Homework 5

1. None
2. 14.6

   (a). $R = \{(x, y) : |x - y| \leq 2\}$

   (b). R is reflexive. Assume $a \in A$,**,NTS** $(a, a) \in R$
   $a - a = 0 \leq 2 \rightarrow (a, a) \in R$

   (c). R is not irreflexive. Assume $a \in A$,**,NTS** $(a, a) \in R$
   $a - a = 0 \leq 2 \rightarrow (a, a) \in R$

   (d). R is symmetric. Assume $(a, b) \in R$**, NTS** $(b, a) \in R$
   $\rightarrow |a - b| \leq 2$
   $\rightarrow |b - a| \leq 2$
   $\rightarrow (b, a) \in R$

   (e). R is not antisymmetric. Counterexample: $(1, 2) \in R, (2, 1) \in R, 1 \neq 2$

   (f). R is not transitive. Counterexample: $(1, 3) \in R, (3, 5) \in R, (1, 5) \notin R$

   14.7(d) $R^{-1} = \{(x, y) : x, y \in \mathbf{N}, y|x\}$
   Prove: let $E = \{(x, y) : x, y \in \mathbf{N}, y|x\}$
   NTS (1) $(x, y) \in R^{-1} \longrightarrow (x, y) \in E$
   (2) $(x, y) \in E \longrightarrow (x, y) \in R^{-1}$
   Prove of (1): Assume $(x, y) \in R^{-1} \longrightarrow (y, x) \in R$
   $\longrightarrow y|x$
   $\longrightarrow (x, y) \in E$
   Prove of (2): Assume $(x, y) \in E \longrightarrow y|x$
   $\longrightarrow (y, x) \in R$
   $\longrightarrow (x, y) \in R^{-1}$

   14.7(e) $R^{-1} = \{(x, y) : x, y \in \mathbf{Z}, yx > 0\}$
   Prove: let $E = \{(x, y) : x, y \in \mathbf{Z}, yx > 0\}$
   NTS (1) $(x, y) \in R^{-1} \longrightarrow (x, y) \in E$
   (2) $(x, y) \in E \longrightarrow (x, y) \in R^{-1}$
   Prove of (1): Assume $(x, y) \in R^{-1} \longrightarrow (y, x) \in R$
   $\longrightarrow yx > 0$
   $\longrightarrow (x, y) \in E$
   Prove of (2): Assume $(x, y) \in E \longrightarrow yx > 0$
   $\longrightarrow (y, x) \in R$
   $\longrightarrow (x, y) \in R^{-1}$

3. 14.12 $R = (\{(x, y) : x, y \in \mathbf{Z}, x \leq y\}$
   $R^{-1} = \{(x, y) : x, y \in \mathbf{Z}, y \leq x\}$

Prove: let $E = \{(x, y) : x, y \in \mathbf{Z}, y \leq x\}$
NTS (1) $(x, y) \in R^{-1} \longrightarrow (x, y) \in E$
(2) $(x, y) \in E \longrightarrow (x, y) \in R^{-1}$
Prove of (1): Assume $(x, y) \in R^{-1} \longrightarrow (y, x) \in R$
$\longrightarrow y \leq x \longrightarrow (x, y) \in E$
Prove of (2): Assume $(x, y) \in E \longrightarrow y \leq x$
$\longrightarrow (y, x) \in R$
$\longrightarrow (x, y) \in R^{-1}$

4. 14.13(a) $A = \{1, 2\}, R = \{(1, 1)\}$
   14.13(b) $A = \emptyset, R = \emptyset$

5. 14.14 Prove: $R\ is\ symmetric \Longleftrightarrow R = R^{-1}$
   NTS (1) $R\ is\ symmetric \longrightarrow R = R^{-1}$
   (2) $R = R^{-1} \longrightarrow R\ is\ symmetric$
   Prove of (1):
   NTS (a) $R \subseteq R^{-1}$
   (b) $R^{-1} \subseteq R$
   Prove of (a): let $(x, y) \in R \longrightarrow (y, x) \in R \longrightarrow (x, y) \in R^{-1}$
   Prove of (b): let $(x, y) \in R^{-1} \longrightarrow (y, x) \in R \longrightarrow (x, y) \in R$
   Prove of (2): let $(x, y) \in R \longrightarrow (y, x) \in R^{-1} \longrightarrow (y, x) \in R \longrightarrow R\ is\ symmetric$

   14.15 Prove: $R\ is\ antisymmetric \Longleftrightarrow R \cap R^{-1} \subseteq \{(a, a) : a \in A\}$
   NTS (1) $R\ is\ antisymmetric \longrightarrow R \cap R^{-1} \subseteq \{(a, a) : a \in A\}$
   (2) $R \cap R^{-1} \subseteq \{(a, a) : a \in A\} \longrightarrow R\ is\ antisymmetric$
   Prove of (1): let $(x, y) \in R \longrightarrow (y, x) \in R \longrightarrow x = y$
   $(y, x) \in R^{-1} \longrightarrow (x, y) \in R^{-1} \longrightarrow (x, y) \in R \cap R^{-1} \wedge (x, y) \in \{(a, a) : a \in A\} \longrightarrow R \cap R^{-1} \subseteq \{(a, a) : a \in A\}$
   Prove of (2): let x R y $\wedge$ y R x $\longrightarrow$ x R y $\wedge$ x R^{-1} y $\longrightarrow (x, y) \in R \cap R^{-1}$
   $(x, y) \in \{(a, a) : a \in A\} \longrightarrow x = y \longrightarrow R\ is\ antisymmetric$

6. 14.17