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# Simulating Innovation:

## Comparing Models of Collective Knowledge, Technological Evolution and Emergent Innovation Networks

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**Abstract.** Computer simulation models have been proposed as a tool for understanding innovation, including models of organisational learning, technological evolution, knowledge dynamics and the emergence of innovation networks. By representing micro-level interactions they provide insight into the mechanisms by which are generated various stylised facts about innovation phenomena. This paper summarises work carried out as part of the *SIMIAN* project and to be covered in more detail in a forthcoming book. A critical review of existing innovation-related models is performed. Models compared include a model of collective learning in networks [1], a model of technological evolution based around percolation on a grid [2, 3], a model of technological evolution that uses Boolean logic gate designs [4], the SKIN model [5], a model of emergent innovation networks [6], and the hypercycles model of economic production [7]. The models are compared for the ways they represent knowledge and/or technologies, how novelty enters the system, the degree to which they represent open-ended systems, their use of networks, landscapes and other pre-defined structures, and the patterns that emerge from their operations, including networks and scale-free frequency distributions. Suggestions are then made as to what features future innovation models might contain.

**Keywords:** Innovation; Novelty; Technological Evolution; Networks

## 1 Introduction

Simulation models of innovation, including organisational learning, knowledge dynamics, technological evolution and the emergence of innovation networks may provide explanations for stylised facts found in the literatures in innovation, science and technology studies. As computer simulation models of social systems, they can provide one with something one cannot easily obtain from the system itself [8]. They offer a third approach to research, combining the ethnographer's interest in complex contexts and causal relations with the quantitative data analyst's interest in large-scale patterns. They can represent rigorously in computer code the micro-level interactions of multiple, heterogeneous parts, then demonstrate the consequences of these interac-

tions and the circumstances in which they occur, including the emergence of macro-level patterns.

There are a number of stylised facts inviting explanation of which some of the most relevant to simulation models of innovation follow. Firstly, innovation can be progressive. New ideas and technologies solve problems, create new capabilities and render obsolete and replace old ideas and technologies. A second stylised fact may be found in the rate of quantitative innovation, that is, the rate at which a type of item becomes better, faster, cheaper, lighter etc. Perhaps the best known example of this is Moore's Law, which holds that the number of circuits that can be fitted on a chip increases exponentially over time, but examples of exponential growth rates exist for many other technologies, such as land and air transport, and the particular growth rates also appear to have grown exponentially over time since 1840 [9, 10]. Thirdly, there is the rate of qualitative innovation, that is, the rate at which qualitatively new types of good or service appear. One rough illustration of this is that humans 10000 years ago had a few hundred types of good available to them, while today in a US city there are barcodes for  $10^{10}$  types of goods [11]. Various stylised facts exist for the frequency distribution of innovation size, where measures of size include the economic returns from innovation [12, 13] and the number of citations received by a particular patent [14]. Given Schumpeter's famous description of the "perennial gale of creative destruction" [15], there is also interest in the size of the web of interdependent technologies and services blown away (rendered obsolete and uncompetitive, thereafter becoming extinct) by the emergence of a particular innovation. The innovation literature typically distinguishes between incremental and radical innovations [16], the former meaning a minor improvement in an existing technological approach, and the latter a switch to a new approach. In addition, it may be recognised that technologies' components are grouped into modules. This leads to the concepts of architectural and modular innovations [17], the former meaning a rearrangement of existing modules, while the latter means a change of a single module. Finally, emergent structures should be mentioned. Networks of firms, suppliers and customers emerge to create and make use of particular interlinked technologies. If empirical studies of such networks can identify regular structural features, then simulation models can investigate the circumstances under which these structures emerge.

This paper summarises some of the main points from a critical survey of several models of organisational learning, knowledge dynamics, technological evolution and innovation networks, undertaken as part of the ESRC-funded SIMIAN project ([www.simian.ac.uk](http://www.simian.ac.uk)) and described in detail in a forthcoming book by the authors. Particular areas for model comparison include the ways these models represent knowledge and/or technologies, how novelty enters the system, the degree to which the models represent open-ended systems, the models' use of networks, landscapes and other pre-defined structures, and the patterns that emerge from the models' operations, primarily network structures and frequency distributions. In addition, based on our experiences with these models and some recent literature, suggestions are made about the form and features that future innovation models might contain.

## 2 The Innovation Models

Simulation models of innovation focus on the production, diffusion and impact of novel ideas, beliefs, technologies, practices, theories, and solutions to problems. Simulation models, especially agent-based models, are able to represent multiple producers, multiple users, multiple innovations and multiple types of interdependency between all of these, leading to some hard-to-predict dynamics. In the case of innovations, some innovations may form the components of further innovations, or they may by their emergence and diffusion alter the functionality and desirability of other innovations. All of the models surveyed below have these aspects.

Space permits only a few models to be surveyed here. Further models of technological evolution, with several points of similarity to the ones included here, may be found in Lane [18]. Also related are science models [19], models of organisational learning, for example March [20], models of strategic decision making, for example Rivkin and Siggelkow [21], and models of language evolution [22, 23]. Treatments of some of these areas can also be found in Watts and Gilbert [24, chapters 7, 5 and 4].

Space also does not permit more than brief indications of the functionality of the models in this survey. For more details the reader is directed to the original source papers and to Watts and Gilbert [24, chapter 7]. The brief descriptions of the models now follow.

Lazer and Friedman [1] (hereafter L&F) simulate an organisation as a network of agents attempting to solve a common complex problem. Each agent has a set of beliefs, represented by a bit string, that encode that agent's solution. Solutions are evaluated using Kauffman's NK fitness landscape definition [25], a moderately "rugged" landscape problem with  $N=20$  and  $K=5$ . Agents attempt seek better solutions through the use of two heuristic search methods: learning from others (copying some of the best solution among the agent's neighbours), and trial-and-error experimentation (trying a solution different from your current solution by mutating one bit). The eventual outcome of searching is that the population converges on a common solution, usually a better solution than any present among the agents initially, and ideally one close to the global optimum for that fitness landscape.

Silverberg and Verspagen [3] (S&V) simulate technological evolution using nodes in a grid lattice to represent interlinked technologies, and percolation up the grid to represent technological progress. Technologies can be in one of four states: impossible, possible but yet-to-be-discovered, discovered but yet-to-be-made-viable, and viable. At initialisation, technologies are set with a fixed chance, to be possible or impossible. Technologies in the first row of the grid are then set to be viable. The best-practice frontier (BPF) is defined as the highest viable technologies in each column. Each time step, from each technology in the BPF, R&D search effort is made over technologies within a fixed radius. As a result of search some possible technologies within the radius may, with a chance dependent on the amount of effort divided

by the number of technologies in the radius, become discovered. Any discovered technologies adjacent to viable technologies become themselves viable. Innovations are defined as any increases in the height of the BPF in one column. Innovation size is defined as the size of the increase. Since technologies may become viable because of horizontal links as well as vertical ones, it is possible for quite large jumps in the BPF, whenever progress in one column has obstructed by an impossible technology while progress continues in other columns from search radiuses can cover the column with the obstruction. The frequency distribution of these innovation sizes is recorded and plotted. For some values of the parameter search radius, this frequency distribution tends towards a scale-free distribution. In their basic [3] model, Silverberg and Verspagen represent the same amount of search as occurring from every column in the grid. Silverberg and Verspagen [2] extend this model with search agent firms who can change column in response to recent progress. The firms' adaptive behaviour has the effect of generating the scale-free distribution of innovation sizes without the need for the modeller to choose a particular value of the search radius parameter, and thus the system represents self-organised criticality [26].

Arthur and Polak [4] (A&P) also simulate technological evolution. Their technologies have a real-world meaning: they are designs for Boolean logic gates, made up of combinations of component technologies, beginning from a base technology, the NAND gate. Each time step a new combination of existing technologies is created and evaluated for how well it generates one of a fixed list of desired logic functions. If it replicates desired functions satisfied by a previously created technology, and is less expensive, where cost is defined as the number of component instances of the base technology, NAND, then the new technology replaces in memory the technology with the equivalent function and higher cost. The replaced technology may have been used as a component technology in the construction of other technologies, in which case it is replaced in them as well. The total number of replacements resulting from the newly created technology is its innovation size. As with the previous model, A&P find example parameter settings in which the frequency distribution of innovation sizes tends towards being scale-free.

The model for Simulating Knowledge dynamics in Innovation Networks (SKIN) [5, 27, 28] simulates a dynamic population of firms. Each firm possesses a set of units of knowledge, called *kenes*, and a strategy, called an innovation hypothesis (IH), for combining several *kenes* to make a product. Input *kenes* not possessed by the firm must be sourced from a market supplied by the other firms. Each *kene* is a triple of numbers, representing a capability, an ability and expertise. Products are created as a normalised sum-product of capabilities and abilities of the *kenes* in the IH, and given a level of quality based on a sum-product of the same *kenes*' abilities and expertise. Firms lacking a market for their products can perform incremental research to adjust their abilities, radical research to swap a *kene* in their IH for another *kene*, or enter an alliance or partnership with another firm to access that firm's *kenes*. Expertise scores increase in *kenes* when they are used, but decrease when not in use, and *kenes* with 0 expertise are forgotten by the firm. Partners are chosen based on past experience of

partnership, customer and supplier relations, and the degree of similarity in genes to the choosing firm. Regular partners may unite to form an innovation network, which then can create extra products in addition to those produced by its members. Products on the markets have dynamic prices reflecting recent supply and demand, research has costs, and firms' behaviour reflects their wealth.

The model of emergent innovation networks of Cowan, Jonard and Zimmermann [6] (CJZ) also simulates a population of firms with knowledge resources. Each firm's knowledge is a vector of continuous variables, representing several dimensions of knowledge. Pairs of firms can collaborate to create new amounts of knowledge, with the amount computed using a constant-elasticity-of-substitution (CES) production function. Each input to this function is a weighted sum of the minimum and maximum values in the corresponding dimension of the collaborating firms' knowledge vectors, the idea here being that if knowledge in each dimension is largely independent of the other dimensions, knowledge is decomposable into subtasks, and firms will be able to choose the best knowledge value for each subtask, but with interdependent knowledge dimensions, both firms may be held back by the weakest firm. If collaboration is by chance successful, the amount output from the production function will be added to one of the variables in a participant's knowledge vector. Evaluating potential collaboration partners is based on experience of recent success, including the evaluating firm's direct experience of the candidate partner (relational credit), and also the evaluator's indirect experience obtained from its other recent collaborators (structural credit). Once all firms have evaluated each other, a set of partnerships is formed using the algorithm for the roommate matching problem. Data on partnerships can be used to draw an innovation network of firms. The structural properties of this network can then be related to the main parameters, the weighting between collaborating firms' minimum and maximum knowledge inputs (representing the decomposability of knowledge) and the weighting between relational and structural credit.

Padgett's hypercycles model of economic production [7, 29, 30] draws upon the ideas from theoretical biology of hypercycles and auto-catalysis [25]. It simulates a population of firms engaged in the transformation and transfer of products. Each firm begins with randomly chosen production skills, called production rules. Inspired by Fontana's algorithmic chemistry these are of the form: given a product of type  $x$ , transform it into an output of type  $y$ . Each time step a new production run attempt is simulated. A product of a random type is drawn from a common environment by one randomly chosen firm and transformed using one of that firm's rules. The output product from transformation is then transferred to a randomly chosen neighbour in a grid network of firms. If a firm lacks a rule suitable for transforming the product it has received, then the product is dumped into the environment and the production run ends. Otherwise, the firm uses a compatible rule to transform it and transfers the output to one of its neighbours, and the production run continues. In addition to processes of transformation and transfer, firms learn by doing. Whenever two firms in succession in the production run have compatible rules to transform products, one of the firms increases its stock of instances of the rule it has just used. Meanwhile, under a

process of rule decay, somewhere in the population of firms a randomly chosen rule is forgotten. Firms that forget all their rules exit the system, leaving gaps in the network. The effect of these four processes (product transformation and transferral, and rule learning by doing and rule decay) is that under various parameter settings a self-maintaining system of firms and rules can emerge over time through self-organisation. This system depends upon there being hypercycles of rules, in which constituent rules are all supplied by other constituent rules.

The models described are all capable of demonstrating that the emergence of some system-level pattern (e.g. convergence on a peak solution, scale-free distributions in change sizes, collaboration networks, self-maintaining systems) is sensitive to various input parameters and structures controlling micro-level behaviour (e.g. initial network structure, search and learning behaviour, knowledge structure).

### 3 Points of Comparison

The models are now compared for the ways they represent knowledge and/or technologies, how novelty enters the system, the degree to which they represent open-ended systems, their use of networks, landscapes and other pre-defined structures, and the patterns that emerge from their operations, including networks and scale-free frequency distributions. A summary is given in **Table 1**.

As may be clear from the above descriptions, the models differ widely in their representation of knowledge and technologies: there were bit strings (L&F), nodes in a grid (S&V), lists of components (A&P), genes (SKIN model), vectors of continuous variables (CJZ) and algorithmic chemistry rules (hypercycles model). These were evaluated using NK fitness (L&F), connection to the base row and height of row (S&V), a list of desired logic functions and cost in terms of number of base components (A&P), and in terms of their ability to take input from and supply output to other model components (SKIN, hypercycles). It seems that later models tended to be better than earlier ones, and represent gains in knowledge.

How novelty enters the system varies in line with how knowledge or technologies are represented. L&F's use of heuristic search methods reflects the idea that there are two sources of novelty: new combination of existing parts, and mutation, during copying or experimentation. This view of novelty in innovation is found not only among Herbert Simon's disciples [31], but is also common among Schumpeterian evolutionary economists [32, 33]. The A&P model recombines existing parts when constructing new logic gate designs. The SKIN model's incremental and radical research processes have effects analogous to mutation of knowledge, while recombination of parts is made through alliance partnerships. The hypercycles model sees a self-maintaining system emerge via a process analogous to ant colony optimisation, as new production runs are constructed out of firms and rules. But novel choices of rule for transforming or route for transferring products can only be made between the rules

and firms still present in the model. Once rules have been forgotten and firms have left the network, they do not return. So in the present version of the model there is no equivalent of a mutation process to reintroduce novel firms and rules. The CJZ model is based on the idea that innovation stems from collaborative use of existing knowledge resources, but it does not specify a mechanism for this. The constant-elasticity-of-substitution production function is a description of the pattern of innovation rather than its explanation. The S&V model also omits an explanation of how a new innovation has been made, beyond the simple concept of “search” from a position at the current best-practice frontier. The regular two-dimensional grid in which technologies lead on to other technologies is relatively simple. In contrast, the relationships between combinations of bits in the L&F model and between complex technologies in the A&P model form much more complex solution spaces / technology spaces.

Another view of novelty is as reinterpretation. In several of these models ideas and technologies can acquire new functions or values due to the appearance or disappearance of ideas and technologies. For example, a kene in the SKIN model can form an input in many different innovation hypotheses, some of which may only appear later in the simulation run. In the A&P model the discovery of a new node can connect other technology nodes and shorten paths to the best-practice frontier. In the hypercycles model the disappearance of a rule or firm alters the amount of production work for other rules and firms.

The hypercycles model simulates the emergence of novel, identifiable self-maintaining structures – a new type of thing with some significant degree of permanence or stability. Thus, whereas some models represent the application of heuristic search methods to combinations of pre-existing objects, models of emergent structures, such as hypercycles and auto-catalytic sets, have the potential to explain how there come to be objects in the first place.

The models vary in the degree to which they represent open-ended systems. L&F’s model is relatively closed. Agents respond to their neighbours, while they differ in beliefs, and to their fitness evaluations. The landscape is static and fixed at the beginning, and does not respond to any events either inside or outside the model. Once a population has converged on a single peak in the fitness landscape, the model’s dynamics come to an end. Change in A&P’s model could also come to an end, since the list of desired functions is static, but in practice the list used by A&P is sufficiently long for the phenomenon of technology extinctions to continue throughout the length of their simulation runs. The models in both S&V and CJZ can permit indefinite extensions / additions of innovation, but S&V’s percolation grid does not gain columns and CJZ’s knowledge vector does not gain dimensions, so in both cases new qualities do not emerge. The SKIN model contains a limit on the number of products, but in practice this limit need not restrict the dynamics of the model. Firms and their kenes come and go throughout a simulation run. The hypercycles model reaches either a dead state in which the firm-rule network becomes too fragmented and no more learn-



ing by doing can occur, or it forms a self-maintaining system of firm-rule hypercycles. With no processes to introduce new firms or rules to the system, dead systems cannot be resurrected and live systems are unable to change much except by a particularly unlucky sequence of rule decay events. (Padgett and Powell [34] however contains proposals for future extensions to the basic hypercycles model that will make it more open-ended.)

**Table 1.** Summary of model comparison

Model & key references	Unit of knowledge or technology	Unit representation	Value: pre-defined or endogenous	Sources of novelty	Emergent patterns
L&F [1]	Workers' beliefs / routines	Bit strings	NK fitness landscape	Recombination; Mutation	Convergence on peak / optimum
S&V [2, 3]	Technologies	Nodes in a grid network	Height & connection to baseline	Chance discovery from nearby nodes	Scale-free distribution of advances of connected nodes
A&P [4]	Technologies	Systems of NAND logic gates	List of desired logic functions	Recombination	Scale-free distribution of obsolescence sizes
SKIN [5, 27, 28]	Firms' "kenes": capabilities, abilities, expertise	Ordered triples of integers	Endogenous demand for inputs	Recombination of kenes; new abilities	Social networks; Scale-free distribution of production network sizes
CJZ [6]	Firms' knowledge	Vector of continuous variables	Endogenous demand for collaboration partners	Not specified	Social networks
Hypercycles [7, 29, 34]	Firms' production skills (rules)	Algorithmic chemistry	Role in hypercycles system	Emergence of self-maintaining system	Self-maintaining system of firms & rules

Nearly all the models assume pre-defined structures. L&F assume a fitness landscape, for which they use Kauffman's NK fitness. S&V assume a network structure for their percolation grid technology space. A&P assume their list of desired functions. The hypercycles model assumes the initial structure for the network of firms, the initial allocation of rules to firms, and the various types of rule themselves (the "chemistry"). For this latter assumption Padgett's studies so far have restricted themselves to a simple cycle of rules, each rule having one input and one output product, and each rule producing the input to the next rule in the cycle, with the final rule producing the input to the first rule. This is clearly a very abstract view of industrial production relations. The SKIN model uses modulo arithmetic (that is, its normalised sum-products) to control the relationships between kenes. This is simple to specify and compute, but remote from real-world applications. The CJZ model relies on its decomposability parameter and its CES production function to control innovation production, and no further description of the underlying knowledge or technological world is needed, but whether knowledge resembles constant elasticity of substitution remains to be demonstrated empirically.

Several models simulate the emergence of a scale-free frequency distribution representing the size of innovations (S&V, A&P and SKIN). In the first two cases innovation size is defined in terms of the number of technologies rendered obsolete or becoming extinct. In the case of SKIN the distribution applies not to the size of new kenes but to the size of new production networks [27]. L&F and A&P simulate systems that show progressive improvements, but with diminishing returns over time, as a peak or optimum is reached in the L&F model and all of A&P's list of desired functions become satisfied. SKIN, CJZ and the hypercycles model simulate the emergence of networks of firms. Only CJZ specifically analyse the structure of these in their paper, though the other models could certainly support analysis. But all three sets of authors are interested in discovering the sensitivity of this emergence to various parameter settings.

## **4 Towards Future Models of Innovation**

Future models of innovation are likely to continue the concepts of innovation as heuristic search and recombination of parts, and to continue to describe the emergent networks and frequency distributions. As more data on industrial eco-systems, innovation networks and other networks of firms become available, the structures of these can be compared with the output from the models. Likewise, empirical frequency distributions can be analysed to find out to what extent they tend towards being scale-free, perhaps log-normal in shape, and with what parameters. Although some of the above models could generate scale-free distributions of change sizes, familiar from the theory of self-organised criticality [26], it is not yet known whether real-world creative destruction follows this distribution of sizes. Stylised facts concerning geometric growth in quality (better, faster, cheaper etc. – i.e. quantitative innovation) and the immense growth in number of types of goods and services (qualitative innovation)

were not explained by the above models. Future models might address these and also the scaling laws relating innovation rates (number of patents per capita, number of entrepreneurs per capita) to city population sizes recently highlighted by Bettencourt, Lobo, Helbing, Kuhnert and West [35]. The chapters and models in Lane [18] represent a first step towards meeting this latter end.

Empirical studies should also inform the inputs and design of these models. For examples, fitness landscapes, the artificial chemistry of rules in the hypercycles model, and the initial firm networks in L&F and the hypercycles models play important roles in the models' behaviour, but abstract or arbitrary choices are currently used for these inputs. Some of the models made a distinction between incremental and radical innovation processes, as is common in innovation literature, but the further identification of modular and architectural innovations [17] was missing. This will require some thought as to how technology spaces and fitness landscapes should be structured. In addition, agent learning behaviour and market mechanisms could also be grounded in real-world cases.

None of the models translated their outputs into familiar economic terms beyond the SKIN model's use of market pricing. To provide insights into the role played by innovation in a whole economy or society, the models surveyed here need more connection between the results of innovation on the one hand, and the future ability of agents to innovate on the other. Knowledge and technologies serve purposes, while both human agents and firms have needs. Failure to meet their basic needs (food, capital) will make it harder to continue to engage in practices such as experimentation, learning from others and trading, thus slowing down the rate at which innovations are generated and diffuse. So far, innovation models have omitted the needs of their human components.

A key feature of technologies and practices is their ability to support more than one attribution of functionality. Technologies and routines developed for one purpose may be reinterpreted and developed to serve a new purpose, one not foreseen during earlier development and present seemingly only by accident. This is the process known in biology as exaptation. Villani, Bonacini, Ferrari, Serra and Lane [36] hold exaptive bootstrapping to be responsible for the continual growth in the number of types of thing, and present an early attempt at a model in which agents' cognitive attributions of functionality to artefacts enables the representation of this process. Several of the models surveyed above can simulate an innovation in the function or value of one thing, due to the appearance or disappearance of another, but they do not distinguish these changes in actual functionality from changes in agents' awareness of functionality. While the models are good at representing innovation as recombination, they are less clear at representing innovation as reinterpretation.

Introducing cognition raises the roles of imagination, analogy and metaphor in generating innovations. The range of logically possible combinations of all our current technologies is vast, and most of them seem nonsensical or unviable. We only

have time to try out a tiny fraction of the combinations, so how do we manage to find any useful ones? Models that treat component technologies or beliefs as indivisible base units will lack a basis for their agents to anticipate how well two never-before-combined base technologies are likely to fare when combined for the first time. Gavetti, Levinthal and Rivkin [37] simulate the use of analogical reasoning in generating new strategies. Attention to reasoning processes and the circumstances in which they work well will be issues for modellers of other forms of innovation as well.

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