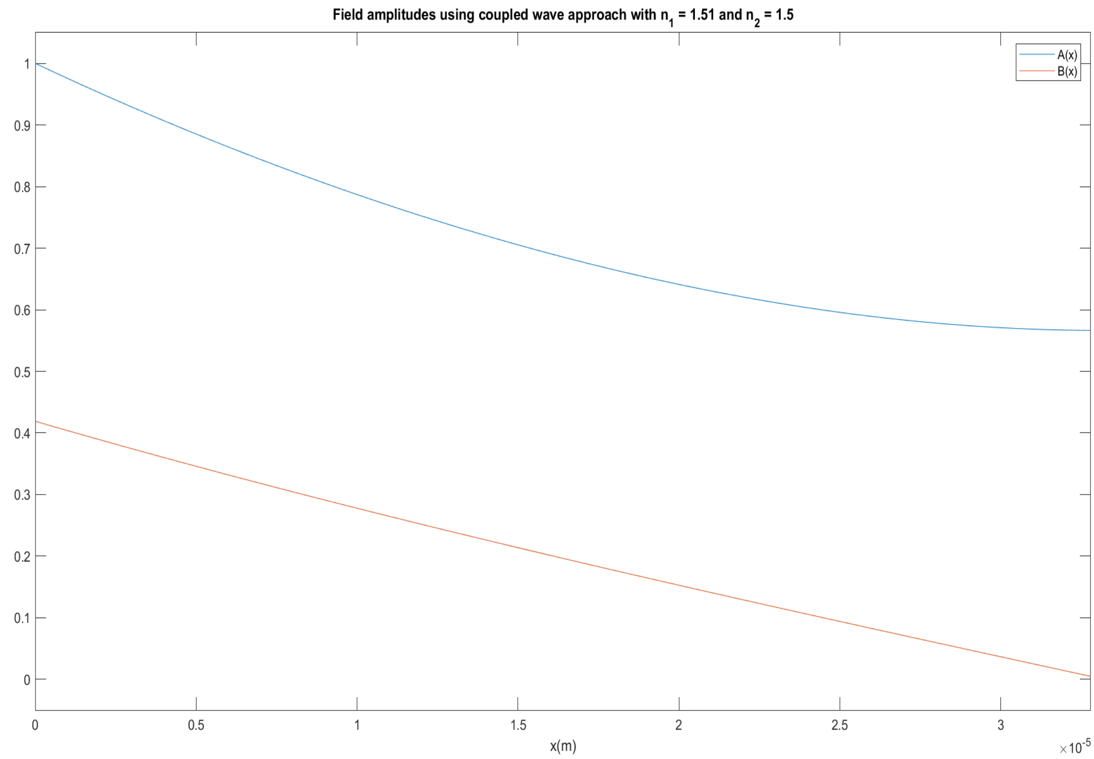
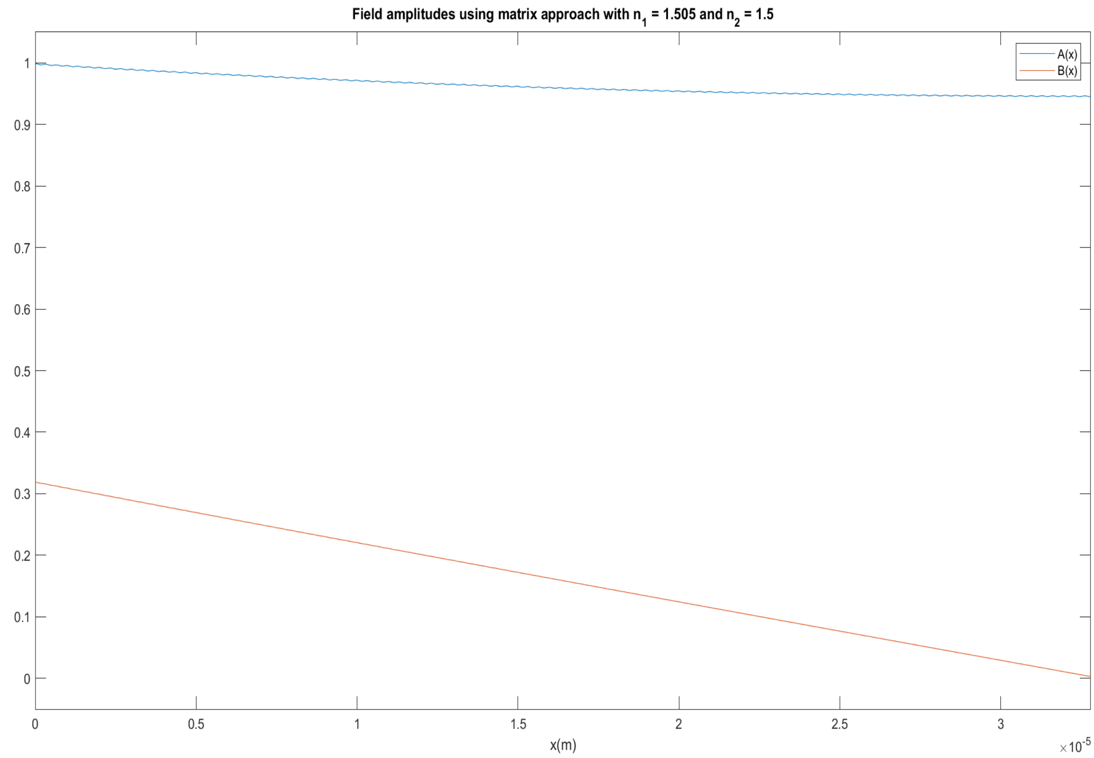


Figure 7: Comparison with $n_1 = 1.51$



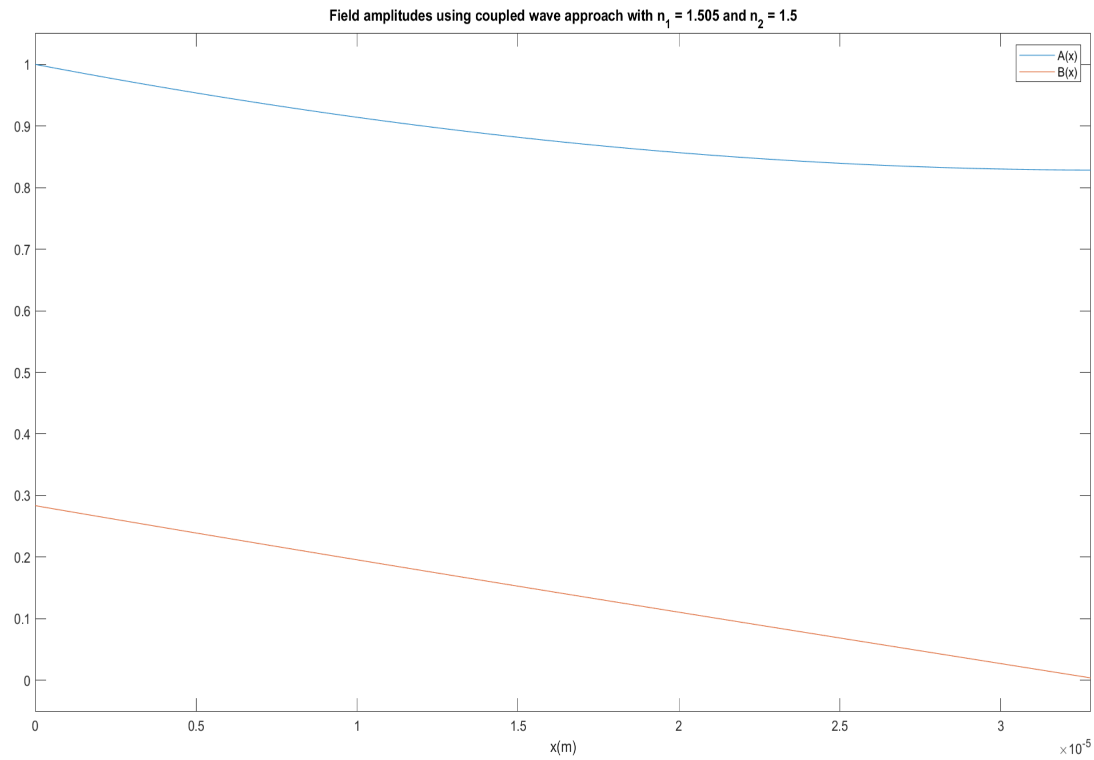
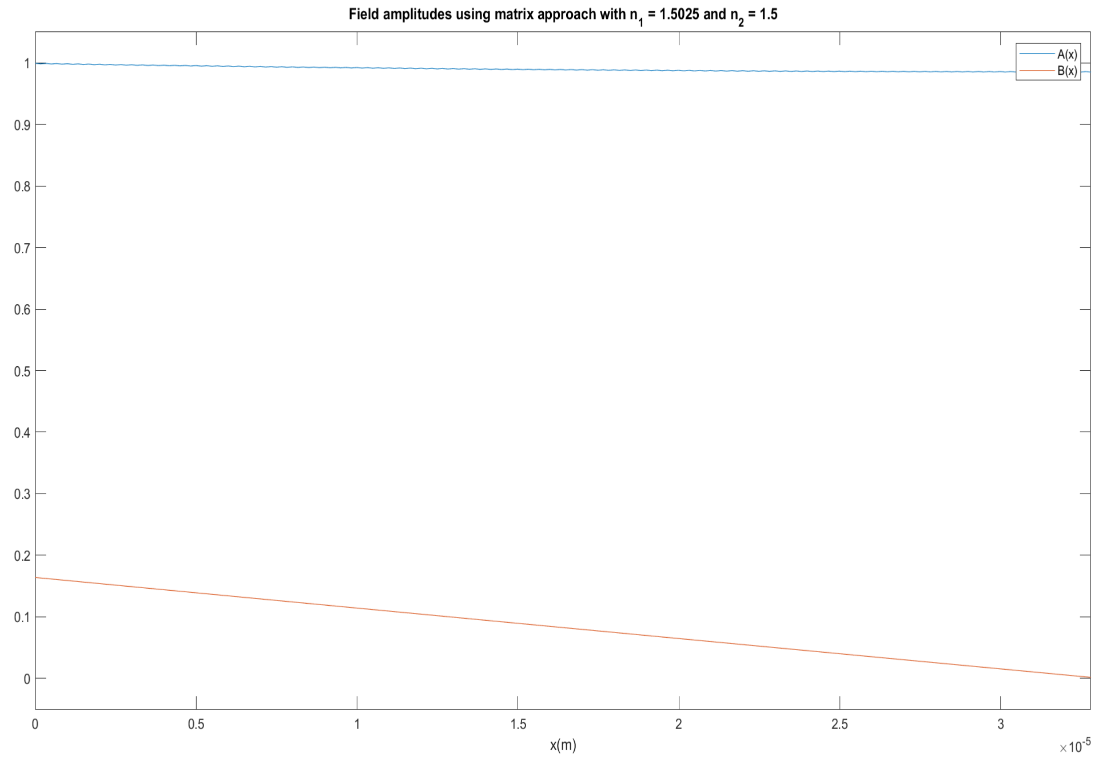


Figure 8: Comparison with $n_1 = 1.505$



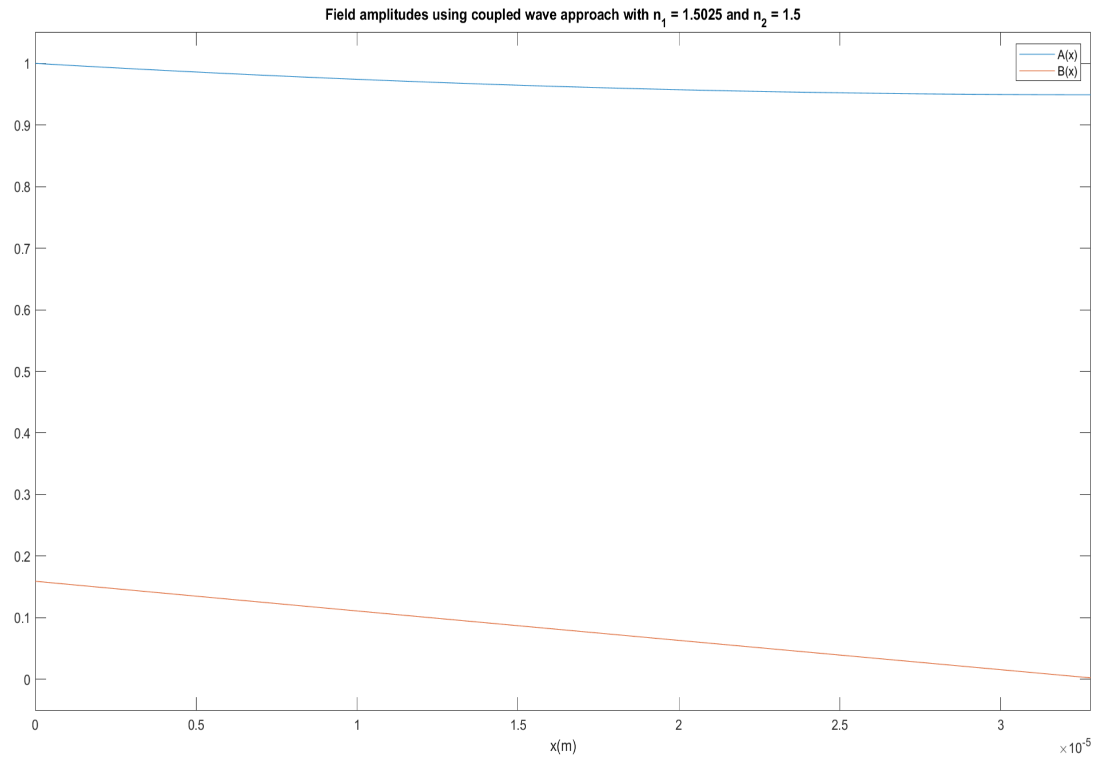
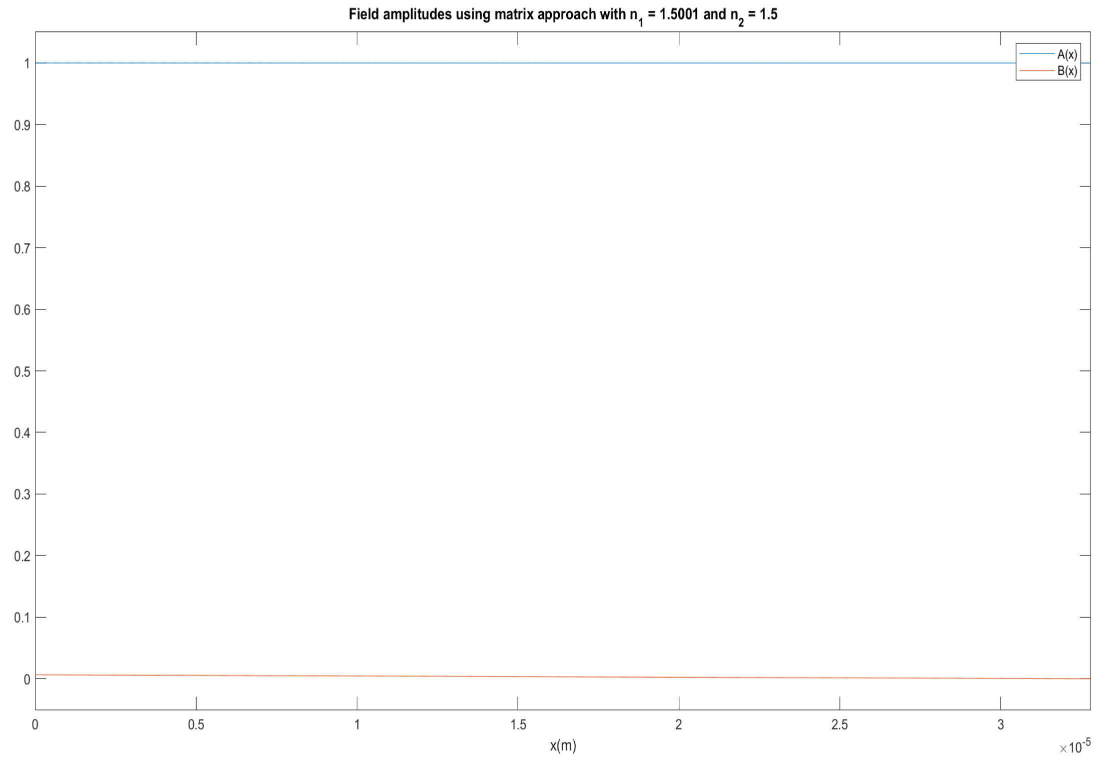


Figure 9: Comparison with $n_1 = 1.5025$



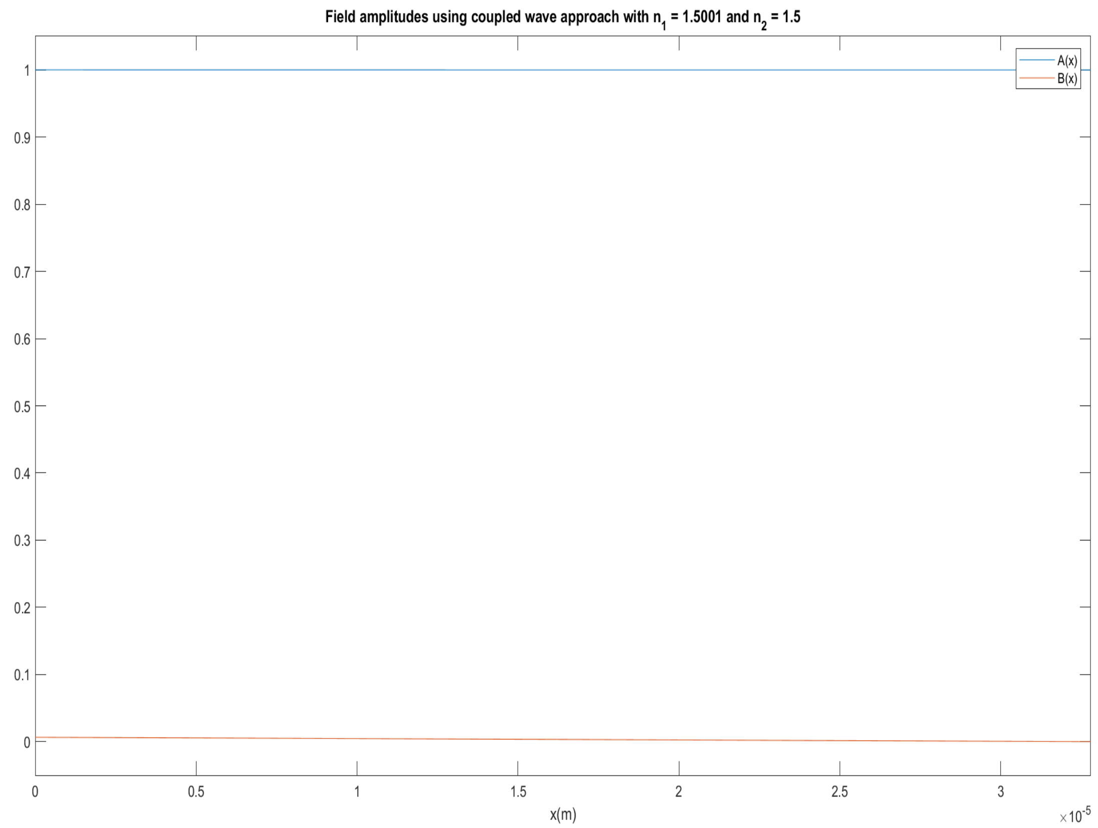
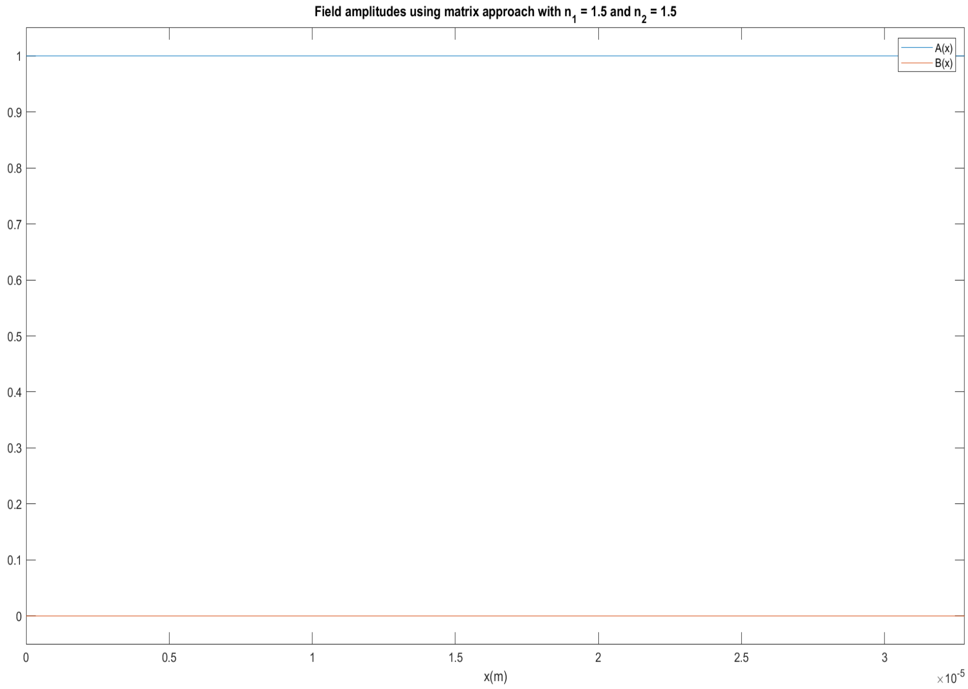


Figure 10: Comparison with $n_1 = 1.5001$

From the above images, we can observe that as the value of n_1 is decreased, the matrix approach and coupled wave approach behave similar to each other. Let us analyze what happens when the values for n_1 and n_2 are exactly equal to each other .i.e $n_1 = n_2 = 1.5$



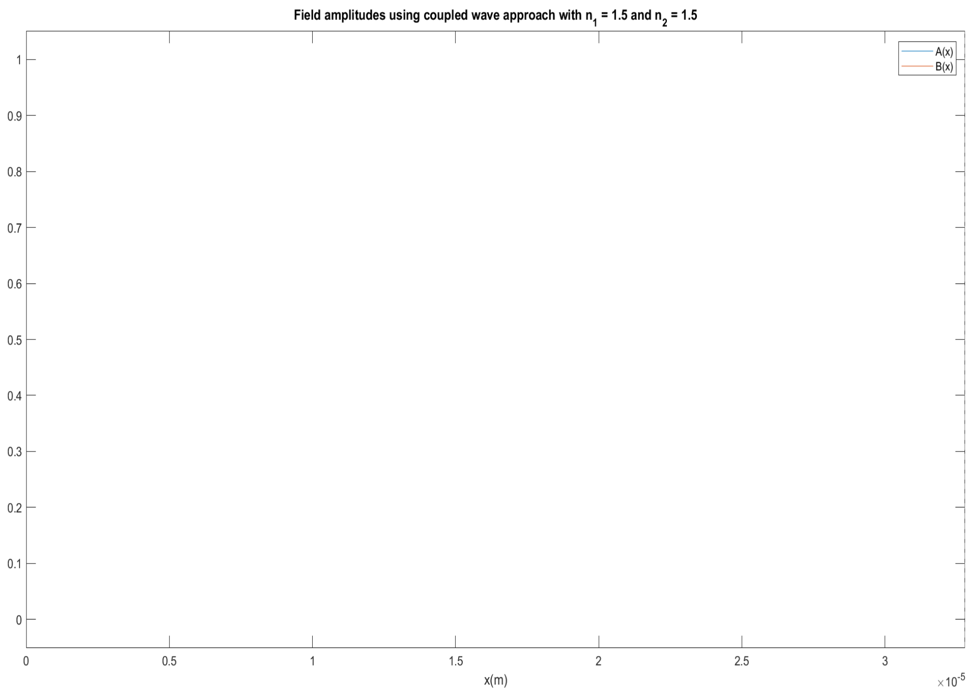


Figure 11: Comparison with $n_1 = n_2 = 1.5$

By observing the above figures, it is understood that the matrix approach gives the expected output. When the values of n_1 and n_2 are exactly equal to each other, there should be no reflection and the transmission co-efficient should be 1 which is exactly obtained in the above image. But when the values of n_1 and n_2 are exactly equal to each other, the coupled wave approach fails. This is because when $n_1 = n_2$, phase matching occurs

$$\Delta k = 2k_x - \frac{2\pi}{\Lambda} = 2 \times \frac{2\pi n_1}{\lambda} - \frac{2\pi}{\frac{\lambda}{2n_1}} = 2 \times \frac{2\pi n_1}{\lambda} - 2 \times \frac{2\pi n_1}{\lambda} = 0$$

Also, when $n_1 = n_2$ and no other parameters are changed, the variation of $\Delta\varepsilon_r$ is no long a square wave and will become a constant zero wave. Since, $\Delta\varepsilon_r = 0$ at all x values,

$$\widetilde{\Delta\varepsilon}_{r,-1} = 0$$

$$\implies \kappa = 0$$

Since, both $\kappa = 0$ and $\Delta k = 0$,

$$s^2 = |\kappa|^2 - \big(\frac{\Delta k}{2}\big)^2 = 0$$

Since, s=0 , $\Delta k = 0$ and $\kappa = 0$, these values when plugged into the equation for A(x) and B(x) , makes the equation unstable and this is why the coupled wave approach fails when the values of n_1 and n_2 are exactly equal to each other

$$A(x) = exp\big(\frac{j\Delta kx}{2}\big)\frac{scosh[s(L-x)] + j\frac{\Delta k}{2}sinh[s(L-x)]}{scosh[s(L)] + j\frac{\Delta k}{2}sinh[s(L)]}A(0) = \frac{0}{0}$$

$$B(x) = exp\big(\frac{-j\Delta kx}{2}\big)\frac{-j\kappa^* sinh[s(L-x)]}{scosh[s(L)] + j\frac{\Delta k}{2}sinh[s(L)]}A(0) = \frac{0}{0}$$

Conclusion

In this report, the field amplitudes of the forward and backward propagating waves in a structure with 200 dielectric layers were numerically solved using matrix approach and coupled wave approach. The corresponding reflection and transmission co-efficients for each method was determined. It was shown the amplitudes decrease exponentially inside the structure for both cases. It was numerically shown that as the refractive index difference is decreased, the two methods behave similar to each other. Finally, it was proven both analytically and numerically that the coupled wave theory fails when the refractive indices match exactly.