Project Ideas: Networks, Order and Centrality

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Overview

The aim of this project is to look at how a key idea in network science, centrality (the importance) of a node, needs to be adapted for a special class of networks – directed acyclic graphs. The behaviour of these measures needs to be tested. This can be done on simple artificial networks created from standard models. Equally, we can look at real data, available from the supervisor or online.

Networks

Networks are a collection of nodes (vertices, points, actors,…) which are connected in pairs by edges (links, arcs, bonds,…). In undergraduate physics, one usually encounters infinite regular networks, such as a crystal lattice. In mathematics, undergraduates study simple random networks in "graph theory". However, in the real world there are many other types of network with widely varying features. The study of networks is now a field in its own right known as "network science", for example see the free book by Coscia (2021).

Centrality

One of the most basic measurements made on a network is to establish the importance of a node using what are called "centrality measures". If a node is a web page (edges are the hyperlinks between web pages) then the most important web page should be listed first in a web search – PageRank is used by Google to do exactly that and is an example of a centrality measure.

Order

Some networks are embedded in time and the edges between the nodes always point in one direction with respect to time. For example, in a "citation network", nodes are academic papers with a time associated with their publication date. The edges only point back in time to older papers as they represent the bibliography. Such a network where the nodes respect an order (e.g. some causality imposed by time) is called a "directed acyclic graph" (DAG). For example, see PhD theses by Clough (2017) or Vasiliauskaite (2020) for an introduction or try my own notes (Evans, 2025).

DAGS are special networks because the edges only every point back in time. If you follow a path in the network, following the direction on the edges, you can only go backwards in time. This order (strictly a "partial order" mathematically), the constraint of causality, leads to some special properties for citation networks. That in turn requires us to adapt or find new ways to analyse such networks.

The most obvious property then is to define centrality measures that respect the order in a DAG. For instance, my papers are often assessed on how many citations they receive, and that is the simplest centrality measure of network science (known as the "degree" of a node).

However, there seems to be relatively little coherent study of centrality measures for DAGs, e.g. see Vasiliauskaite (2020). Recently I put together a list of some possible centrality measures (Evans 2025), some little known outside their fields such as ecology, social science, some adapted from those appropriate for ordinary networks.

Additional Reading

M.Coscia 2021. “The Atlas for the Aspiring Network Scientist”, https://arxiv.org/pdf/2101.00863

JR.Clough 2017.“Causal Structure In Networks”, PhD Thesis Imperial College London. https://doi.org/10.25560/50162

V.Vasiliauskaite, 2020. “Paths and Directed Acyclic Graphs,” PhD Thesis Imperial College London https://doi.org/10.25560/81811

T.S.Evans, 2025. “DAGology”, private notes.