3.1(a) $3|100 =$ **F**. There is no $k \in \mathbb{Z}$ such that $100 = 3k$.

3.1(b) $3|99=$ **T**. Notice that $99 = 3k$ where $k = 33$.

3.1(c) $3|-3 =$ **T**. Notice that $-3 = 3k$ where $k = -1$.

3.1(d) $-5|-5 =$ **T**. Notice that $-5 = -5k$ where $k = 1$.

3.1(e) $-2|-7 =$ **F**. There is no $k \in \mathbb{Z}$ such that $-7 = -2k$.

3.1f) $0|4 =$ **F**. There is no $k \in \mathbb{Z}$ such that $4 = 0k$.

3.1(g) $-4|-0 =$ **T**. Notice that $0 = 4k$ where $k = 0$.

3.1(h) $-0|-0 =$ **T**. Notice that $0 = 0k$ where $k \in \mathbb{Z}$.

3.5
Prove: Every integer is a rational number
Assume $(a \in \mathbb{Z})$
NTS $a = kb$ where $(b \in \mathbb{Z}) \wedge (b \neq 0)$
Let $b = 1$ such that $(b \in \mathbb{Z})$
Notice that $a = ab$ where $b = 1$
Disprove: All rational numbers are integers
Counterexample: $a = 2$, $b = 3$, $a/b \in \mathbb{Z} =$ **F**

3.12(e) 100 has 9 positive divisors: 1,2,4,5,10,20,25,50,100.

3.12(f) 1000000 has 49 positive divisors.

3.12(k) $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8$ has 96 positive divisors.

3.12(l) 0 has infinite positive divisors.

7.10(c)

x	y	x ∨ y	-x	-y	x ∧ (-y)	(-x) ∧ y	x ∧ (-y) ∨ (-x) ∧ y
T	T	T	F	F	F	F	F
T	F	T	F	T	T	F	T
F	T	T	T	F	F	T	T
F	F	F	T	T	F	F	F

The truth table of x ∨ y and x ∧ (-y) ∨ (-x) ∧ y is not the same
x ∨ y is not logically equivalent to x ∧ (-y) ∨ (-x) ∧ y

7.5

x	y	x ↔ y	-x	-y	(-x) ↔ (-y)
T	T	T	F	F	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

7.8

x	y	z	x ∨ y	x ∨ y → z	x → z	y → z	(x → z) ∧ (y → z)
T	T	T	T	T	T	T	T
T	F	F	T	F	F	T	F
F	T	F	T	F	T	F	F
F	F	T	F	T	T	T	T
F	T	T	F	T	T	T	T
T	F	T	T	T	T	T	T
T	T	F	T	F	F	F	F
F	F	F	F	T	T	T	T

7.11(a)

$$
\begin{aligned}
&(x \vee y) \vee (x \vee (-y)) \\
&\qquad = (x \vee x) \vee (y \vee (-y)) \\
&\qquad = T \vee T \\
&\qquad = T
\end{aligned}
$$

7.11(b)

$$
\begin{aligned}
(x \wedge (x \rightarrow y)) \rightarrow y &= (x \wedge ((-x) \vee y)) \rightarrow y \\
&= (((-x) \wedge x) \vee (y \wedge x)) \rightarrow y \\
&= (F \vee (y \wedge x)) \rightarrow y \\
&= (y \wedge x) \rightarrow y \\
&= -(y \wedge x) \vee y \\
&= (-x) \vee ((-y) \vee y) \\
&= (-x) \vee T \\
&= T
\end{aligned}
$$

7.11(h)

$$\begin{aligned}
&\textit{Let } A = (x \to y) \textit{ and } B = (x \to -y)\\
(A \wedge B) \to (-x) &= -(A \wedge B) \vee (-x)\\
&= ((-A) \vee (-B)) \vee (-x)\\
&= (-A) \vee ((-B) \vee (-x)))\\
&= (-A) \vee ((x \to -y) \vee (-x))\\
&= (-A) \vee (-((-x) \vee (-y)) \vee (-x))\\
&= (-A) \vee (-(x \wedge y) \vee (-x))\\
&= (-A) \vee (-x \vee x) \wedge (-x \vee y)\\
&= (-A) \vee (T \wedge (-x \vee y)\\
&= (-A) \vee A\\
&= T
\end{aligned}$$

7.13(a)

$$\begin{aligned}
&(x \vee y) \wedge (x \vee -y) \wedge -x\\
&= (x \vee y) \wedge ((x \wedge -x) \vee (-x \wedge -y))\\
&= (x \vee y) \wedge (F \vee (-x \wedge -y))\\
&= (x \vee y) \wedge (-x \wedge -y)\\
&= (x \wedge (-x \wedge -y)) \vee (y \wedge (-x \wedge -y))\\
&= (F \wedge y) \vee (F \wedge -x)\\
&= F \vee F\\
&= F
\end{aligned}$$

7.13(c)

$$\begin{aligned}
((x \to y) \wedge (-x \to -y)) \wedge -y &= ((x \to y) \wedge ((x \vee y) \wedge -y)\\
&= ((x \to y) \wedge ((x \wedge -y) \vee (y \wedge -y))\\
&= ((x \to y) \wedge ((x \wedge -y) \vee F)\\
&= ((x \to y) \wedge (x \wedge -y)\\
&= ((-x \vee y) \wedge x) \wedge -y\\
&= ((-x \wedge x) \vee (y \wedge x)) \wedge -y\\
&= (F \vee (y \wedge x)) \wedge -y\\
&= (y \wedge x) \wedge -y\\
&= (y \wedge -y) \wedge x\\
&= F \wedge x\\
&= F
\end{aligned}$$