ECO394D Probability and Statistics Homework 2

Sibo Ding

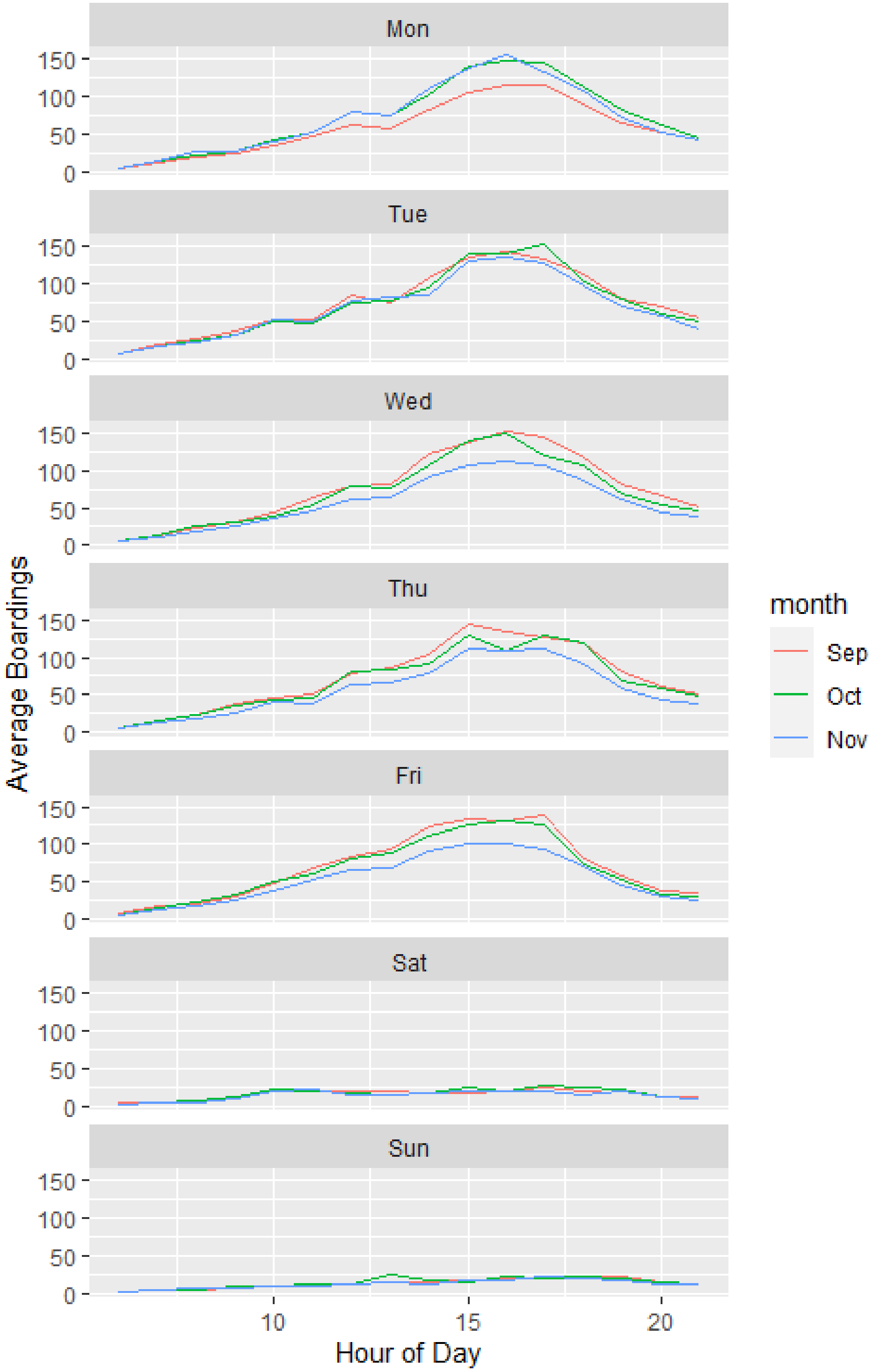
Summer 2023

```
library(dplyr)

##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union

library(ggplot2)
```

Problem 1

1

```
metro <- read.csv("capmetro_UT.csv")

# Recode the categorical variables in sensible, rather than
alphabetical, order
metro <- metro |> mutate(
  day_of_week = factor(day_of_week, levels = c("Mon", "Tue", "Wed",
"Thu", "Fri", "Sat", "Sun")),
  month = factor(month, levels = c("Sep", "Oct", "Nov")))

metro |> group_by(month, day_of_week, hour_of_day) |>
  summarize(avg_boarding = mean(boarding)) |>
  ggplot(aes(hour_of_day, avg_boarding, col = month)) +
  geom_line() +
  facet_wrap(~ day_of_week, nrow = 7) +
  labs(x = "Hour of Day",
       y = "Average Boardings",
       caption = "y-axis is average boardings, x-axis is hour of day
       Each plot is 1 day of week, showing 3 months")

## `summarise()` has grouped output by 'month', 'day_of_week'. You can
override
## using the `.groups` argument.
```

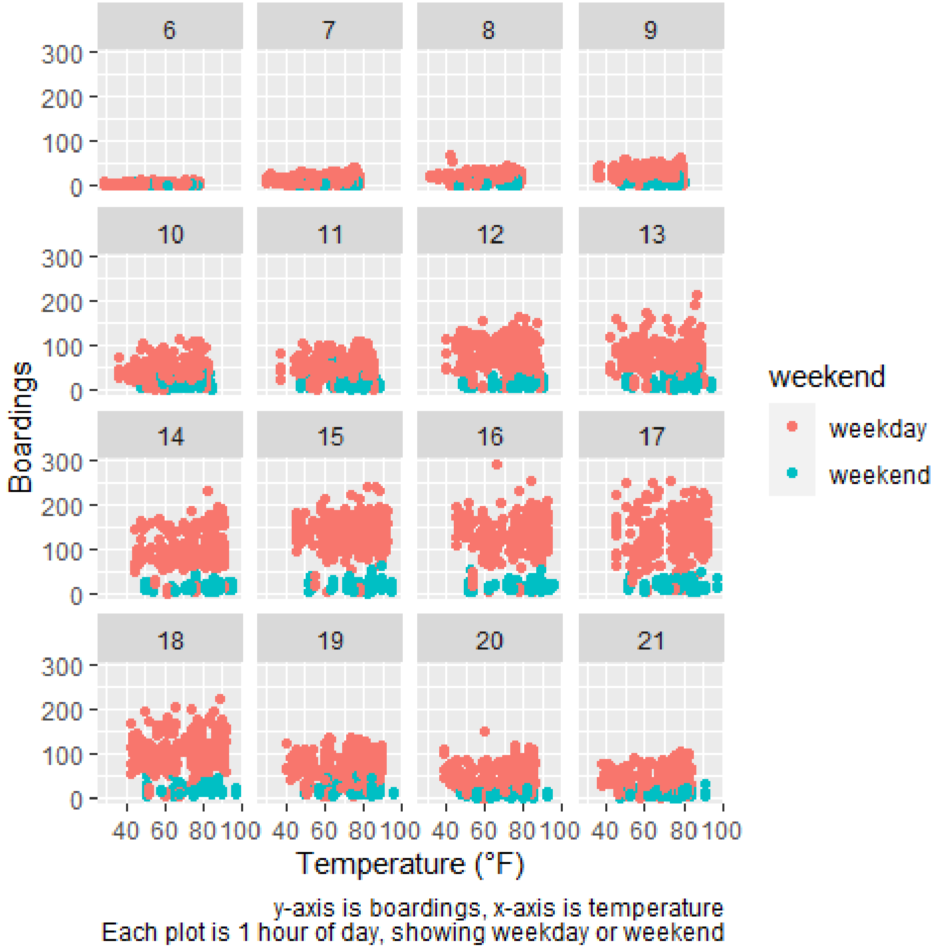


y-axis is average boardings, x-axis is hour of day
Each plot is 1 day of week, showing 3 months

The hour of peak boardings changes from day to day, as workdays have peaks but weekends do not.
UT students have fewer class on Monday in September.
Similarly, UT students have fewer class on Wed/Thur/Fri in November.

2

```
metro |> ggplot(aes(temperature, boarding, col = weekend)) +
  geom_point() +
  facet_wrap(~ hour_of_day) +
  labs(x = "Temperature (°F)",
       y = "Boardings",
       caption = "y-axis is boardings, x-axis is temperature
       Each plot is 1 hour of day, showing weekday or weekend")
```



y-axis is boardings, x-axis is temperature
Each plot is 1 hour of day, showing weekday or weekend

Holding hour of day and weekend status constant, temperature does not have a noticeable effect on the number of UT students riding the bus, as different temperatures have similar boardings (data is somewhat rectangular).

Problem 2

Part A

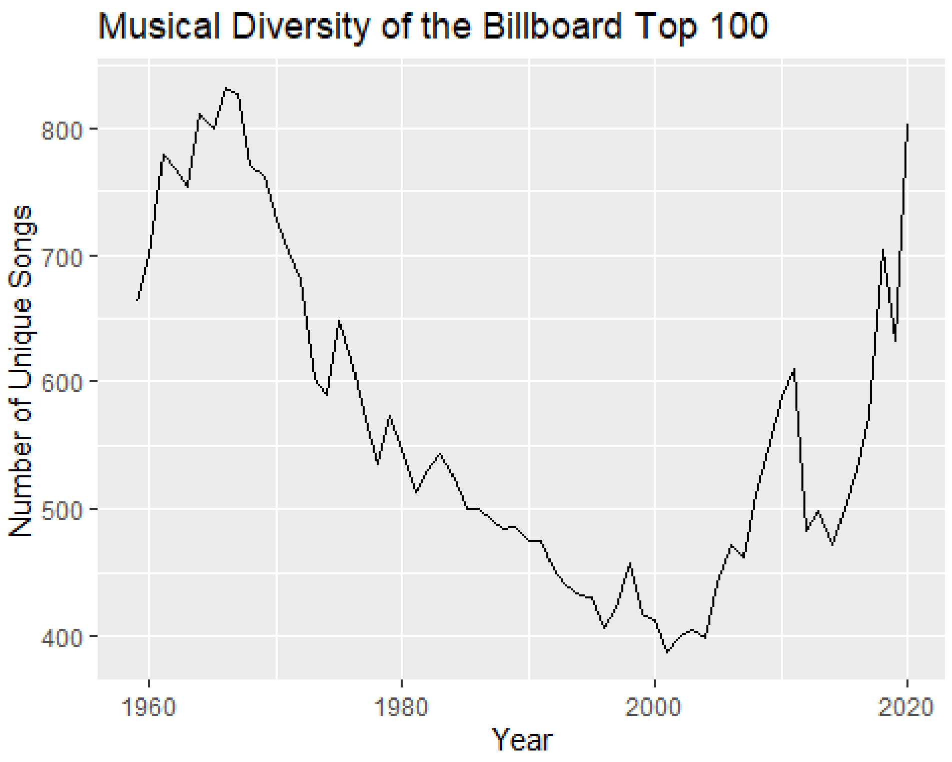
```
bb <- read.csv("billboard.csv")

# 10 songs with highest total number of weeks
bb |> group_by(performer, song) |> count() |> arrange(desc(n)) |>
head(10)
```

```
## # A tibble: 10 × 3
## # Groups:   performer, song [10]
##    performer                                  song
n
##    <chr>                                      <chr>
<int>
##  1 Imagine Dragons                            Radioactive
87
##  2 AWOLNATION                                 Sail
79
##  3 Jason Mraz                                 I'm Yours
76
##  4 The Weeknd                                 Blinding Lights
76
##  5 LeAnn Rimes                                How Do I Live
69
##  6 LMFAO Featuring Lauren Bennett & GoonRock Party Rock Anthem
68
##  7 OneRepublic                                Counting Stars
68
##  8 Adele                                      Rolling In The Deep
65
##  9 Jewel                                      Foolish Games/You Were
Meant…    65
## 10 Carrie Underwood                           Before He Cheats
64
```

10 songs with highest total number of weeks on Billboard Top 100.

Part B

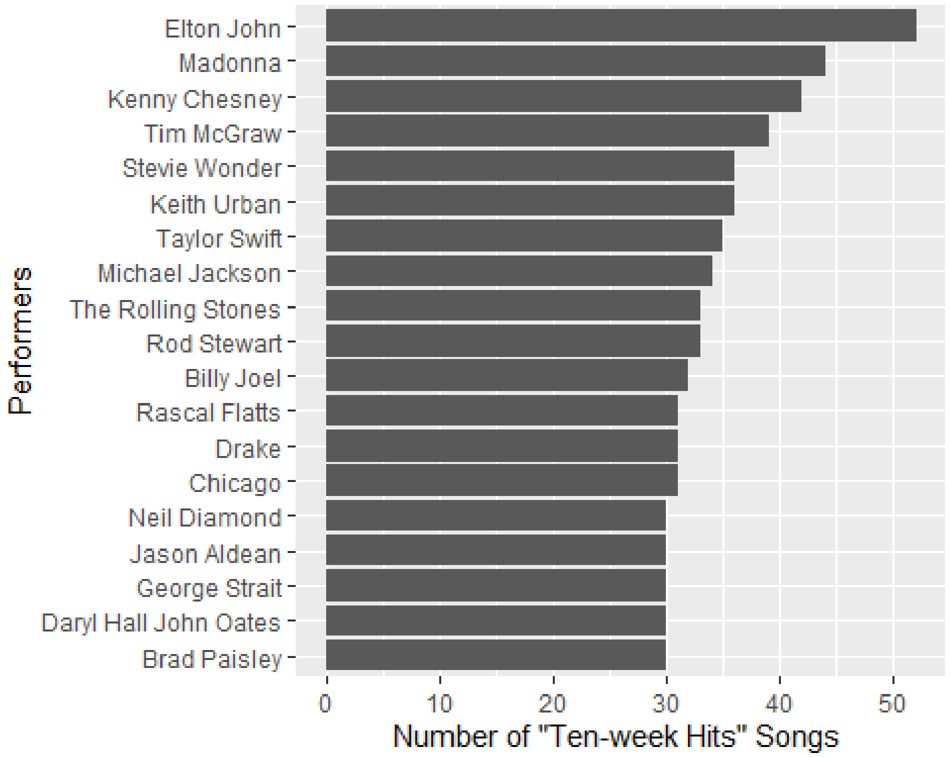
```
# The number of unique songs in each year
bb |> filter(!year %in% c(1958, 2021)) |>
  distinct(year, song_id) |>
  group_by(year) |>
  count() |>
  ggplot(aes(year, n)) +
  geom_line() +
  xlab("Year") +
  ylab("Number of Unique Songs") +
  ggtitle("Musical Diversity of the Billboard Top 100")
```



The line graph shows the number of unique songs each year.
The number of unique songs decreased from 1970s to 2000s, increased from 2000s to 2020s.

Part C

```
bb |> group_by(performer, song) |>
  count() |>
  filter(n >= 10) |>  # Songs on Billboard for at least 10 weeks
  group_by(performer) |>
  count() |>
  filter(n >= 30) |>  # Performers who have at least 30 "10-week" songs
  # Reorder performers based on "n"
  ggplot(aes(forcats::fct_reorder(performer, n), n)) +
  geom_col() +
  coord_flip() +
  xlab("Performers") +
  ylab("Number of \"Ten-week Hits\" Songs ")
```



The bar plot shows performers since 1958 who have at least 30 songs that are "ten-week hits", with corresponding number of songs shown in descending order.

Problem 3

A

```
creat <- read.csv("creatinine.csv")

lin <- lm(creatclear ~ age, data = creat)

coef(lin)[1] + coef(lin)[2] * 55

## (Intercept)
##     113.723
```

B

```
coef(lin)[2]

##        age
## -0.6198159
```

If age increases by 1 year, creatinine clearance rate decrease by 0.62 on average.

C

```
# Prediction for 40-year-old
coef(lin)[1] + coef(lin)[2] * 40
```

```
## (Intercept)
##    123.0203

# Prediction for 60-year-old
coef(lin)[1] + coef(lin)[2] * 60

## (Intercept)
##     110.624
```

The 40-year-old is healthier, as it far exceeds the prediction of regression for its age.

Problem 4

Part A

$$P(At\ least\ 1\ lemon) = 1 - P(No\ lemon) = 1 - \left(\frac{2}{3}\right)^3 = 0.70$$

Part B

```
dice_sum <- gtools::permutations(6, 2, repeats.allowed = TRUE) |>
  rowSums()

# P(Sum is odd)
sum(dice_sum %% 2 == 1) / length(dice_sum)

## [1] 0.5

# P(Sum is less than 7)
sum(dice_sum < 7) / length(dice_sum)

## [1] 0.4166667

# P(Sum is less than 7 | Sum is odd)
sum(dice_sum %% 2 == 1 & dice_sum < 7) / sum(dice_sum %% 2 == 1)

## [1] 0.3333333
```

These two events are not independent, as $P(Sum\ is\ less\ than\ 7|Sum\ is\ odd) \neq P(Sum\ is\ less\ than\ 7)$.

Part C

$$\begin{aligned}&P(Yes) = P(Yes|RC) \cdot P(RC) + P(Yes|TC) \cdot P(TC)\\&0.65 = 0.5 \times 0.3 + P(Yes|TC) \times (1 - 0.3)\\&P(Yes|TC) = \frac{5}{7}\end{aligned}$$

Part D

Bayes' Theorem

$$\begin{aligned}&P(Positive|Disease) = 0.993\\&P(Negative|No\ disease) = 0.9999\end{aligned}$$

$$\begin{aligned} &P(Positive|No\ disease) = 1 - 0.9999 = 0.0001 \\ &P(Disease) = 0.000025 \\ &P(No\ disease) = 1 - 0.000025 = 0.999975 \end{aligned}$$

$$\begin{aligned} P(Disease|Positive) &= \frac{P(Disease\ and\ positive)}{P(Positive)} \\ &= \frac{0.000025 \times 0.993}{0.000025 \times 0.993 + 0.999975 \times 0.0001} = 0.1989 \end{aligned}$$

Part E

Bayes' Theorem

$$\begin{aligned} &P(R|A) = 0.99 \\ &P(R|A') = 0.10 \\ &P(A) = 0.05 \\ &P(A') = 1 - 0.05 = 0.95 \end{aligned}$$

$$P(A|R) = \frac{P(A \cap R)}{P(R)} = \frac{0.99 \times 0.05}{0.99 \times 0.05 + 0.10 \times 0.95} = 0.3426$$

Problem 5

```
home <- read.csv("epl_2018_19_home.csv")
away <- read.csv("epl_2018_19_away.csv")

# GF: goals for; GA: goals against; GP: games played
avg_goal_by_team <- mean(home$GF + away$GF)
avg_goal_home <- mean(home$GF) / mean(home$GP)
avg_goal_away <- mean(away$GF) / mean(away$GP)
```

1

```
liv_attack <- (
  home |> filter(Team == "Liverpool") |> select(GF) +
  away |> filter(Team == "Liverpool") |> select(GF)) / avg_goal_by_team

tot_defense <- (
  home |> filter(Team == "Tottenham") |> select(GA) +
  away |> filter(Team == "Tottenham") |> select(GA)) / avg_goal_by_team

lambda_liv <- avg_goal_home * liv_attack * tot_defense

tot_attack <- (
  home |> filter(Team == "Tottenham") |> select(GF) +
  away |> filter(Team == "Tottenham") |> select(GF)) / avg_goal_by_team

liv_defense <- (
  home |> filter(Team == "Liverpool") |> select(GA) +
  away |> filter(Team == "Liverpool") |> select(GA)) / avg_goal_by_team
```

```r
lambda_tot <- avg_goal_away * tot_attack * liv_defense

# Monte Carlo simulations of Poisson distribution
set.seed(42)
n_sim <- 100000
liv <- rpois(n_sim, as.numeric(lambda_liv))
tot <- rpois(n_sim, as.numeric(lambda_tot))

# Liverpool win
sum(liv > tot) / n_sim

## [1] 0.67117

# Draw
sum(liv == tot) / n_sim

## [1] 0.21034

# Liverpool lose
sum(liv < tot) / n_sim

## [1] 0.11849
```

Question: I want to predict the probabilities of win/lose/draw results between Liverpool (home) and Tottenham (away).

Approach: Monte Carlo simulation

p.m.f. of Poisson distribution: $P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$
Take Liverpool for example, I estimate its λ based on seasonal scores. Then I simulate every number of Liverpool's goals based on the probability calculated by the p.m.f. of Poisson distribution. I do the same thing for Tottenham, assuming teams' scores are independent. Liverpool win if Liverpool's goals are more than Tottenham's in a certain pair (liv, tot), vice versa.

Results: Liverpool have 67% probability to win, 21% probability to draw, 12% probability to lose.

Conclusion: I predict Liverpool have 67% probability to win, using Monte Carlo simulations on Poisson distribution.

2

```r
mac_attack <- (
  home |> filter(Team == "Manchester City") |> select(GF) +
  away |> filter(Team == "Manchester City") |> select(GF)) /
avg_goal_by_team

ars_defense <- (
  home |> filter(Team == "Arsenal") |> select(GA) +
  away |> filter(Team == "Arsenal") |> select(GA)) / avg_goal_by_team
```

```r
lambda_mac <- avg_goal_home * mac_attack * ars_defense

ars_attack <- (
  home |> filter(Team == "Arsenal") |> select(GF) +
  away |> filter(Team == "Arsenal") |> select(GF)) / avg_goal_by_team

mac_defense <- (
  home |> filter(Team == "Manchester City") |> select(GA) +
  away |> filter(Team == "Manchester City") |> select(GA)) /
avg_goal_by_team

lambda_ars <- avg_goal_away * ars_attack * mac_defense

# Monte Carlo simulations of Poisson distribution
set.seed(42)
n_sim <- 100000
mac <- rpois(n_sim, as.numeric(lambda_mac))
ars <- rpois(n_sim, as.numeric(lambda_ars))

# Manchester City win
sum(mac > ars) / n_sim

## [1] 0.77992

# Draw
sum(mac == ars) / n_sim

## [1] 0.1411

# Manchester City lose
sum(mac < ars) / n_sim

## [1] 0.07898
```

Question, Approach, and Conclusion are similar to previous question.

Results: Manchester City have 78% probability to win, 14% probability to draw, 8% probability to lose.