Example Solutions for Assignment 5

Question 1

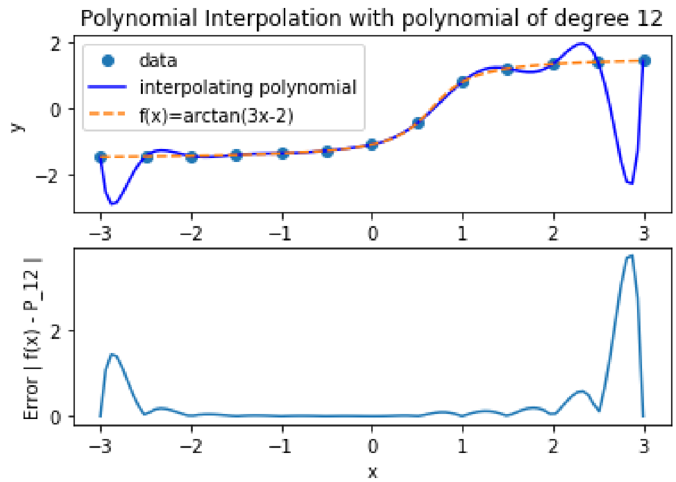

Figure 1: Figure for question 1(a). Polynomial interpolation of $f(x)$ through 13 equally spaced points between $x = -3$ and 3. Top panel: The data, interpolating polynomial $P_{12}(x)$ and the function $f(x)$ are plotted. Bottom panel: The error $|f(x) = P_{12}$ is plotted.

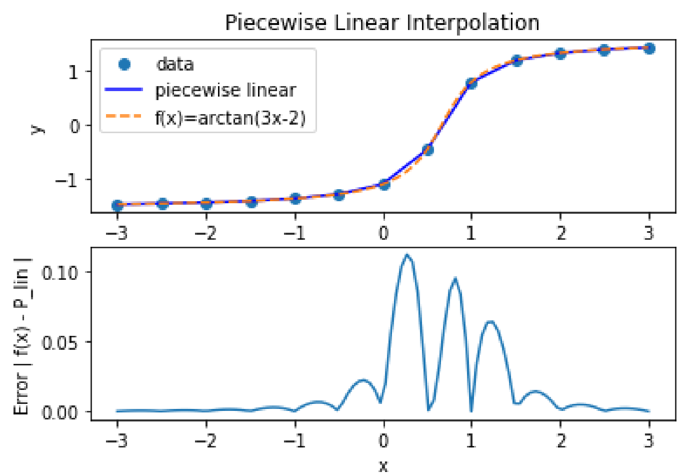

Figure 2: Figure for question 1(b). Piecewise linear interpolation of $f(x)$ through 13 equally spaced points between $x = -3$ and 3. Top panel: The data, the interpolant $P_{lin}(x)$ and the function $f(x)$ are plotted. Bottom panel: The error $|f(x) = P_{lin}$ is plotted.

(d) Some observations: The error of the polynomial interpolant is largest near the boundaries ($x = -3$ and 3), and is significantly larger in magnitude than the other interpolants. This is likely due to the non-smoothness of $f(x)$, i.e. higher-order derivatives are large. The error of the piecewise interpolants are largest near $x = 0.5$; this is where the derivative of $f(x)$ is largest. For the piecewise interpolants, since lower-order polynomials are being used, the error is associated with lower-order derivatives. The error for the cubic spline interpolant is significantly lower than that of the piecewise linear interpolant. Furthermore, aesthetically, the cubic spline interpolant just looks nicer. Therefore, at least in this case, the cubic spline interpolant is the best (although this is often the case, it is not always).

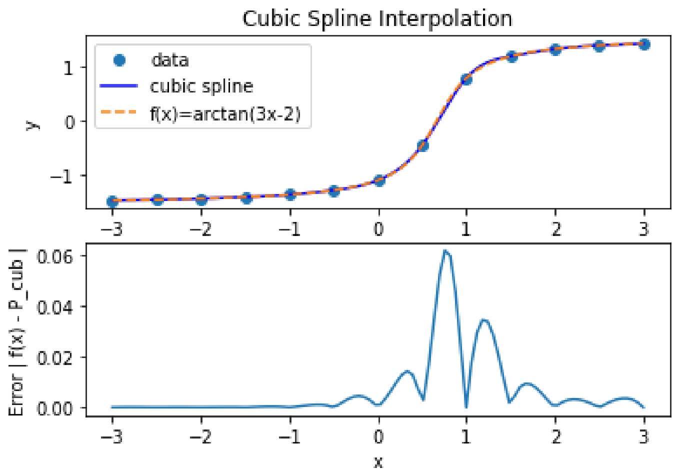

Figure 3: Figure for question 1(c). Cubic spline interpolation of $f(x)$ through 13 equally spaced points between $x = -3$ and 3. Top panel: The data, the interpolant $P_{cub}(x)$ and the function $f(x)$ are plotted. Bottom panel: The error $|f(x) = P_{cub}$ is plotted.

Question 2

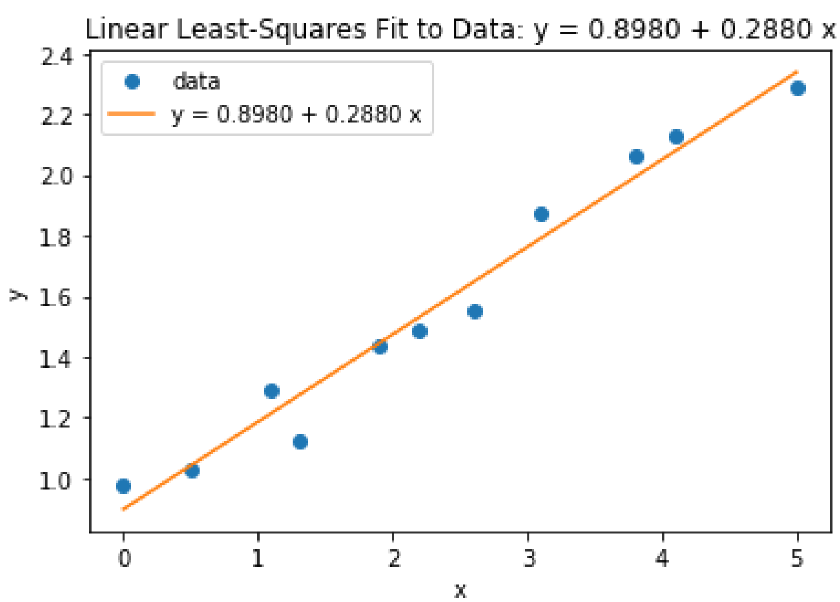

Figure 4: Linear least squares fit to noisy data. Figure for question 2(b).

Question 3

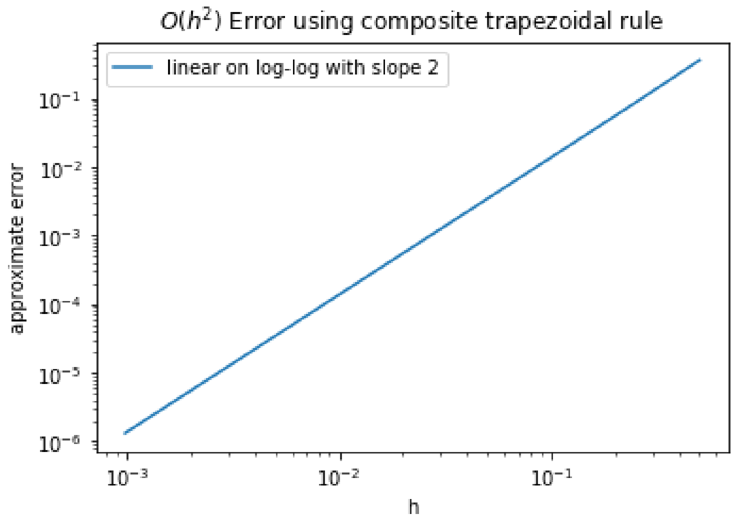

Figure 5: Figure for question 3(b). Approximate error for composite trapezoidal rule for varying length of subinterval.

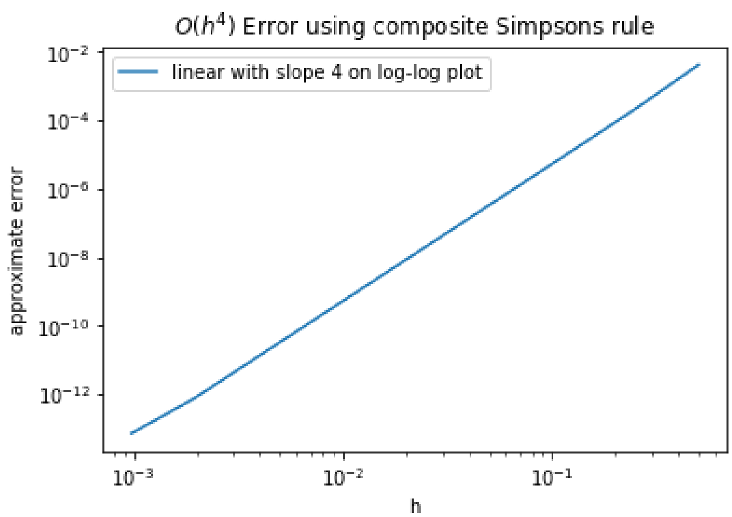

Figure 6: Figure for question 3(d). Approximate error for composite Simpson's rule for varying length of subinterval.

(e) Here we are using $|I_k - I_{k-1}|$ as an approximation of the error of I_{k-1}. The idea is that we are assuming that the error in I_k is significantly less than the error in I_{k-1}, and therefore, we can use the value of I_k as a surrogate for the true value of the integral.
As such, we expect the error of composite trapezoidal rule to be $O(h^2)$. Thus, if we plot the approximate error on a log-log plot, we should get a straight line with slope 2, which is indeed the case. For composite Simpson's rule, we expect the error to be $O(h^4)$. Thus, if we plot the approximate error on a log-log plot, we should get a straight line with slope 4, which is indeed the case.