Homework 4

1. 11.2(a) There is an integer that is not prime, $\exists x \in \mathbb{Z}$; x is not prime
   11.2(b) All integers are either prime or composite, $\forall x \in \mathbb{Z}$; x is either prime or composite
   11.2(c) All integers have a square that is not 2, $\forall x \in \mathbb{Z}$; $x^2 \neq 2$
   11.2(d) There is an integer that is not divisible by 5, $\exists x \in \mathbb{Z}$; $5|x = F$
   11.2(e) All integers are not divisible by 7, $\forall x \in \mathbb{Z}$; $7|x = F$
   11.2(f) There is an integer that has a square that is negative, $\exists x \in \mathbb{Z}$; $x^2 < 0$
   11.2(g) There is an integer that the products with any integers are not 1,
   $\exists x \in \mathbb{Z}$;$\forall y \in \mathbb{Z}$; $xy \neq 1$
   11.2(h) For any two integers, the quotient of them is not equal to 10,
   $\forall x \in \mathbb{Z}$;$\forall y \in \mathbb{Z}$; $x/y \neq 10$
   11.2(i) For any integers, there exists another integer that the product of them is not 0,
   $\forall x \in \mathbb{Z}$;$y \in \mathbb{Z}$; $xy \neq 0$
   11.2(j) There is an integer that any integers are smaller or equal to it.
   $\exists x \in \mathbb{Z}$;$\forall y \in \mathbb{Z}$; $y \leq x$
   11.2(k) Sombody does not love everybody,
   $exists\ a\ person\ x$;$\forall$ $person\ y$; x does not love y
   11.4(a) F (b) T (c) F (d) T (e) F (f) T (g) T (h) T

2. 11.5(a) $\exists x \in \mathbb{Z}$; $x \geq 0$
   There is an integer that is larger than or equal to 0
   11.5(b) $\forall x \in \mathbb{Z}$; $x \neq x + 1$
   All integers are not equal to the next integer
   11.5(c) $\forall x \in \mathbb{N}$; $x \leq 10$
   All national numbers are less than or equal to 10
   11.5(d) $\exists x \in \mathbb{N}$; $x + x \neq 2x$
   There is a national number which the sum of it with itself is not equal to the twice of it
   11.5(e) $\forall x \in \mathbb{Z}$; $\exists y \in \mathbb{Z}$;$x \leq y$
   For all integers, there exists an integer less than or equal to it
   11.5(f) $\exists x \in \mathbb{Z}$; $\exists y \in \mathbb{Z}$; $x \neq y$
   There exist an integer that is not equal to another integer
   11.5(g) $\exists x \in \mathbb{Z}$; $\forall y \in \mathbb{Z}$; $x + y \neq 0$
   There exist an integer that the sum of it and any other integer is zero
   11.6 Both of the statements are the same thing. The order of them does not matter.
   11.7(a) True, x can only be 2
   11.7(b) False, x can be 2 or -2
   11.7(c) True, x can only be $\sqrt{3}$
   11.7(d) True, x can only be 0
   11.7(e) True, x can only be 1

3. 12.1(a) $\{1, 2, 3, 4, 5, 6, 7\}$
   12.1(b) $\{4, 5\}$
   12.1(c) $\{1, 2, 3\}$
   12.1(d) $\{6, 7\}$
   12.1(e) $\{1, 2, 3, 6, 7\}$
   12.1(f) $\{(1, 4), (1, 5), (1, 6), (1, 7), (2, 4), (2, 5), (2, 6), (2, 7), (3, 4), (3, 5), (3, 6), (3, 7), (4, 4), (4, 5), (4, 6),$
   12.1(g) $\{(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5),$

4. **Proof:** Assume that $x \in (A \cup B) - (A \cap B)$.
   NTS $x \in A \triangle B$
   $\rightarrow (x \in A \vee x \in B) \wedge (x \notin A \vee x \notin B)$
   $\rightarrow (x \in A) \wedge (x \notin A \vee x \notin B)) \vee (x \in B) \wedge (x \notin A \vee x \notin B)$
   $\rightarrow ((x \in A \wedge x \notin A) \vee (x \in A \wedge x \notin B)) \vee ((x \in B \wedge x \notin B) \vee (x \in B \wedge x \notin A))$
   $\rightarrow (F \vee (x \in A \wedge x \notin B)) \vee (F \vee (x \in B \wedge x \notin A))$
   $\rightarrow (x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin A)$
   $\rightarrow (x \in A - B) \vee (x \in B - A)$
   $\rightarrow x \in A \triangle B$

5. 12.14 To prove $A - \emptyset = A$

   (a). $x \in A - \emptyset \longrightarrow x \in A$

   (b). $x \in A \longrightarrow x \in A - \emptyset$

   Proof of (a): Assume that $x \in A - \emptyset$.
   $\rightarrow x \in A \wedge x \notin \emptyset$
   $\rightarrow x \in A$
   Proof of (b): Assume that $x \in A$**.**
   NTS $x \in A \wedge x \notin \emptyset$
   $\rightarrow x \notin \emptyset$
   $\rightarrow x \in A \wedge x \notin \emptyset$
   To prove $\emptyset - A = \emptyset$ $\emptyset - A \subseteq \emptyset \wedge \emptyset \subseteq \emptyset - A$
   $\rightarrow \emptyset - A = \emptyset$

   12.15 To prove $A \triangle A = \emptyset$

   (a). $x \in A \triangle A \longrightarrow x \in \emptyset$

   (b). $x \in \emptyset \longrightarrow x \in A \triangle A$

   Proof of (a): Assume that $x \in A \triangle A$.
   $\rightarrow x \in A - A$
   $\rightarrow x \in \emptyset$
   Proof of (b): Assume that $x \in \emptyset$.

$\rightarrow x \in A - A$
$\rightarrow x \in A \triangle A$
To prove $A \triangle \emptyset = A$

(a). $x \in A \triangle \emptyset \longrightarrow x \in A$

(b). $x \in A \longrightarrow x \in A \triangle \emptyset$

Proof of (a): Assume that $x \in A \triangle \emptyset$.
$\rightarrow x \in (A - \emptyset) \cup (\emptyset - A)$
$\rightarrow x \in A \cup \emptyset$
$\rightarrow x \in A$
Proof of (b): Assume that $x \in A$.
NTS $x \in (A - \emptyset) \cup (\emptyset - A)$
$A = A \cup \emptyset \rightarrow x \in A \cup \emptyset$
$\rightarrow x \in A \triangle \emptyset$

6. 12.21(a) F $A = 1, 2, 3, B = 2, C = 3, A - (B - C) = 1, 3, (A - B) - C = 1$
   12.21(b) T
   12.21(c) F $A = 1, 2, 3, B = 2, C = 3, A - (B - C) = 1, 3, (A \cup B) - C = 1, 2, (A - C) \cap (B - C) = 2$
   12.21(d) F $A = 1, 2, 3, B = 1, 2, 3, C = 4, B - C = 1, 2, 3 = A, A \cup C = 1, 2, 3, 4 \neq B$
   12.21(e) F $A = 1, 2, 3, B = 1, 2, 3, C = 3, A \cup C = 1, 2, 3 = B, B - C = 1, 2 \neq A$
   12.21(f) F $A = 1, 2, B = 2, 3, |A - B| = 1, |A| - |B| = 0$
   12.21(g) F $A = 1, 2, B = 2, 3, A - B = 1, (A - B) \cup B = 1, 2, 3 \neq A$
   12.21(h) F $A = 1, 2, B = 2, 3, (A \cup B) - B = 1 \neq A$

7. **Proof of (a):** Assume that $x \in A - (B \cup C)$.
   NTS $x \in (A - B) \cap (A - C)$
   $\rightarrow x \in A \wedge x \notin (B \cup C)$
   $\rightarrow x \in A \wedge x \notin B \wedge x \notin C$
   $\rightarrow (x \in A \wedge x \notin B) \wedge (x \in A \wedge x \notin C$
   $\rightarrow x \in (A - B) \wedge X \in (A - C)$
   $\rightarrow x \in (A - B) \cap (A - C)$
   Proof of (b): Assume that $x \in (A - B) \cap (A - C)$

   NTS $x \in A - (B \cup C)$. $\rightarrow x \in (A - B) \wedge X \in (A - C)$
   $\rightarrow (x \in A \wedge x \notin B) \wedge (x \in A \wedge x \notin C)$
   $\rightarrow x \in A \wedge (x \notin B \wedge x \notin C)$
   $\rightarrow x \in A \wedge x \notin (B \cup C)$
   $\rightarrow x \in A - (B \cup C)$

8. Let $D = A \cup B$
   LHS=$|Z \cup C|$

$|Z \cup C| = |Z| + |C| - |Z \cap C|$
$= |A \cup B| + |C| - |A \cup B| \cap C$
$= |A| + |B| - |A \cap B| + |C| - |(A \cap C) \cup (B \cap C)|$
$= |A| + |B| - |A \cap B| + |C| - |A \cap C| - |B \cap C| + |(A \cap C) \cap (A \cap B)|$
$= |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| = RHS$

9. 14.1(a) $(1,2),(1,3),(1,4),(1,5),(2,3),(2,4),(2,5),(3,4),(3,5),(4,5)$
14.1(b) $(1,1),(1,2),(1,3),(1,4),(1,5),(2,2),(2,4),(3,3),(4,4),(5,5)$
14.1(c) $(1,1),(2,2),(3,3),(4,4),(5,5)$
14.1(d) $(1,1),(1,3),(1,5),(2,2),(2,4),(3,1),(3,3),(3,5),(4,2),(4,4),(5,1),(5,3),(5,5)$
14.2(a) For $a,b \in \{1,2,3,4,5\},(a,b) \in R$ iff $b-a=1$
14.2(b) For $a,b \in \{1,2,3,4,5\},(a,b) \in R$ iff $a \geq b$
14.2(c) For $a,b \in \{1,2,3,4,5\},(a,b) \in R$ iff $a+b=6$
14.2(d) For $a,b \in \{1,2,3,4,5\},(a,b) \in R$ iff $a|b$

10. 14.3(a) reflexive, symmetric, antisymmetric, transitive
For irreflexive, x=1,1 R 1=T

14.3(b) irreflexive, antisymmetric
For reflexive, x=1, 1 R 1=F
For symmetric, $(1,2) \in R,(2,1) \notin R$
For transitive, $(1,2),(2,3) \in R,(2,3) \notin R$

14.3(c) antisymmetric, transitive
For reflexive, x=2, 2 R 2=F
For irreflexive, x=1,1 R 1=T
For symmetric, $(1,2) \in R,(2,1) \notin R$

14.3(d) symmetric
For reflexive, x=2, 2 R 2=F
For irreflexive, x=1,1 R 1=T
For antisymmetric, $(1,2) \in R,(2,1) \in R, 1 \neq 2$
For transitive, $(1,1),(1,2) \in R,(1,2) \notin R$

14.3(e) reflexive, symmetric, transitive
For irreflexive, x=1,1 R 1=T
For antisymmetric, $(1,2) \in R,(2,1) \in R, 1 \neq 2$

14.4(a) reflexive, symmetric, transitive
14.4(b) irreflexive, antisymmetric
14.4(c) reflexive, symmetric, transitive
14.4(d) reflexive, symmetric
14.4(e) irreflexive, symmetric
14.4(f) irreflexive, antisymmetric