Homework 3

1. 8.7(a) For every photo, there are 2 websites to post.
   There are 30 photos.
   Therefore, the total ways are: 2^{30}

   8.9 Assume the black rook is placed first.
   For every square of the black rook, the white rook has $64 - 15$ squares to place.
   There are 64 squares for black rock to place, so the total ways are:
   $(64 - 15) \times 64 = 3136$

2. 8.16 The first digit has 10 choices of digits.
   The second, third and last one have 7 choices.
   $10 \times 7^3 = 3430$

   8.19 The first card has 52 choices.
   The second one has $(52/4 - 1) \times 3 = 36$ choices.
   The third one has $(52/4 - 2) \times 2 = 22$ choices.
   The last one has $(52/4 - 3) = 10$ choices.
   $52 \times 36 \times 22 \times 10 = 411840$

3. $2655! + 2, 2655 + 3, \ldots 2655! + 2655, 2655! + 2656$
   $2|(2655! + 2), 3|(2655! + 3), \ldots 2655|(2655! + 2655), 2|(2655 + 2656)$

4. $B = \{\emptyset\}$, $C = \{\emptyset, \{\emptyset\}\}$, $D = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$
   $2^D = \{\emptyset, \{\emptyset, \{\emptyset\}\}, \{\emptyset, \{\emptyset, \{\emptyset\}\}\}, \{\{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}, \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}\}$

5. 10.1(g) $\{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}\}$
   10.3(c) $A = \{x \in \mathbb{Z} : x \in \emptyset\}, \quad |A| = 0$
   10.3(d) $A = \{x \in \mathbb{Z} : \emptyset \in x\}, \ |A| = 0$
   10.3(e) $A = \{x \in \mathbb{Z} : \emptyset \subseteq \{x\}\}, \ |A| = \infty$
   10.3(f) $A = 2^{2^{\{1,2,3\}}}, \ |A| = 2^8$
   10.3(g) $A = \{x \in 2^{\{1,2,3,4\}} : |x| = 1\}$, $A = \{\{1\}, \{2\}, \{3\}, \{4\}\}, \ |A| = 4$

6. 10.4(a) $2 \in \{1, 2, 3\}$
   10.4(b) $2 \subseteq \{1, 2, 3\}$
   10.4(c) $\{2\} \in \{\{1\}, \{2\}, \{3\}\}$
   10.4(d) $\emptyset \subseteq \{1, 2, 3\}$
   10.4(e) $\mathbb{N} \subseteq \mathbb{Z}$
   10.4(f) $\{2\} \subseteq \mathbb{Z}$
   10.4(g) $\{2\} \in 2^{\mathbb{Z}}$

7. 10.5(a) $A = \{1\}$, $B = \{1, 2\}$, $C = \{1, 2, 3\}$
   10.5(b) $A = \{1\}$, $B = \{\{1\}, 2\}$, $C = \{\{1\}, 2, 3\}$
   10.5(c) $A = \{1\}$, $B = \{\{1\}, 2\}$, $B = \{\{\{1\}, 2\}, 3\}$
   10.5(d) $A = \{1\}$, $B = \{1, 2\}$, $C = \{\{1, 2\}, 3\}$
   10.6(a) $A = \{1\}$
   10.6(b) $A = \{1\}$
   10.6(c) $A = \emptyset$
   10.6(d) No solution, since no set belongs to empty set.

8. 10.12 Let $x \in C$, $kx = 12$, where $k \in \mathbb{Z}$
   NTS $mx = 36$, where $m \in \mathbb{Z}$
   $kx = 12 \rightarrow (3k)x = 36$
   $mx = 36$ when $m = 3k$
   $x|36 \rightarrow x \in D \rightarrow C \subseteq D$

   10.14 $x = \emptyset$, $\emptyset \subseteq \{\emptyset\}$

   10.15 $(3, 4, -5) \in P$, since $3^2 + 4^2 = (-5)^2$
   There are no x any y such that $-5 = x^2 + y^2$ when $x \in \mathbb{Z}$ and $y \in \mathbb{Z}$
   $(3, 4, -5) \in T = F \rightarrow P \neq T$

9. (a) Assume $a \in A \rightarrow a = 6k - 5$
   $a = 6(k - 1) + 1 = 3(2k - 2) + 1 = 3m + 1$, when $m = 2k - 2$
   $\rightarrow a \in B \rightarrow A \subseteq B$ (b) Counterexample: Assume $b = 10 = 3 \times 3 + 1 \rightarrow b \in B$
   There is no value of k that $6k - 5 = 10$ when $k \in \mathbb{Z}$
   $\rightarrow b \in A = F \rightarrow B \subseteq A = F$