#33: Graph Vocabulary + Implementation

April 11, 2018 · *Wade Fagen-Ulmschneider*

Motivation:

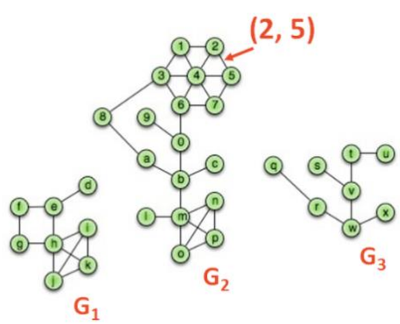
Graphs are awesome data structures that allow us to represent an enormous range of problems. To study these problems, we need:

1. A common vocabulary to talk about graphs
2. Implementation(s) of a graph
3. Traversals on graphs
4. Algorithms on graphs

Graph Vocabulary

Consider a graph **G** with vertices **V** and edges **E**, **G=(V,E)**.

|V| = n
|E| = m



Incident Edges:
I(v) = { (x, v) in E }

Degree(v): **|I|**

Adjacent Vertices:
A(v) = { x : (x, v) in E }

Path(G_2): Sequence of vertices connected by edges

Cycle(G_1): Path with a common begin and end vertex.

Simple Graph(G): A graph with no self loops or multi-edges.

Subgraph(G): **G' = (V', E')**:
V' ∈ V, E' ∈ E, and (u, v) ∈ E → u ∈ V', v ∈ V'

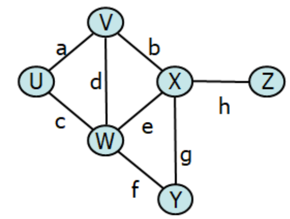
Graphs that we will study this semester include:

Complete subgraph(G)
Connected subgraph(G)
Connected component(G)
Acyclic subgraph(G)
Spanning tree(G)

Size and Running Times

Running times are often reported by **n**, the number of vertices, but often depend on **m**, the number of edges.

For arbitrary graphs, the minimum number of edges given a graph that is:



Not Connected: m = 0

Minimally Connected:* m = n - 1

The maximum number of edges given a graph that is:

Simple: n(n-1) / 2 ~ O(n^2)

Not Simple: Infinite
unbounded number of edges

The relationship between the degree of the graph and the edges:

Sum(deg(v)) = 2m

Proving the Size of a Minimally Connected Graph

Theorem: Every minimally connected graph **G=(V, E)** has **|V|-1** edges.

Proof of Theorem

Consider an arbitrary, minimally connected graph **G=(V, E)**.

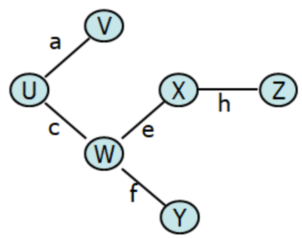
Lemma 1: Every connected subgraph of **G** is minimally connected.
(Easy proof by contradiction left for you.)

Inductive Hypothesis: For any **j** < **|V|**, any minimally connected graph of **j** vertices has **j-1** edges.

Suppose |V| = 1:
Definition: A minimally connected graph of 1 vertex has 0 edges.
Theorem: |V|-1 edges ➔ 1-1 = 0.

Suppose |V| > 1:
Choose any vertex **u** and let **d** denote the degree of **u**.

Remove the incident edges of **u**, partitioning the graph into ____ components: $C_0 = (V_0, E_0)$, …, $C_d = (V_d, E_d)$.



By Lemma 1, every component C_k is a minimally connected subgraph of **G**.

By our ___________________________:

Finally, we count edges:

Graph ADT

Data	Functions

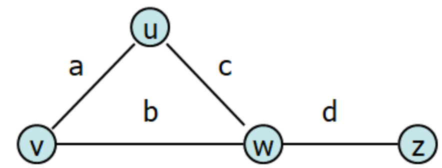
Vertices
 	
Edges
 	
Some data structure maintaining the structure between vertices and edges.
	`insertVertex(K key);`
	`insertEdge(Vertex v1, Vertex v2,`
	`          K key);`
	
	`removeVertex(Vertex v);`
	`removeEdge(Vertex v1, Vertex v2);`
	
	`incidentEdges(Vertex v);`
	`areAdjacent(Vertex v1, Vertex v2);`
	
	`origin(Edge e);`
	`destination(Edge e);`

Graph Implementation #1: Edge List

Vert. | **Edges**

u
v
w
z

		a
		b
		c
		d



Operations:
insertVertex(K key):

removeVertex(Vertex v):

areAdjacent(Vertex v1, Vertex v2):

incidentEdges(Vertex v):

Graph Implementation #2: Adjacency Matrix

Vert. | **Edges** | **Adj. Matrix**

u
v
w
z

		a
		b
		c
		d

	u	**v**	**w**	**z**
u				
v				
w				
z				

CS 225 – Things To Be Doing:

1. Topic list for Programming Exam C available; starts Tuesday 4/17
2. lab_puzzles released today
3. MP6 released due on Monday, April 16th
4. Daily POTDs are ongoing!