Recovering a complex object using near-field HIO algorithm and structured illumination

Phase Imaging

- An X-ray imaging technique that utilizes sinusoidal gratings and modified HIO algorithm is presented.
- The algorithm can recover the amplitude and the phase profile of a complex object using only two diffraction patterns
- No mechanical movement of the imaging system is required
- Numerical simulations using test images are presented to demonstrate the algorithm.

- Consider an input object

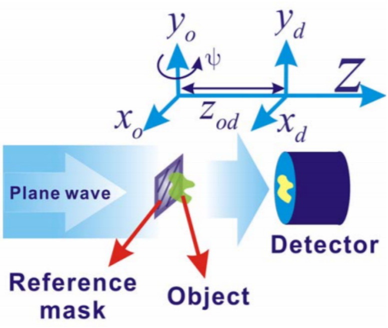
$$f(x, y) = T(x, y)e^{j\phi(x,y)}$$

- Objective is to estimate $T(x, y)$ and $\phi(x, y)$

- Traditional iterative algorithms use two images at the focal and defocal planes to recover the object profile.

- To get the focused and defocused images , focusing lenses are required which limits the resolution of the image. Also,a mechanical system is required to defocus the image.

- The presented algorithm uses lens free X-ray imaging . It can also be applied to use visible light lasers.
- Two sinusoidal gratings are placed with the object to get two different diffraction patterns.



Experimental Setup

Sinusoidal phase gratings

The sinusoidal phase grating induces a phase change in the incident light and is represented by ,

$$P(x, y) = exp\left[i\frac{m}{2}sin\{2\pi f_m(xcos\theta + ysin\theta)\}\right]$$

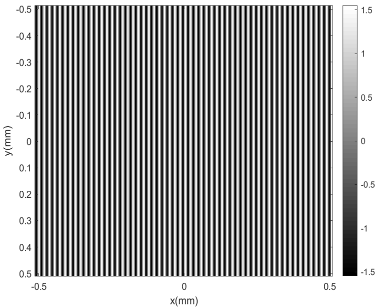
Where,

f_m = spatial frequency

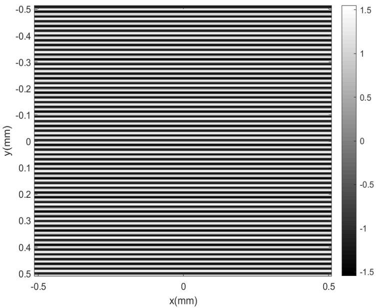
θ = angle of orientation

m= $\frac{\pi}{2}$ radians

Sinusoidal phase gratings



Sinusoidal phase grating, A



Sinusoidal phase grating, B

Oversampling criteria

- Let the intensity and the object domain have N pixels in each direction. This makes the total number of pixels in each domain N^2.
- For a complex valued image with M^2 pixels, the total number of unknown values are $2M^2$.

$$N^2 > 2M^2$$

$$\implies \frac{N}{M} > 2^{\frac{1}{2}}$$

- Hence, the image is padded with zeros to satisfy the oversampling criteria. The simulations are carried out with N=1024 and M=512. To provide better visibility, only the central 512 by 512 pixels are displayed

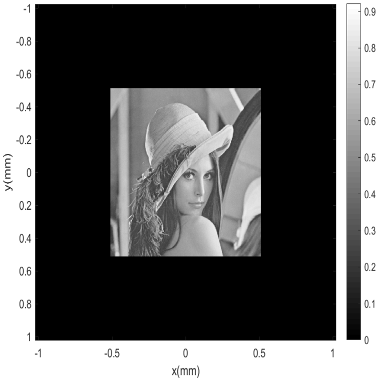
Simulation parameters

λ	0.10125 nm
z	0.8m
Pixel size, d_{pix}	2μm
Noise variance, σ^2	10^{-4}
f_m	55 cycles per mm
θ	$0^\circ/90^\circ$

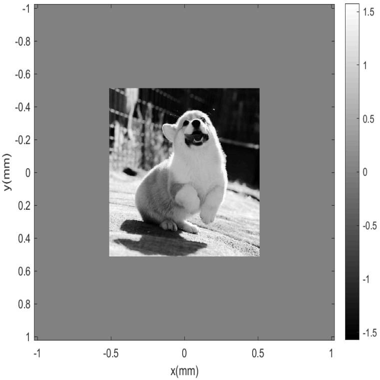
Input object

The complex object is,

$$U(x_1, y_1) = T(x_1, y_1) \times e^{j\phi(x_1, y_1)}$$

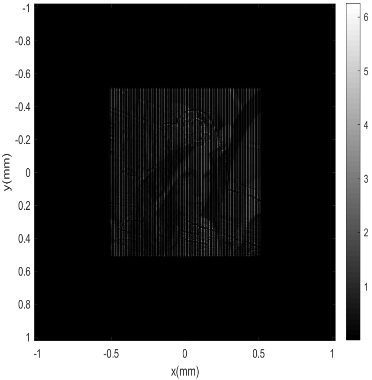


$$T(x_1, y_1) = |U(x_1, y_1)|$$

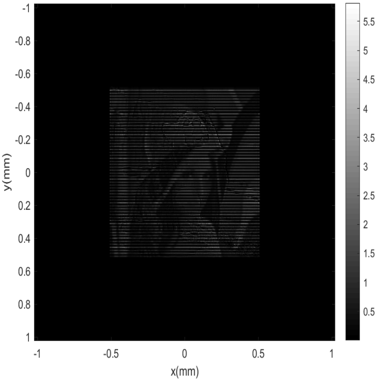


$$\phi(x_1, y_1) = arg\{U(x_1, y_1)\}$$

Step 1: Obtain the intensity patterns



Intensity pattern with mask A

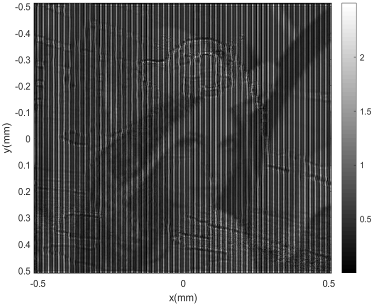


Intensity pattern with mask B

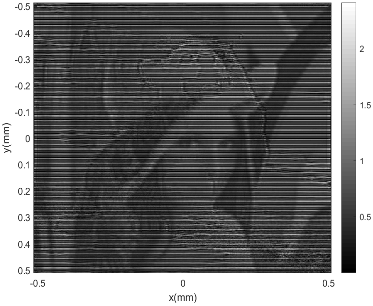
Step 2: Obtain the magnitude of the complex amplitude

The magnitude of the complex field in the diffraction planes are obtained using the relation,

$$|U(x_2, y_2)| = \sqrt{I(x_2, y_2)}$$



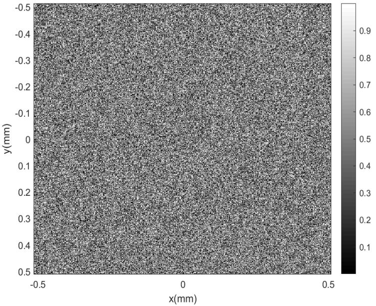
$|U_A(x_2, y_2)|$



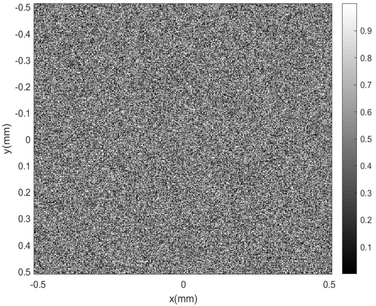
$|U_B(x_2, y_2)|$

Step 3: Initialize the random transmission and phase profile

The input for the iteration program is a N by N matrix of random numbers for both phase and transmission profile of the object



$T_{est}(x_1, y_1)$

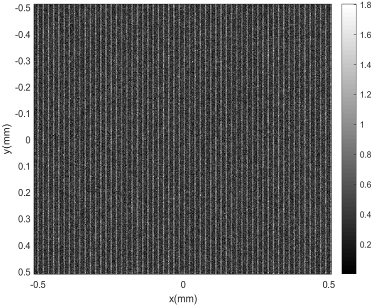


$\phi_{est}(x_1, y_1)$

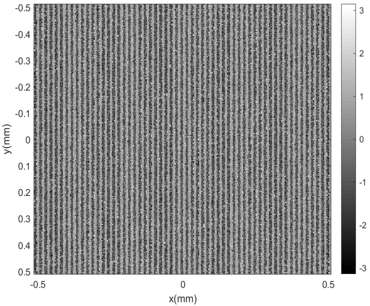
Step 4: Propagate the initial estimate of the object with grating A to diffraction plane

$$U_{A_{est}}(x_2, y_2) = FRT\left\{ T_{est}(x_1, y_1) e^{j\phi_{est}(x_1, y_1)} e^{jA} \right\}$$

FRT stands for forward Fresnel transform using angular spectrum approach.



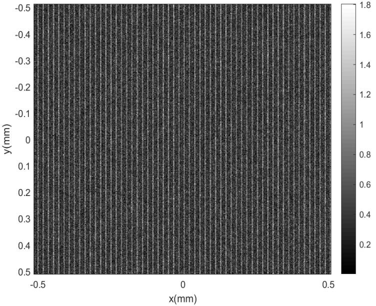
$|U_{A_{est}}(x_2, y_2)|$



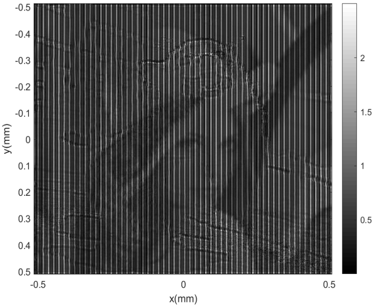
$\phi_{A_{est}}(x_2, y_2)$

Step 5: Replace the magnitude of the complex field $U_{A_{est}}(x_2, y_2)$

The magnitude $|U_{A_{est}}(x_2, y_2)|$ is replaced by the measured magnitude $|U_A(x_2, y_2)|$. The phase $\phi_{A_{est}}(x_2, y_2)$ is retained.



$|U_{A_{est}}(x_2, y_2)|$



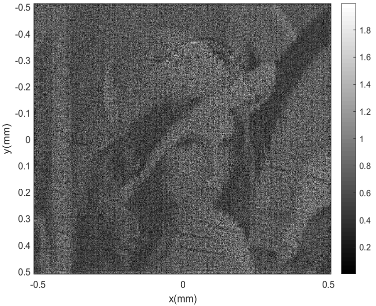
$|U_A(x_2, y_2)|$

$$U_{A_{est}}(x_2, y_2) = |U_A(x_2, y_2)| e^{j\phi_{A_{est}}(x_2, y_2)}$$

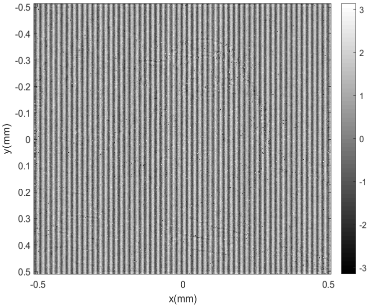
Step 5: Back propagate $U_{A_{est}}(x_2, y_2)$

$$U_{A_{est}}(x_1, y_1) = |U_{A_{est}}(x_1, y_1)| e^{j\phi_{A_{est}}(x_1, y_1)} = IFRT\{U_{A_{est}}(x_2, y_2)\}$$

IFRT stands for Inverse Fresnel transform



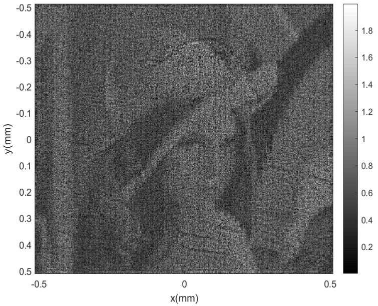
$|U_{A_{est}}(x_1, y_1)|$



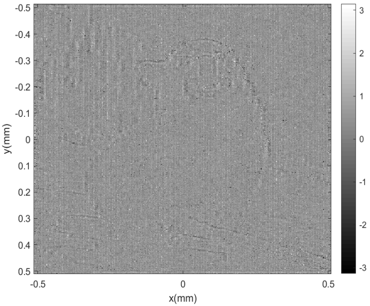
$\phi_{A_{est}}(x_1, y_1)$

Step 6: Divide $U_{A_{est}}(x_1, y_1)$ by mask A

$$U_{est}(x_1, y_1) = |U_{est}(x_1, y_1)| e^{j\phi_{est}(x_1, y_1)} = \frac{U_{A_{est}}(x_1, y_1)}{e^{jA}}$$



$|U_{est}(x_1, y_1)|$

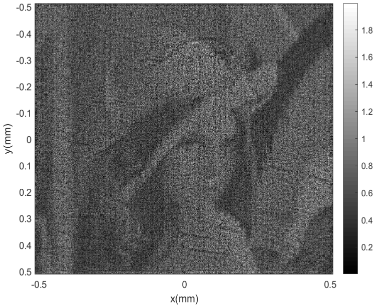


$\phi_{est}(x_1, y_1)$

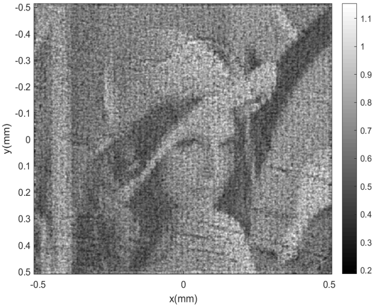
Step 7: Apply Gaussian smoothing and HIO algorithm on $|U_{est}(x_1, y_1)|$

$$|U_{est}^{'}(x_1, y_1)| = \begin{cases} |U_{est}(x_1, y_1)| & \text{if } x_1, y_1 \notin \gamma \\ |U_{est}(x_1, y_1)| - \beta \times |T_{est}(x_1, y_1)| & \text{if } x_1, y_1 \in \gamma \end{cases}$$

Where γ is the pixels with a value 0 in $|U_{est}(x_1, y_1)|$; $\beta = 0.9$



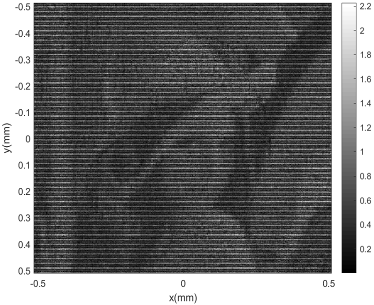
$|U_{est}(x_1, y_1)|$



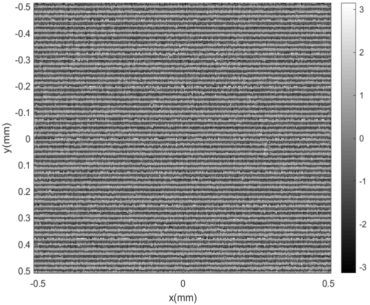
$|U_{est}^{'}(x_1, y_1)|$

Step 8: Propagate the estimate of the object with grating B to diffraction plane

$$U_{B_{est}}(x_2, y_2) = FRT\left\{|U'_{est}(x_1, y_1)|e^{j\phi_{est}(x_1,y_1)}e^{jB}\right\}$$



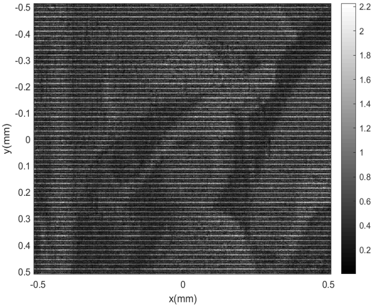
$|U_{B_{est}}(x_2, y_2)|$



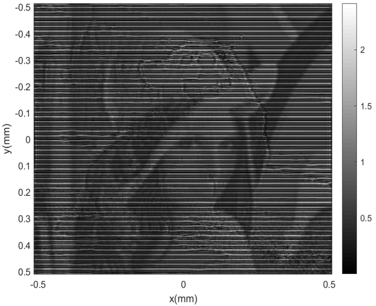
$\phi_{B_{est}}(x_2, y_2)$

Step 9: Replace the magnitude of the complex field $U_{B_{est}}(x_2, y_2)$

The magnitude $|U_{B_{est}}(x_2, y_2)|$ is replaced by the measured magnitude $|U_B(x_2, y_2)|$. The phase $\phi_{B_{est}}(x_2, y_2)$ is retained.



$|U_{B_{est}}(x_2, y_2)|$

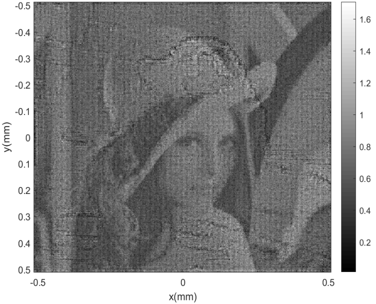


$|U_B(x_2, y_2)|$

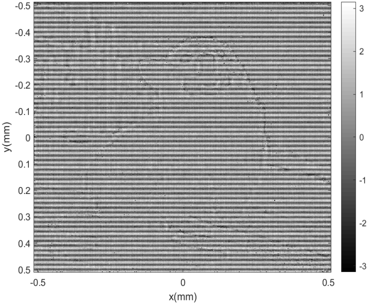
$$U_{B_{est}}(x_2, y_2) = |U_B(x_2, y_2)| e^{j\phi_{B_{est}}(x_2, y_2)}$$

Step 10: Back propagate $U_{B_{est}}(x_2, y_2)$

$$U_{B_{est}}(x_1, y_1) = |U_{B_{est}}(x_1, y_1)| e^{j\phi_{B_{est}}(x_1, y_1)} = IFRT\{U_{B_{est}}(x_2, y_2)\}$$



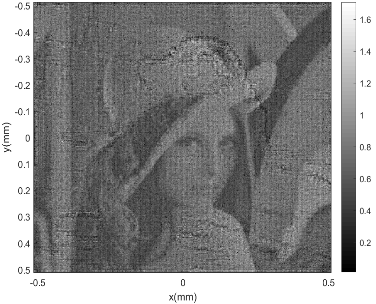
$|U_{B_{est}}(x_1, y_1)|$



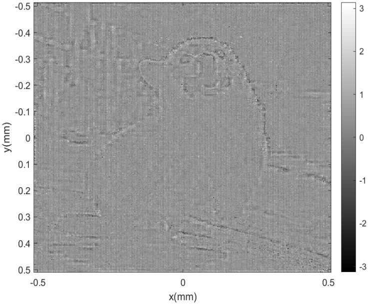
$\phi_{B_{est}}(x_1, y_1)$

Step 11: Divide $U_{B_{est}}(x_1, y_1)$ by mask B

$$U_{est}(x_1, y_1) = |U_{est}(x_1, y_1)| e^{j\phi_{est}(x_1, y_1)} = \frac{U_{B_{est}}(x_1, y_1)}{e^{jB}}$$



$|U_{est}(x_1, y_1)|$

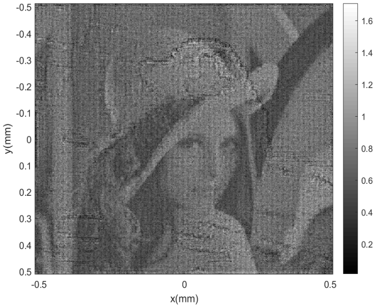


$\phi_{est}(x_1, y_1)$

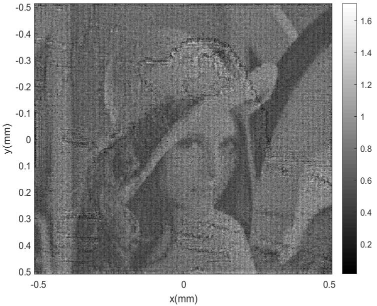
Step 12: Apply Gaussian smoothing and HIO algorithm on $|U_{est}(x_1, y_1)|$

$$|U_{est}^{'}(x_1, y_1)| = \begin{cases} |U_{est}(x_1, y_1)| & \text{if } x_1, y_1 \notin \gamma \\ |U_{est}(x_1, y_1)| - \beta \times |T_{est}(x_1, y_1)| & \text{if } x_1, y_1 \in \gamma \end{cases}$$

Where γ is the pixels with a value 0 in $|U_{est}(x_1, y_1)|$; $\beta = 0.9$



$|U_{est}(x_1, y_1)|$



$|U_{est}^{'}(x_1, y_1)|$

Iterative process

The above steps indicate the process involved in a single iteration. After 200 iterations, the following output is obtained,

Estimated $T(x_1, y_1)$

Estimated $\phi(x_1, y_1)$

Comparison

$T(x_1, y_1)$

Estimated $T(x_1, y_1)$

Comparison

$\phi(x_1, y_1)$

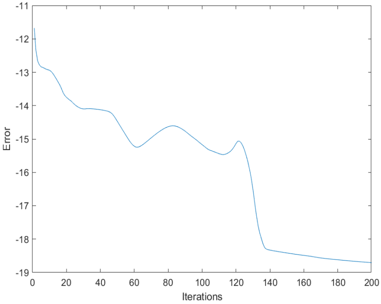
Estimated $\phi(x_1, y_1)$

Error plot

The error function is defined as

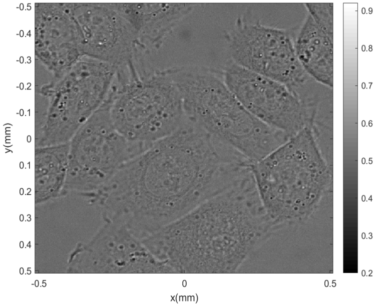
$$\xi = log\left\{ \sum \sum \left(\frac{I_{est} - I_{exp}}{I_{exp}} \right)^2 d_x d_y \right\}$$

I_{exp} is the expected intensity distribution over the detector plane and I_{est} is the estimated intensity.

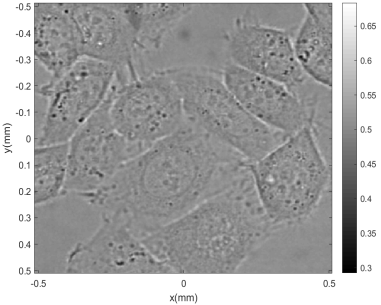


Logarithmic Error plot

Verification with cell image

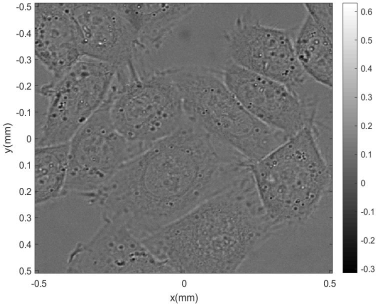


$T(x_1, y_1)$

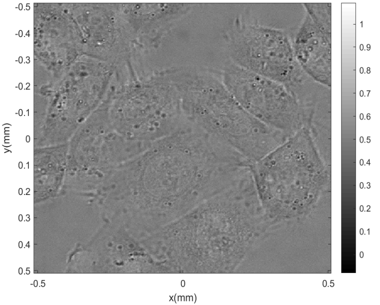


Estimated $T(x_1, y_1)$

Verification with cell image



$\phi(x_1, y_1)$

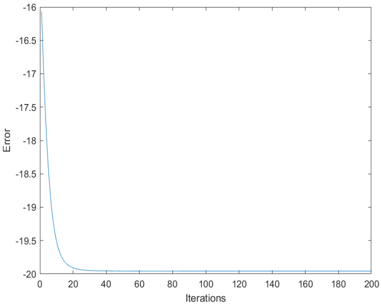


Estimated $\phi(x_1, y_1)$

Error plot for cell image

The error function is defined as

$$\xi = log\left\{ \sum \sum \left(\frac{I_{est} - I_{exp}}{I_{exp}} \right)^2 d_x d_y \right\}$$



Logarithmic Error plot

Advantages of the algorithm

- No mechanical movement of the system required
- No prior information of the object required
- Works good for near field applications and also applicable for visible light applications
- Excellent noise performance

Machine learning solutions

- Model based approach to remove residue of the structured light from the estimated object
- To provide better estimate and noise suppression in the estimated object