

1. Вычисление коэффициентов уравнения $y = a + bx$ при помощи МНК, для зависимости L от k в углекислом газе.

Для 2909 Гц:

Formula for $\langle xy \rangle$:

$$(1 \cdot 3 + 2 \cdot 47 + 3 \cdot 96 + 4 \cdot 146 + 5 \cdot 196)/5$$

Formula for $\langle x \rangle$:

$$(1 + 2 + 3 + 4 + 5)/5$$

Formula for $\langle y \rangle$:

$$(3 + 47 + 96 + 146 + 196)/5$$

Formula for $\langle x^2 \rangle$:

$$(1^2 + 2^2 + 3^2 + 4^2 + 5^2)/5$$

Formula for $\langle y^2 \rangle$:

$$(3^2 + 47^2 + 96^2 + 146^2 + 196^2)/5$$

$$y = a + bx$$

$$b = \frac{\langle xy \rangle - \langle x \rangle \langle y \rangle}{\langle x^2 \rangle - \langle x \rangle^2} = \frac{389.80 - 3.00 \cdot 97.60}{11.00 - 9.00} = 48.50$$

$$a = \langle y \rangle - b \langle x \rangle = 97.60 - 48.50 \cdot 3.00 = -47.90$$

$$\sigma_b = \frac{1}{\sqrt{n}} \sqrt{\frac{\langle y^2 \rangle - \langle y \rangle^2}{\langle x^2 \rangle - \langle x \rangle^2} - b^2} = \frac{1}{2.236} \sqrt{\frac{14233.200195 - 9525.759766}{11.000000 - 9.000000} - 48.500} = 0.542325$$

$$\sigma_a = \sigma_b \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = 0.542325 \sqrt{11.000000 - 9.000000} = 0.766964$$

Для 2096 Гц:

Formula for $\langle xy \rangle$:

$$(1 \cdot 0 + 2 \cdot 58 + 3 \cdot 117 + 4 \cdot 181 + 5 \cdot 240)/5$$

Formula for $\langle x \rangle$:

$$(1 + 2 + 3 + 4 + 5)/5$$

Formula for $\langle y \rangle$:

$$(0 + 58 + 117 + 181 + 240)/5$$

Formula for $\langle x^2 \rangle$:

$$(1^2 + 2^2 + 3^2 + 4^2 + 5^2)/5$$

Formula for $\langle y^2 \rangle$:

$$(0^2 + 58^2 + 117^2 + 181^2 + 240^2)/5$$

$$y = a + bx$$

$$b = \frac{\langle xy \rangle - \langle x \rangle \langle y \rangle}{\langle x^2 \rangle - \langle x \rangle^2} = \frac{478.20 - 3.00 \cdot 119.20}{11.00 - 9.00} = 60.30$$

$$a = \langle y \rangle - b \langle x \rangle = 119.20 - 60.30 \cdot 3.00 = -61.70$$

$$\sigma_b = \frac{1}{\sqrt{n}} \sqrt{\frac{\langle y^2 \rangle - \langle y \rangle^2}{\langle x^2 \rangle - \langle x \rangle^2} - b^2} = \frac{1}{2.236} \sqrt{\frac{21482.800781 - 14208.639648}{11.000000 - 9.000000} - 60.300} = 0.444810$$

$$\sigma_a = \sigma_b \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = 0.444810 \sqrt{11.000000 - 9.000000} = 0.629056$$

Для 2506 Гц:

Formula for $\langle xy \rangle$:

$$(1 \cdot 5 + 2 \cdot 52 + 3 \cdot 105 + 4 \cdot 157 + 5 \cdot 208)/5$$

Formula for $\langle x \rangle$:

$$(1 + 2 + 3 + 4 + 5)/5$$

Formula for $\langle y \rangle$:

$$(5 + 52 + 105 + 157 + 208)/5$$

Formula for $\langle x^2 \rangle$:

$$(1^2 + 2^2 + 3^2 + 4^2 + 5^2)/5$$

Formula for $\langle y^2 \rangle$:

$$(5^2 + 52^2 + 105^2 + 157^2 + 208^2)/5$$

$$y = a + bx$$

$$b = \frac{\langle xy \rangle - \langle x \rangle \langle y \rangle}{\langle x^2 \rangle - \langle x \rangle^2} = \frac{418.40 - 3.00 \cdot 105.40}{11.00 - 9.00} = 51.10$$

$$a = \langle y \rangle - b \langle x \rangle = 105.40 - 51.10 \cdot 3.00 = -47.90$$

$$\sigma_b = \frac{1}{\sqrt{n}} \sqrt{\frac{\langle y^2 \rangle - \langle y \rangle^2}{\langle x^2 \rangle - \langle x \rangle^2} - b^2} = \frac{1}{2.236} \sqrt{\frac{16333.400391 - 11109.160156}{11.000000 - 9.000000} - 51.100} = 0.426770$$

$$\sigma_a = \sigma_b \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = 0.426770 \sqrt{11.000000 - 9.000000} = 0.603544$$

Для 3038 Гц:

Formula for $\langle xy \rangle$:

$$(1 \cdot 6 + 2 \cdot 51 + 3 \cdot 96 + 4 \cdot 140 + 5 \cdot 185)/5$$

Formula for $\langle x \rangle$:

$$(1 + 2 + 3 + 4 + 5)/5$$

Formula for $\langle y \rangle$:

$$(6 + 51 + 96 + 140 + 185)/5$$

Formula for $\langle x^2 \rangle$:

$$(1^2 + 2^2 + 3^2 + 4^2 + 5^2)/5$$

Formula for $\langle y^2 \rangle$:

$$(6^2 + 51^2 + 96^2 + 140^2 + 185^2)/5$$

$$y = a + bx$$

$$b = \frac{\langle xy \rangle - \langle x \rangle \langle y \rangle}{\langle x^2 \rangle - \langle x \rangle^2} = \frac{376.20 - 3.00 \cdot 95.60}{11.00 - 9.00} = 44.70$$

$$a = \langle y \rangle - b \langle x \rangle = 95.60 - 44.70 \cdot 3.00 = -38.50$$

$$\sigma_b = \frac{1}{\sqrt{n}} \sqrt{\frac{\langle y^2 \rangle - \langle y \rangle^2}{\langle x^2 \rangle - \langle x \rangle^2} - b^2} = \frac{1}{2.236} \sqrt{\frac{13135.599609 - 9139.359375}{11.000000 - 9.000000} - 44.700^2} = 0.076638$$

$$\sigma_a = \sigma_b \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = 0.076638 \sqrt{11.000000 - 9.000000} = 0.108382$$

2. Вычисление коэффициентов уравнения $y = a + bx$ при помощи МНК, для зависимости L от k в воздухе.

Для 3475 Гц:

Formula for $\langle xy \rangle$:

$$(1 \cdot 5 + 2 \cdot 48 + 3 \cdot 99 + 4 \cdot 148 + 5 \cdot 198)/5$$

Formula for $\langle x \rangle$:

$$(1 + 2 + 3 + 4 + 5)/5$$

Formula for $\langle y \rangle$:

$$(5 + 48 + 99 + 148 + 198)/5$$

Formula for $\langle x^2 \rangle$:

$$(1^2 + 2^2 + 3^2 + 4^2 + 5^2)/5$$

Formula for $\langle y^2 \rangle$:

$$(5^2 + 48^2 + 99^2 + 148^2 + 198^2)/5$$

$$y = a + bx$$

$$b = \frac{\langle xy \rangle - \langle x \rangle \langle y \rangle}{\langle x^2 \rangle - \langle x \rangle^2} = \frac{396.00 - 3.00 \cdot 99.60}{11.00 - 9.00} = 48.60$$

$$a = \langle y \rangle - b \langle x \rangle = 99.60 - 48.60 \cdot 3.00 = -46.20$$

$$\sigma_b = \frac{1}{\sqrt{n}} \sqrt{\frac{\langle y^2 \rangle - \langle y \rangle^2}{\langle x^2 \rangle - \langle x \rangle^2} - b^2} = \frac{1}{2.236} \sqrt{\frac{14647.599609 - 9920.160156}{11.000000 - 9.000000} - 48.600} = 0.593212$$

$$\sigma_a = \sigma_b \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = 0.593212 \sqrt{11.000000 - 9.000000} = 0.838929$$

Для 4052 Гц:

Formula for $\langle xy \rangle$:

$$(1 \cdot 0 + 2 \cdot 40 + 3 \cdot 81 + 4 \cdot 120 + 5 \cdot 160)/5$$

Formula for $\langle x \rangle$:

$$(1 + 2 + 3 + 4 + 5)/5$$

Formula for $\langle y \rangle$:

$$(0 + 40 + 81 + 120 + 160)/5$$

Formula for $\langle x^2 \rangle$:

$$(1^2 + 2^2 + 3^2 + 4^2 + 5^2)/5$$

Formula for $\langle y^2 \rangle$:

$$(0^2 + 40^2 + 81^2 + 120^2 + 160^2)/5$$

$$y = a + bx$$

$$b = \frac{\langle xy \rangle - \langle x \rangle \langle y \rangle}{\langle x^2 \rangle - \langle x \rangle^2} = \frac{320.60 - 3.00 \cdot 80.20}{11.00 - 9.00} = 40.00$$

$$a = \langle y \rangle - b \langle x \rangle = 80.20 - 40.00 \cdot 3.00 = -39.80$$

$$\sigma_b = \frac{1}{\sqrt{n}} \sqrt{\frac{\langle y^2 \rangle - \langle y \rangle^2}{\langle x^2 \rangle - \langle x \rangle^2}} - b^2 = \frac{1}{2.236} \sqrt{\frac{9632.200195 - 6432.039551}{11.000000 - 9.000000}} - 40.000 = 0.126263$$

$$\sigma_a = \sigma_b \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = 0.126263 \sqrt{11.000000 - 9.000000} = 0.178563$$

Для 5111 Гц:

Formula for $\langle xy \rangle$:

$$(1 \cdot 0 + 2 \cdot 34 + 3 \cdot 69 + 4 \cdot 101 + 5 \cdot 135)/5$$

Formula for $\langle x \rangle$:

$$(1 + 2 + 3 + 4 + 5)/5$$

Formula for $\langle y \rangle$:

$$(0 + 34 + 69 + 101 + 135)/5$$

Formula for $\langle x^2 \rangle$:

$$(1^2 + 2^2 + 3^2 + 4^2 + 5^2)/5$$

Formula for $\langle y^2 \rangle$:

$$(0^2 + 34^2 + 69^2 + 101^2 + 135^2)/5$$

$$y = a + bx$$

$$b = \frac{\langle xy \rangle - \langle x \rangle \langle y \rangle}{\langle x^2 \rangle - \langle x \rangle^2} = \frac{270.80 - 3.00 \cdot 67.80}{11.00 - 9.00} = 33.70$$

$$a = \langle y \rangle - b \langle x \rangle = 67.80 - 33.70 \cdot 3.00 = -33.30$$

$$\sigma_b = \frac{1}{\sqrt{n}} \sqrt{\frac{\langle y^2 \rangle - \langle y \rangle^2}{\langle x^2 \rangle - \langle x \rangle^2}} - b^2 = \frac{1}{2.236} \sqrt{\frac{6868.600098 - 4596.840332}{11.000000 - 9.000000}} - 33.700 = 0.195245$$

$$\sigma_a = \sigma_b \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = 0.195245 \sqrt{11.000000 - 9.000000} = 0.276118$$

Для 3890 Гц:

Formula for $\langle xy \rangle$:

$$(1 \cdot 0 + 2 \cdot 40 + 3 \cdot 81 + 4 \cdot 120 + 5 \cdot 160)/5$$

Formula for $\langle x \rangle$:

$$(1 + 2 + 3 + 4 + 5)/5$$

Formula for $\langle y \rangle$:

$$(0 + 40 + 81 + 120 + 160)/5$$

Formula for $\langle x^2 \rangle$:

$$(1^2 + 2^2 + 3^2 + 4^2 + 5^2)/5$$

Formula for $\langle y^2 \rangle$:

$$(0^2 + 40^2 + 81^2 + 120^2 + 160^2)/5$$

$$y = a + bx$$

$$b = \frac{\langle xy \rangle - \langle x \rangle \langle y \rangle}{\langle x^2 \rangle - \langle x \rangle^2} = \frac{320.60 - 3.00 \cdot 80.20}{11.00 - 9.00} = 40.00$$

$$a = \langle y \rangle - b \langle x \rangle = 80.20 - 40.00 \cdot 3.00 = -39.80$$

$$\sigma_b = \frac{1}{\sqrt{n}} \sqrt{\frac{\langle y^2 \rangle - \langle y \rangle^2}{\langle x^2 \rangle - \langle x \rangle^2} - b^2} = \frac{1}{2.236} \sqrt{\frac{9632.200195 - 6432.039551}{11.000000 - 9.000000} - 40.000} = 0.126263$$

$$\sigma_a = \sigma_b \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = 0.126263 \sqrt{11.000000 - 9.000000} = 0.178563$$

3. Вычисление коэффициентов уравнения $y = a + bx$ при помощи МНК, для зависимости f от k .

Для воздуха:

Formula for $\langle xy \rangle$:

$$(6 \cdot 1071 + 7 \cdot 1280 + 8 \cdot 1501 + 9 \cdot 1713 + 10 \cdot 1922 + 12 \cdot 2312 + 14 \cdot 2598 + 15 \cdot 2805) / 8$$

Formula for $\langle x \rangle$:

$$(6 + 7 + 8 + 9 + 10 + 12 + 14 + 15) / 8$$

Formula for $\langle y \rangle$:

$$(1071 + 1280 + 1501 + 1713 + 1922 + 2312 + 2598 + 2805) / 8$$

Formula for $\langle x^2 \rangle$:

$$(6^2 + 7^2 + 8^2 + 9^2 + 10^2 + 12^2 + 14^2 + 15^2) / 8$$

Formula for $\langle y^2 \rangle$:

$$(1071^2 + 1280^2 + 1501^2 + 1713^2 + 1922^2 + 2312^2 + 2598^2 + 2805^2) / 8$$

$$y = a + bx$$

$$b = \frac{\langle xy \rangle - \langle x \rangle \langle y \rangle}{\langle x^2 \rangle - \langle x \rangle^2} = \frac{21027.75 - 10.13 \cdot 1900.25}{111.88 - 102.52} = 191.1$$

$$a = \langle y \rangle - b \langle x \rangle = 1900.25 - 191.01 \cdot 10.13 = -33.71$$

$$\sigma_b = \frac{1}{\sqrt{n}} \sqrt{\frac{\langle y^2 \rangle - \langle y \rangle^2}{\langle x^2 \rangle - \langle x \rangle^2} - b^2} = \frac{1}{2.828} \sqrt{\frac{3953733.500000 - 3610950.000000}{111.875000 - 102.515625} - 191.008^2} = 4.189659$$

$$\sigma_a = \sigma_b \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = 4.189659 \sqrt{111.875000 - 102.515625} = 12.817463$$

Для углекислого газа:

Formula for $\langle xy \rangle$:

$$(5 \cdot 1003 + 7 \cdot 1322 + 8 \cdot 1501 + 9 \cdot 1673 + 10 \cdot 1803 + 12 \cdot 2350 + 13 \cdot 2505 + 17 \cdot 3159) / 8$$

Formula for $\langle x \rangle$:

$$(5 + 7 + 8 + 9 + 10 + 12 + 13 + 17)/8$$

Formula for $\langle y \rangle$:

$$(1003 + 1322 + 1501 + 1673 + 1803 + 2350 + 2505 + 3159)/8$$

Formula for $\langle x^2 \rangle$:

$$(5^2 + 7^2 + 8^2 + 9^2 + 10^2 + 12^2 + 13^2 + 17^2)/8$$

Formula for $\langle y^2 \rangle$:

$$(1003^2 + 1322^2 + 1501^2 + 1673^2 + 1803^2 + 2350^2 + 2505^2 + 3159^2)/8$$

$$y = a + bx$$

$$b = \frac{\langle xy \rangle - \langle x \rangle \langle y \rangle}{\langle x^2 \rangle - \langle x \rangle^2} = \frac{21729.00 - 10.13 \cdot 1914.50}{115.13 - 102.52} = 185.95$$

$$a = \langle y \rangle - b \langle x \rangle = 1914.50 - 185.95 \cdot 10.13 = 31.78$$

$$\sigma_b = \frac{1}{\sqrt{n}} \sqrt{\frac{\langle y^2 \rangle - \langle y \rangle^2}{\langle x^2 \rangle - \langle x \rangle^2} - b^2} = \frac{1}{2.828} \sqrt{\frac{4104154.500000 - 3665310.250000}{115.125000 - 102.515625} - 185.948^2} = 5.319453$$

$$\sigma_a = \sigma_b \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = 5.319453 \sqrt{115.125000 - 102.515625} = 18.889210$$

4. Вычисление коэффициентов уравнения $y = a + bx$ при помощи МНК, для зависимости f от k .

Для $T = 303,2 \text{ K}$:

Formula for $\langle xy \rangle$:

$$(1 \cdot 256 + 2 \cdot 502 + 3 \cdot 752 + 4 \cdot 1005 + 5 \cdot 1246)/5$$

Formula for $\langle x \rangle$:

$$(1 + 2 + 3 + 4 + 5)/5$$

Formula for $\langle y \rangle$:

$$(256 + 502 + 752 + 1005 + 1246)/5$$

Formula for $\langle x^2 \rangle$:

$$(1^2 + 2^2 + 3^2 + 4^2 + 5^2)/5$$

Formula for $\langle y^2 \rangle$:

$$(256^2 + 502^2 + 752^2 + 1005^2 + 1246^2)/5$$

$$y = a + bx$$

$$b = \frac{\langle xy \rangle - \langle x \rangle \langle y \rangle}{\langle x^2 \rangle - \langle x \rangle^2} = \frac{2753.20 - 3.00 \cdot 752.20}{11.00 - 9.00} = 248.30$$

$$a = \langle y \rangle - b \langle x \rangle = 752.20 - 248.30 \cdot 3.00 = 7.30$$

$$\sigma_b = \frac{1}{\sqrt{n}} \sqrt{\frac{\langle y^2 \rangle - \langle y \rangle^2}{\langle x^2 \rangle - \langle x \rangle^2} - b^2} = \frac{1}{2.236} \sqrt{\frac{689117.000000 - 565804.875000}{11.000000 - 9.000000} - 248.300^2} = 0.799214$$

$$\sigma_a = \sigma_b \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = 0.799214 \sqrt{11.000000 - 9.000000} = 1.130260$$

Для $T = 313,2 \text{ K}$:

Formula for $\langle xy \rangle$:

$$(1 \cdot 267 + 2 \cdot 514 + 3 \cdot 768 + 4 \cdot 1024 + 5 \cdot 1280)/5$$

Formula for $\langle x \rangle$:

$$(1 + 2 + 3 + 4 + 5)/5$$

Formula for $\langle y \rangle$:

$$(267 + 514 + 768 + 1024 + 1280)/5$$

Formula for $\langle x^2 \rangle$:

$$(1^2 + 2^2 + 3^2 + 4^2 + 5^2)/5$$

Formula for $\langle y^2 \rangle$:

$$(267^2 + 514^2 + 768^2 + 1024^2 + 1280^2)/5$$

$$y = a + bx$$

$$b = \frac{\langle xy \rangle - \langle x \rangle \langle y \rangle}{\langle x^2 \rangle - \langle x \rangle^2} = \frac{2819.00 - 3.00 \cdot 770.60}{11.00 - 9.00} = 253.60$$

$$a = \langle y \rangle - b \langle x \rangle = 770.60 - 253.60 \cdot 3.00 = 9.80$$

$$\sigma_b = \frac{1}{\sqrt{n}} \sqrt{\frac{\langle y^2 \rangle - \langle y \rangle^2}{\langle x^2 \rangle - \langle x \rangle^2} - b^2} = \frac{1}{2.236} \sqrt{\frac{722457.000000 - 593824.312500}{11.000000 - 9.000000} - 253.600^2} = 0.820387$$

$$\sigma_a = \sigma_b \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = 0.820387 \sqrt{11.000000 - 9.000000} = 1.160203$$

Для $T = 323,2 \text{ K}$:

Formula for $\langle xy \rangle$:

$$(1 \cdot 270 + 2 \cdot 520 + 3 \cdot 773 + 4 \cdot 1034 + 5 \cdot 1283)/5$$

Formula for $\langle x \rangle$:

$$(1 + 2 + 3 + 4 + 5)/5$$

Formula for $\langle y \rangle$:

$$(270 + 520 + 773 + 1034 + 1283)/5$$

Formula for $\langle x^2 \rangle$:

$$(1^2 + 2^2 + 3^2 + 4^2 + 5^2)/5$$

Formula for $\langle y^2 \rangle$:

$$(270^2 + 520^2 + 773^2 + 1034^2 + 1283^2)/5$$

$$y = a + bx$$

$$b = \frac{\langle xy \rangle - \langle x \rangle \langle y \rangle}{\langle x^2 \rangle - \langle x \rangle^2} = \frac{2836.00 - 3.00 \cdot 776.00}{11.00 - 9.00} = 254.00$$

$$a = \langle y \rangle - b \langle x \rangle = 776.00 - 254.00 \cdot 3.00 = 14.00$$

$$\sigma_b = \frac{1}{\sqrt{n}} \sqrt{\frac{\langle y^2 \rangle - \langle y \rangle^2}{\langle x^2 \rangle - \langle x \rangle^2} - b^2} = \frac{1}{2.236} \sqrt{\frac{731214.812500 - 602176.000000}{11.000000 - 9.000000} - 254.000} = 0.825379$$

$$\sigma_a = \sigma_b \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = 0.825379 \sqrt{11.000000 - 9.000000} = 1.167262$$

Для $T = 333,2 \text{ K}$:

Formula for $\langle xy \rangle$:

$$(1 \cdot 275 + 2 \cdot 524 + 3 \cdot 780 + 4 \cdot 1027 + 5 \cdot 1301)/5$$

Formula for $\langle x \rangle$:

$$(1 + 2 + 3 + 4 + 5)/5$$

Formula for $\langle y \rangle$:

$$(275 + 524 + 780 + 1027 + 1301)/5$$

Formula for $\langle x^2 \rangle$:

$$(1^2 + 2^2 + 3^2 + 4^2 + 5^2)/5$$

Formula for $\langle y^2 \rangle$:

$$(275^2 + 524^2 + 780^2 + 1027^2 + 1301^2)/5$$

$$y = a + bx$$

$$b = \frac{\langle xy \rangle - \langle x \rangle \langle y \rangle}{\langle x^2 \rangle - \langle x \rangle^2} = \frac{2855.20 - 3.00 \cdot 781.40}{11.00 - 9.00} = 255.50$$

$$a = \langle y \rangle - b \langle x \rangle = 781.40 - 255.50 \cdot 3.00 = 14.90$$

$$\sigma_b = \frac{1}{\sqrt{n}} \sqrt{\frac{\langle y^2 \rangle - \langle y \rangle^2}{\langle x^2 \rangle - \langle x \rangle^2} - b^2} = \frac{1}{2.236} \sqrt{\frac{741186.187500 - 610586.000000}{11.000000 - 9.000000} - 255.500} = 1.993737$$

$$\sigma_a = \sigma_b \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = 1.993737 \sqrt{11.000000 - 9.000000} = 2.819570$$