Homework 2

1.(a) **Pf:** Assume x and y are consecutive perfect squares.
Counterexample:
$x = 0$, $x = a^2$, $a = 0$
$y = (a+1)^2 \to 1^2$
$\to y = 1$
$x - y = 0 - 1 = -1 \to -1$ is not even

(b) **Pf:** Assume x and y are consecutive perfect squares. $x = a^2, y = (a+1)^2$
NTS $x - y = 2k - 1$ where $k \in \mathbb{Z}$

$$\begin{aligned} x - y &= a^2 - (a+1)^2 \\ &= a^2 - a^2 - 1 - 2a \\ &= -2a - 1 \\ &= 2k - 1 \; when \; k = -a \end{aligned}$$

2. **Pf:** Assume $a|d$ and $a|e$. $d = ma, e = na$ where $m \in \mathbb{Z} \wedge$, where $n \in \mathbb{Z}$
NTS $db + ec = ka$ where $k \in \mathbb{Z}$

$$\begin{aligned} db + ec &= ma \times b + na \times c \\ &= a \times mb + anc \\ &= a(mb + nc) \\ &= ka \; when \; k = mb + nc \end{aligned}$$

3. **Pf:** Assume a is an integer.
NTS $2|(a^2 - a)$
$a^2 - a = a(a-1)$
case 1:if a is even $\to a = 2m$, where $m \in \mathbb{Z}$
$a^2 - a = 2m(2m-1) \to 2|2m(2m-1)$
$\to 2|(a^2 - a)$, when a is even
case 2:if a is odd $\to a = 2n + 1$, where $n \in \mathbb{Z}$
$a^2 - a = (2n+1)(2n+1-1) = 2n(2n+1) \to 2|2n(2n+1)$
$\to 2|(a^2 - a)$, when a is odd
Therefore, $2|(a^2 - a)$

4. counterexample:
$x = F, y = T$
$x \to y = F \to T = T, -y \to -x = F \to T$
$x \leftrightarrow y = F \leftrightarrow T = F$

5. **Pf:** Assume that x and y are integers with the same parity.
NTS $x + y = 2k$ where $k \in \mathbb{Z}$ when x and y are even
$\rightarrow x = 2m, y = 2n$ where $m \in \mathbb{Z} \wedge n \in \mathbb{Z}$

$$\begin{aligned} x + y &= 2m + 2n \\ &= 2(m + n) \\ &= 2k \; when \; k = m + n \end{aligned}$$

when x and y are odd
$\rightarrow x = 2m + 1, y = 2n + 1$ where $m \in \mathbb{Z} \wedge n \in \mathbb{Z}$

$$\begin{aligned} x + y &= (2m + 1) + (2n + 1) \\ &= 2m + 2n + 2 \\ &= 2(m + n + 1) \\ &= 2k \; when \; k = m + n + 1 \end{aligned}$$

6. **Pf:** Assume that x, y, z, w are consecutive integers.
$\rightarrow x = a.y = a + 1, z = a + 2, w = a + 3$ where $a \in \mathbb{Z}$
NTS $x \times y \times \; zw = k^2 - 1$ where $k \in \mathbb{Z}$

$$\begin{aligned} x \times y \times \; zw &= a(a + 1)(a + 2)(a + 3) \\ &= a(a + 3) \times (a + 1)(a + 2) \\ &= (a^2 + 3a)(a^2 + 3a + 2) \\ m = a^2 + 3a & \\ m(m + 2) &= (m + 1)^2 - 1 \\ &= k^2 - 1 \; where \; k = m + 1 \end{aligned}$$

7. **Pf:** Assume that a, b, c are integers.
counterexample: $a = 2, b = 3, c = 12$
$a|c, b|c, a + b =, 5|12 = F \rightarrow (a + b)|c = F$

8. **Pf:** Assume that $n \in \mathbb{Z}$
counterexample: $n = 5, n^2 - n + 1 = 25 - 5 + 1 = 21,$
$3|21, 21 isnotprime$

9. **Pf:** Assume that x is an odd integer.
$x = 2k + 1$ where $k \in \mathbb{Z}$
NTS $x^2 = 8m + 1$

$$\begin{aligned} x^2 &= (2k + 1)^2 \\ &= 4k^2 + 4k + 1 \end{aligned}$$

NTS $4k^2 + 4k + 1 = 8m + 1$
$4k^2 + 4k = 8m$
$k^2 + K = 2m$

$k(k+1) = 2m$
case 1: if k is even $\rightarrow k = 2n$ where $n \in \mathbb{Z}$
$2n(2n+1) = 4n^2 + 2n = 2(n^2 + n) = 2m$ when $m = n^2 + n$
case 2: if k is odd $\rightarrow k = 2n + 1$ where $n \in \mathbb{Z}$
$(2n+1)(2n+2) = 4n^2 + 6n + 2 = 2(n^2 + 3n + 1) = 2m$ when $m = n^2 + 3n + 1$
Therefore, $x^2 = 8m + 1$