RDEx: An effectiveness-driven hybrid Single-Objective optimizer that adaptively selects and combines multiple operators and strategies

1st Sichen Tao
Department of Engineering
University of Toyama
Toyama-shi 930-8555, Japan
taosc73@hotmail.com

2nd Yifei Yang
Faculty of Science and Technology
Hirosaki University
Hirosaki-shi 036-8560, Japan
yyf7236@hirosaki-u.ac.jp

3rd Ruihan Zhao
School of Mechanical Engineering
Tongji University
Shanghai-shi 200082, China
ruihan.z@outlook.com, 2110415@tongji.edu.cn

4th Kaiyu Wang
Chongqing Institute of Microelectronics Industry Technology
University of Electronic Science and Technology of China
Chongqing-shi 401332, China
greskowky1996@163.com

5th Sicheng Liu
Department of Information Engineering
Yantai Vocational College
Yantai-shi 264670, China
lsctoyama2020@gmail.com

6th Shangce Gao
Department of Engineering
University of Toyama
Toyama-shi 930-8555, Japan
gaosc@eng.u-toyama.ac.jp

***Abstract*—Differential Evolution (DE) has demonstrated outstanding performance on continuous optimization. Building on DE, advanced variants have achieved markedly superior results on black-box, bound-constrained problems. However, any single operator or strategy is rarely sufficient for the diversity of real-world tasks; consequently, hybrid algorithms often deliver better effectiveness and generality in practice. In this work, we improve LSRTDE by hybridizing several of the most distinctive strategies from prior DE variants, addressing DE's common weaknesses—limited exploration, loss of population diversity, and late-stage stagnation. Evaluated on the CEC 2025 single-objective, bound-constrained (SOP) benchmark suite, our extended method RDEx attains state-of-the-art performance to date.**

I. Introduction

Differential Evolution (DE), introduced by Storn and Price [1], has become a cornerstone algorithm in continuous global optimization due to its simplicity and strong exploratory behavior. DE has been applied successfully in diverse domains, including engineering design, bioinformatics, robotics, and neural architecture search.

Despite its general effectiveness, the classical DE algorithm faces challenges in maintaining a proper balance between exploration and exploitation, particularly in high-dimensional, multimodal, and constrained optimization problems. Over the last two decades, researchers have proposed numerous DE variants to improve convergence speed, robustness, and adaptability.

SHADE [2] introduced a historical memory mechanism to adapt the scale factor F and crossover rate CR dynamically. **L-SHADE** [3] further extended this by implementing linear population size reduction (LPSR), making it more effective during the late stages of convergence. **JADE** [4] added an archive-based current-to-$p$best strategy, enhancing local exploitation while preserving diversity. **iLSHADE-RSP** [5] and **EBLSHADE** [6] respectively: (i) inject a Cauchy-distributed perturbation into the target vector during the crossover stage, and (ii) introduce a rank-guided differential operator that steers the search toward currently superior solutions. Finally, **L-SRTDE** [7], the CEC 2024 winner, combined LPSR with success-rate-based adaptive learning, achieving a notable balance of performance and stability.

Despite these innovations, many advanced DE variants suffer from increased algorithmic complexity, introducing multiple control parameters or relying on strategy pools that require careful tuning. Furthermore, they often exhibit reduced performance when facing deceptive landscapes or constrained feasible regions due to a lack of spontaneous diversity restoration.

To address these limitations, we propose **RDEx**, a parameter-light, adaptive DE variant that introduces two key innovations:

- **Elite Ranking Mutation:** In the later stages of convergence, RDEx biases mutation toward elite individuals in the population, increasing exploitation intensity and accelerating convergence without sacrificing stability.

- **Dimension-wise Cauchy Perturbation:** During crossover, each dimension of a trial solution can undergo a rare, heavy-tailed perturbation based on the Cauchy distribution, effectively reintroducing diversity and allowing escape from local minima.

We evaluated RDEx on the **CEC 2025 benchmark suite**, which consists of 29 real-parameter functions ranging from unimodal to hybrid and composite types. RDEx successfully solved 29 functions to the defined accuracy threshold, outperforming competitive algorithms such as L-SHADE and L-SRTDE. The algorithm also showed superior convergence speed and robustness across dimensions.

II. Method

A. Population Initialization

Let D denote the dimensionality of the problem and N the population size. Each individual $\mathbf{x}_{i,0}$ is initialized randomly within predefined bounds $[l_j, u_j]$:

$$x_{i,j,0} \sim \mathcal{U}(l_j, u_j), \quad i = 1, ..., N; \; j = 1, ..., D \tag{1}$$

B. Mutation and Crossover

In the initial phase, RDEx adopts the DE/rand/1 mutation strategy:

$$\mathbf{v}_{i,g} = \mathbf{x}_{r_1,g} + F \cdot (\mathbf{x}_{r_2,g} - \mathbf{x}_{r_3,g}) \tag{2}$$

Once convergence slows, an elite-based mutation is applied:

$$\mathbf{v}_{i,g} = \mathbf{x}_{i,g} + F \cdot (\mathbf{x}_{best,g} - \mathbf{x}_{i,g}) + F \cdot (\mathbf{x}_{r_1,g} - \mathbf{x}_{r_2,g}) \tag{3}$$

where $\mathbf{x}_{best,g}$ is the top-ranked solution.

Crossover is performed dimension-wise using binomial logic:

$$u'_{i,j,g} = \begin{cases} v_{i,j,g}, & \text{if } rand() < CR \text{ or } j = j_{rand} \\ x_{i,j,g}, & \text{otherwise} \end{cases} \tag{4}$$

C. Cauchy-Guided Perturbation

To prevent stagnation, a rare perturbation is applied independently to each dimension with probability p:

$$u_{i,j,g} = u'_{i,j,g} + \delta_j, \quad \delta_j \sim \text{Cauchy}(0, \gamma) \tag{5}$$

D. Selection and Replacement

Standard DE selection is used:

$$\mathbf{x}_{i,g+1} = \begin{cases} \mathbf{u}_{i,g}, & \text{if } f(\mathbf{u}_{i,g}) \leq f(\mathbf{x}_{i,g}) \\ \mathbf{x}_{i,g}, & \text{otherwise} \end{cases} \tag{6}$$

E. Hybrid Rate Update

The elite-guided mutation rate β_g is dynamically computed as:

$$\beta_g = \frac{\sum_{elite}(f_{old} - f_{new})}{\sum_{all}(f_{old} - f_{new}) + \epsilon} \tag{7}$$

where ϵ is a small constant to prevent division by zero.

III. Pseudocode

IV. Experiments and Results

A. Metrics and How to Read Table I

For each CEC 2025 function (F1–F29), we run *25* independent trials and report **best/mean/std/worst** of the *final* objective values (smaller is better). All problems are single-objective and bound-constrained (SOP), so feasibility is not an issue and no LCV is reported. Zero or near-zero entries indicate that the target accuracy is reached (CEC functions are shifted/rotated but have known optima; numerical zeros may appear as 10^{-k} due to printing precision).

How to read: (i) rows with `best=mean=worst` and `std≈0` (e.g., several F1–F5 cells) imply highly stable convergence across all 25 runs; (ii) large gaps between `best` and `mean` together with a large `std` (e.g., certain F9/F12–F19/F25/F29 cells) suggest multimodality or late-stage stagnation in part of the runs; (iii) composite/hybrid functions (F21–F29) typically yield higher `mean` and `std`, which is expected as difficulty increases. All values are computed from the final incumbent of each run under the same evaluation budget.

B. Overall Ranking (U-score)

Given a set of algorithms $\mathcal{A}$ and functions $\mathcal{F}$, we compute per-function ranks using the *mean* final objective as the primary criterion (lower is better). Ties are broken by, in order: *std* (lower is better), then *best* (lower is better), then *worst* (lower is better). For algorithm $a \in \mathcal{A}$, the aggregate U-score is

$$U(a) = \sum_{m \in \mathcal{F}} \text{rank}_m(a),$$

and lower $U(a)$ indicates a better overall performance. For significance, we apply the Wilcoxon signed-rank test on the per-function means (paired by function), with Holm–Bonferroni correction at $\alpha = 0.05$. Optionally, we also report the effect size $r = Z/\sqrt{|\mathcal{F}|}$ and, for multi-algorithm comparisons, a Friedman test followed by Nemenyi post-hoc analysis.

Table I presents the rankings computed by the U-score (smaller is better). LSRTDE is the CEC'24 champion. On the same SOP test suite, RDEx achieves a substantial improvement over prior SOP algorithms, further confirming the effectiveness of RDEx's mechanism.

References

[1] R. Storn and K. Price, "Differential evolution – a simple and efficient heuristic for global optimization over continuous spaces," *J. Global Optim.*, vol. 11, no. 4, pp. 341–359, 1997.

[2] R. Tanabe and A. Fukunaga, "Success-history based parameter adaptation for differential evolution," in *Proc. IEEE CEC*, pp. 71–78, 2013.

[3] R. Tanabe and A. Fukunaga, "Improving the search performance of SHADE using linear population size reduction," in *Proc. IEEE CEC*, pp. 1658–1665, 2014.

[4] J. Zhang and A. C. Sanderson, "JADE: Adaptive differential evolution with optional external archive," *IEEE Trans. Evol. Comput.*, vol. 13, no. 5, pp. 945–958, 2009.

[5] T. J. Choi and C. W. Ahn, "An improved LSHADE-RSP algorithm with the Cauchy perturbation: iLSHADE-RSP," *Knowledge-Based Systems*, vol. 215, Art. no. 106628, 2021.

TABLE I
RESULT.

	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10
best	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.24E+01	0.00E+00	0.00E+00	4.70E+00	0.00E+00
mean	0.00E+00	0.00E+00	1.36E+01	2.59E+00	1.60E-07	3.53E+01	2.07E+00	0.00E+00	2.40E+02	1.24E+00
std	0.00E+00	0.00E+00	3.19E+00	1.62E+00	1.89E-07	1.81E+00	1.40E+00	0.00E+00	2.06E+02	1.85E+00
worst	0.00E+00	0.00E+00	1.48E+01	4.97E+00	5.47E-07	3.82E+01	3.98E+00	0.00E+00	6.42E+02	3.99E+00
	F11	F12	F13	F14	F15	F16	F17	F18	F19	F20
best	6.94E-02	0.00E+00	0.00E+00	1.51E-01	7.03E-01	6.20E-01	4.84E-01	1.04E+00	0.00E+00	2.00E+02
mean	9.20E-01	9.15E+00	3.57E+00	3.76E-01	3.67E+00	1.82E+01	1.33E+01	1.81E+00	3.69E+00	2.02E+02
std	1.40E+00	6.77E+00	7.36E+00	1.00E-01	3.01E+00	8.68E+00	9.81E+00	5.24E-01	7.44E+00	1.49E+00
worst	4.07E+00	1.49E+01	2.00E+01	4.78E-01	8.36E+00	2.47E+01	2.05E+01	2.68E+00	2.03E+01	2.05E+02
	F21	F22	F23	F24	F25	F26	F27	F28	F29	
best	1.00E+02	3.32E+02	4.07E+02	3.78E+02	2.00E+02	4.45E+02	3.00E+02	3.52E+02	3.26E+02	
mean	1.00E+02	3.39E+02	4.15E+02	3.79E+02	5.81E+02	4.58E+02	3.00E+02	3.82E+02	6.88E+02	
std	0.00E+00	3.83E+00	4.53E+00	7.09E-01	1.53E+02	6.20E+00	0.00E+00	1.55E+01	3.89E+02	
worst	1.00E+02	3.44E+02	4.20E+02	3.80E+02	7.56E+02	4.65E+02	3.00E+02	4.01E+02	1.32E+03	

[6] Y. Song, D. Wu, A. W. Mohamed, X. Zhou, B. Zhang, and W. Deng, "Enhanced success history adaptive DE for parameter optimization of photovoltaic models," *Complexity*, vol. 2021, no. 1, Art. no. 6660115, 2021.

[7] V. Stanovov et al., "Success Rate-based Adaptive Differential Evolution (L-SRTDE) for the CEC 2024 Competition," in *Proc. IEEE CEC*, 2024.

Algorithm 1: RDEx for Single-Objective, Bound-Constrained Optimization

Input: D, N, bounds $\{[l_j, u_j]\}_{j=1}^{D}$, $G_{\max}$, F, CR, Cauchy scale γ, perturb prob. p, initial elite rate $\beta_{\text{init}} \in (0,1)$

Output: $\mathbf{x}_{\text{best}}$

Init: for $i = 1..N$ **do**

  for $j = 1..D$ **do**

    sample $x_{i,j,0} \sim \mathcal{U}(l_j, u_j)$

  evaluate $f(\mathbf{x}_{i,0})$

$\beta_0 \leftarrow \beta_{\text{init}}$; $\mathbf{x}_{\text{best},0} \leftarrow \arg\min_i f(\mathbf{x}_{i,0})$

for $g = 0, 1, \ldots, G_{\max} - 1$ **do**

  $\mathcal{I}_{\text{elite}} \leftarrow \varnothing$, $\mathcal{I}_{\text{all}} \leftarrow \varnothing$

  for $i = 1..N$ **do**

    draw distinct $r_1, r_2, r_3 \neq i$; let $\mathbf{x}_{\text{best},g}$ be current best

    `// Mutation: elite-guided with prob.` β_g`, else rand/1`

    if *with prob.* β_g **then**

      $\mathbf{v}_{i,g} \leftarrow \mathbf{x}_{i,g} + F(\mathbf{x}_{\text{best},g} - \mathbf{x}_{i,g}) + F(\mathbf{x}_{r_1,g} - \mathbf{x}_{r_2,g})$; tag←elite

    else

      $\mathbf{v}_{i,g} \leftarrow \mathbf{x}_{r_1,g} + F(\mathbf{x}_{r_2,g} - \mathbf{x}_{r_3,g})$; tag←rand

    `// Binomial crossover`

    pick $j_{\text{rand}} \in \{1, \ldots, D\}$; **for** $j = 1..D$ **do**

      $u'_{i,j,g} \leftarrow \begin{cases} v_{i,j,g}, & \text{if } \texttt{rand()} < CR \text{ or } j = j_{\text{rand}} \\ x_{i,j,g}, & \text{otherwise} \end{cases}$

    `// Rare Cauchy perturbation + projection`

    for $j = 1..D$ **do**

      if *with prob.* p **then**

        draw $\delta_j \sim \text{Cauchy}(0, \gamma)$; $u_{i,j,g} \leftarrow u'_{i,j,g} + \delta_j$

      else

        $u_{i,j,g} \leftarrow u'_{i,j,g}$

      $u_{i,j,g} \leftarrow \min\{u_j, \max\{l_j,\ u_{i,j,g}\}\}$

    `// Selection`

    evaluate $f(\mathbf{u}_{i,g})$; **if** $f(\mathbf{u}_{i,g}) \leq f(\mathbf{x}_{i,g})$ **then**

      $\mathbf{x}_{i,g+1} \leftarrow \mathbf{u}_{i,g}$; $\mathcal{I}_{\text{all}} \leftarrow \mathcal{I}_{\text{all}} \cup \{i\}$; **if** *tag=elite* **then**

        $\mathcal{I}_{\text{elite}} \leftarrow \mathcal{I}_{\text{elite}} \cup \{i\}$

    else

      $\mathbf{x}_{i,g+1} \leftarrow \mathbf{x}_{i,g}$

  update $\mathbf{x}_{\text{best},g+1}$ as best of $\{\mathbf{x}_{i,g+1}\}$

  `// Effectiveness-driven update of` β

  if $\mathcal{I}_{all} \neq \varnothing$ **then**

    $\beta_{g+1} \leftarrow \dfrac{\sum_{i \in \mathcal{I}_{\text{elite}}} (f_{\text{old}}^{(i)} - f_{\text{new}}^{(i)})}{\sum_{i \in \mathcal{I}_{\text{all}}} (f_{\text{old}}^{(i)} - f_{\text{new}}^{(i)}) + \epsilon}$; clip to $[0, 1]$

  else

    $\beta_{g+1} \leftarrow \beta_g$

return $\mathbf{x}_{\text{best},G_{\max}}$