EC427 - Optimization Techniques
IIIT Vadodara

End-Semester Examination
50 Marks

Autumn 2022
November 16, 2022

Note:

1. This is a closed book test. Cheat sheets and calculators are not allowed.
2. Answers directly written without proper derivation/justification/computation will not be awarded any marks, irrespective of whether they are correct or not.
3. You will not get any marks for writing unnecessary theory.

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1. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be the objective function defined as $f(x,y) = x^2 + (y-3)^2 + 10$. The set of feasible points are $\Omega = \{(x,y) \in \mathbb{R}^2 \mid g_1(x,y) \leq 0, g_2(x,y) \leq 0\}$, where $g_1 : \mathbb{R}^2 \to \mathbb{R}, g_2 : \mathbb{R}^2 \to \mathbb{R}$ defined as $g_1(x,y) = x^2 - y, g_2(x,y) = y - x - 2$. **[10]**
   (a) At the points of minima, which of the constraints are active, and why?
   (b) Using the Karush-Kuhn-Tucker conditions, find the point of minima.
   (c) Find the Lagrange multipliers, μ_1, μ_2 associated with the constraints μ_1, μ_2.

2. Let x, y, z denote the height, width and breadth of a closed box made out of a flat card-board. Minimize the surface area of the required cardboard sheet needed to construct the closed box that has a volume of $10m^3$. Also compute the height, width and breadth of the optimal cardboard box, and compute the Lagrange multiplier associated with the constraint. **[10]**

3. Let $f : \mathbb{R}^n \to \mathbb{R}$ be defined as $f(x) = \frac{1}{2}x^TQx - b^Tx$, where $Q \in SPD(n)$ and $b \in \mathbb{R}^n$ is a fixed vector. Let V_1, V_2 be two subspaces of $\mathbb{R}^n$ such that there is a non-zero vector $d \in V_1 \cap V_2$. If $x_1 = \arg\min_{x \in V_1} f(x)$ and $x_2 = \arg\min_{x \in V_2} f(x)$ with $f(x_1) < f(x_2)$, then show that $x_1 - x_2$ is Q-conjugate to d? Is this not true if $f(x_1) = f(x_2)$. **[5]**

4. Prove as applicable whether the following functions f are convex, concave, neither or both. **[12]**
   (a) Let $A \in \mathbb{R}^{m \times n}$, and $f : \mathbb{R}^n \to \mathbb{R}, f(x) = ||Ax - b||^2$.
   (b) $f : \mathbb{R}^2 \to \mathbb{R}, f(x,y) = x^2 + 2y^3 - 2xy + y^2 + 10x + 100$.
   (c) $f : \Omega \subset \mathbb{R}^2 \to \mathbb{R}, f(x,y) = x^2 - y^2, \Omega = \{(x,y) \in \mathbb{R}^2 \mid x + y = 0\}$
   (d) $f : \Omega \subset \mathbb{R}^2 \to \mathbb{R}, f(x,y) = x^2 + y^2, \Omega = \{(x,y) \in \mathbb{R}^2 \mid x^2 - y^2 = 0\}$

f'' [illegible]

5. Let $f\mathbb{R} \to \mathbb{R}$ be defined as $f(x) = 10x - \log(x)$, where log denotes the natural logarithm. **[8]**
   (a) Write the Newton's iteration for computing the minimizer for f.
   (b) Find the minima of f.
   (c) Will the Newton's method converge to the minima for any initialization? Why?

6. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as $f(x,y) = (x-y)^4 + x^2 - y^2 - 2x + 2y + 10$. Find all points in $\mathbb{R}^2$ satisfying the First order necessary conditions. Which of these points satisfy the Second order necessary conditions? **[5]**