Numericals Methods

MA202 | Numerical Differentiation

202051088 HRITIK KUMAR SECTION_1

QUESTIONS

Q.1 (a) Write a MATLAB script to calculate numerical derivative of tan−1 (x) at x = 1.

(b) Find the error using forward difference, backward difference, and central difference methods. Comment on order of accuracy.

(c) Plot the error using log-log scale for the step size h = 1e − 04.

(d) Use step sizes ranging from 10−1 to 10−8 . Plot the error using log-log scale for each of your step sizes.(Note: Use array operations not a for loop).

(e) Comment on trade-off between truncation error and roundoff error, i.e., look at the minima of different methods in the graphs.

Q. 2: Write a MATLAB script to calculate first order as well as second order numerical derivative of 2 − x + ln(x) at x = 1.

Repeat the steps (b), (c), (d), and (e) of Q.1

Q. 3: (a) Write a MATLAB script to calculate partial derivative of
f(x) = sin(x1)exp(−x2)

at x1 = 0.5 and x2 = 1.

(b) Find the error using central difference method.

(c) Plot the error using log-log scale for the step size h = 1e − 06.

SOLUTIONS

Q.1

```
clc;
clear;
close all;
a = 1;
truVal = 1/(1 + a.^2);
h = 1e-4;
% Forward difference formula
fwdDiff = (f(a + h) - f(a))/h;
errFwd = abs(truVal - fwdDiff);
disp(['Error in forward difference: ', num2str(errFwd)]);
% Backward difference formula
bckDiff = (f(a) - f(a - h))/h;
errBck = abs(truVal - bckDiff);
disp(['Error in backward difference: ',
num2str(errBck)]); % Central difference formula
cntrDiff = (f(a + h) - f(a - h))/(2*h);
errCntr = abs(truVal - cntrDiff);
disp(['Error in central difference: ', num2str(errCntr)]);
% Improved Forward difference formula
iFwdDiff = (-f(a + 2*h) + 4*f(a + h) -
3*f(a))/(2*h); errIFwd = abs(truVal - iFwdDiff);
disp(['Error in improved forward difference: ',
num2str(errIFwd)]); % Improved Backward difference formula
iBckDiff = (3*f(a) - 4*f(a - h) + f(a -
2*h))/(2*h); errIBck = abs(truVal - iBckDiff);
disp(['Error in improved backward difference: ',
num2str(errIBck)]); % Improved Central difference formula
iCntrDiff = (8*f(a + h) - 8*f(a - h) - f(a + 2*h) + f(a -
2*h))/(12*h); errICntr = abs(truVal - iCntrDiff);
disp(['Error in improved central difference: ',
num2str(errICntr)]); % PLoting graph for diffrent method
semilogy([errFwd, errBck, errCntr, errIFwd, errIBck, errICntr], 'o')
title('Error for the step size h = 1e # 04')
ylabel('Error')
xlabel('Different error methods')
% Defining function for tan
inverse(x) function fx = f(x)
 fx = atan(x);
```

```
end
```

Output

```
Error in forward difference: 2.4999e-05

Error in backward difference: 2.5001e-05

Error in central difference: 8.3317e-10

Error in improved forward difference: 1.6665e-09

Error in improved backward difference: 1.6682e-09

Error in improved central difference: 2.4014e -13
```

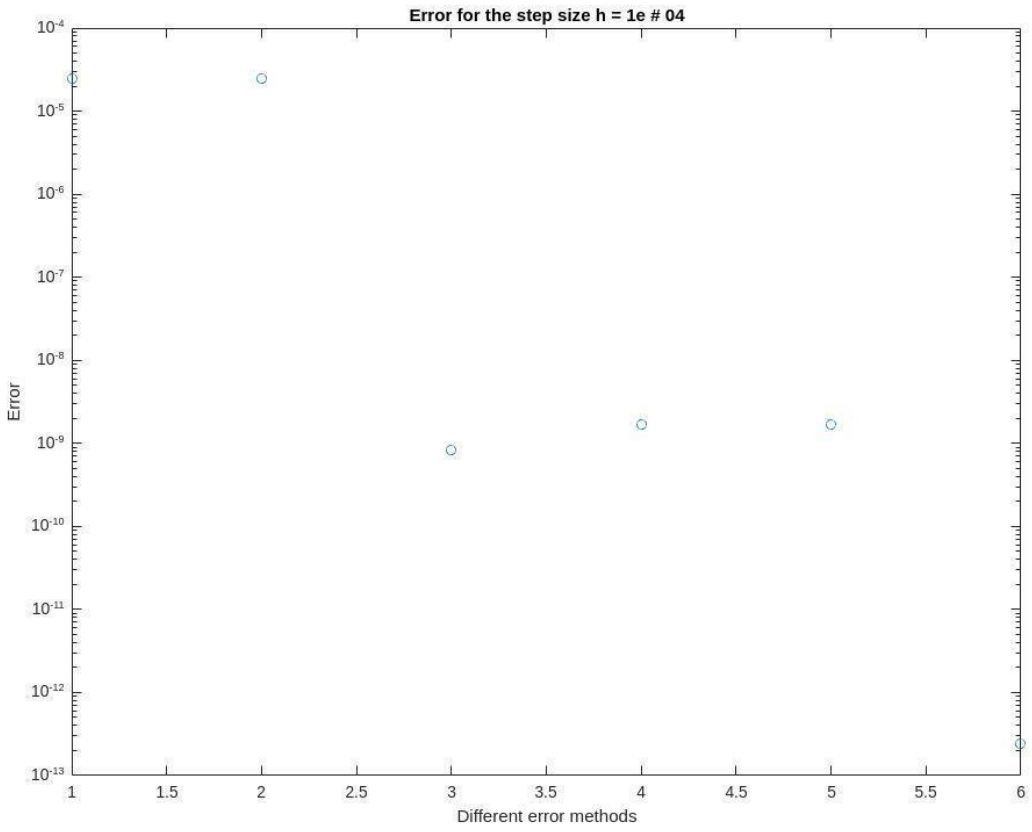


Figure: 1

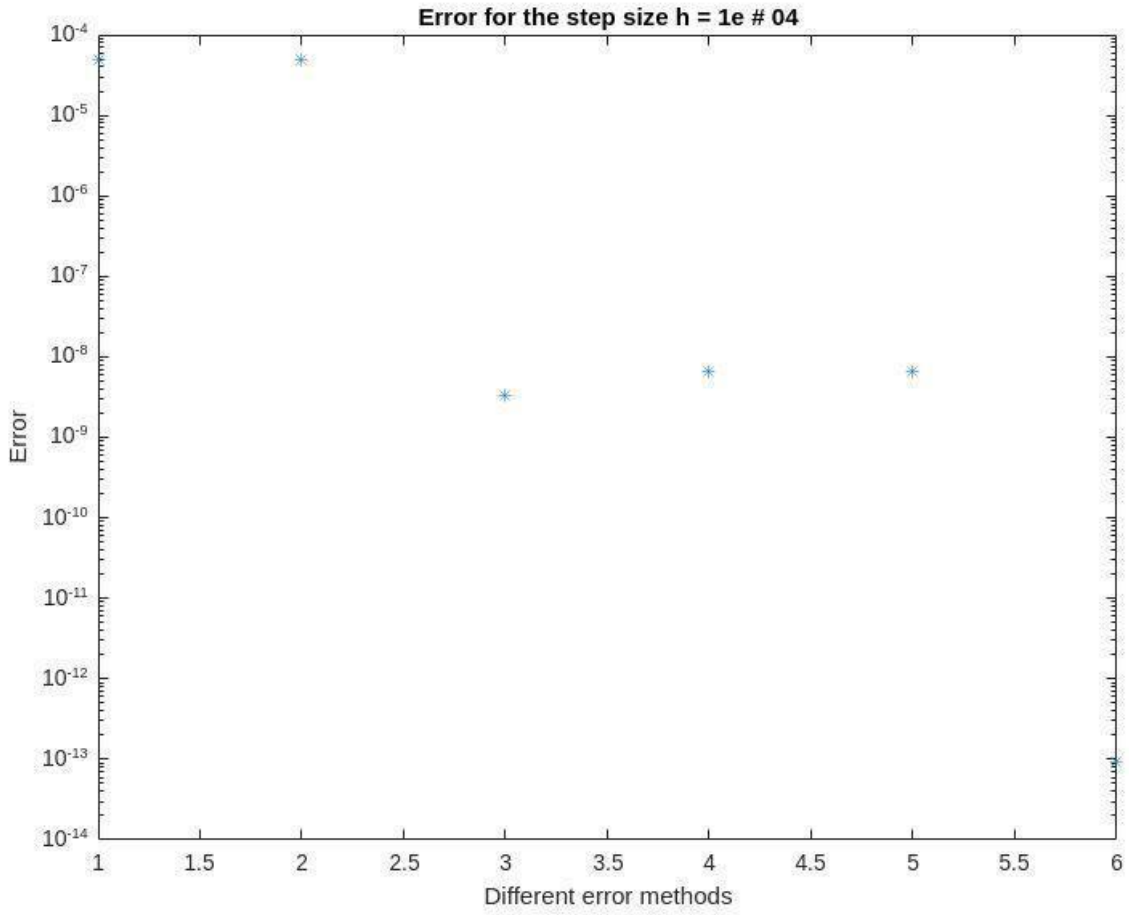
Q.2.

```
clc;
clear;
close all;
a = 1;
truVal = 0;
h = 1e-4;
% Forward difference formula
fwdDiff = (f(a + h) - f(a))/h;
errFwd = abs(truVal - fwdDiff);
disp(['Error in forward difference: ', num2str(errFwd)]);
% Backward difference formula
bckDiff = (f(a) - f(a - h))/h;
errBck = abs(truVal - bckDiff);
disp(['Error in backward difference: ',
num2str(errBck)]); % Central difference formula
cntrDiff = (f(a + h) - f(a - h))/(2*h);
errCntr = abs(truVal - cntrDiff);
disp(['Error in central difference: ', num2str(errCntr)]);
% Improved Forward difference formula
iFwdDiff = (-f(a + 2*h) + 4*f(a + h) -
3*f(a))/(2*h); errIFwd = abs(truVal - iFwdDiff);
disp(['Error in improved forward difference: ',
num2str(errIFwd)]); % Improved Backward difference formula
iBckDiff = (3*f(a) - 4*f(a - h) + f(a -
2*h))/(2*h); errIBck = abs(truVal - iBckDiff);
disp(['Error in improved backward difference: ',
num2str(errIBck)]); % Improved Central difference formula
iCntrDiff = (8*f(a + h) - 8*f(a - h) - f(a + 2*h) + f(a - 2*h))/
(12*h); errICntr = abs(truVal - iCntrDiff);
disp(['Error in improved central difference: ',
num2str(errICntr)]); % Plots
semilogy([errFwd, errBck, errCntr, errIFwd, errIBck, errICntr], '*')
title('Error for the step size h = 1e # 04')
ylabel('Error')
xlabel('Different error methods')
% Function function
fx = f(x) fx = 2 - x
+ log(x); end
```

Output

```
Error in forward difference: 4.9997e-05

Error in backward difference: 5.0003e-05

Error in central difference: 3.3334e-09

Error in improved forward difference: 6.6658e-09
```

```
Error in improved backward difference: 6.668e-09

Error in improved central difference: 9.2519e-14
```



Q.3:

```
clc;
clear;
close all;
a = [0.5; 1];
h = [10e-6; 10e-6];
truVal = [cos(a(1))*exp(-a(2)); -sin(a(1))*exp(-a(2))];
% Central difference formula
% Partial differentiation w.r.t x
cntDiff(1) = (f(a(1) + h(1), a(2)) - f(a(1) - h(1),
a(2)))./(2.*h(1)); % Partial differentiation w.r.t y
cntDiff(2) = (f(a(1), a(2) + h(2)) - f(a(1), a(2) -
h(2)))./(2.*h(2)); errCnt = abs(truVal - cntDiff.');
disp('Error in cntDiff is:');
disp(errCnt)
% The errCnt has two values: The first one is the differentiation w.r.t x
% and second is differentiation w.r.t y.
% Plot
semilogy(errCnt, 'r*')
% Function
function fx = f(x1, x2)
 fx = sin(x1)*exp(-x2);
end
```

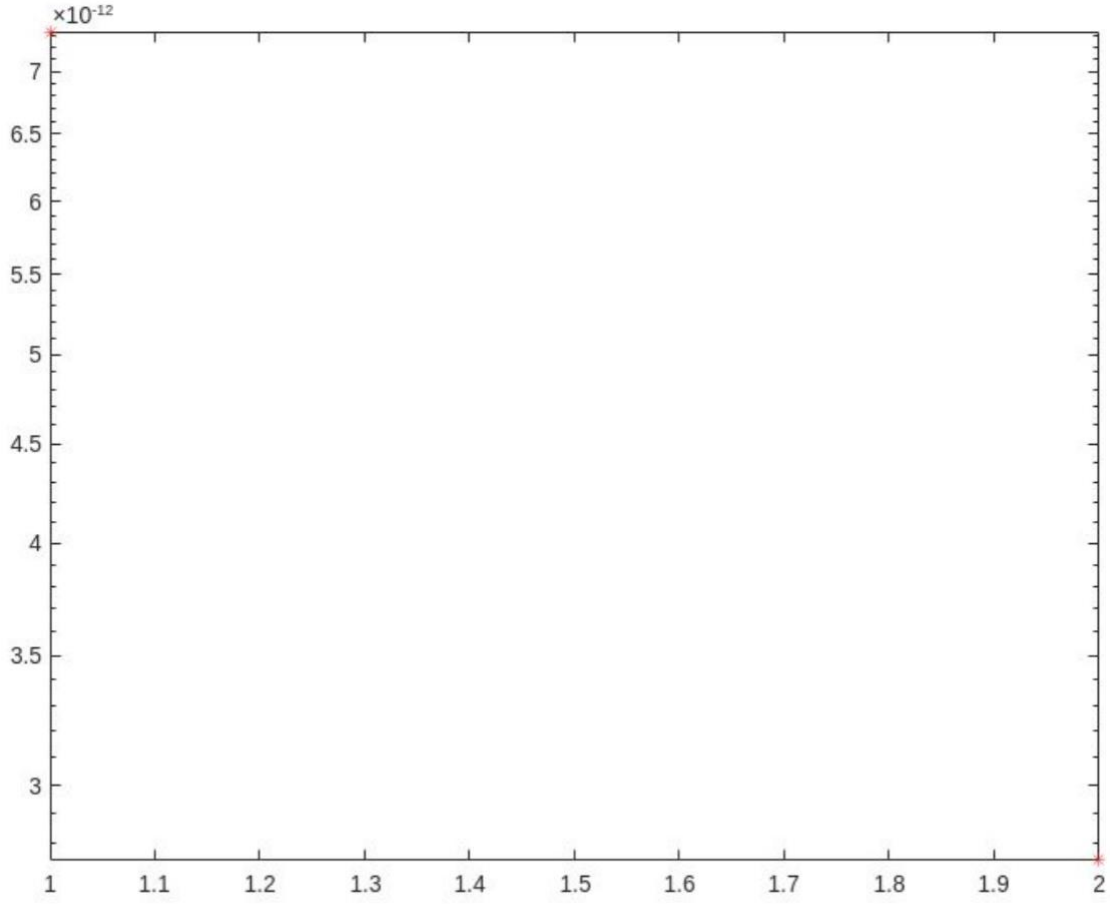
Output

Error in cntDiff is:

1.0e-11 *

0.7339

0.2744


×10-12
7
6.5
6
5.5
5
4.5
4
3.5
3
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2