Homework 6

1. 15.3

   (a). equivalent relation
   (b). not equivalent relation $(3,3) \notin R$
   (c). not equivalent relation $(1,3) \in R, (3,1) \notin R$
   (d). not equivalent relation $(1,3) \in R, (3,1) \notin R$
   (e). equivalent relation
   (f). not equivalent relation $(4,4) \notin R$
   (g). equivalent relation

2. 15.7

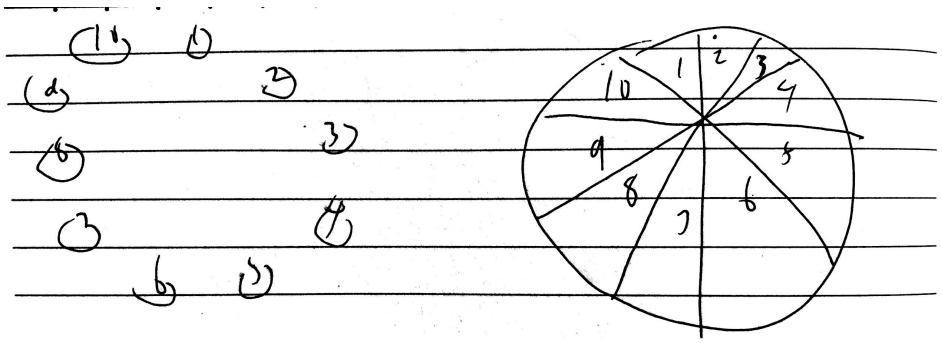
   (a). $\{1,2\}$
   (b). $\{4\}$
   (c). $\{120,121,122,123,124,125,126,127,128,129\}$
   (d). $\{you, brothers, sisters\}$
   (e). $\{allpeoplewithsamebirthdayasme\}$
   (f). $\{\{1,2\},\{1,3\},\{1,4\},\{1,5\},\{2,3\},\{2,4\},\{2,5\},\{3,4\},\{3,5\},\{4,5\}\}$

3. 15.8

   (a). 10
   (b). 366
   (c). 8
   (d). 50

4. 15.9
   NTS $(1) x \in [1] \longrightarrow x \in [3]$
   NTS $(2) x \in [3] \longrightarrow x \in [1]$
   Prove of (1): $x \in [1] \longrightarrow x = 2k+1$ where $k \in \mathbf{z} \longrightarrow x = 2(k-1)+3 \longrightarrow x \in [3]$
   Prove of (2): $x \in [3] \longrightarrow x = 2k+3$ where $k \in \mathbf{z} \longrightarrow x = 2(k+1)+1 \longrightarrow x \in [1]$

5. 15.15



6. 16.1(a) $\{\{1\},\{2\},\{3\}\},\{\{1\},\{2,3\}\},\{\{2\},\{1,3\}\},\{\{3\},\{1,2\},\{1,2,3\}\}$

   16.1(b) $\{\{1\},\{2\},\{3\},\{4\}\}.\{\{1\},\{2,3,4\}\},\{\{2\},\{1,3,4\}\},$
   $\{\{3\},\{1,2,4\}\},\{\{4\},\{1,2,3\}\},\{\{1,2\},\{3,4\}\},\{\{1,3\},\{2,4\}\},\{\{1,4\},\{2,3\}\},\{1,2,3,4\}$

   16.3 $5!/2! = 60$

7. 16.8 $6! \times 5!$ $5!$ is the ways that men can form a circle, $6!$ is the ways that women can be arranged between men
   16.9 $19!$

   16.12 $40!/((2!)^{20}20!)$ $20!$ ways to arrange the 20 groups, in 20 pairs, players can be arranged in 2^{20} ways
   $40!3^{10}/((4!)^{10}10!)$ $10!$ ways to arrange the 10 groups, in 10 pairs, players can be arranged in 4^{10} ways, inside the group, 3 ways to arrange players

8. 16.14 $10!^3$ $10!$ for each position and $10!$ ways are repeated. $10!^4/10!$
   16.15 $2^3 - 1 = 7$,$2^{99} - 1$
   16.16(a) 3^{100}
   16.16(b) 3^{97}
   16.17 $100!/(20!(5!)^{20}) > 100!/(5!(20!)^5)$
   $\longrightarrow$ The number of partitions of A into 20 part of size 5 is greater.