MA102: Mathematics II: Intro. to Discrete Mathematics

Midsem (Remote) Marks: 10

Course Instructor: Dr. Swapnil Lokhande Date: June 28, 2021

Instructions: Clearly write your name and roll number on the top of each page. Solutions must be written clearly.

Scan solutions in sequence.

Problem 1 **1 mark**

Let f be a mapping from X to Y, and $A \subseteq X$. Prove that

(a) if A is countable, so is $f(A)$;

(b) if f is one to one and A is uncountable, so is $f(A)$.

Problem 2 **2 marks**

Let the function composition $f \circ f \circ \cdots \circ f$ (i numbers of composition) be denoted by f^i. If $i = 0$, then f^i is the identity function. Let f be a bijection from finite set X to same set X. For each $a \in X$ define $S(a) := \{f^i(a); i = 0, 1, 2, \ldots\}$. Define the relation R on X by the rule: for all $a, b \in X$, $a\ R\ b$ if and only if $b \in S(a)$. Prove that R is an equivalence relation on X.

Problem 3 **1 mark**

Using Well Ordering principle prove that $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n \cdot (n+1) = \frac{n(n+1)(n+2)}{3}$.

Problem 4 **2 marks**

Let n be an odd number. Let $f : S \to S$ be any bijection where $S = \{1, 2, \ldots, n\}$. Show that the following number $R = \prod_{i=1}^{n} (i - f(i))$ will always be an even number.

Problem 5 **1 mark**

Show that $\{(x, y) | x - y \in \mathbb{R}\}$ is an equivalence relation on the set of complex numbers-$\mathbb{C}$. What are $[i], [\sqrt{2} + 1]$?

Problem 6 **1 mark**

Let f be a function from A to B. We define $S_f : P(A) \to P(B)$ as $S_f(X) = f(X)$, image of X under f, for every subset X of A. If f is one-to-one/injective then show that S_f is one-to-one/injective. P(X) denotes power set of a set X.

Problem 7 **1 mark**

Prove that there is no positive integer n such that $n^2 + n^3 = 100$. Which method did you use?

Problem 8 **1 mark**

Whether the argument given below is correct or incorrect and explain why. All sparrows like fruit. My pet bird is not a sparrow. Therefore, my pet bird does not like fruit.