

YAKEEN 2.0

- SUBJECT - PHYSICS
- CHAPTER - BASIC MATHS AND CALCULUS

Lecture No.- 04

NEET +2024



By-Dr. MR SIR

Today's targets



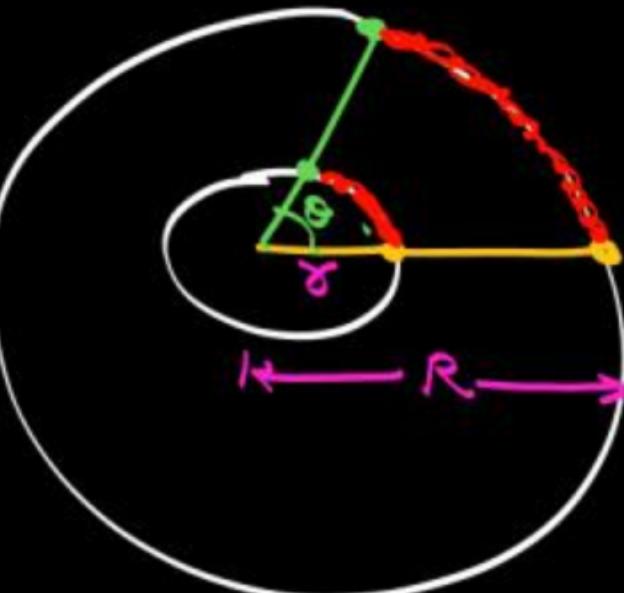
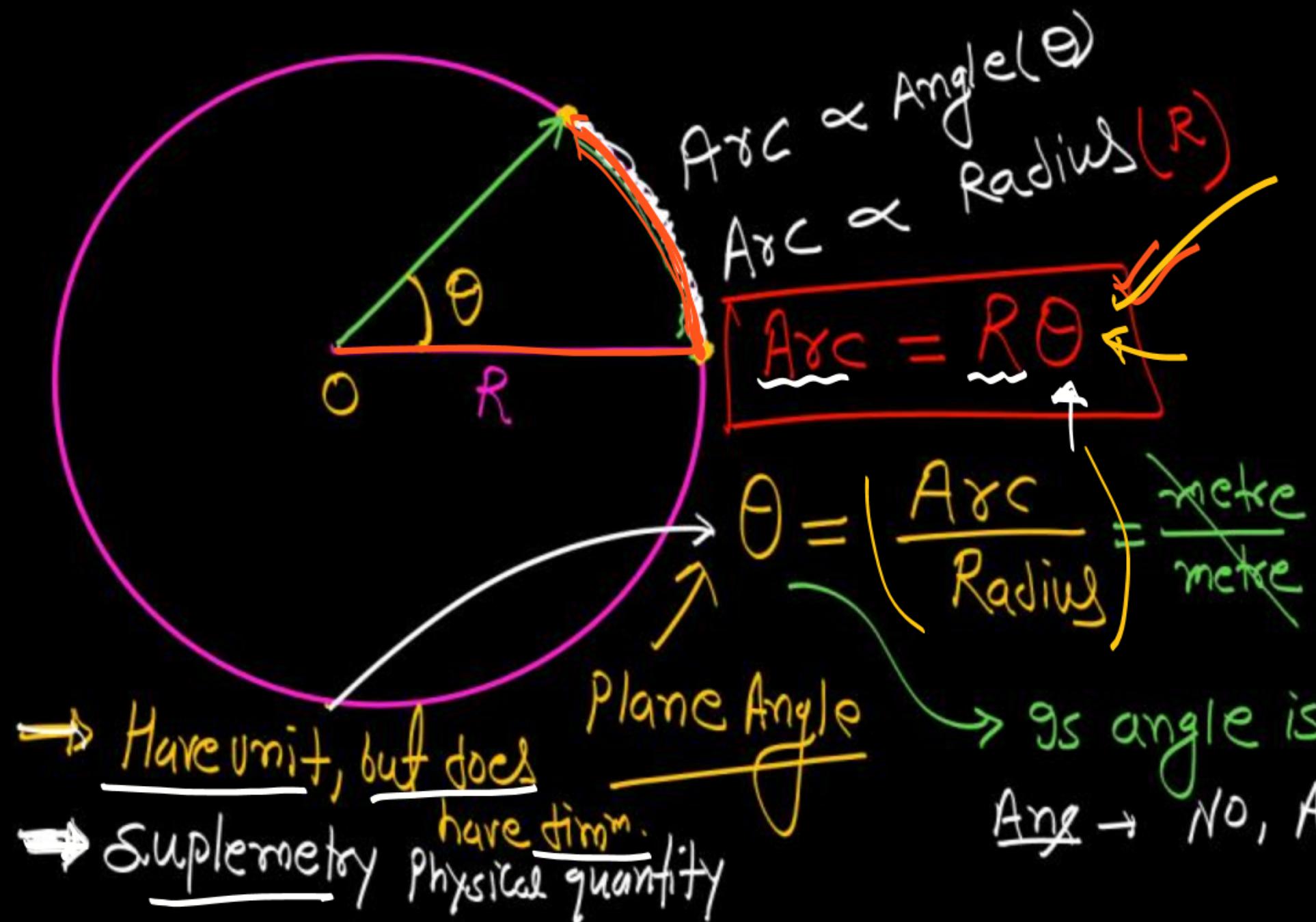
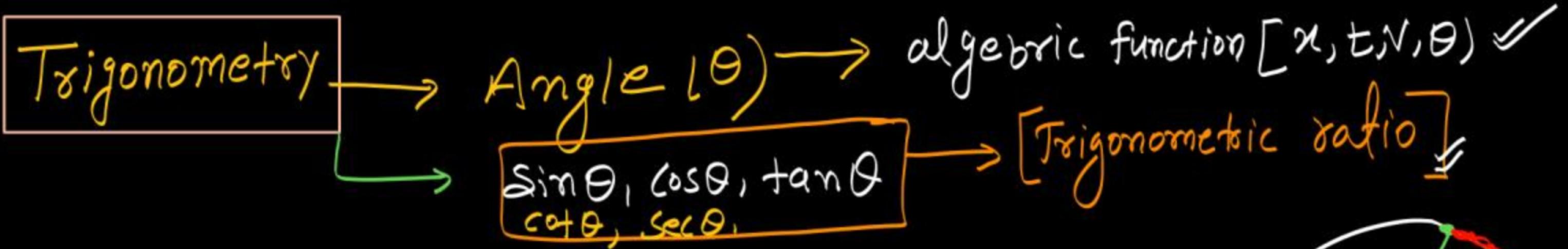
- 1 # Revision of last class
- 2 H/W
- 3 Trigonometry - II
- G.P. series

7

$$\frac{2^{m+2} - 2^m}{2^m}$$



$$\text{Volume} = \pi R^2 H$$



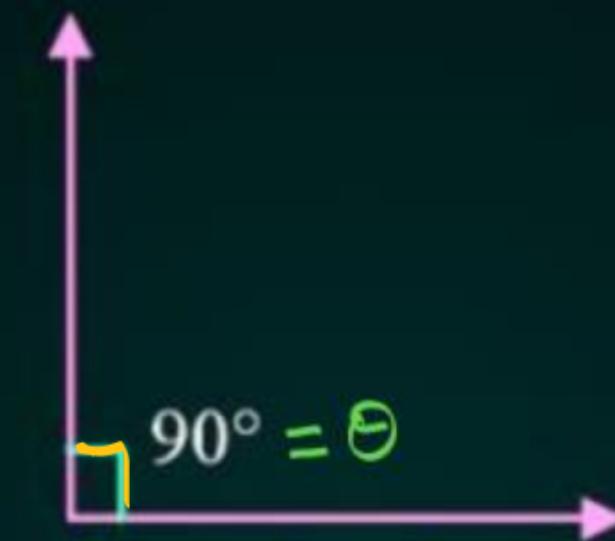
Is angle is unitless ??

Ans → No, Angle have unit →

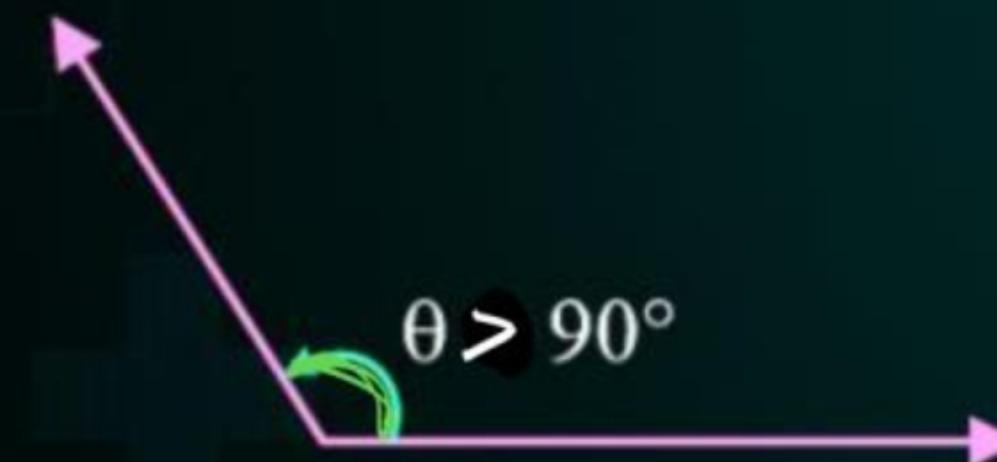
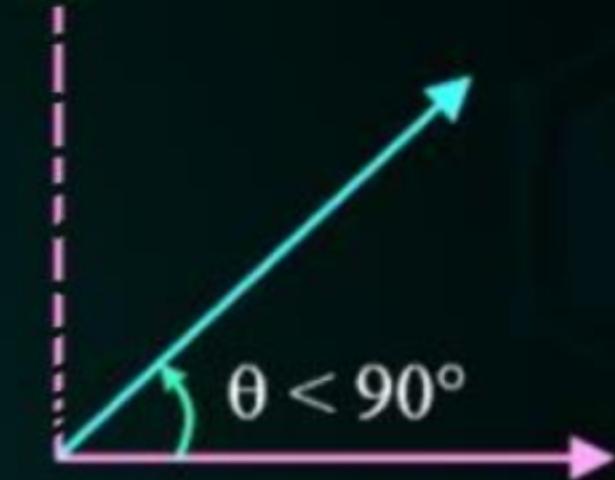
Radian (S.I. unit)



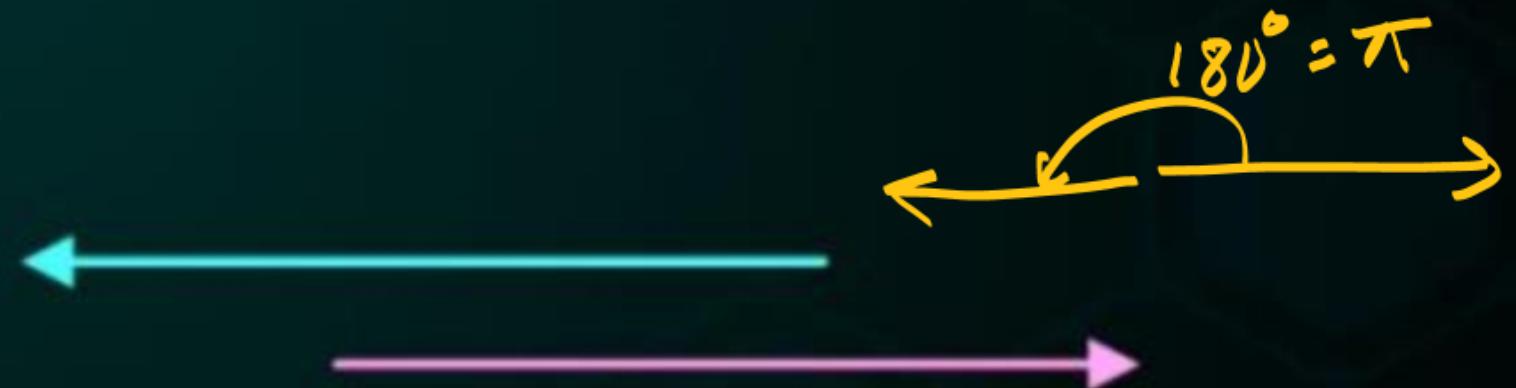
Angle $\theta = 0^\circ, 360^\circ, 720^\circ$



Perpendicular to each other



Ex- $\theta = 120^\circ$,
may be



$\theta = 180^\circ$ ✓



UNIT OF ANGLE

S.I. units
C.G.S = Radian

Other unit (Practical unit) = degree, minute, second

$$\pi \text{ rad} = 180^\circ$$

$$1 \text{ rad} = \frac{180^\circ}{\pi}$$

$$1^\circ = \frac{\pi \text{ rad}}{180}$$

$$1^\circ = 60'$$

$$\text{Arc} = R\theta$$

$$\sin(\theta), \cos(\theta)$$

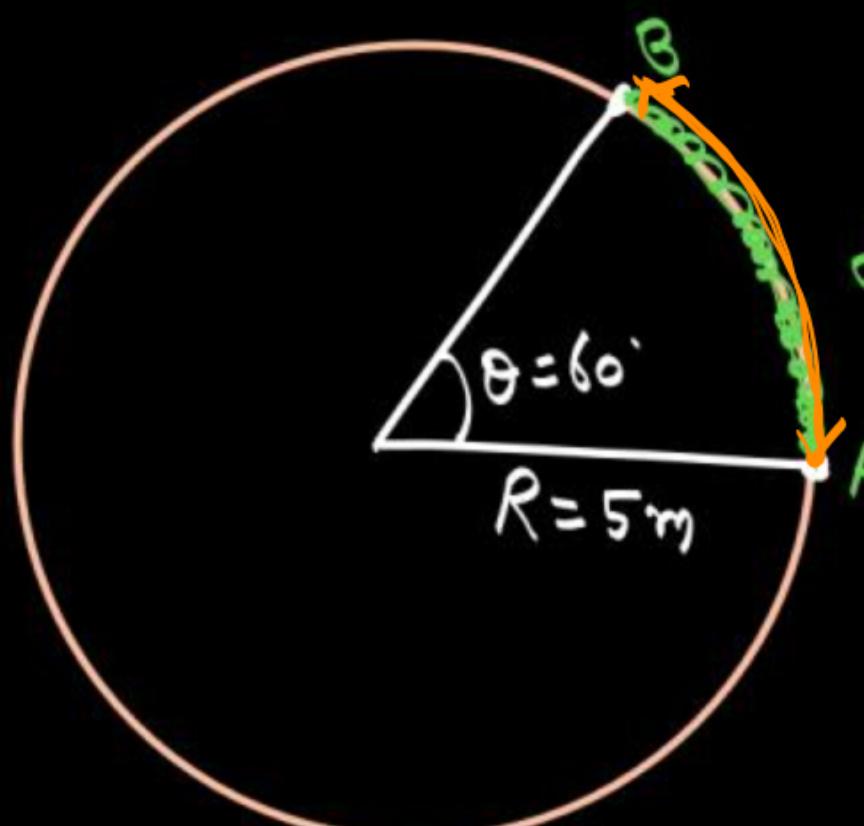
algebraic fun
must be in radian

Trigonometric function
degree, radian

* For algebraic function (θ) we always use radian, for trigonometric ratio we can use both degree or radian

When object moves 60° on circular path of radius 5m then distance travelled by object.

50m



distⁿ

$$\text{dist}^n = \text{Arc} = R\theta$$

$$= 5 \times 60 = 300 \text{m} \times \times$$

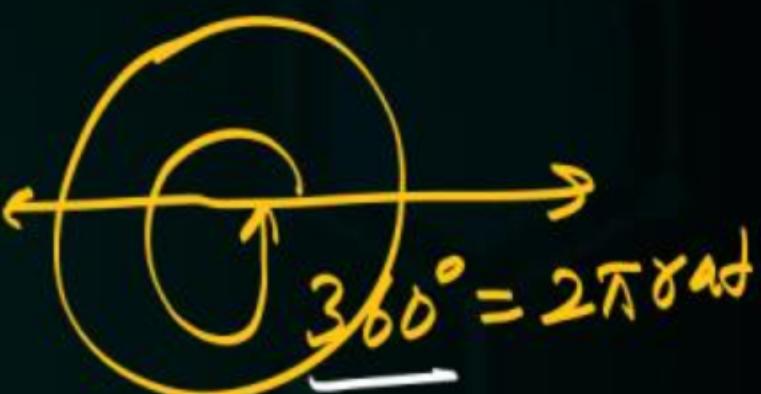
$$= 5 \times 60^\circ = 5 \times 60 \times \frac{\pi \text{ rad}}{180^\circ}$$

$$\boxed{\text{dist}^n = \frac{5\pi}{3} \text{m}}$$

$$60^\circ = \cancel{60} \times \frac{\pi \text{ rad}}{180^\circ}$$
$$= \frac{\pi \text{ rad}}{3}$$

Question

When a clock shows 4 O'clock, how much angle do its minute and hour hand make.

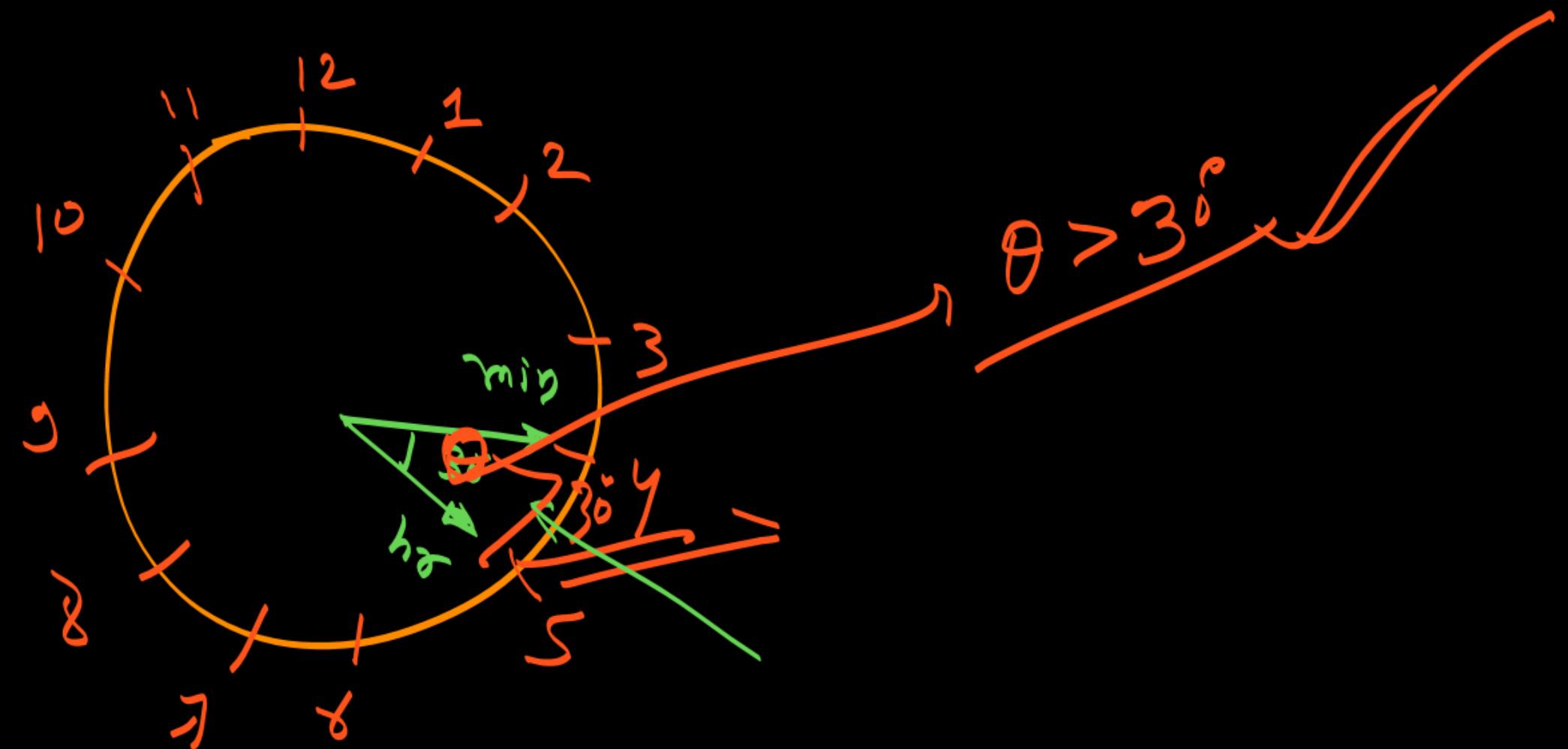


$$12\alpha = 360^\circ$$

$$\alpha = 30^\circ$$

$$\begin{aligned}4\alpha &= \theta \\ \theta &= 4 \times 30 \\ &= 120^\circ\end{aligned}$$

5 hr : 20 mint



find total Angle moved by object in π rotation ??

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Soln

Angle in one rotⁿ = $(2\pi \text{ rad})$

|| || π rotⁿ = $(2\pi) \pi \text{ rad}$

= $2\pi^2 \text{ rad}$

Ans

$\pi \text{ rad} \rightarrow \text{Angle}$

$\pi \text{ Newton} = F$

$\pi \text{ Joule} = E$



TRIGONOMETRIC RATIO



$$H^2 = P^2 + B^2 \quad -\star$$



Unitless, Dimensionless

$$\sin \theta = \frac{P}{H}$$

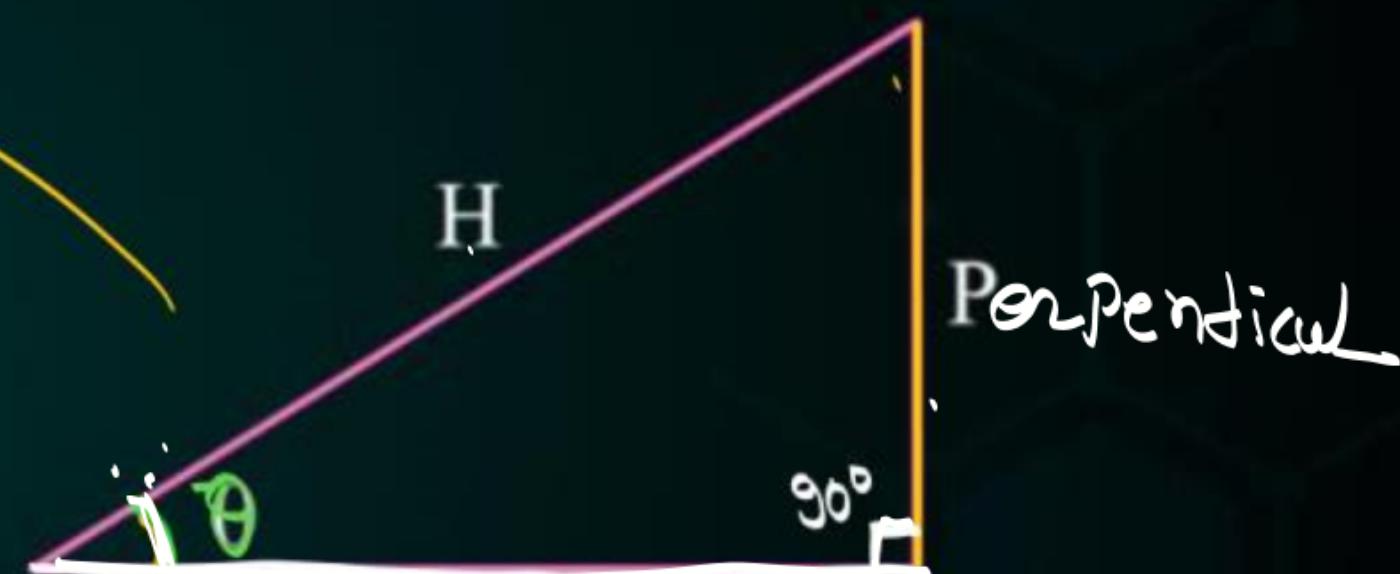
$$\frac{H}{P} = \csc \theta$$

$$\cos \theta = \frac{B}{H}$$

$$\frac{H}{B} = \sec \theta$$

$$\tan \theta = \frac{P}{B}$$

$$\frac{B}{P} = \cot \theta$$



$$\begin{bmatrix} \cos \theta \cdot \sec \theta = 1 \\ \tan \theta \cdot \cot \theta = 1 \\ \sin \theta \cdot \csc \theta = 1 \end{bmatrix}$$

$$H = \sqrt{P^2 + B^2}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

P
W

	0°	30°	45°	60°	90°	120°	135°	150°	180°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\frac{1}{\sqrt{3}} = \infty$	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

$$\theta \rightarrow 0^\circ \text{ to } 90^\circ$$

$\sin \theta \uparrow$

$\cos \theta \downarrow$

$$\tan \theta = \frac{\sin \theta \uparrow}{\cos \theta \downarrow} = \uparrow$$

$$\leq \sin \theta \leq$$

$$\leq \cos \theta \leq$$

$$\leq \tan \theta \leq$$

Maxima



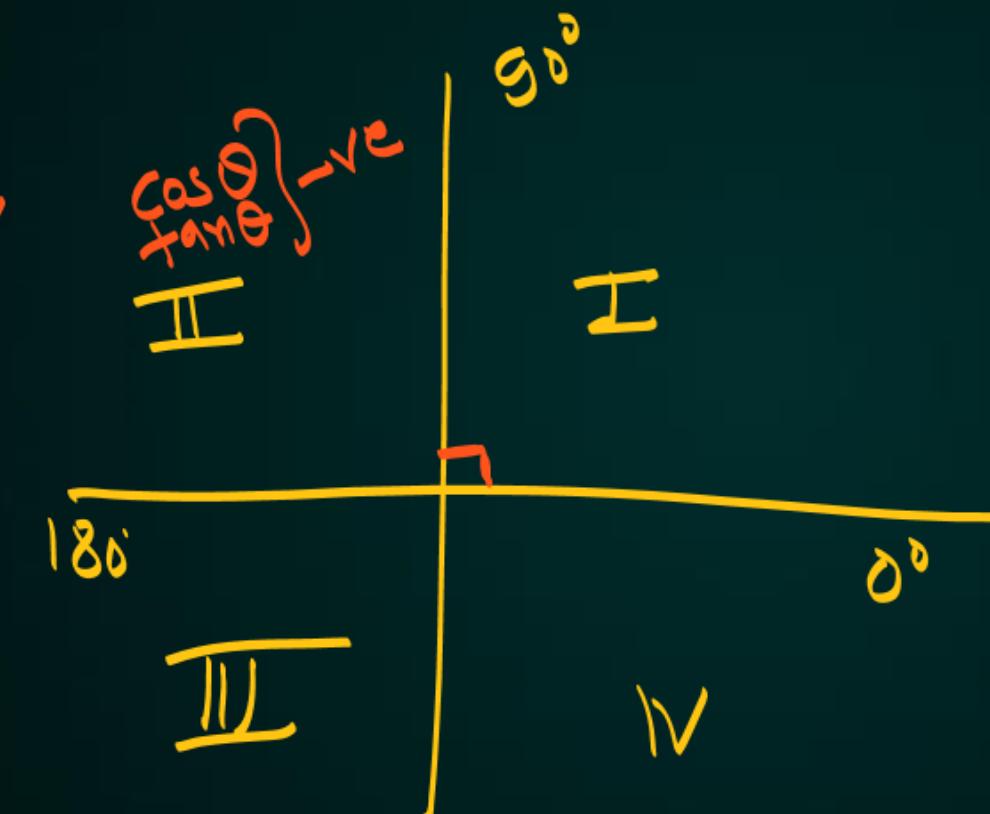
0°	30°	45°	60°	90°	120°	135°	150°	180°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	-1

...

Question

Find value of $\cos(120^\circ)$, $\sin(150^\circ)$ and $\tan(135^\circ)$ are

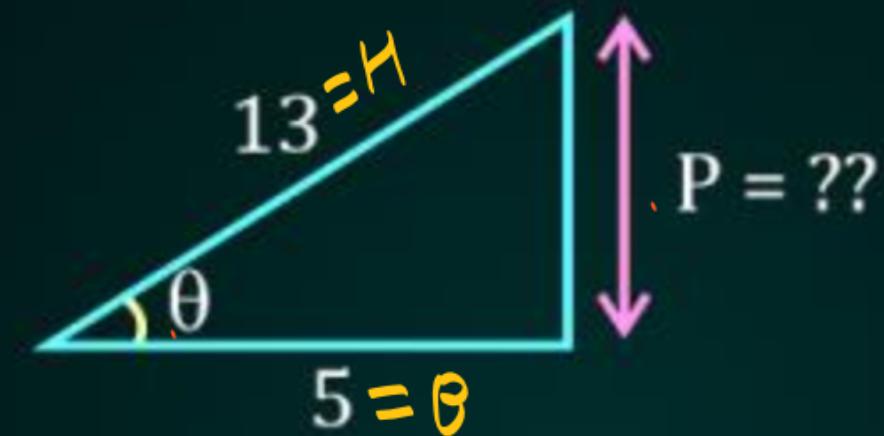
- 1 $-\frac{\sqrt{3}}{20}, \frac{1}{2}, 1$ ~~\times~~
- 2 $\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}, -1$ ~~\times~~
- 3 $-\frac{1}{2}, \frac{1}{2}, -1$ $-\frac{1}{2}, \frac{1}{2}, -1$
- 4 $-\frac{1}{2}, \frac{\sqrt{3}}{2}, -1$



$$\cos(120^\circ) = -\frac{1}{2}$$
$$\sin(150^\circ) = \frac{1}{2}$$
$$\tan(135^\circ) = -1$$

Question

Find angle and perpendicular



$$\tan \theta = \frac{12}{5}$$

$$\rightarrow \theta = \tan^{-1} \left(\frac{12}{5} \right)$$

$$\sin \theta = \frac{12}{13}$$

$$\rightarrow \theta = \sin^{-1} \left(\frac{12}{13} \right)$$

$$P^2 + B^2 = H^2$$

$$P^2 + (5)^2 = (13)^2$$

$$P^2 = 13^2 - 5^2$$

$$P = \sqrt{144} = \underline{\underline{12}}$$

Question

If $\frac{A_1}{A_2} = \frac{7}{3}$ then find $\frac{A_1 + A_2}{A_1 - A_2}$

$$\frac{A_1}{A_2} = \frac{7}{3}$$

$$\frac{A_1 + A_2}{A_1 - A_2} = \frac{7 + 3}{7 - 3} = \frac{10}{4} = 2.5$$

Question

If $\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} = \frac{5}{3}$ then find $\frac{I_1}{I_2}$.

H/W

Let $\sqrt{I_1} = x$ $\sqrt{I_2} = y$ $\frac{x}{y} = \frac{8}{7} = \frac{1}{1}$

$\frac{x}{y} = \frac{5+3}{5-3}$

$\frac{x}{y} = \frac{8}{2}$

$\frac{x}{y} = 4$

$\frac{x+y}{x-y} = \frac{5}{3}$

$3x + 3y = 5x - 5y$

$8y = 2x$

$$\sqrt{\frac{I_1}{I_2}} = \frac{1}{1}$$

$$\frac{I_1}{I_2} = \frac{16}{1}$$

Ans

Question

P
W

A & B are angle

Find value

(i) $\sin(A + B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

(ii) $\sin(A - B) = \sin A \cos B - \cos A \cdot \sin B$

(iii) $\cos(A + B) = \cos A \cdot \cos B - \sin A \cdot \sin B$ →

(iv) $\cos(A - B) = \cos A \cdot \cos B + \sin A \cdot \sin B$ ✓

(v) $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

→ Not useful

$\sin(A + B) = \sin A \cdot \cos B + \sin B \cdot \cos A$

$$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$$

$$gf [A=B=\theta] \leftarrow$$

$$\sin(2\theta) = 2 \sin \theta \cdot \cos \theta$$

$$gf [2\theta = \alpha]$$

$$\sin \alpha = 2 \sin\left(\frac{\alpha}{2}\right) \cdot \cos\left(\frac{\alpha}{2}\right)$$

half Angle formula

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\rightarrow \sin^2 \theta = 1 - \cos^2 \theta$$

MR Raffiq

$$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$$

$$gf [A=B=\theta]$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$\cos \theta = \cos^2 \theta/2 - \sin^2 \theta/2$$

half angle

$$\begin{aligned} \cos(2\theta) &= \cos^2 \theta - (1 - \cos^2 \theta) \\ &= \cos^2 \theta - 1 + \cos^2 \theta \end{aligned}$$

$$\cos 2\theta = 2\cos^2 \theta - 1$$

$$2\cos^2 \theta = 1 + \cos(2\theta)$$

Question

Find value of

(i)

$$\sin(75^\circ) = \sin(45^\circ + 30^\circ) = \sin 45^\circ \cos 30^\circ + \sin 30^\circ \cos 45^\circ = \frac{1}{\sqrt{2}} \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{\sqrt{2}}$$

(ii)

$$\sin(105^\circ) = \sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \sin 45^\circ \cos 60^\circ$$

(iii)

$$\cos(120^\circ) = -\frac{1}{2}$$

60°

$$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3} + 1}{2\sqrt{2}} \quad *$$



$$\begin{cases} \cos(75^\circ) = x \\ \cos(105^\circ) = -x \end{cases}$$

$$\cos(15^\circ) = \cos(60^\circ - 45^\circ) = \cos 60^\circ \cdot \cos 45^\circ + \sin 60^\circ \sin 45^\circ = \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{1 + \sqrt{3}}{2\sqrt{2}}$$

$$= \cos(45^\circ - 30^\circ) = \cos 45^\circ \cdot \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3} + 1}{2\sqrt{2}}$$



Question

Not for all.

$\sqrt{1 - \sin \theta}$ is equal to

- 1 $(\sin \theta + \cos \theta)$
- 2 $(\sin \theta - \cos \theta)$
- 3 $\sin \frac{\theta}{2} + \cos \frac{\theta}{2}$
- 4 $\sin \frac{\theta}{2} - \cos \frac{\theta}{2}$

Sol

$$\sqrt{1 - \sin \theta} = \sqrt{\left(\sin \frac{\theta}{2} + \cos \frac{\theta}{2}\right)^2 - 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}$$
$$= \sqrt{\left(\sin \frac{\theta}{2} - \cos \frac{\theta}{2}\right)^2}$$
$$= \underline{\underline{\sin \frac{\theta}{2} - \cos \frac{\theta}{2}}}$$

A.B

$(a-b)^2 = a^2 + b^2 - 2ab$

$(a+b)^2 = a^2 + b^2 + 2ab$

$(a+b)(a-b) = a^2 - b^2$

hind

$$\sin 2\theta = 2 \sin \theta \cdot \cos \theta$$
$$\sin \theta = 2 \sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2}$$

$$\# \quad \sqrt{1 + \sin \theta} = \sqrt{\underbrace{\sin^2 \frac{\theta}{2}}_{\text{ }} + \underbrace{\cos^2 \frac{\theta}{2}}_{\text{ }} + 2 \sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2}}$$

$$= \sqrt{\left(\sin \frac{\theta}{2} + \cos \frac{\theta}{2}\right)^2}$$

$$= \left[\sin \frac{\theta}{2} + \cos \frac{\theta}{2} \right]$$

Question

Not for all

$\sqrt{1 + \cos \theta}$ is equal to

$$= \sqrt{2 \cos^2 \frac{\theta}{2}}$$

$$= \sqrt{2} \cos \frac{\theta}{2}$$

1 $\sqrt{2} \sin \frac{\theta}{2}$

2 $\sqrt{2} \cos \frac{\theta}{2}$

3 $\frac{1}{\sqrt{2}} \sin \frac{\theta}{2}$

4 $\frac{1}{\sqrt{2}} \cos \frac{\theta}{2}$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

half angle

$$\cos \theta = \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}$$

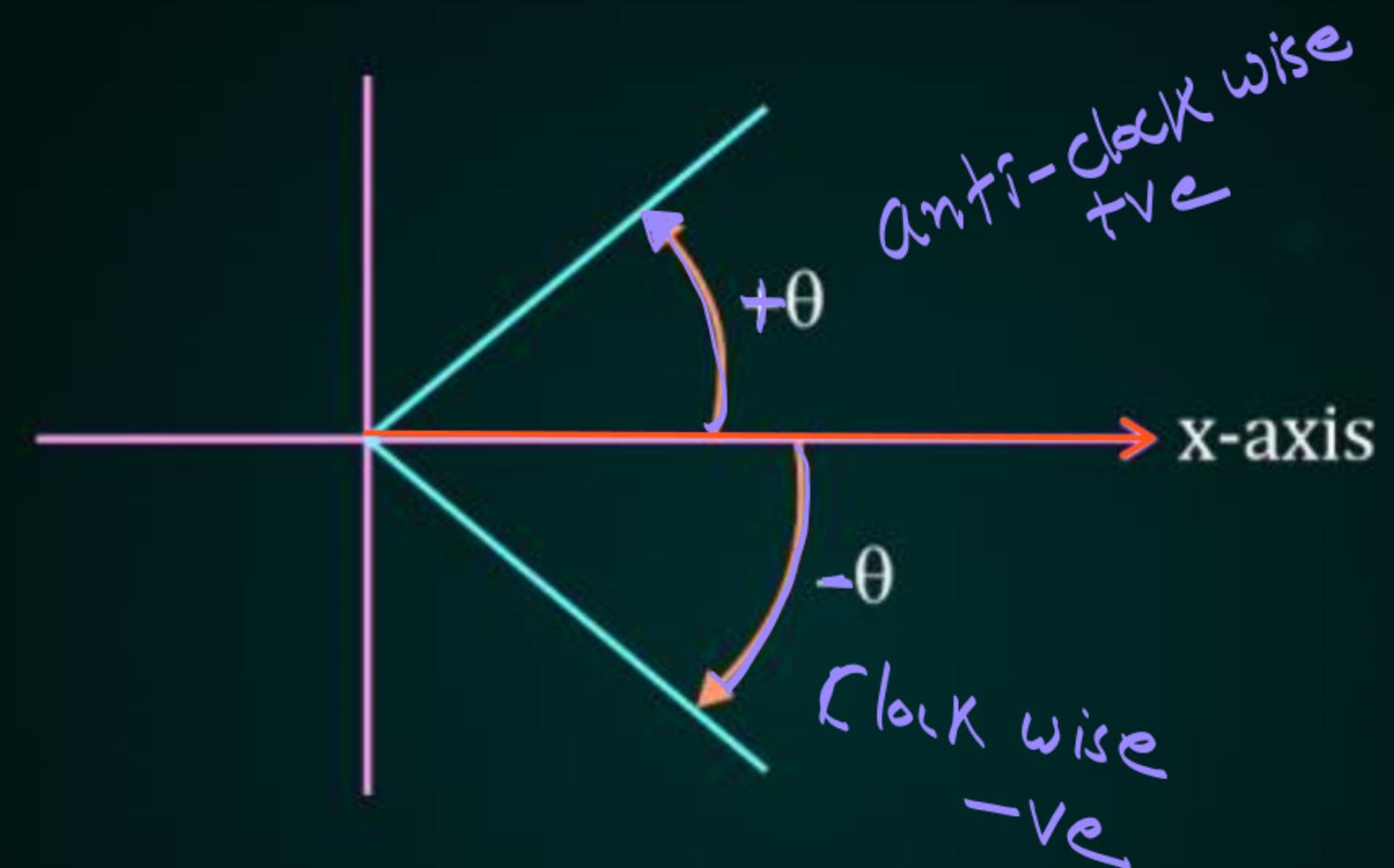
$$\cos \theta = \cos^2 \frac{\theta}{2} - (1 - \cos^2 \frac{\theta}{2})$$

$$\cos \theta = 2 \cos^2 \frac{\theta}{2} - 1$$

$$1 + \cos \theta = 2 \cos^2 \frac{\theta}{2}$$

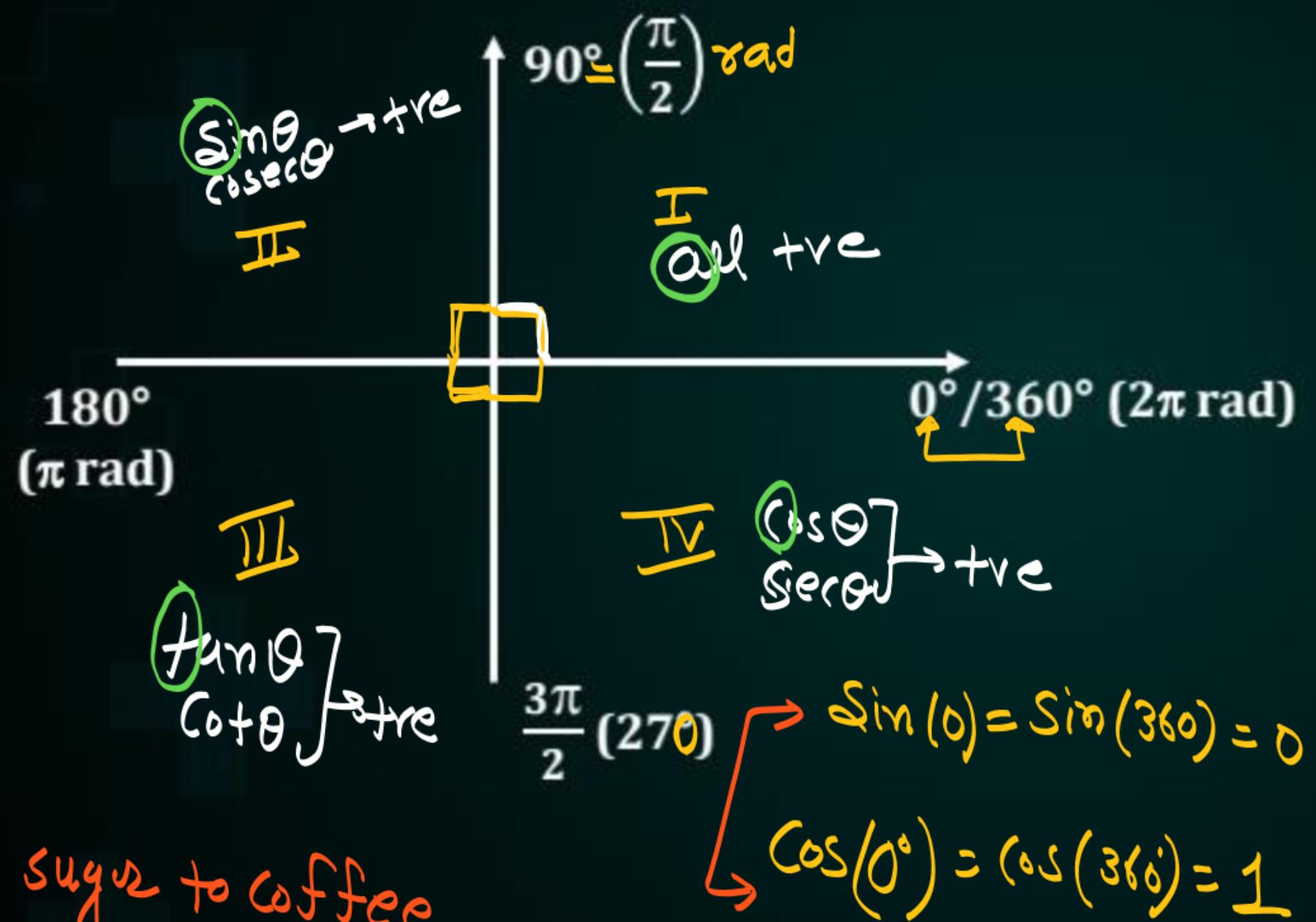
Question

Find..





TRIGONOMETRY FUNCTION CHARGE



$$\sin(120^\circ) = \sin(90^\circ + 30^\circ)$$

$$= \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$\sin(90^\circ - \theta) = \sin 90^\circ \cos \theta - \cos 90^\circ \sin \theta$

$= \cos \theta$ ✓

* $\cos(90^\circ - \theta) = \cos 90^\circ \cos \theta + \sin 90^\circ \sin \theta$

$\sin(90^\circ + \theta) = \cos \theta$ ←

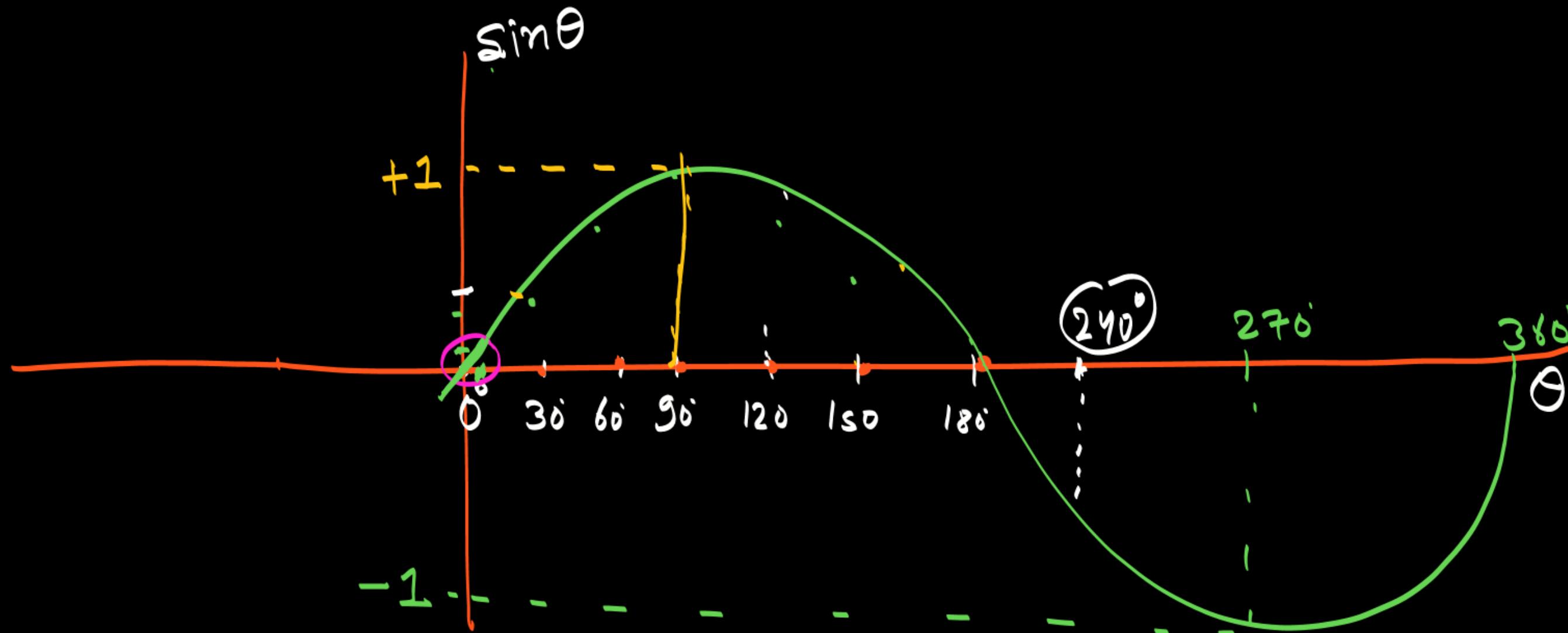
* $\sin(180^\circ - \theta) = \sin 180^\circ \cos \theta - \cos 180^\circ \sin \theta$

$\cos(180^\circ - \theta) =$

*

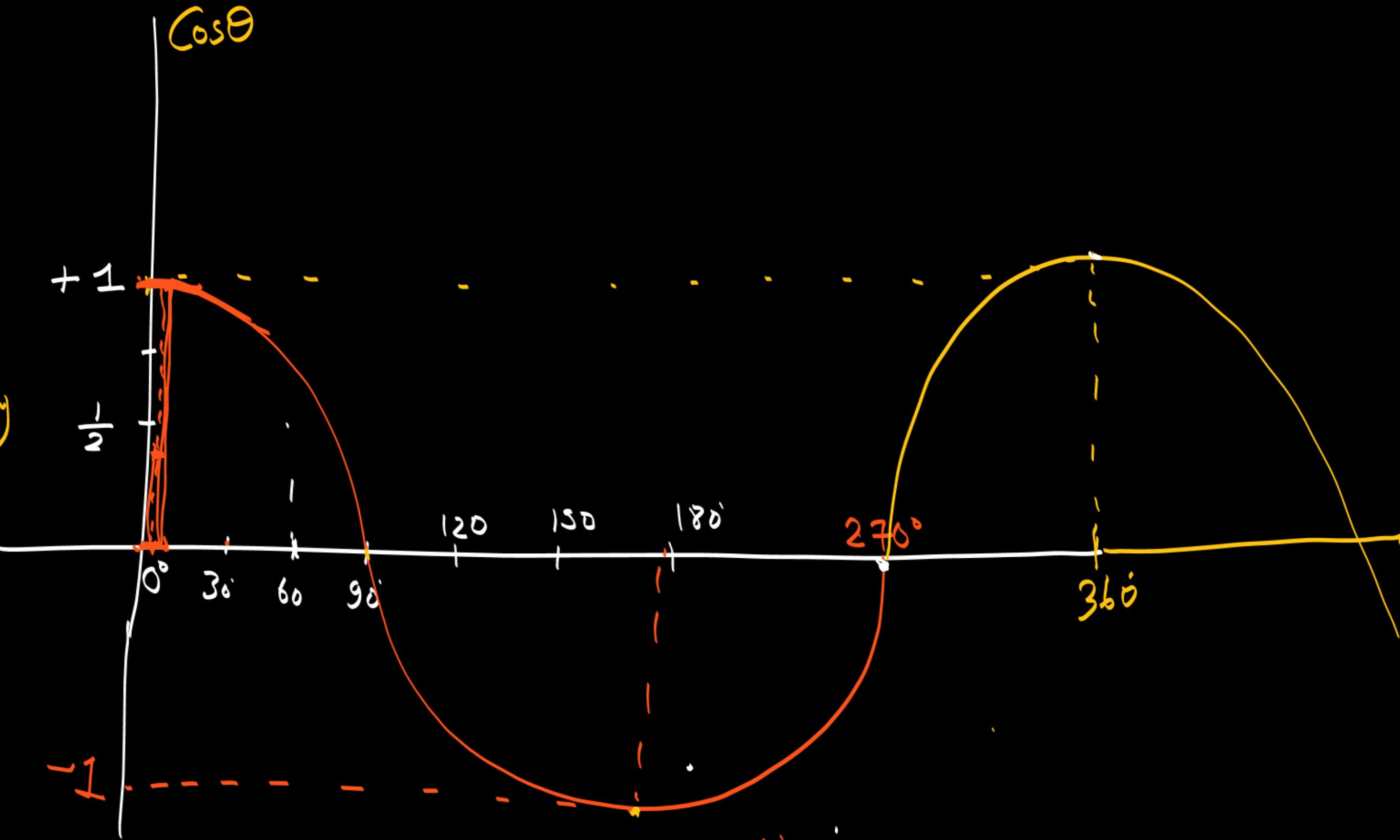
$$\begin{aligned}\sin(180 - \theta) &= \sin 180^\circ \cos \theta - \cos 180^\circ \sin \theta \\ &= -(-1) \sin \theta \\ &= +\sin \theta\end{aligned}$$

$$\begin{aligned}\rightarrow \cos(90 + \theta) &= \cos 90^\circ \cos \theta - \sin 90^\circ \sin \theta \\ &= -\sin \theta\end{aligned}$$



$$\cos 0^\circ = 1$$
$$\cos \theta = 1$$

small ($up \rightarrow \theta$)



$\checkmark \sin(90 - \theta) =$

$\checkmark \cos(90 - \theta) =$

$\checkmark \boxed{\sin(90 + \theta) = \sin 90 \cos \theta + \cos 90 \sin \theta}$
 $\quad \quad \quad = \cos \theta$

$\checkmark \cos(90 + \theta) =$

$\checkmark \sin(180 - \theta) =$

$\times \boxed{\cos(180 - \theta) = \cos(180) \cdot \cos \theta + \sin 180 \sin \theta}$
 $\quad \quad \quad = -\cos \theta$

$\cos(270 - \theta) = \cos(270) \cos \theta + \sin 270 \sin \theta$

$\boxed{\cos(270 - \theta) = -\sin \theta}$

$\cos 0^\circ = 1$
 $\cos 180^\circ = -1$

$\boxed{\cos 90^\circ = \cos 270^\circ = 0}$

$\sin 0^\circ = \sin 180^\circ = 0$

$\sin 90^\circ = +1$

$\sin(270) = -1$

Question

Find value of

$$(i) \sin(-30^\circ) = \sin(0-30^\circ) = \sin 0^\circ \cos 30^\circ - \cos 0^\circ \sin 30^\circ = -\sin 30^\circ = -\frac{1}{2}$$

$$(ii) \cos(-60^\circ) = \cos(0-60^\circ) = \cos 0^\circ \cos 60^\circ + \sin 0^\circ \sin 60^\circ = \cos 60^\circ = \frac{1}{2}$$

$$(iii) \sin(120^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$(iv) \sin(390^\circ) = \sin(360^\circ + 30^\circ) = \sin 30^\circ = \frac{1}{2}$$

$$(v) \tan(150^\circ) = -\frac{1}{\sqrt{3}}$$

$$(vi) \tan(-30^\circ) = \tan(0-30^\circ) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B} = \frac{\tan 0^\circ - \tan 30^\circ}{1 + \tan 0^\circ \tan 30^\circ} = \frac{-\tan 30^\circ}{1} = -\tan 30^\circ$$

odd function

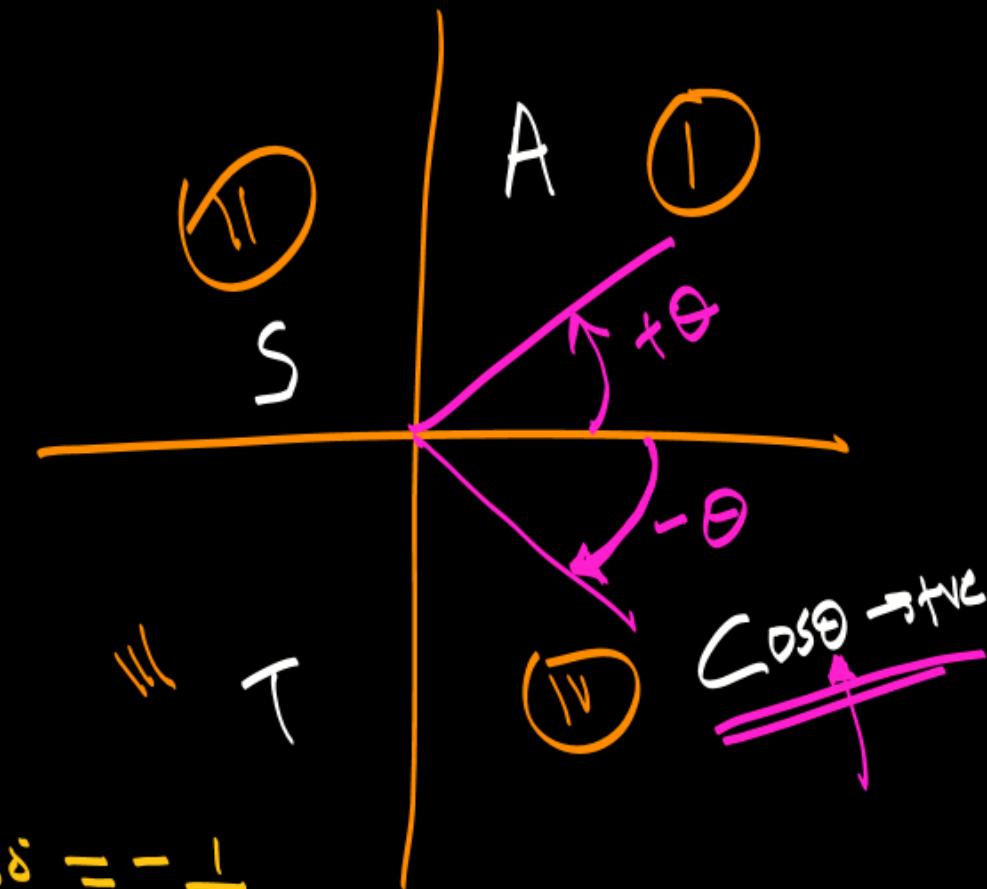
$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta \leftarrow \text{even funct.}$$

$$\tan(-\theta) = -\tan\theta$$

$$\begin{bmatrix} \cos(-30^\circ) = \cos 30^\circ = \sqrt{3}/2 \\ \cos(-45^\circ) = \cos 45^\circ = \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\begin{array}{||} \begin{array}{l} \sin(-30^\circ) = -\sin 30^\circ = -\frac{1}{2} \\ \tan(-45^\circ) = -\tan 45^\circ = -1 \end{array} \end{array}$$



Question

$$0^\circ - 180^\circ$$

Find value of different trigonometric function.

$$\Rightarrow \sin(240^\circ) = \sin(360^\circ - 120^\circ) = \sin(-120^\circ) = -\sin(120^\circ) = -\frac{\sqrt{3}}{2} \quad \text{Ans}$$

1 $\sin(300^\circ) = \sin(360^\circ - 60^\circ) = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$

2 $\cos(240^\circ) = \cos(360^\circ - 120^\circ) = \cos(-120^\circ) = \cos(120^\circ) = -\frac{1}{2}$

$\sin(180^\circ + 60^\circ) = \sin 180^\circ + \sin 60^\circ \cos 180^\circ$

$= -\frac{\sqrt{3}}{2}$

3 $\sin(-120^\circ) = -\sin(120^\circ) = -\frac{\sqrt{3}}{2}$

4 $\cos(330^\circ) = \cos(360^\circ - 30^\circ)$

$= \cos(-30^\circ)$

$= \cos 30^\circ = \frac{\sqrt{3}}{2}$

$\sin(240^\circ) = \sin(2 \times 120^\circ) = 2 \sin 120^\circ \cos 120^\circ$

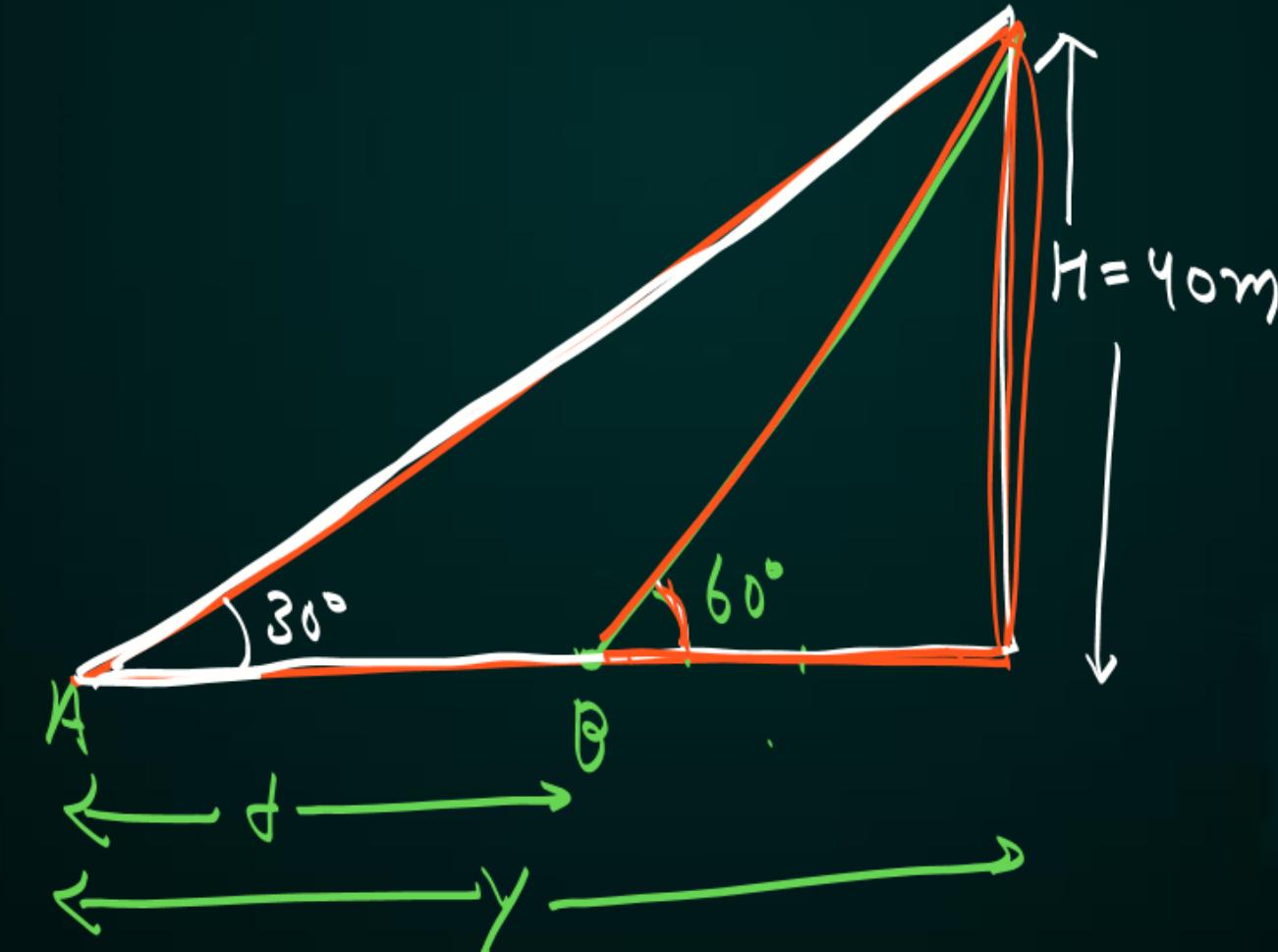
$= 2 \frac{\sqrt{3}}{2} \times -\frac{1}{2} = -\frac{\sqrt{3}}{2}$

$\sin(270^\circ - 30^\circ) =$

Question

A person, at any instant finds angle of elevation of 40 m high building as 30° . After some time the angle of elevation becomes 60° , then value of distance, the person moved towards the building will be :

- 1 $d = \frac{40}{\sqrt{3}}$
- 2 $d = \frac{80}{\sqrt{3}}$
- 3 $d = \frac{60}{\sqrt{3}}$
- 4 $d = \frac{120}{\sqrt{3}}$

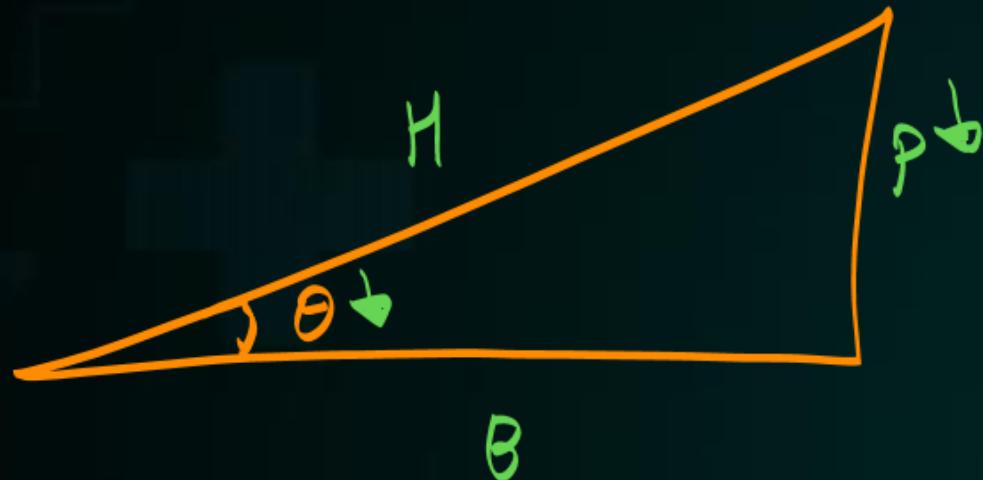


$$\tan 30^\circ = \frac{H_0}{y} - 10$$

$$\tan 60^\circ = \frac{H_0}{y-d} - 0$$

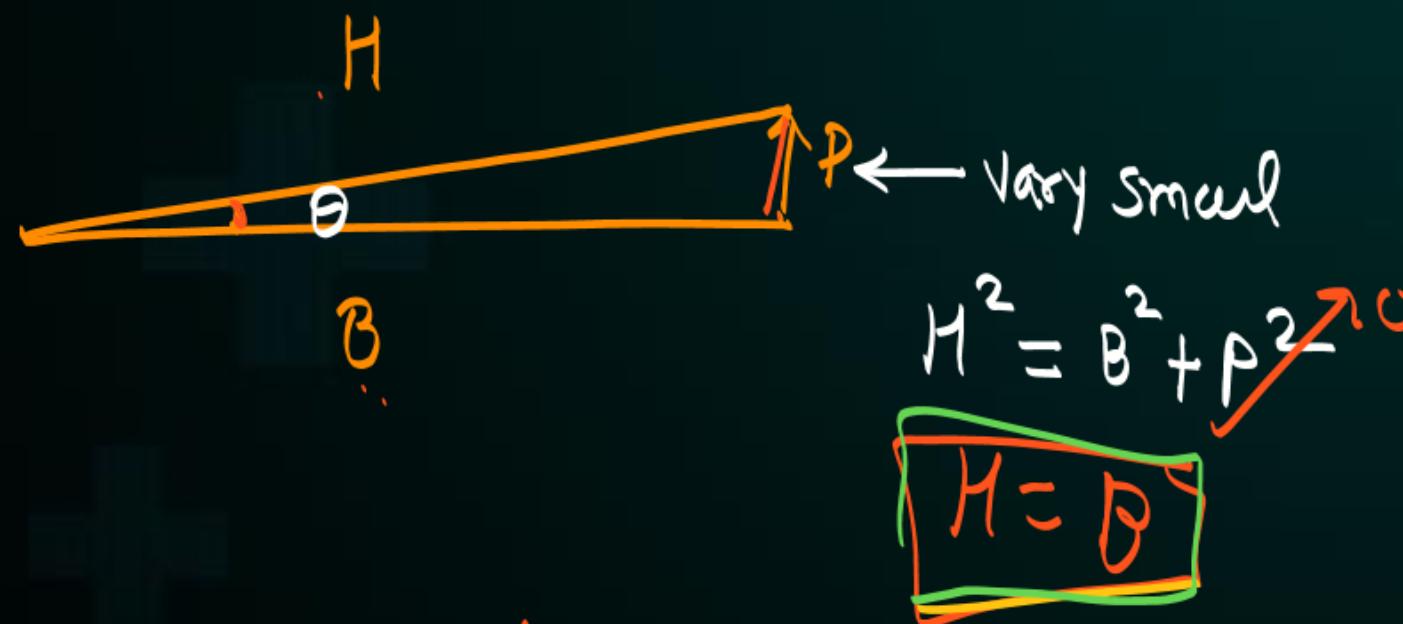


SMALL ANGLE APPROXIMATION



$$\cos \theta = \frac{B}{H}$$

if θ is very small [$\theta \leq 7^\circ$]



$$H = B$$

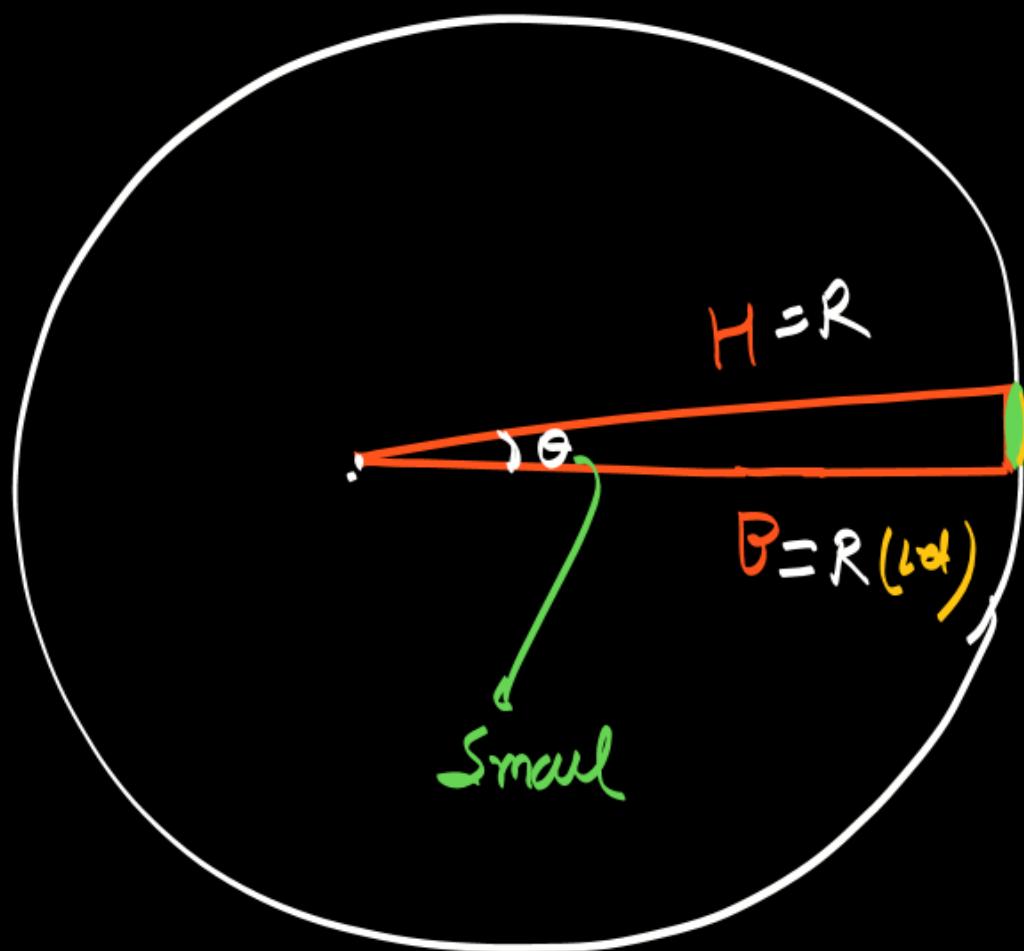
✖ $\cos \theta = \frac{B}{H} = 1$

$$\left. \begin{array}{l} \cos 1^\circ = 1 \\ \cos 2^\circ = 1 \\ \cos 4^\circ = 1 \\ \cos 5^\circ = 1 \end{array} \right\}$$

$$\sec 3^\circ = 1$$

$$\cancel{\cos 360^\circ} = \cos 0^\circ$$

$$\sec 3^\circ = \frac{1}{\cos 3^\circ} = \frac{1}{1}$$



$$\sin\theta = \frac{P}{R} = \frac{R\theta}{R}$$

$$\sin\theta = \theta$$

Angle is very small

$$\tan\theta = \frac{P}{B} = \frac{R\theta}{R} = \theta$$

\uparrow Small

$$\frac{\sin\theta}{\theta} = \frac{\theta}{\theta} = 1$$

for small angle $\sin\theta = \tan\theta$

\uparrow Small

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\theta}{1}$$

Question

P
W

Find value :

(i) $\sin 2^\circ = 2^\circ \times \frac{\pi \text{ rad}}{180} = \frac{\pi}{90^\circ} \checkmark$

(ii) $\tan 3^\circ = 3^\circ \times \frac{\pi \text{ rad}}{180} = \frac{\pi}{60} \checkmark$

(iii) $\cos 3^\circ = 1$

(iv) $\sin (88.5^\circ) = 1$



$\sin 2^\circ = 2^\circ \times \frac{\pi \text{ rad}}{180} = \frac{\pi}{90^\circ} \checkmark$

$\sin \theta = \theta$ \checkmark

Question

Correct value of $\cos(2^\circ)$

1 2°

2 1

3 $\pi/50$

4 0

Question

Find value of $\tan (3^\circ)$

- 1 3°
- 2 $\sin (3^\circ)$
- 3 $\frac{\pi}{60}$ rad
- 4 Both (2) and (3)

H/ω

Thank You