$$S_n = \sum_{k=0}^{N-1} x_k e^{-jnk\frac{2\pi}{N}}, \quad n = 0,1, ..., N-1$$

$$x_k = \sin(\omega t + \varphi) = \sin(n_0 \Delta\omega \cdot k \Delta t + \varphi), \quad k = 0,1, ..., N-1$$

$$\Delta\omega \cdot \Delta t = \frac{2\pi}{N}$$

$$x_k = \frac{1}{2j}\left(e^{j\left(n_0 k \frac{2\pi}{N} + \varphi\right)} - e^{-j\left(n_0 k \frac{2\pi}{N} + \varphi\right)} \right)$$

$$S_n = \frac{1}{2j}\sum_{k=0}^{N-1} e^{j\left(n_0 k \frac{2\pi}{N}+\varphi\right) - jnk\frac{2\pi}{N}} - \frac{1}{2j}\sum_{k=0}^{N-1} e^{-j\left(n_0 k \frac{2\pi}{N}+\varphi\right) - jnk\frac{2\pi}{N}}$$

$$= \frac{1}{2j} e^{+j\varphi} \sum_{k=0}^{N-1} e^{j(n_0 - n)k\frac{2\pi}{N}} - \frac{1}{2j} e^{-j\varphi} \sum_{k=0}^{N-1} e^{-j(n_0 + n)k\frac{2\pi}{N}}$$

$$= \frac{1}{2j} e^{+j\varphi} \sum_{k=0}^{N-1} \left(e^{j(n_0 - n)\frac{2\pi}{N}} \right)^k - \frac{1}{2j} e^{-j\varphi} \sum_{k=0}^{N-1} \left(e^{-j(n_0 + n)\frac{2\pi}{N}} \right)^k$$

$$a_1, a_2, ..., a_N$$

$$a_l = a_1 \cdot q^{l-1}$$

$$s_N = \sum_{l=1}^{N} a_l = \frac{a_1 \cdot \left(q^N - 1\right)}{q - 1}$$

При $n = n_0$:

$$S_n = \frac{1}{2j} e^{+j\varphi} \sum_{k=0}^{N-1} \left(e^0\right)^k - \frac{1}{2j} e^{-j\varphi} \sum_{k=0}^{N-1} \left(e^{-j\cdot 2n_0 \cdot \frac{2\pi}{N}} \right)^k$$

$$= \frac{1}{2j} e^{+j\varphi} \cdot N - \frac{1}{2j} e^{-j\varphi} \cdot \frac{1 \cdot \left[\left(e^{-j\cdot 2n_0 \cdot \frac{2\pi}{N}} \right)^N - 1 \right]}{e^{-j\cdot 2n_0 \cdot \frac{2\pi}{N}} - 1}$$

$$= \frac{1}{2j} e^{+j\varphi} \cdot N - \frac{1}{2j} e^{-j\varphi} \cdot \frac{e^{-j\cdot 4\pi \cdot n_0} - 1}{e^{-j\cdot 2n_0 \cdot \frac{2\pi}{N}} - 1}$$

$$= \frac{1}{2j} e^{+j\varphi} \cdot N - \frac{1}{2j} e^{-j\varphi} \cdot \frac{1 - 1}{e^{-j\cdot 2n_0 \cdot \frac{2\pi}{N}} - 1}$$

$$= \frac{1}{2j} e^{+j\varphi} \cdot N$$

При $n=-n_0+N$:

$$S_n=\frac{1}{2j}e^{+j\varphi}\sum_{k=0}^{N-1}\left(e^{j(n_0-n)\frac{2\pi}{N}}\right)^k-\frac{1}{2j}e^{-j\varphi}\sum_{k=0}^{N-1}\left(e^{-j(n_0+n)\frac{2\pi}{N}}\right)^k$$

$$=\frac{1}{2j}e^{+j\varphi}\sum_{k=0}^{N-1}\left(e^{j(2n_0-N)\frac{2\pi}{N}}\right)^k-\frac{1}{2j}e^{-j\varphi}\sum_{k=0}^{N-1}\left(e^{-j\cdot N\cdot\frac{2\pi}{N}}\right)^k$$

$$=\frac{1}{2j}e^{+j\varphi}\sum_{k=0}^{N-1}\left(e^{j\cdot 2n_0\cdot\frac{2\pi}{N}}\right)^k\cdot\left(e^{-j\cdot 2\pi}\right)^k-\frac{1}{2j}e^{-j\varphi}\sum_{k=0}^{N-1}\left(e^{-j\cdot 2\pi}\right)^k$$

$$=\frac{1}{2j}e^{+j\varphi}\sum_{k=0}^{N-1}\left(e^{j\cdot 2n_0\cdot\frac{2\pi}{N}}\right)^k\cdot 1^k-\frac{1}{2j}e^{-j\varphi}\sum_{k=0}^{N-1}1^k$$

$$=\frac{1}{2j}e^{+j\varphi}\cdot\sum_{k=0}^{N-1}\left(e^{j\cdot 2n_0\cdot\frac{2\pi}{N}}\right)^k-\frac{1}{2j}e^{-j\varphi}\cdot N$$

$$=\frac{1}{2j}e^{+j\varphi}\cdot 0-\frac{1}{2j}e^{-j\varphi}\cdot N$$

$$=-\frac{1}{2j}e^{-j\varphi}\cdot N$$

При $n\neq n_0$ и $n\neq -n_0+N$

$m_1=n-n_0$ и $m_2=n+n_0$

– целые числа, не равные ни 0, ни N,

поэтому :

$$S_n=\frac{1}{2j}e^{+j\varphi}\sum_{k=0}^{N-1}\left(e^{j(n_0-n)\frac{2\pi}{N}}\right)^k-\frac{1}{2j}e^{-j\varphi}\sum_{k=0}^{N-1}\left(e^{-j(n_0+n)\frac{2\pi}{N}}\right)^k$$

$$=\frac{1}{2j}e^{+j\varphi}\sum_{k=0}^{N-1}\left(e^{jm_1\frac{2\pi}{N}}\right)^k-\frac{1}{2j}e^{-j\varphi}\sum_{k=0}^{N-1}\left(e^{-jm_2\frac{2\pi}{N}}\right)^k$$

$$=\frac{1}{2j}e^{+j\varphi}\cdot\frac{1\cdot\left[\left(e^{jm_1\frac{2\pi}{N}}\right)^N-1\right]}{e^{jm_1\frac{2\pi}{N}}-1}-\frac{1}{2j}e^{-j\varphi}\cdot\frac{1\cdot\left[\left(e^{-jm_2\frac{2\pi}{N}}\right)^N-1\right]}{e^{-jm_2\frac{2\pi}{N}}-1}$$

$$=\frac{1}{2j}e^{+j\varphi}\cdot\frac{e^{j\cdot m_1\cdot 2\pi}-1}{e^{jm_1\frac{2\pi}{N}}-1}-\frac{1}{2j}e^{-j\varphi}\cdot\frac{e^{-j\cdot m_2\cdot 2\pi}-1}{e^{-jm_2\frac{2\pi}{N}}-1}$$

$$=\frac{1}{2j}e^{+j\varphi}\cdot\frac{1-1}{e^{jm_1\frac{2\pi}{N}}-1}-\frac{1}{2j}e^{-j\varphi}\cdot\frac{1-1}{e^{-jm_2\frac{2\pi}{N}}-1}$$

$$=0$$